अध्याय 7(B)

निश्चित समाकलन
[DEFINITE INTEGRAL]

दीर्घ उत्तरीय प्रश्न-II

प्रश्न 1. समाकलन $\int_2^3 x^5 dx$ का मान ज्ञात कीजिए।

(म.प्र. 2022)

हल :
$$I = \int_2^3 x^5 dx$$

$$\int x^n dx = \frac{x^n + 1}{n+1} = \left[\frac{x^6}{6}\right]_2^3$$

$$= \frac{1}{6}[x^6]_2^3 = \frac{1}{6}[3^6 - 2^6] = \frac{1}{6}[729 - 64] = \frac{665}{6}.$$

उत्त

प्रश्न 2. सिद्ध कीजिए कि

$$\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \frac{\pi}{12}.$$

(NCERT)

हल : माना
$$I = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}} = \int_{\pi/6}^{\pi/3} \frac{dx}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}$$

$$= \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx \qquad ...(I)$$

अतः $$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}+\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}dx,$$

[प्रगुण $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$ से]

$\Rightarrow$ $$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

$\Rightarrow$ $$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx \qquad ...(2)$$

समी. (1) और (2) को जोड़ने पर,

$$I+I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$\Rightarrow$ $$2I=\int_{\pi/6}^{\pi/3}1.dx=[x]_{\pi/6}^{\pi/3}=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}$$

$\therefore$ $$I=\frac{\pi}{12}.$$

यही सिद्ध करना था।

प्रश्न 3. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}e^x(\log\sin x+\cot x)\,dx$ का मान ज्ञात कीजिए।

हल : $\int e^x(\log\sin x+\cot x)\,dx$

$$=\int \underset{\text{II}}{e^x}\,\underset{\text{I}}{\log\sin x}\,dx+\int e^x\cot x\,dx$$

$$=e^x\log\sin x\,dx-\int\frac{\cos x}{\sin x}.e^x\,dx+\int e^x\cot x\,dx$$

$$=e^x\log\sin x\,dx-\int\cot x.e^x\,dx+\int e^x\cot x\,dx\ =e^x\log\sin x$$

$$\therefore\ \int_{\frac{\pi}{4}}^{\frac{\pi}{2}}e^x[\log\sin x+\cot x]\,dx=[e^x\log\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$=\left[e^{\frac{\pi}{2}}\log\sin\frac{\pi}{2}-e^{\frac{\pi}{4}}\log\sin\frac{\pi}{4}\right]=\left[e^{\frac{\pi}{2}}\log 1-e^{\frac{\pi}{4}}\log\frac{1}{\sqrt{2}}\right]$$

$$=0-e^{\frac{\pi}{4}}\log\frac{1}{\sqrt{2}}=e^{\frac{\pi}{4}}\log\sqrt{2}.$$

उत्तर

प्रश्न 4. सिद्ध कीजिए— $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx = \frac{\pi}{4}$.

(म.प्र. 20

हल : $$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}} dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

समीकरण (1) व (2) को जोड़ने पर,

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$$

$$\Rightarrow \quad 2I = \int_0^{\pi/2} dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0$$

$$\Rightarrow \quad I = \frac{\pi}{4}.$$

यही सिद्ध करना था।

प्रश्न 5. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$ का मान ज्ञात कीजिए।

(म.प्र. 2020)

हल : मान लीजिए कि $I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$ ...(1)

$$I = \int_0^{\pi/2} \frac{\sin^4\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)} dx$$

$$I = \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx \quad ...(2)$$

समी. (1) और (2) को जोड़ने पर,

$$I + I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx + \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx$$

$$2I = \int_0^{\pi/2} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x} dx$$

$$2I = \int_0^{\pi/2} 1 dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2} - 0$$

$$\therefore \quad I = \frac{\pi}{4}.$$

उत्तर

प्रश्न 6. सिद्ध कीजिए कि

$$\int_0^{\pi/2} \frac{x \sin x \cos x}{\cos^4 x + \sin^4 x} dx = \frac{\pi^2}{16}.$$

हल : माना

$$I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\cos^4 x + \sin^4 x} dx \quad ...(1)$$

$$I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right) \sin\left(\frac{\pi}{2} - x\right) \cos\left(\frac{\pi}{2} - x\right)}{\cos^4\left(\frac{\pi}{2} - x\right) + \sin^4\left(\frac{\pi}{2} - x\right)} dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{\left(\frac{\pi}{2} - x\right) \cos x \sin x}{\sin^4 x + \cos^4 x} dx \quad ...(2)$$

समी. (1) और (2) को जोड़ने पर,

$$2I = \int_0^{\pi/2} \frac{\left(x + \frac{\pi}{2} - x\right) \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$= \int_0^{\pi/2} \frac{\frac{\pi}{2} \cdot \sin x \cos x}{\sin^4 x + \cos^4 x} dx = \frac{\pi}{2} \int_0^{\pi/2} \frac{\tan x \sec^2 x}{1 + \tan^4 x} dx$$

$$= \frac{\pi}{2} \int_0^{\infty} \frac{t\, dt}{1 + t^4}, \qquad [\text{माना } \tan x = t \Rightarrow \sec^2 x\, dx = dt]$$

$$= \frac{\pi}{2} \cdot \frac{1}{2} \int_0^{\infty} \frac{du}{1 + u^2}, \qquad [\text{माना } t^2 = u \Rightarrow 2t\, dt = du]$$

$$= \frac{\pi}{4} \cdot [\tan^{-1} u]_0^{\infty} = \frac{\pi}{4}\left(\frac{\pi}{2} - 0\right) = \frac{\pi^2}{8}$$

$$\therefore \quad I = \frac{\pi^2}{16}.$$

यही सिद्ध करना था।

प्रश्न 7. सिद्ध कीजिए—

$$\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} dx = \frac{\pi}{4}.$$

हल : माना

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} dx \quad ...(1)$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan\left(\frac{\pi}{2} - x\right)}}{1 + \sqrt{\tan\left(\frac{\pi}{2} - x\right)}} dx = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{1 + \sqrt{\cot x}} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\frac{1}{\sqrt{\tan x}}}{1+\frac{1}{\sqrt{\tan x}}} dx = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\tan x}} \times \frac{\sqrt{\tan x}}{\sqrt{\tan x}+1} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}} dx$$

समी. (1) और (2) को जोड़ने पर,

$$I + I = \int_0^{\frac{\pi}{2}} \left[\frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} + \frac{1}{1+\sqrt{\tan x}}\right] dx$$

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}+1}{1+\sqrt{\tan x}} dx \Rightarrow 2I = \int_0^{\frac{\pi}{2}} 1\, dx = [x]_0^{\frac{\pi}{2}}$$

$\Rightarrow$ $$2I = \left[\frac{\pi}{2} - 0\right] \Rightarrow I = \frac{\pi}{4}.$$

यही सिद्ध करना था

प्रश्न 8. सिद्ध कीजिए कि

$$\int_0^{\frac{\pi}{4}} \log_e(1+\tan x)\, dx = \frac{\pi}{8} \log_e 2.$$

हल : माना $$I = \int_0^{\frac{\pi}{4}} \log_e(1+\tan x)\, dx$$

$$= \int_0^{\frac{\pi}{4}} \log_e\left[1+\tan\left(\frac{\pi}{4}-x\right)\right] dx$$

$$= \int_0^{\frac{\pi}{4}} \log_e\left[1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}.\tan x}\right] dx = \int_0^{\frac{\pi}{4}} \log_e\left[1+\frac{1-\tan x}{1+\tan x}\right] dx$$

$$= \int_0^{\frac{\pi}{4}} \log_e\left[\frac{1+\tan x+1-\tan x}{1+\tan x}\right] dx = \int_0^{\frac{\pi}{4}} \log_e\left[\frac{2}{1+\tan x}\right] dx$$

$$= \int_0^{\frac{\pi}{4}} \log 2\, dx - \int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx$$

$\therefore$ $$I = \log_e 2 \int_0^{\frac{\pi}{4}} dx - I$$

$\Rightarrow$ $$2I = \log_e 2 [x]_0^{\frac{\pi}{4}} = \log_e 2\left(\frac{\pi}{4}\right)$$

$\therefore$ $$I = \frac{\pi}{8} \log_e 2.$$

यही सिद्ध करना था।

प्रश्न 9. $\int_0^\pi \frac{x}{1+\sin x} dx$ का मान ज्ञात कीजिए।

हल : $$I = \int_0^\pi \frac{x}{1+\sin x} dx \quad \text{...(1)}$$

$\Rightarrow$ $$I = \int_0^\pi \frac{(\pi - x)}{1+\sin(\pi - x)} dx$$

$\Rightarrow$ $$I = \int_0^\pi \frac{\pi - x}{1+\sin x} dx \quad \text{...(2)}$$

समी. (1) और (2) को जोड़ने पर,

$$2I = \int_0^\pi \frac{x+\pi-x}{1+\sin x} dx = \pi \int_0^\pi \frac{dx}{1+\sin x}$$

$$= \pi \int_0^\pi \frac{1-\sin x}{(1+\sin x)(1-\sin x)} dx = \pi \int_0^\pi \frac{1-\sin x}{1-\sin^2 x} dx$$

$$= \pi \int_0^\pi \frac{1-\sin x}{\cos^2 x} dx = \pi \int_0^\pi \frac{1}{\cos^2 x} dx - \pi \int_0^\pi \frac{\sin x}{\cos^2 x} dx$$

$$= \pi \int_0^\pi \sec^2 x \, dx - \pi \int_0^\pi \sec x \tan x \, dx = \pi[\tan x - \sec x]_0^\pi$$

$$= \pi[0 - (-1) - 0 + 1] = 2\pi$$

$\therefore$ $$I = \pi.$$

उत्तर

प्रश्न 10. $\int_0^\pi \frac{dx}{5+4\cos x}$ का मान ज्ञात कीजिए।

हल : माना $$I = \int_0^\pi \frac{dx}{5+4\cos x}$$

$$= \int_0^\pi \frac{dx}{5\left(\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}\right) + 4\left(\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}\right)}$$

$$= \int_0^\pi \frac{dx}{9\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}} = \int_0^\pi \frac{\sec^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx$$

माना $\tan \frac{\pi}{2} = t$,

$\therefore$ $$\frac{1}{2} \sec^2 \frac{\pi}{2} dx = dt$$

जब $x = 0$, तब $t = \tan 0 = 0$

तथा जब $x = \pi$, तब $t = \tan \frac{\pi}{2} = \infty$

$\therefore$ $$I = 2\int_0^{\infty}\frac{dt}{9+t^2} = \frac{2}{3}\left[\tan^{-1}\frac{t}{3}\right]_0^{\infty} = \frac{2}{3}[\tan^{-1}\infty - \tan^{-1}0]$$

$$= \frac{2}{3}\left(\frac{\pi}{2}-0\right) = \frac{\pi}{3}.$$

प्रश्न 11. सिद्ध कीजिए—

$$\int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x} = \frac{\pi^2}{2ab}.$$

हल : माना $$I = \int_0^{\pi}\frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$\Rightarrow$ $$I = \int_0^{\pi}\frac{(\pi-x)\,dx}{a^2\cos^2(\pi-x) + b^2\sin^2(\pi-x)} \quad ...(1)$$

$\Rightarrow$ $$I = \int_0^{\pi}\frac{(\pi-x)\,dx}{a^2\cos^2 x + b^2\sin^2 x} \quad ...(2)$$

समी. (1) और (2) को जोड़ने पर,

$$2I = \int_0^{\pi}\frac{(x+\pi-x)}{a^2\cos^2 x + b^2\sin^2 x}dx$$

$$= \int_0^{\pi}\frac{\pi\,dx}{a^2\cos^2 x + b^2\sin^2 x} = 2\int_0^{\pi/2}\frac{\pi\,dx}{a^2\cos^2 x\left(1+\frac{b^2}{a^2}\tan^2 x\right)}$$

$$= \frac{2\pi}{a^2}\int_0^{\pi/2}\frac{\sec^2 x\,dx}{\left(1+\frac{b^2}{a^2}\tan^2 x\right)}, \quad \left[\text{माना } \frac{b}{a}\tan x = t \Rightarrow \sec^2 x\,dx = \frac{a}{b}dt\right]$$

$$= \frac{2\pi}{a^2}\cdot\frac{a}{b}\int_0^{\infty}\frac{dt}{1+t^2} = \frac{2\pi}{ab}[\tan^{-1}t]_0^{\infty}$$

$$= \frac{2\pi}{ab}(\tan^{-1}\infty - \tan^{-1}0) = \frac{2\pi}{ab}\cdot\frac{\pi}{2} = \frac{\pi^2}{ab}$$

$\therefore$ $$I = \frac{\pi^2}{2ab}.$$

यही सिद्ध करना था।

प्रश्न 12. सिद्ध कीजिए—

$$\int_0^{\pi/2}\log_e \sin x\,dx = -\frac{\pi}{2}\log_e 2.$$

हल : माना $$I = \int_0^{\pi/2}\log_e \sin x\,dx \quad ...(1)$$

$\Rightarrow$ $$I = \int_0^{\pi/2}\log\sin\left(\frac{\pi}{2}-x\right)dx$$

$\Rightarrow$ $$I = \int_0^{\pi/2}\log\cos x\,dx \quad ...(2)$$

समी. (1) और (2) को जोड़ने पर,

$$2I = \int_0^{\pi/2} (\log \sin x + \log \cos x)\, dx$$

$$= \int_0^{\pi/2} \log \sin x \cos x\, dx = \int_0^{\pi/2} \log \frac{2 \sin x \cos x}{2} dx$$

$$= \int_0^{\pi/2} (\log \sin 2x - \log 2)\, dx$$

$$= \int_0^{\pi/2} \log \sin 2x\, dx - \int_0^{\pi/2} \log 2\, dx$$

$$= \int_0^{\pi} \log \sin t \frac{dt}{2} - \log 2 \int_0^{\pi/2} dx, \qquad [2x = t \Rightarrow 2dx = dt$$

जब $x = 0$ तब $t = 0$, जब $x = \frac{\pi}{2}$ तब $t = \pi$]

$$= 2 \cdot \frac{1}{2} \int_0^{\pi/2} \log \sin t\, dt - \log 2 [x]_0^{\pi/2} = \int_0^{\pi/2} \log \sin t\, dt - \log 2 \cdot \frac{\pi}{2}$$

$$= I - \frac{\pi}{2} \log 2$$

$\Rightarrow$ $$I = -\frac{\pi}{2} \log 2$$ यही सिद्ध करना था।

प्रश्न 13. सिद्ध कीजिए कि—

$$\int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x)\, dx = -\frac{\pi}{2} \log 2.$$ **(NCERT)**

हल : माना $$I = \int_0^{\frac{\pi}{2}} (2 \log \sin x - \log \sin 2x)\, dx$$

$$= \int_0^{\frac{\pi}{2}} [2 \log \sin x - \log(2 \sin x \cos x)]\, dx$$

$$= \int_0^{\frac{\pi}{2}} [2 \log \sin x - \log 2 - \log \sin x - \log \cos x]\, dx$$

$$= \int_0^{\frac{\pi}{2}} [\log \sin x - \log 2 - \log \cos x]\, dx$$

$$= \int_0^{\frac{\pi}{2}} \log \sin x\, dx - \int_0^{\frac{\pi}{2}} \log 2\, dx - \int_0^{\frac{\pi}{2}} \log \cos x\, dx$$

$$= \int_0^{\frac{\pi}{2}} \log \sin x\, dx - \log 2 \int_0^{\frac{\pi}{2}} 1\, dx - \int_0^{\frac{\pi}{2}} \log \cos\left(\frac{\pi}{2} - x\right) dx$$

$$= \int_0^{\frac{\pi}{2}} \log \sin x\, dx - \log 2 [x]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \log \sin x\, dx$$

$$= -\log 2 \left[\frac{\pi}{2} - 0\right]$$

$\Rightarrow$ $$I = -\frac{\pi}{2} \log 2.$$ यही सिद्ध करना था।

प्रश्न 14. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$ का मान ज्ञात कीजिए। (NCERT)

हल : माना $$I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$$

$\Rightarrow$ $$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx, \quad \left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$\Rightarrow$ $$I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx$$

समी. (1) और (2) को जोड़ने पर,

$$I + I = \int_0^{\pi} \left[\frac{x \sin x}{1+\cos^2 x} + \frac{(\pi - x)\sin x}{1+\cos^2 x}\right] dx$$

$\Rightarrow$ $$2I = \int_0^{\pi} \frac{x \sin x + \pi \sin x - x \sin x}{1+\cos^2 x} dx = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} dx$$

माना $\cos x = t$, तब $-\sin x dx = dt$

जब $x = 0$, तब $t = 1$ तथा जब $x = \pi$, तब $t = -1$

$\therefore$ $$2I = -\int_1^{-1} \frac{\pi\, dt}{1+t^2} = \pi \int_{-1}^{1} \frac{dt}{1+t^2} = \pi \left[\tan^{-1} t\right]_{-1}^{1}$$

$$= \pi\left[\tan^{-1} 1 - \tan^{-1}(-1)\right] = \pi\left[\frac{\pi}{4} + \frac{\pi}{4}\right] = \pi \times \frac{2\pi}{4}$$

$\Rightarrow$ $$I = \frac{\pi^2}{2}.$$

प्रश्न 15. $\int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9 \sin 2x} dx$ का मान ज्ञात कीजिए। **(NCERT; CBSE 2018)**

हल : माना $$I = \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9 \sin 2x} dx$$

$$= \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9[1 - (1 - \sin 2x)]} dx$$

$$= \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9[1 - (\cos^2 x + \sin^2 x - 2 \sin x \cos x)]} dx$$

$$= \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9[1 - (\sin x - \cos x)^2]} dx$$

[$\sin x - \cos x = t$ रखने पर, $(\cos x + \sin x)dx = dt$

$x = 0, t = -1; x = \frac{\pi}{4}, t = 0$]

$$= \int_{-1}^{0} \frac{dt}{16 + 9(1 - t^2)} = \int_{-1}^{0} \frac{dt}{16 + 9 - 9t^2} = \int_{-1}^{0} \frac{dt}{25 - 9t^2}$$

$$= \int_{-1}^{0} \frac{dt}{9\left(\frac{25}{9} - t^2\right)} = \frac{1}{9} \cdot \int_{-1}^{0} \frac{dt}{\left(\frac{5}{3}\right)^2 - t^2}$$

$$=\frac{1}{9}\cdot\frac{1}{2\cdot\frac{5}{3}}\left[\log\left(\frac{\frac{5}{3}+t}{\frac{5}{3}-t}\right)\right]_{-1}^{0}=\frac{1}{30}\left[\log\frac{5/3}{5/3}-\log\frac{2/3}{8/3}\right]$$

$$=\frac{1}{30}\left[\log 1-\log\frac{1}{4}\right]$$

$\therefore$ $$I=-\frac{1}{30}\log\frac{1}{4}=\frac{1}{30}\log 4.$$ उत्तर

प्रश्न 16. $\int_0^1 \frac{\tan^{-1} x}{1+x^2}dx$ का मान ज्ञात कीजिए। **(म. प्र. 2019)**

हल : माना $$I=\int_0^1 \frac{\tan^{-1} x}{1+x^2}dx$$

$\tan^{-1} x = t$ रखने पर,

$$\frac{d}{dx}\tan^{-1} x=\frac{dt}{dx} \Rightarrow \left(\frac{1}{1+x^2}\right)dx=dt$$

जब $x=0$, तब $t=\tan^{-1} 0=0$

तथा जब $x=1$, तब $t=\tan^{-1} 1=\frac{\pi}{4}$

$\therefore$ $$I=\int_0^{\pi/4} t\,dt=\left[\frac{t^2}{2}\right]_0^{\pi/4}=\frac{1}{2}\times\frac{\pi^2}{16}$$

$$I=\frac{\pi^2}{32}.$$ उत्तर