Chapter - 12

Nature of Light

12.1 Nature of light

What is light? The question was of great concern for philophers and scientists for a long time. Descartes in 1637 gave particle (corpuscular) model of light, according to which light consist of tiny particles orignating from the source of light. This model was further developed by Isaac Newton. Newton was able to explain some optical phenomen, such as rectiliuar propogation of light, reflection and dispersion. In explaining refraction of light, Newton attributed imaginary properties to the refracting medium. He conculded that the speed of light in denser medium should be greater than the speed of light in the rarer medium. But the experiment in this regard by Foucault in 1850 proved converse of it.

In 1678, the Dutch physicist Christian Huyens a contemporay of Newton gave wave theory of light. This wave theory of light is the matter of concers to us in this chapter. The wave theory clearly expalined many optical phenomenon and moreover it was in agreement with Foucault's experimental results that speed of light in denser medium should be less than the speed of light in rarer medium. Even than the theory was not readily accepted, because of Newton's authority, and also that a wave always requires a medium for propogative, while light can travel in vaccum.

The interference experiment by Thomas Young in 1801 firmly established wave theory of light. The wave length of light was measured and the wave length of yellow light was found to be = 0.5 μm. Because of smallness of wave length, rectilinear propogation of light was asumed. After the experiment of Thomes Young, many experiments involing interference and diffraction were carried out which firmly established wave nature of light.

The only difficulty about the requirment of medium for propogation of light was removed by the elctromagnetic wave theory of light given by Maxwell. According to this light is an electro magnetic wave, and electromagnetic wave can travel in the vacuum.

12. 2 Huygen's Wave Theory and Wavefront

It was the Huygen who proposed the wave theory of light. Before using wave theory to explain the optical phenomenon let us first know some basic properties of a wavefront. According to wave theory, we first define a wavefront. If we drop a stone in still water, we see that circular rings spread outwards from the center (the point where stone hits the surface). We see that all the points on a ring oscillate in the same phase we can consider the snapshot of the ring as a wavefront oscillate in the same, phase and produces secondary wavelets.

A wave front can be defined as a locus of points which oscillate in the same phase. It is a surface of constant phase. All the points on a wavefront are source of secondary wavelets. The direction in which the wavefront travels is called a ray. Ray and wave front are always perpendicular to each other.

Let us discuss, the source and shape of wave front. If the source is a point source, than the shape of the wavefront will be spherical, as the medium is asumed to be homogenous (isotropic) and speed of light is same in all directions fig. 12 (a).If the source is a line source (a linear), the shape of the wave front will be cylendrical as shown in fig. 12.1 (b). The source is at a great distance, the rays are approimately parallel because the radius of curvature becomes very high and the wavefront is practically plane. Shown in fig. 12.1 (c).

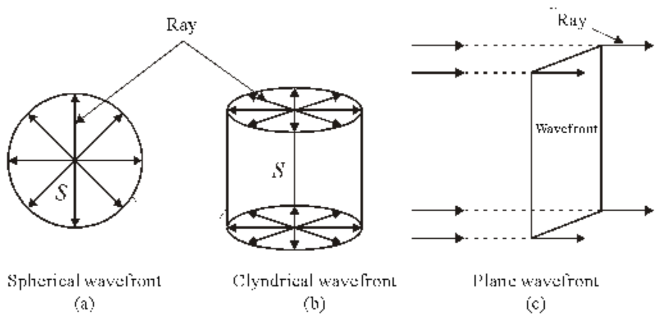


Fig. 12.1 (a) Spherical (b) Clyndcrical (c) Plane wavefront

To show the propagation of a wavefront, let us consider a part AB of a wave front. According to wave theory all the points on this wave front behave as secondary sources and produce secondary wavelets. If C is the speed of light, then $C\Delta t$ will be the radius of the secondary wavelet in time Δt. The envelops of all those secondary wavelets is the position of wave front at that instant. Fig. 12.2 B and B' shows the position of the wavefront at an instant t and $t + \Delta t$. For spherical and plane wavefront. The backwards motion was logically rejected by Fresnel, later on.

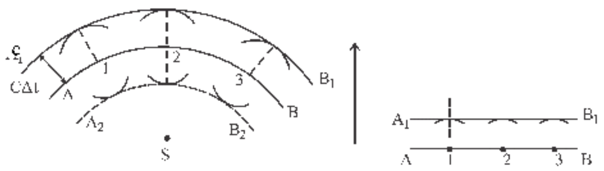


Fig. 12.2

12.3 Reflection and refraction at a plane surface

In the previous section we defined the construction and propagation of a wavefront based Huygen's principle. We will prove the experimental optical phenomenon of reflection and refraction and the laws govering them using Huygens theory.

12.3.1 Reflection at a plane surface

Fig. 12.3 shows MN as a reflecting surface, having a medium above it. Speed of wave is υ in this medium. AB is the incident wavefront and rays 1 and 2 are corresponding waves to it. All the points on the incident wavefront emits secondary wavelets. When the secondary wavelet from point B reaches the surface at C in time Δt ($BC = v\Delta t$), the secondary wavelet after reflection from A reaches point E. As the wavefront move forwards all the secondary wavelets on wave front AB get reflected and reach CE. Such that $AE = v\Delta t$.

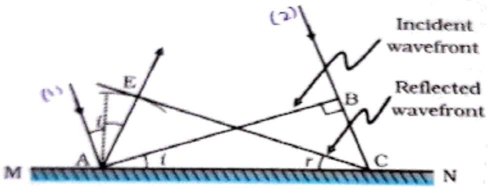


Fig. 12.3 : Reflection of the plane wave by a plane surface

Consider the Δ ABC and Δ AEC ; (i) AE = BC = $v\Delta t$, AC is common, and $\angle$AEC = $\angle$ABC = 90 (ray is always perpendicular to wavefront). The two triangles are congruant hence $\angle i = \angle r$. This is the law of reflection. Since incident ray, reflected ray and normal all lie in the same plane (plane of paper) II law of reflection is also varified.

12.3.2 Refraction at a Plane Surface

First we consider reflection from a rarer medium to a denser medium. Fig. 12.4 shows MN as an interface of two media velocity of the wave in medium 1 and medium 2 is v_1 and v_2 respectively. AB is the incident wavefront A'A is incident ray. $\angle$BAC = $\angle i$. All the points on incident wavefront AB emits secondary waves lets. While the secondary wave lets from A, enters second medium and reaches E (such that $AE = v_2 t$); the secondary wavelet from B still moves in first medium and reaches C (BC = $v_1 t$) during the same time interval t. The envelop of all secondary wavelets is CE (called refracted wavefront) and angle ACE = $\angle r$.

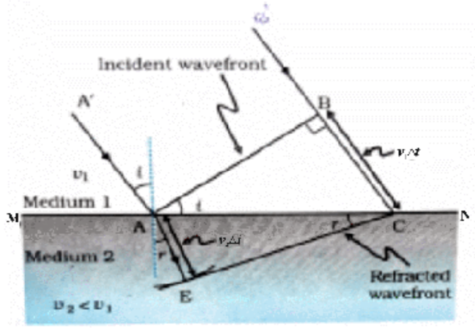


Fig. 12.4 : Refraction from a rarer medium to denser medium

Considering right angles ABC and AEC we get

$$\sin i = \frac{BC}{AC} = \frac{v_1 \Delta t}{AC} \quad \ldots (12.1)$$

and

$$\sin r = \frac{AD}{AC} = \frac{v_2 \Delta t}{AC} \quad \ldots (12.2)$$

dividing eq. 12.1 by eq. 12.2 we get

$$\frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1}{v_2} \quad \ldots (12.3)$$

$n_1 = \frac{c}{v_1}$ and $n_2 = \frac{c}{v_2}$ (for first and second median respectively)

We get $\frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21}$ (where n_{21} is refractive index of medium 2, with respect to medium 1)

$n_1 \sin i = n_2 \sin r$

since $n_2 > n_1$; $\angle i = \angle r$. The refracted ray bends twoards the normal. From the geometry of the figure it is also verified (valid) for incident and refracted rays. The second law is also verified since the incident ray, refracted ray and the normal lie in the same plane as that plane of paper.

If λ_1 and λ_2 are the wave lengths in the media respectively - from $\frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{\lambda_1}{\lambda_2}$ hence it is clear that during refraction, only wavelength changes due to change in velocity and frequency remains unchanged.

Now we consider Fig. 12.5 for refraction from denser medium to a rarer medium. From the diagram we see that $\angle i < \angle r$ hence the refracted ray bends away from the normal. In this case we see that

$$n_1 \sin i = n_2 \sin r \text{ and } \frac{\sin i}{\sin r} = \frac{n_2}{n_1} = n_{21}$$

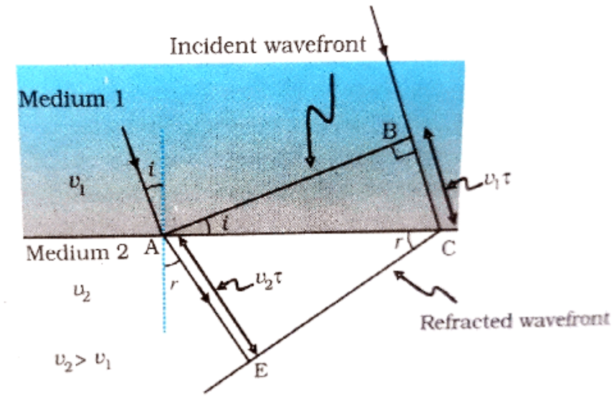
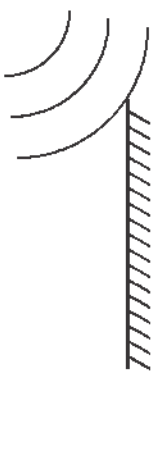
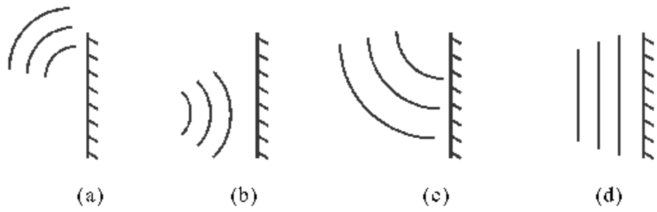


Fig. 12.5 : Refraction from a denser medium to rarer medium

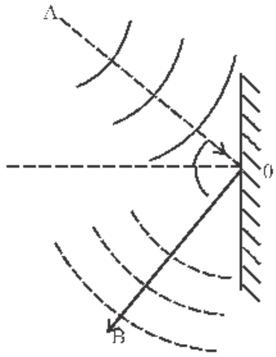
If $r = 90^0$, $n_1 \sin i = n_2 \sin r$; at this angle no refrated ray will be obtained and $i = i_c$ (called critical angle for total internal reflection) hence $\sin i = \frac{n_2}{n_1} \sin r$.

Example 12.1 : A spherical wavefront is incident on a reflecting surface. Which will be the refracted wavefront out of the 4 given.

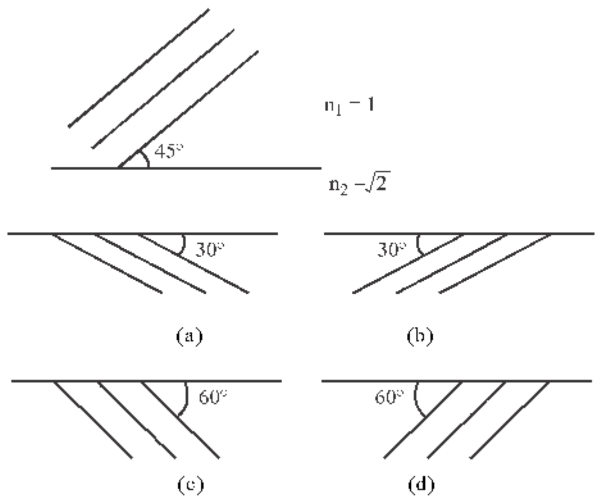




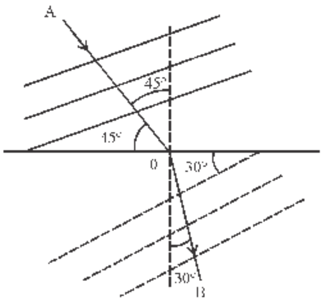
Solution : In a homogenous medium the wavefront is always perpendicular to the ray. In the above problem, take the centre of spherical wavefront as A and draw incident ray AO (perpendicular to wavefront) and normal at O. From geometry draw reflected ray (using $i = r$) OB. Now construct spherical wavefront taking O as a centre, which comes out as given in fig. (c).



Example 12.2 : Fig. shows the incident wavefront at the interface of two media. Which of the given four diagrams, represents the refracted wavefront.



Solution : The incident ray is given by AO. From Snell's law



$n_1 \sin i = n_2 \sin r$

$$1\sin 45^\circ = \sqrt{2}\sin r$$

$$\frac{1}{\sqrt{2}} = \sqrt{2}\sin r$$

$$\sin r = \frac{1}{2}$$

$$r = 30^\circ$$

hence option (a) is the correct representation of the refracted wavefront.

12.4 Interference of Light and Coherent Sources

When two exactly similar wave meet (superpose) each other in space, the resultant effect on intensity is called interference. This phenomenon is based on principle of supperposition, according to which when two or more waves superpose each other at a point in space, then the resultant amplitude will be vector sum of individual amplitude. You have studied this effect for general waves in class XI.

Now we know that light energy also propagates in the form of waves, hence when two or more light waves superpose in space, the sustained effect of maximum and minimum intensity is observed. This phenomenon is called interference of light. The maximum effect is called constructive, while the minimum effect is called destructive interference.

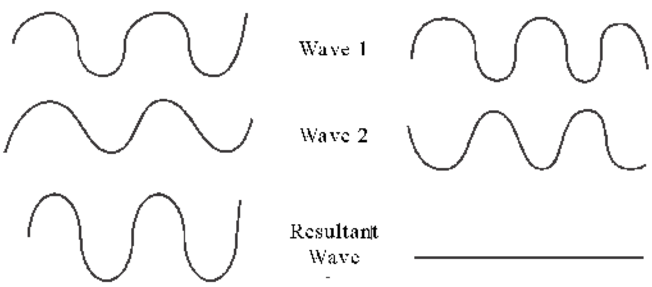


Fig. 12. 6 : To stages of super position of waves of equal amplitude

Fig. 12.6 shows the time dependance of two waves. Fig. 12.6 (a) shows that the two waves reach a point in same phase, i.e. crest of one meet the crest of other and trough of one meets the trough of other. This is possible when phase difference between the two waves is $\phi = 0, 2\pi, 4\pi ... 2n\pi$ Radian. It is constructive interference.

In fig. 12.6 (b) the two waves meet each other in opposite phase i.e the crest of one meet the trough of the other. Such type of effect is called destructive effect. For this effect the phase difference will be $\pi, 3\pi, 5\pi ...$ $(2n+1)\pi$ Radian.

The two waves are said to be coherent if their phase difference ϕ is not a function of time, which means ϕ should remain constant with time. This is the important condition for sustained effect, of interference.

Two separate sources like two candles or two bulbs (even of the same frequency operated by single switch) can never be coherent. Because the emission of light is an atomic phenomenon and it can't be synchronised. Emission of light from an atom takes place in 10^{-8} s. The different groups of atoms emitt light during theis period, which can't be synchronised with the emission from other source.

Fig. 12. 7 (a) Wave of finite length
Fig. 12.7 (b) Wave of infinite length

The phase of wave changes randomly and hence the phase difference between two waves also changes randomly.

Hence two independent sources can't produce interference. Intensity distribution is uniform in space.

It is not easy to obtain two coherent sources, unless we obtain two sources from a single source. Our such method is division of wave front into two, which is obtained in Young's double slit experiment.

Now a days coherent sources like LASER source are available which are monochromatic and intense sources. We can obtain interference using two such sources.

12.5 Necessary Conditions for Interference

For clear and sustained effect of interference the following conditions are there -

(1) The two sources should be coherent, i.e. the phase difference between the waves from the sources should remain constant with time.

(2) The two sources should have same freqeuncy.

(3) To obtain a better constrast between the maximum and minimum effect and that of average effect, the amplitude of the two wave should be nearly equal.

(4) Both the waves should move in the same direction and superpose.

(5) The two sources should be very near to each other.

(6) The slits used as a source should be narrow. Other wise a broad source can be treated as group of many point sources, which produce interference effect separately. And what we see is the average effect on screen which is uniform distribution of light and not interference.

(7) The path difference between the two waves should be very small, otherwise the waves from two sources that reach a point simultaneously to superpose, they may reach one after the other. The path difference must not exceed a few centimeters.

(8) If the light from the two sources is polarized, then they should have the same plane of polarization.

12.6 Young's Double Slit Experiment

In the orignal experiment by Thomas Young in 1801, the sunlight was passed through single pin hole and it is allowed to pass through two symetrical holes in a cord board. The interference pattern was obtained on a screen which was in the form of bright and dark areas. Here we will use a monochromatic source of light, a single slit and a double slit. The arrangement is shown in the figure 12.8.

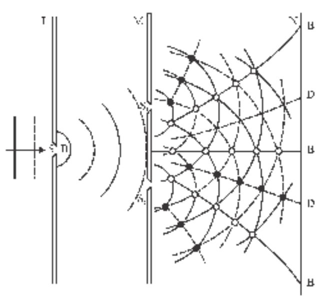


Fig. 12.8 : Young's double slit experiment

As shown in the above figure we have a screen L with a single slit. There is another screen M which has two slits very near to each other. On another screen N where we can see alternate dark and bright bands. The pattern of these bands are called interference and the bands are called dark and bright frings. Shown in fig. 12.9.

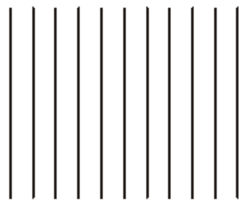


Fig. 12.9 : Interference fringes

A spherical wavefront which originate from a point source i.e. slit s spreads and reaches double slit screen. Since all the points on this wavefront are in same phase, the two slits S_1 and S_2 behave like two coherent sources having same phase. To show the superposition of the two waves in the space ahead of the double slit, we draw the waves in the form of bold line (showing crest) and dotted line (showing trough). The points where crest of one wave meet the crest of other, and where trough of one meet trough of other waves are shown as white circles and black dots indicate the points where a crest meets a trough, in fig. 12.8. The line joining these points are called nodal and antinodal lines. The colour of the bright strips is that of the monochromatic source used in the experiemnt.

12.6.1 Analytical Treatment of Interference

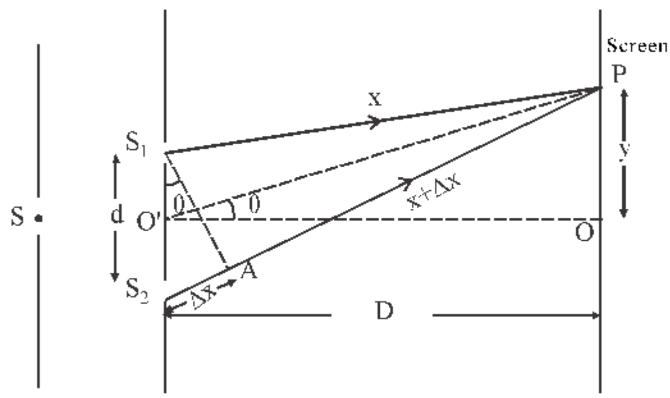


Fig. 12.10 Geometrical construction for anlytical treatment

For the analytical treatment of interference we take the light as an electromagnic wave, and only the electrical field vector E produce the whole visual effect. From fig.

12.10 we see that the light waves from the two sources S_1 and S_2 which reach a point P on the screen has covered two different path lengths x and $x+\Delta x$. The electric field due to these waves at P is E_1 and E_2 respectively given as

$$E_1 = E_{m1}\sin(kx-\omega t) \qquad \ldots(12.7)$$

and
$$E_2 = E_{m2}\sin\left(k(x+\Delta x)-\omega t\right) \qquad \ldots(12.8a)$$

$$= E_{m2}\sin(kx-\omega t+\phi) \qquad \ldots(12.8b)$$

where
$$\phi = k\Delta x = \frac{2\pi}{\lambda}(\Delta x) \qquad \ldots(12.9)$$

Here ω is angular frequency, λ = wave length and ϕ is the phase difference produced by path difference Δx.

From principle of superposition the resultant electric field at point is given by -

$$E = E_1 + E_2 = E_{m1}\sin(kx-\omega t) + E_{m2}\sin(kx-\omega t+\phi)$$

$$= E_{m1}\sin(kx-\omega t) + E_{m2}\sin(kx-\omega t)\cos\phi$$

$$+ E_{m2}\cos(kx-\omega t)\sin\phi$$

$$= (E_{m1}+E_{m2}\cos\phi)\sin(kx-\omega t) + (E_{m2}\sin\phi)\cos(kx-\omega t) \qquad \ldots(12.10)$$

if
$$(E_{m1}+E_{m2}\cos\phi) = E_m\cos\alpha \qquad \ldots(12.11)$$

and
$$E_{m2}\sin\phi = E_m\sin\alpha \qquad \ldots(12.12)$$

$$E = E_m\left[\sin(kx-\omega t)\cos\alpha + \cos(kx-\omega t)\sin\alpha\right]$$

$$= E_m\sin(kx-\omega t+\alpha)$$

Hence the resultant wave will also be a sine curve of freqeuncy ω, whose amplitude is given by

$$E_m = \sqrt{E_{m1}^2 + E_{m2}^2 + 2E_{m1}E_{m2}\cos\phi} \qquad \ldots(12.13)$$

and the phase angle

$$\alpha = \tan^{-1}\frac{E_{m2}+\sin\phi}{E_{m1}+E_{m2}\cos\phi} \qquad \ldots(12.14)$$

From definition the intensity is equal to square of the amplitude i.e. $I \propto E_m^2$ or $I = KA^2$ where K is a constant hence the intensity

$$I = K E_m^2 = K\left[E_{m1}^2 + E_{m2}^2 + 2E_{m1}E_{m2}\cos\phi\right]$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi \qquad \ldots(12.15)$$

where I_1 and I_2 are intensities from the sources S_1 and S_2 and $\frac{I_1}{I_2} = \left(\frac{E_{m1}}{E_{m2}}\right)^2$. From eq. 12.15 it is clear that the resultant intensity at P is different from the sum of individual intensities ($I_1 + I_2$) and depends on ϕ(the phase difference).

The resultant intensity will be maximum when $\cos\phi = 1$ i.e. $\phi = 0, \pm 2\pi, \pm 4\pi$................

or
$$\phi = \pm 2n\pi \quad n = 0, 1, 2 \qquad \ldots(12.16)$$

or
$$\frac{2\pi}{\lambda}\Delta x = \pm 2n\pi$$

or λ the the path difference $\Delta x = \pm n\lambda$ where

$$n = 0, 1, 2 \qquad \ldots(12.17)$$

and the maximum intensity

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2 \qquad \ldots(12.18)$$

If the amplitude of the waves are equal i.e.

$$I_{max} = \left(2\sqrt{I_0}\right)^2 = 4I_0$$

Hence if the phase difference and path difference between the rays reaching point P is $\pm 2n\pi$ and integral multiple of λ the intensity will be more than ($I_1 + I_2$) and these points are called maxima. And if $I_1 = I_2 = I_0$ the intensity at maxima will be $4I_O$.

The resultant intensity will be minimum when $\cos\phi = 1$

or $\phi = \pm\pi, \ \pm 3\pi, \ 5\pi$

$$\phi = \pm(2n-1)\pi \ \text{ where } n = 1, 2, 3 \qquad \ldots(12.19)$$

or path difference

$$\Delta x = \pm(2n-1)\frac{\lambda}{2} \ \text{ where } n = 1, 2, 3 \qquad \ldots(12.20)$$

and the minimum intensity will be -

$$I_{min} = \left(I_1 + I_2 - 2\sqrt{I_1 I_2}\right) = \left(\sqrt{I_1}-\sqrt{I_2}\right)^2 \qquad \ldots(12.21)$$

hence when the path difference and phase difference between the waves reaching point are odd multiple of (λ/2) and ϕ = I (2 n - 1) π respectively, the resultant intensity will be minimum. Moreover when $I_1 - I_2 - I_0$ the resultant intensity will be zero. The equation 12.15 can be written as -

$$\cos\phi = 2\cos^2(\phi/2) - 1$$

$$I_1 + I_2$$

$$\therefore \qquad I = 4I_0\cos^2(\phi/2) \qquad \ldots(12.22)$$

(using identity $\cos\phi = 2\cos^2(\phi/2) - 1$)

From the above analysis we can conclude that all the maxima are equidistant, distance between them and intensity $I_{max} = \left[\left(\sqrt{I_1} + \sqrt{I_2}\right)^2\right]$. The same is true for the minima *i.e.* $I_{min} = \left[\left(\sqrt{I_1} \sim \sqrt{I_2}\right)^2\right]$. The maxima and minima are alternatively situated because $\Delta x = 0, \pm\lambda, \pm 2\lambda \ldots$ for maxima.

Where as for minima, $\Delta x = \pm\frac{\lambda}{2}, \pm 3\frac{\lambda}{2} \pm 5\frac{\lambda}{2} \ldots$

Fig. 12.11 shows the graph between phase difference ϕ and the position of maxima and minima. It also show that I_{max} is greater than $I_1 + I_2$ by an amount $2\sqrt{I_1 I_2}$ and for I_{max} it is less than ($I_1 + I_2$) by the same amount.

The law of energy conservation holds goods in this phenomenon, the amount of energy gained at maxima is exactly equal to the energy lost at minima. Energy is only redistributed.

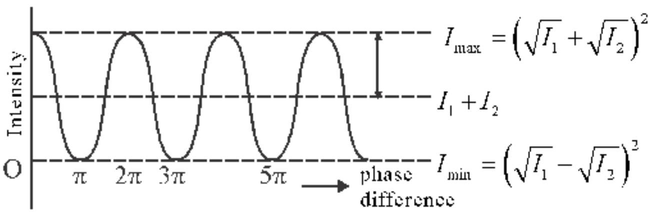


Fig. 12.11 : Intensity distribution in interference (for $I_1 = I_2$)

Here we see what happens when the sources are in coherent, i.e. ϕ when changes with ture. In this case $<\cos^2\phi/2> = 1$ (i.e. average for one cycle for $<\cos^2\phi/2> = 1/2$). From eq. 12.22 . $I = 4I_0 \times \frac{1}{2} = 2I_0$ Which shows that intensity at all points will be sum of intensity of each other.

12.6.2 Expression for Fringe Width

As we have seen earlier that the dark and bright fringes are equidistanly situated. The distance between the two consicutive bright or two dark finges is called fringe width.

We cosider Fig. 12.10 for determination of fringe width. If d = separation of two slits and D = distance of the screen from double slit. We find central bright fringe at point O, since path difference in this case is zero. Now consider the fringe at point P on the screen situated at a distance y from central bright fringe. The path difference between the waves originating from S_1 and S_2 and reaching P will be $\Delta x = S_2P - S_1P$, since D >> d, S_1P and S_2P can be taken as parallel. And S_1A is a perpendicular on S_2P; S_1A can be taken as perpendicular to O'P also

hence $\angle S_2S_1A = \angle OO'P = \theta$. From the diagram 12.10; from $\Delta OO'P$, $\tan\theta = \frac{OP}{OO'} = \frac{y}{D}$ and from ΔS_1S_2A $\sin\theta = \frac{S_2A}{S_1S_2} = \frac{S_2A}{d} = \frac{\Delta x}{d}$ since θ can be taken as very small and for very small angle in radian $\sin\theta \approx \theta$.

$$\text{We get } \frac{y}{D} = \frac{\Delta x}{d} \qquad \ldots(12.23a)$$

$$\text{and} \qquad y = \Delta x\left(\frac{D}{d}\right) \qquad \ldots(12.23b)$$

If there is n[th] bright fringe at P, then $\Delta x = n\lambda$

$$\text{and } (y_n)_{Br} = n\lambda\left(\frac{D}{d}\right) \qquad \ldots(12.24)$$

Similarly for $(n-1)^{th}$ bright fringe

$$(y_{n+1})_{Br} = (n+1)\lambda\left(\frac{D}{d}\right) \qquad \ldots(12.25)$$

Hence the fringe width of a dark fringe will be

$$\beta = y_{n\ 1} - y_n = \frac{D\lambda}{d} \qquad \ldots(12.26)$$

similarly for a dark fringe at P -

$$(y_n)_{dark} = (2n-1)\frac{\lambda}{2}D \qquad \ldots(12.27)$$

$$(y_{n-1})_{dark} = (2n+1)\frac{\lambda}{2}D \qquad \ldots (12.28)$$

Again the fringe width for a bright fringe will be -

$\beta = \frac{D}{d}\lambda$ we conclude that all the fringes are of the same width and independent of n.

The angular width of a fring is given by -

$$\theta_o = \Delta\theta = \frac{\beta}{D} = \frac{\lambda}{d} \text{ (independent of D)}$$

We can draw some important conclusion from, the above result.

(i) For a constant value of D and d; $\beta \propto \lambda$; $\lambda_{red} > \lambda_{blue}$ hence $\beta_{red} > \beta_{blue}$

(ii) $\beta \propto \frac{1}{d}$ (for D and λ constant) which indicates that d is kept small to get wide fringes as discussed in see. 12.5 point 5.

(iii) $\beta \propto D$ show that increases with screen distance. But it D is too large the intensity of the fringes diminishes and observation of interference pattern will be difficult.

(iv) We can find wave length of a monochromatic source by this method.

(v) If the whole experiment set up is placed in a medium of reflective index n. Then λ will be reduced to λ / n and $\beta' = \frac{\lambda' D}{d} = \frac{1}{n}\left(\frac{\lambda D}{d}\right) = \frac{1}{n}\beta_{air}$; we can find n by measuring β_{air} and β' .

12.6.3 Interference Fringes Produced by White Light

White light consists of the wavelengths ranging from 3800 Å to 7800 Å. The interference pattern obtained consits of central bright white fring followed by bands of colours in the sequence red to violet. Beyond violet a constant illumination is seen. This property is used to locate central bright fringe in the experiment using monochromatic light where location of central bright fringe is impossible. We just replace the monochromatic source with white light and mark the central bright fringe.

Example 12.3 : In Young's double slit experiment the ratio of the amplitude of the sources is 3 : 2. Find (a) Ratio of amplitude (b) Ratio of intensity for bright and dark fringes.

Solution : The resultant amplitude for maxima is $E_{max} = E_1 + E_2$ and for minima $E_{min} = E_1 - E_2$

$$\text{given } \frac{E_1}{E_2} = \frac{3}{2} \text{ hence } \frac{E_1 + E_2}{E_1 - E_2} = \frac{3+2}{3-2} = 5$$

$$\text{and } \frac{I_{max}}{I_{min}} = \left(\frac{E_{max}}{E_{min}}\right)^2 = 25$$

Example 12.4 : In Young's double slit experiment d = 0.2 mm for λ = 8000 Å . Find fringe width on a screen placed at 1 m from double slit.

Solution : $\beta = \frac{\lambda D}{d}$;

$\lambda = 8000\,\text{Å} = 8\times10^{\;7}\,\text{m},\ D = 1\,\text{m}$,

Substituting we get $d = 0.2mm = 2\times10^{\;4}m$

$$\beta = \frac{8\times10^{\;7}\times1}{2\times10^{-4}} = 4\times10^{-3}m = 4mm$$

Example 12.5 : In Young's double slit experiment 60 fringes are seen in a given space when λ = 6600 Å is used. What will be be number of fringes in that area when the source is replaced by λ = 4400 Å .

Solution : If n fringes of width β are situated in area w then $w = n\beta$

$$w = n\beta = \frac{n\lambda D}{d} \quad \text{hence}$$

$$\text{and } n_2 = \frac{n_1\lambda_1}{\lambda_2} = \frac{60\times6600}{4400} = 90$$

Example 12.6 : A dichromatic source of wavelengths 4200 Å and 4800 Å is used in young's double slit experiment in which the separation between the slit is 2.0 mm and the screen is at 1 m from double slit. Find the minimum distance from central maxima, where the bright fringes of both the wavelengths coincide.

Solution : The distance of n^{th} bright fringe from central maxima is given by $y = \frac{n\lambda D}{d}$. If the two bright fringes of different colours coincide at a point then

$$y = \frac{n_1 \lambda_1 D}{d} = \frac{n_2 \lambda_2 D}{d} \quad \text{hence} \quad \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{4800}{4200} = \frac{8}{7}$$

hence n = 7 for 4200 Å and n = 8 for 4800 Å

$$\text{hence } y = \frac{8 \times 4800 \times 10^{-10} m \times 1.0m}{2 \times 10^{-3} m}$$

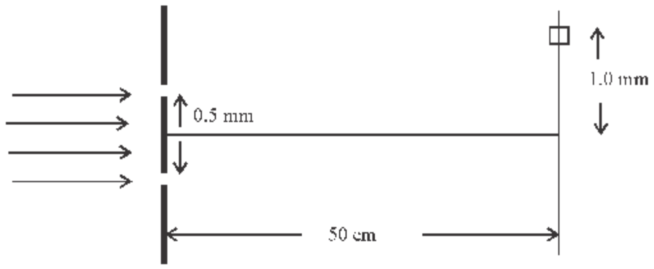
$$= 19200 \times 10^{-7} m$$

$$= 1.92 \times 10^{-3} m = 1.92mm$$

Example 12.7 : In choherent white light (400 mm to 700 mm) is used in a double slit experiment. The separation of the slits is 0.5 mm and screen is at a distance 50 cm from slits. There is hole on the screen at a distance 1.0 mm from the central line.

(a) Which wavelength will be absent in the light passing through the hole?

(b) Which wavelength (wavelengths) passing through the hole will have maximum intensity?



Solution : (a) The wavelength which cannot pass through the hole, has destructive interference at that point -

$$y = \frac{(2n-1)\lambda D}{2d}$$

$$\lambda = \frac{2dy}{(2n-1)d} = \frac{2(0.5 \times 10^{-3} m) \times 10^{-3} m}{(2n-1) \times 50 \times 10^{-2} m}$$

$$= \frac{2000}{(2n-1)} nm$$

$$n = 1 \quad \lambda_1 = \frac{2000}{1} = 2000$$

$$n = 2 \quad \lambda_2 = \frac{2000}{3} = 667nm$$

$$n = 3 \quad \lambda_3 = \frac{2000}{5} = 400nm$$

$$n = 4 \quad \lambda_4 = \frac{2000}{7} = 286nm$$

Similarly λ_1 and λ_2 are not a part of incident light, hence the absent wavelength are 667 nm and 400 nm.

(b) The wavelength which has maximum intnesity will have a constructive interference at the hole. For this -

$$y = \frac{n\lambda D}{d}$$

$$\lambda = \frac{yd}{nD} = \frac{0.5 \times 10^{-3} \times 10^{-3}}{n(0.5)} = \frac{1000nm}{n}$$

$$n = 1 \qquad \lambda_1 = 1000nm$$

$$n = 2 \qquad \lambda_2 = 500nm$$

$$n = 3 \qquad \lambda_3 = 333.33nm$$

Only the length λ_2 = 500 nm present in the incident light hence it will have maximum intensity in the outgoing light from the hole.

12.7 Differaction

Diffraction is a characterstic property of the waves. When light passes through an obstacle it bends at the edges of the obstacle. The rectilinear propogation of light seems to fail in this case. When light passes through an obstacle, the part of the wavefront which passes, spreads in the geometrical shadow or the incident rays bends at the edges of the obstacle in geometrical shadow. This phenomenon of bending of the rays or spreading of the wavefront passing through an obstacle is called diffraction.

The effect is pronounced if the condition is λ = d is fulfilled i.e. size of the obstacle should be of the order of wavelength. In daily life this condition is fulfilled for sound waves as the size of doors and windows are of the order of wavelength of sound. That is the reason that we can hear the conversation in the room, even if we are out of it. But diffraction of light is not commonly observed in daily life, because the size of the obtascle (hole) should be of the order of 0.0005 mm.

The detailed explanation of diffraction was given by

Fresnel. It would be very interesting to note an event in the history of science. A top apponent of wave theory. Poision ridiculed Fresnel; that if the explanation of Fresnel is correct, then we should find a bright spot at the center of the shadow of an opaque disc. The experiment was performed by poision and the Fresnel was proved right.

12.7.1 Composition of Diffraction of Sound and Light

As mentioned in the previous section it is easier to observe diffraction of sound in daily life, since the basic condition of clear diffraction is fulfilled.

To observe light diffraction in daily life the order of obstacle is = 10^{-7}m. For ultrasonic waves the required obstacle is = 1 cm. For diffraction of short radio waves, medium waves, the obstacle size is very large, hence these waves can diffract and bend by buildings and hills. For diffraction of X-rays, an obstacle of the order of 1Å is required. This is the dimension of crystal lattice and atomic spacing, this is the basis of crytallograhic study.

It can be understood by Huygen's principle that, if the size of an opening/obstacle is very large compared to wave length of light, the light wave will pass through it without bending which shows the rectilinear propagation of light. And if $\lambda = d$, the most part of the incident wave front is stopped by the obstacle and the small part passing through it will emittit secondary wavelets, and the wavelet spreads in geometrical shadow. (Actually it is the diffraction that occur at the slits in double slit experiment).

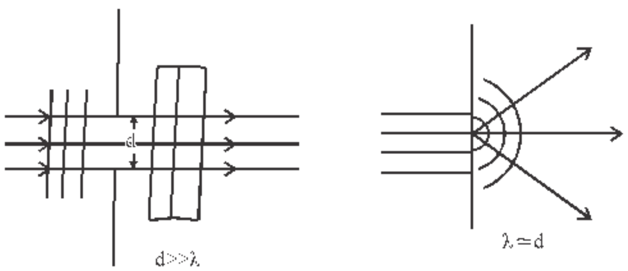


Fig. 12.12 Explanation of diffraction by Hutgen's principle

12.7.2 Types of Diffraction : Fresnel and Fraunhofer Diffraction

The study of diffraction can be devided into two classes.

(a) Fresnel's diffraction : When the source and the screen are at finite distance from the obstacle/ apparture, then it is called Fresnel's diffraction. In Fresnel's diffraction the incident wavefront is either spherical or cylinderical.

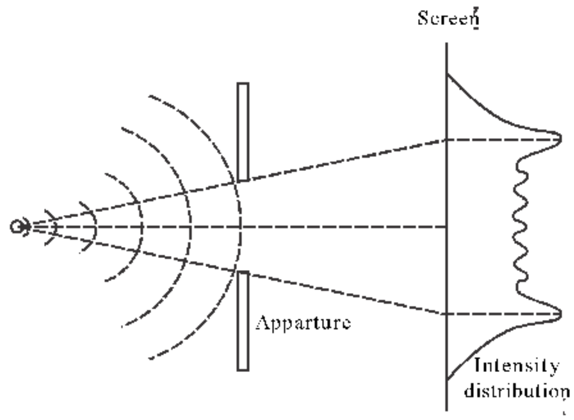


Fig. 12.13 Fresnel's diffraction

(b) Fraunhofer Dffraction : If the effective distances of source and the screen is infinite, the diffraction is called Fraunhofer diffraction. The incident and diffracted wavefronts are plane.

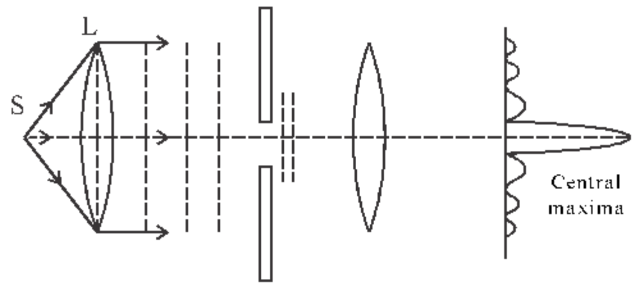


Fig. 12.14 Fraunhofer diffraction

Fresnel's explanation is based in certain hypothesis and the result is somewhat approximate. While in Fraunhofer diffraction the explanation is simpler because of use of plane wave front. We will confine ourself to Fraunhofer diffraction in this chapter.

12.8 Fraunhofer Diffraction Due to Single Slit

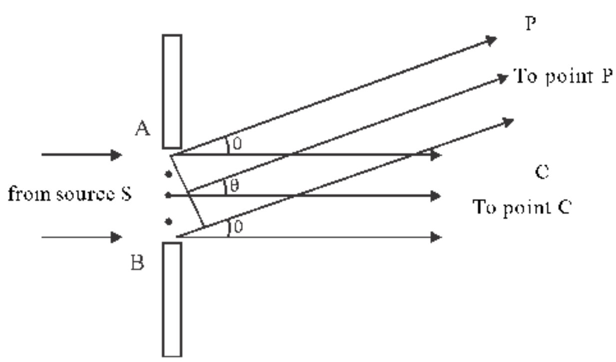


Fig. 12.15 Diffraction by a single slit

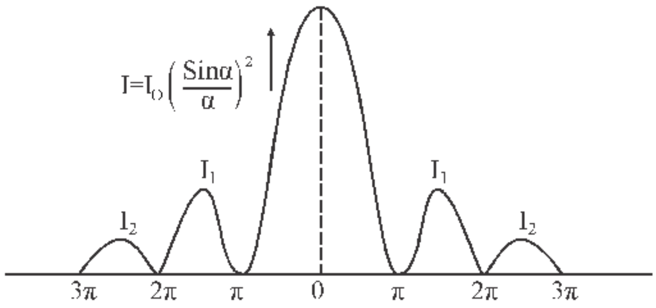


Fig. 12.16 The intensity distribution in Fraunhofer diffraction

When the size of the obstacle is of the order of wave length of light and the incident wavefront is plane, we obtain a broad pattern. This pattern consist of central bright spot and alternate dark and bright bands of decreasing intensity. This pattern is called Fraunhofers diffraction.

To understand it consider fig. 12.15. A plane wave front is incident on a slit AB of size a. M is mid point of AB. MC is a perpendicular to slit, where C is the center of the screen. All the secondary wave lets from different part of incident wavefront AB meet at C in the same phase, hence it is a bright spot of high intensity. To find the intensity on any other point on screen, we point to P from M, the angle between MP and MC gives the angular position of point P. Draw two lines from A and B, which are parallel to MP. The path difference between the two rays starting from A and B and meeting at P is

$$BP - AP = AQ = a\sin\theta \qquad \ldots (12.30)$$

(can be taken as $sin\theta \approx \theta$ for small angles)

The path difference between the above two rays reaching P may be taken as a θ. But there are large number of secondary wavelets on wavefront AB, which contribute differently at P due to their different path difference. To understand it we devide the wave front AB into two parts AM and MB. A wave starting from A and reacting P will be destroyed by a wave originating from M. So all the waves from AB which contribute at P; are destroyed by the same number of waves originating from MB and reaching at P, by one to one mapping. Hence the net intensity at P will be zero. We get a dark band at P. For this condition the path difference $a\sin\theta = \lambda$.

$$\sin\theta = \frac{\lambda}{a} \qquad \ldots (12.31)$$

Similarly for n^{th} minima we get

$$\sin\theta_n = n\left(\frac{\lambda}{a}\right) \qquad \ldots (12.32)$$

The above equation gives the angular position of n^{th} minima.

If the path difference between two waves originating from A and B and reaching a point on the screen is odd multiple of $\lambda/2$; say $= \frac{3\lambda}{2}$. In this case the wavefront AB can be devided in 3 - parts. All the secondary wavelets from one part, that reach a point Q on the screen, are canncelled/destroyed by the same number of wavelets orginating from the next second part of AB and reacting Q. Now the contribution of the next third part of AB will produce some intensity on Q. Hence Q will be a maxima, but of reduced intensity.

In this way for n^{th} order bright band

$$a\sin\theta_n = (2n+1)\frac{\lambda}{2} \qquad \ldots (12.33)$$

$$\sin\theta_n = \frac{(2n+1)\lambda}{2a} \qquad \ldots (12.34)$$

Equations 12.32 and 12.34 gives the angular positions of minima and maxima; which is symetrical about the central maxima, width of central maxima will be greater than a (due to diffraction). The intensity of a minima is never zero, but extreamly small.

12.9 Difference between Interference and Diffraction

Now we will compare the pattern obtained in young's double slit experiment (interference pattern) and pattern by a single slit (diffraction pattern).

(i) In interference there is superposition of wave originating from two narrow slits. The diffraction pattern is due to super position of group of waves originating from each point of a single slit.

(ii) In terference pattern there are many bright and dark bands of same intensity and equidistant. In diffraction pattern there is central maxima whose width is twice in comparison to other maximas.

The intensity falls sucessively as we move to other maxima on either side of centre.

(iii) In identical experimental set up of the points at which there is maxima in interference, there is minima in diffraction. Its opposite is also correct.

(iv) The intensity at minima in interference is zero where as in diffraction the intensity at minima is non zero.

Example 12.8 : In single slit diffraction pattern the second order bright fringe is at a distance 1.4 mm from centre of central maxima. Screen is at a distance 80 cm from a slit of width 0.80 mm. Consdering monochromatic incident light find out its wave length.

Solution : Here $y_2 = 1.4\,mm = 1.4\times10^{-3}\,m$

$$D = 80\,cm = 0.8\,m$$

$$a = 0.80\,mm = 8\times10^{-4}\,m$$

For second order bright fringe

$$y_2 = \frac{5}{2}\frac{\lambda D}{a} \Rightarrow \lambda = \frac{2y_2 a}{5D}$$

$$= \frac{2\times1.4\times10^{3}\times8\times10^{4}}{5\times0.8}$$

$$= 5.6\times10^{7} = 560\,\text{nm}$$

Example 12.9 : In single slit diffraction experiment the first order minima for red colour (λ = 660 nm) concides with first order maxima for their colour whose wave length is λ'. Find out λ'.

Solution : The position of minima in single slit diffraction experiment is

Given by $\sin\theta = \frac{n\lambda}{a}$

For red colour the position of first minima

$$\sin\theta_1 = 1\left(\frac{\lambda_R}{a}\right)$$

For position of nth maxima $\sin\theta_n = (2n+1)\frac{\lambda}{2a}$

For wavelength λ' the position of first maxima

$$\sin\theta_1' = \frac{3\lambda'}{2a}$$

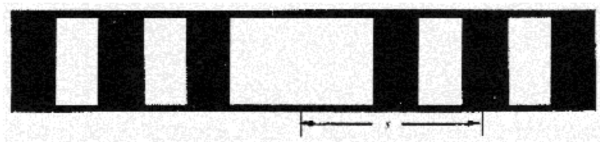
For condition given question $\sin\theta_1 = \sin\theta_1'$

$$\frac{\lambda_R}{a} = \frac{3\lambda'}{2a}$$

$$\lambda' = \frac{2}{3}\lambda_R$$

$$\lambda' = \frac{2}{3}\times660 = 440\,\text{nm} = 4400\,\text{Å}$$

Example 12.10 : Light of wave length 600 mm is incident single slit of width 4×10^{4} m. Diffraction pattern is observed on a screen placed at 2 m from slit which is shown in fig. Find out from the fig.



Solution : In fig. the distance if second order minima from central maxima is 'S'.

$$\text{Hence } s = \frac{2\lambda D}{a} = \frac{2\times600\times10^{9}\times2}{4\times10^{-4}} = 0.006\,\text{m}$$

12.10 Resolving Power

The smallest angular separation done by an apparatures is called its resoltuion. Reciprocal of angular resolution is called resoltuion power. Generally the apparture of camera, microscope and telescope is circular. Hence bright circular pattern due to diffraction is observed around central maxima by detailed analysis of circular apparture it is found that the minimum angular separation between images of two sources of just resolution is

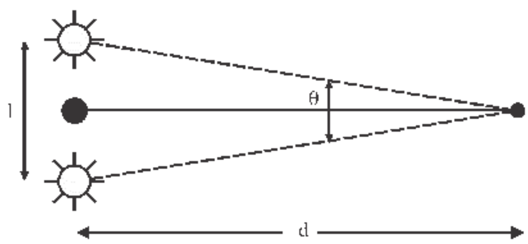


Fig. 12.17 Resolution Power

$$\theta_{min} = \frac{1.22\lambda}{D} \qquad \ldots(12.35)$$

(minimum angle of resolution for circular aperture)

Here d is diameter of aperture and θ_{min} in radian

If distance between two stars is ℓ and telescope is at a distance *d* from them than angle subtended by both points on objective lens of telescope is $\theta = \frac{\ell}{d}$.

For resolution

$$\frac{\ell}{d} = \frac{1.22\lambda}{D} \qquad \ldots (12.36)$$

For a microscope it is easy to take actual separation (S) between two points because points near to focal point of objective lens hence approximately

$$\theta_{\min} = \frac{s}{f} \text{ and } s = f\theta_{\min}$$

Here *f* is focal length of lens than using in equation 12.36 we have

$$s = \frac{1.22\lambda f}{D} \qquad \ldots (12.37)$$

(limit of resolution for a microscope)

Resolving Power

$$= \frac{1}{\text{resolving limit}} = \frac{D}{1.22\lambda f} \qquad \ldots (12.38)$$

$$\text{Resolving Power} \propto \frac{1}{\lambda}$$

From above equation it is clear that the resolving power of an optical instrument is reciprocal of wave length. Hence resolving power is more for light of short wave length.

Example 12.11 : Diameter of objective lens of telescope situated at mount Palomar is 5.00 m. Find our minimum angle of resolution for light of 600 nm wavelength.

Solution : Diameter of lens $D - 5.00\, m$

and $\lambda = 600 \text{ nm} = 6 \times 10^{7}$ m

$$\theta_{\min} = \frac{1.22\lambda}{D} = \frac{1.22 \times 6 \times 10^{7}}{5} = 1.46 \times 10^{-7} \text{ rad}$$

Example 12.12 : Distance between two narrow holes is 1.525 mm which are placed in front of a light source of wave length 5.00×10^{-5} cm. They are seen by a telescope having objective lens of diameter 0.400 cm. Find out the maximum distance from telescope so that holes are just resolved.

Solution : $\theta_{\min} = \frac{\ell}{d} = \frac{1.22\lambda}{D}$ (From eq. 12.36)

here $\ell = 1.525$ mm, $\lambda = 5.00 \times 10^{5}$ cm, $D = 0.400$ cm

$$\frac{\ell}{d_{\max}} = \frac{1.22\lambda}{D}$$

$$d_{\max} = \frac{\ell D}{1.22\lambda}$$

$$= \frac{1.525 \times 10^{3} \times 0.4 \times 10^{2}}{1.22 \times 5 \times 10^{-7}}$$

$$= 0.1 \times 10^{12} \text{ m} = 10 \text{ m}$$

Example 12.13 : Two paints seperated by 0.1 mm are just seen by a micorscope when light of 6000 Å wave length is used. If light of 4800 Å is used than what is the limit of resolution.

Solution : Limit of resolution for a microscope is

$$s = \frac{1.22\lambda f}{D} \propto \lambda$$

Hence $s_2 = s_1 \frac{\lambda_2}{\lambda_1} = 0.1 \times \frac{4800}{6000} = 0.08$ mm

12.11 Polarisation of Light

We know that in transverse wave, displacement is perpendicular to the direction of propogation of wave. If we think about a wave formed in a string, the vibration of string always remains in a plane it is called plane polarised wave. Hence polarisation is intrinsic property of transverse wave.

In year 1864 James Clark Maxwell showed theoritcally that light waves are electromagnetic waves. Electric and magnetic field vectors have wave motion in same phase in perpendicular planes to the direction of propogation of wave. In electromagnetic wave electric field vector E, magnetic field vector B and direction of propogation of wave all three are mutually perpendicular hence light waves are trasverse waves. In electrocmagnetic wave the plane which contains the plane of vibrations of electric field vectors and direction of propogation of wave is called plane of vibration. the electric field vector of electro magnetic wave is mainly responsible for all optical events. Hence electric vector E is called light vector.

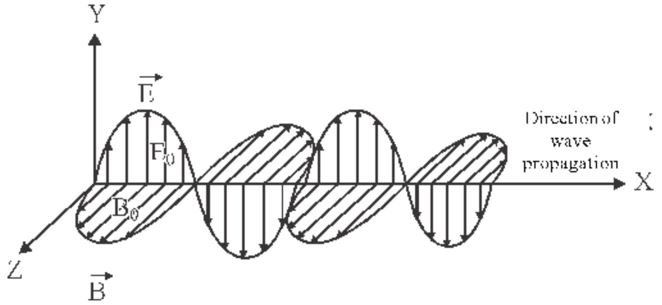


Fig. 12.18 Electromagnetic wave

Emission of ligth is due to transition of electrons in atoms from excited state to lower states. Hence in light vibrations of electric vectors of different waves are found in all possible directions perpendicular to direction of propogation. This type of light ray is called unpolarised light. It is nature of normal light. In this type of ray the vibrations of electric field are perpendicularly symmetrical to the direction of propogation. If direction of propogation is taken perpendicular to the plane of paper than unpolarised light can be shown by fig. 12.19 (a, b).

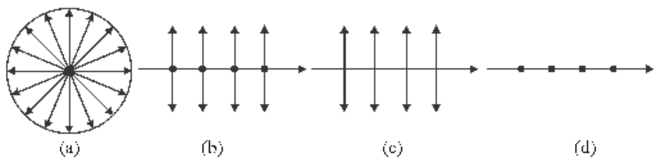


Fig. 12.19 (a,b) Unpolarised light
(c,d) Polarised light

If by some method the vibration of electric field vector in light ray are confine in a definite direction or directions than this event is called polarisation of light and light is called polarised light.

This type of light is shown in fig. 12.19 (c). In this electric field vectors are confine parallel to plane of paper. If vibration are confine perpendicular to plane of paper than they are shown by dot (•) as shown in fig. 12.19 (d). In plane polarised light the plane in which electric field vectors and directions of propogation of wave both are situated is called plane of vibration.

A plane perpendicular to plane of vibration in which there is only direction of propogation of wave and components of electric field vectors are zero is called plane of polarisation.

Hence plane of polarisation and plane of vibration are perpendicular to each other. In fig. 12.20 plane ABCD and EFGH are plane of vibration and plane of polarisation respectively.

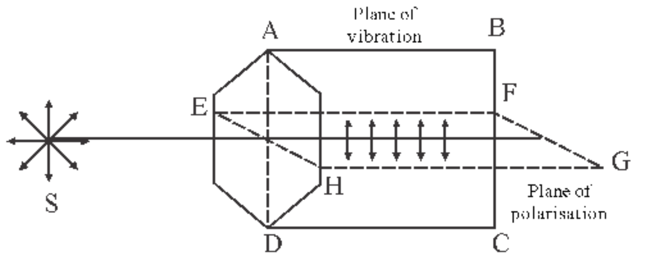


Fig. 12.20 Plane of vibration and plane of polarisation

12.12 Methods of Production of Plane Polarised Light

Following are the methods for production of plane polarised light

(i) By reflection (ii) By refraction

(iii) By double refraction (iv) By Dichroism

(v) By scattering

12.12.1 Polarisation of Light by Reflection and Brewster's Law

Scientist Brewster found that when unpolarised light is incident on a transparent medium (glass, water, etc.) at an specific incident angle i_p than reflected light is completely plane polarised. In this situation the reflected light and refracted light is perpendicular to each other. It is called Brewster's law and angle incident i_p is called Brewsters angle.

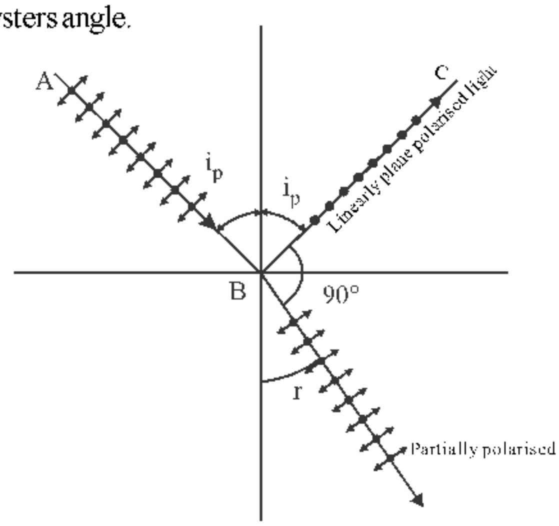


Fig. 12.21 Polarisation by reflection

According to fig. 12.21 when angle of incidence is equal to Brewsters angle i_p then angle of refraction

$$r = 90^\circ - i_p$$

from Snell's law

$$n = \frac{\sin i_p}{\sin r} = \frac{\sin i_p}{\sin\left(90^\circ - i_p\right)}$$

or $$n = \frac{\sin i_p}{\cos i_p}$$

$$n = \tan i_p$$

$$i_p = \tan^{-1}(n) \qquad \ldots (12.39)$$

from eq. (12.39) it is clear that Brewsters angle only depends on refractive index of reflecting surface.

12.2.2 Polarisation of Light by Refraction

When unpolarised light is incident at Brewser's angle on a parallel glass plate (slab) than reflected light from upper and inner surfaces of plate is totally polarised but the refracted and emergent light is partially polarised. If many such same glass plates are placed parallel to each other and if unpolarised light is incident on first plate at Brewster's angle than reflected part of unpolarised light after reflection from the plates is totally polarised but amount of polarisation gradually increases in refracted part as it advance through the plates. If number of plates is large than emergent light is plane polarised such type of arragement of plates is called pile of plates.

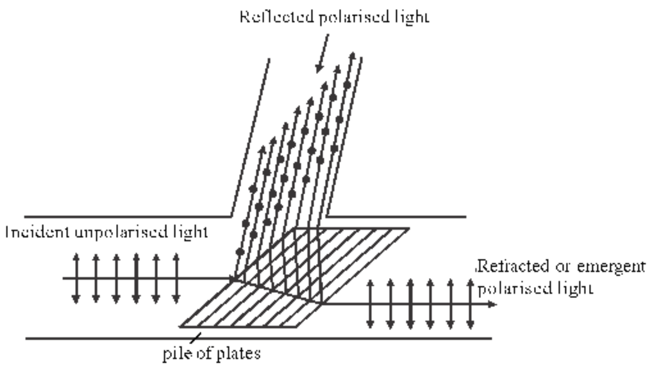


Fig. 12.22 Polarisation by refraction

12.12.3 Polaridsation by Double Refraction

When a light ray is incident on calcite or iceland spar crystle than after refraction two refracted rays are observed. Such type of action of light is called double refraction and crystle is called double refractive crystle. Light rays from double refractive crystle is plane polarisation.

To understand the process of double refraction we make ane ink point on white paper and a calcite crystle is placed above the point. On seeing from above two points are seen instead of one.

If crystle is rotated around the direction of incident light than one of the image remains stationary and second image revolve around the stationary image. The stationary image is formed according to general laws of refraction. The ray due to which stationary image is formed is called ordinary ray or O - ray and the image is called O-image or ordinary image ordinary ray follows the general laws of refraction hence always remains in incident plane and inside crystle its velocity remains same in all directions.

But the moving image is formed by extra ordinary ray or E-ray hence it is called extra ordinary image E-image . This ray does not follow the general laws of refraction and its velocity inside the srystle is different in different directions hence it is called E - ray or extra ordinary ray. E and O rays are plane polarised and vibrations of E and O ray are perpendicular to each other. For separation of E and O ray Nicol prism is used which is made up of calcite crystle.

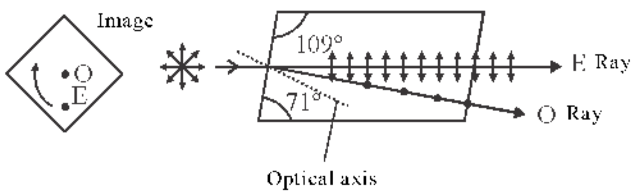


Fig. 12.23

12.12.3.1 Nicol Prism

It is an optical applience by which plane polarised light can be produced and also used for its analysis. Nicol prism works on the property of double refraction. The ordinary ray obtained by double refraction in nicol prism is reflected and separated by total internal reflection and only extra ordinary ray is allowed to emerge out of crystle which is plane polarised . Hence we get plane polarised light from Nicol prism.

Construction : For construction of Nicol prism a piece of calcite crystle is taken whose length is there times of its width. Its corner faced are cut such that the angles in the principle section becomes 66^0 and 112^0 in place of 71^0 and 109^0. The crystle is then cut diagonally into two parts. The surfaces of these parts are grinded to make optically flat and than they are polished. The polished surfaces are joined together with a special cement known as Canada balsam as shown in fig. 12.24. The upper and lower surfaces of this crystal are painted black.

Working : According to fig. 12.24 unpolarised light incident on surface AB of Nicol prism splits in O ray and E ray. For O ray the refractive index of calcite 1.658 is greater than the refractive index of Canada balsam 1.55. Hence O-ray travels from denser medium calcite in rarer medium Canada balsam. Because the incident angle for O-ray on Canada balsam surface is greater than critical angle hence this ray is reflected by total internal reflection and absorbed by black surface.

The refractive index of calcite for E-ray is 1.468 which is less than the refractive index of Canada baslam hence this ray passes through Canada baslam medium and emerges as plane polarised light from Nicol.

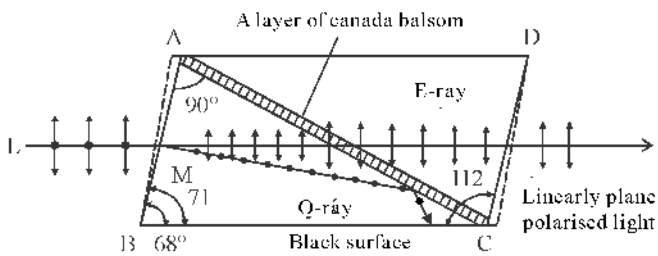


Fig. 12.24 Nicol Prism

12.12.4 Polarisation by Dichroism

When unpolarised light is incident on tourmaline crystle it splits in two plane polarised rays. Tourmaline cristal relectively absorbs one of the ray out of two refractive rays and other ray emerges out of crystal without absorption. Above action of crystle is called dichroism.

Hence transmitted light from tourmaline crystal is plane polarised. Like tourmaline crystle the property of dichroism is found in some or gave come pounds. For commerical use polaroid are made from organic compound on the basis of dichroism.

12.12.4.1 Polaroid

Polaroid is an cheap device to produce plane polarised light on commercial basis which work on dichroism. For construction of polaroid film micro crystal of organic compound herphethite or Iodosulfate of quinine are spread over a thin film of nitrocelulose and fixed such that their optical axis are parallel to each other. These crystals are highly dichroic and absorbs completely one of the two double refracted rays. This film is secured between two glass plates. It is a polaroid.

Working : When unpolarised light passes through a polariod it splits in two plane polarised rays. In which vibration of electric vector in one of the ray are parallel to the axis of herphethaite srystal and in other ray perpendicular to axis. According to fig. 12.25 the ray whose vibrations are perpendicular to the axis of herphethaite crystal is completely absorbed. The ray whose vibrations are parallel emerges from the polaroid. Hence emergent light is totally plane polarised light.

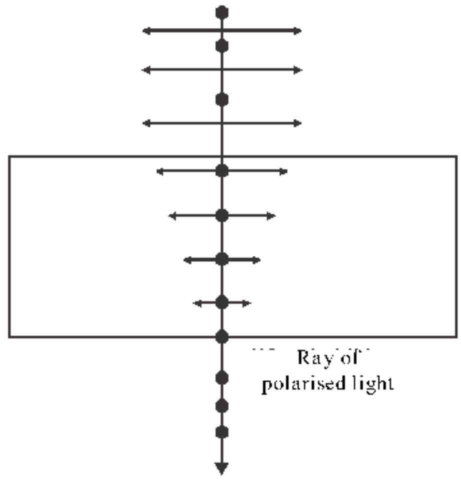


Fig. 12.25 Polarisation by polaroid

Uses of Polaroids

(i) Polaroid are used to produce and analyse plane polarised light.

(ii) A polaroid layer is fix on wind screen galss of car or other vehicle. So the intensity of reflected light from road and other surfaces is reduced for driver.

(iii) They are used to see three dimensional pictures.

(iv) Polaroid are used in head lights of car, truck etc. so that the intensity of bright light from front vehicles which falls on the eyes of driver can be reduced.

(v) Polaroids are used to study the optical properties of metals and structure of optically active materials.

(vi) Polarimeter is used to measure the concentration of optically active mateials like sugar syrup.

12.13 Identification of Plane Polarised and Unpolarised Light

By normal eye we can not study wheather a ray is polarised, partically polarised or unpolarised. To find wheather light is polarised or unpolarised we need calcite crystal, tourmaline crystal, nicol prism or polaroid.

(i) Unpolarised Light : We see emergent E ray from nicol prism or polaroid. When crystal is rotated, if there is no change in intensity of emergent say in any position than light ray is un polarised, because in unpolarised light vibrations are found in all possible directions. Hence intensity remains same in each situation.

(ii) Partially Polarised Light : On rotating the nicol prism or polaroid if intensity of emergent light changes but it is not zero in any situation than light is partially polarised.

(iii) Plane Polarised light : When the polariod in rotated, and the intensity of the emergent light in maximum at one position of the polaroid and zero when polariod in rotated by 90^0 from the position of maximum intensity. Then the incident light is plane polarised.

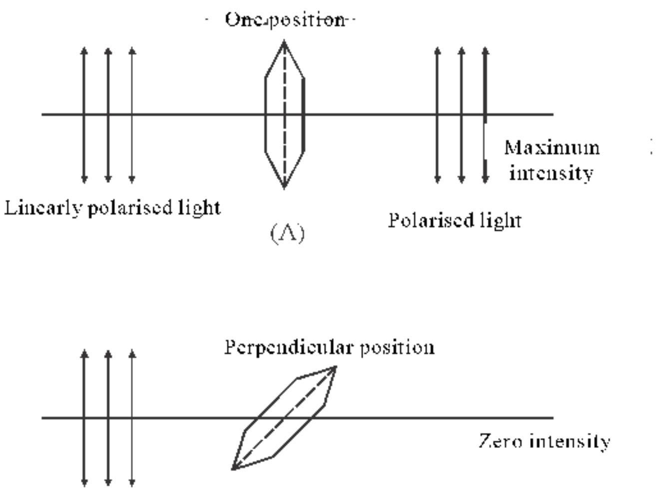


Fig. 12.26 : Detection of polarised light

12.13.1 Malus Law

When light ray is seen from two nicol prism or two polaroids when their axis are parallel than intensity of emergent light is maximum. And when polarised are is crossed position than intensity of emergent light is zero. First nicol prism or polaroid is called polariser and second nicol prism or polaroid is called analyser. For intensity of emergent ray scientist Malus gave a law which is called Malus law.

According to Malus law - When unpolarised light is transmitted through polariser and analyser than intnesity I of light transmitted by analyser is directly proportional to the square of the cosin of angle between the transmission axis of the analyser and the polariser.

$$I \propto \cos^2\theta$$

here θ is angle between axis of polariser and analyser.

or $I = I_0 \cos^2\theta$ . . . (12.40)

here I_O is maximum intensity of emergent ray and is equal to the intensity transmitted by polariser (incident on analizer)

(i) If $\theta = 0^\circ$

$I = I_0$ maximum value (parallel arrangement)

(ii) $\theta = 90^\circ$

$I = 0$ minimum value (crossed arrangement)

Example 12.14 : If critical angle for a material is 45^0 than calculate its angle of polarisation.

Solution : Critical angle $\theta_c = 45^\circ$

refractive index of material

$$n = \frac{1}{\sin\theta_c} = \frac{1}{\sin 45^\circ}$$

Hence $n = \sqrt{2}$

From Brewster's law

$$\tan i_p = \mu = \sqrt{2} = 1.414$$

$$i_p = 54.7^\circ$$

Example 12.15 : For a transparent material slab when incident angle is 60^0 than reflected light is totally polarised. Find out refractive index and angle of refraction for material.

Solution : Here angle of polarisation $i_p = 60^\circ$

From Brewster;s law

$$n = \tan i_p = \tan 60^\circ$$

$$n = \sqrt{3} = 1.732$$

$$i_p + r = 90^\circ$$

angle of refraction

$$r = 90^\circ - i_p = (90^\circ - 60^\circ) = 30^\circ$$

Example 12.16 : When sunlight is incident at an angle 37^0 the reflected ray is completely plane polarized. Find the (i) refractive index of water and (2) angle of polarization.

Solution : When light is incident at 37^0 on surface of water -

$$\theta_p = 90 - 37 = 53^\circ$$

also $\quad n = \tan\theta_p = \tan 53^\circ = \frac{4}{3}$

also $\quad \theta_p + r = 90$

$$r = 90 - 58 = 37^\circ$$

Example 12.17 : Two polaroids are oriented in such a way that their planes are perpendicular to incident light. Their axis are at 30^0 with each other. What part of the incident unpolarized light will pass through?

Solution : Let the intensity of incident light be I_O. The intneisty after passing through first polarizer will be $I_O/2$. The out going intensity after passing through second polaroid will be -

$$I' = \frac{I_0}{2}\cos^2(30) = \frac{I_0}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8}I_0$$

$$\frac{I'}{I_0} = \frac{3}{8} = 37.5\%$$

Example 12.18: When the axis of polarizer and analyser are parallel the emergent (out going) intensity is I_O. If the analyser is rotated by 45^0, what will be the intensity of emergent light?

Solution : From Mallus law

$$I = I_0\cos^2\theta$$

$$\theta = 45^\circ$$

$$I = I_0\cos^2 45^\circ = I_0\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_0}{2}$$

Important Points

1. Huygens principle - Each point situated on wavefront behaves as a source of new disturance. It is called source of secondary disturbance. Each point source due to its vibrations emit shperical waves in all directions.

2. Inteference - When two or more than two light waves of same frequency and constant phase difference with time, propogates in same direction, they super impose with each other, the resultant intensity in the super imposion region is different from the sum of intensity of each wave due to superposition region the change in the distribution of light intensity in spuperposition region in called inteference.

3. Young's double slit experinment - Conditions for constructive interference - two waves are in same phase at a point if their path difference is zero or integral multiple of wave length then there is constructive interference between the waves.

   Position of bright fringe $y = \frac{n\lambda D}{d}$ condition for distructive interference -

   When path difference between waves at a point is old multiple of half wave length the waves are in opposite phase at the point then there is distruction interference between them.

   Position of dark fringe is given by

   $$y = \frac{(2n-1)\lambda D}{2d}$$

   fringe width $\beta = \frac{\lambda D}{d}$

   angular fringe width $\omega = \frac{\beta}{D} = \frac{\lambda}{d}$

4. Coherent source and stable interference pattern - For stable interference pattern phase difference between the waves must remain same. In this situation the sources are called coherent. The frequency of coherent sources is same and phase difference between them remains constant with time.

5. Intensity distribution in Young's double slit experiment - If two waves of amplitude E_1 and E_2 and intensities I_1, I_2 respectively interference with each other, than resultant amplitude and intnesity are as follows -

   $$E_{max} = \sqrt{E_1^2 + E_2^2 + 2E_1E_2\cos\phi}$$

   and $$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

   For constructive interference the resultant amplitude and intensity aremaximum and their expressions are as follows -

   $$E_{max} = E_1 + E_2$$

   $$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 \quad [\text{for } \phi = 2n\pi]$$

   For distructive interference the resultant amplitude and intensity are minimum and given by following expressions-

   $$E_{min} = |E_1 - E_2|$$

and $I_{max} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ [for $\phi = (2n-1)\pi$]

Ratio of maximum and minimum intensities

6. Differection - Bending or spreading of light from the edges of an obstacles is called diffraction. For sharp diffraction pattern the size of obstacle should be in the range of incident wave length.

7. Fraun hofer diffraction through single slit - Minima in diffraction pattern - The angular position of minima on screen in diffraction pattern is given by following formula -

$$\theta = \sin^{-1}\left(\frac{n\lambda}{a}\right) \approx \frac{n\lambda}{a} \quad \text{(for smaller values of } \theta\text{)}$$

Maxima in diffraction pattern -

For mixima in diffraction pattern

$$\sin\theta = (2n-1)\frac{\lambda}{2a} \quad \text{or} \quad \theta \simeq (2n-1)\frac{\lambda}{2a}$$

8. Resolving power of telescope and microscope. The smallest angular separation done by an apparatures is called resolution that apparatus and inverse of angular resolution is called resolution power.

$$\theta_{min} = \frac{1.22\lambda}{D} \quad \text{(For circular operture minimum angle of resolution)}$$

Here D is diameter of aperture and θ_{min} in radian.

9. Polarisation - If the vibration's of light are confined in a specific plane than light is called plane polarised.

10. Methods to produce polarised light -

Following are the methods to produce plane polarised light - (i) By reflection (ii) By refraction (iii) By double refraction (iv) By dichromatism (v) By scattering

11. Brewster's law - When unpolarised light is incident on a reflecting transparent surface then at a specific incident angle, reflected light is completely plane polarised. In this situation

$\tan\theta_P = n$ and $\theta_p = \tan^{-1} n$ and

12. When unpolarised light transmitts through polariser and analyser than intensity of emergent light is directly propotional to square of cosine of angle between the pass axis of polariser and analyser.

Questions For practice

Multiple Choice Type Questions -

1. For demonstration of interference we require two sources which emits readiations of -

(a) Equal frequency and constant phase difference

(b) Approximately same frequency

(c) Equal frequency

(d) Different wave length

2. Monochromatic source of light is used in Young's double slit experinment. Shape of interference fringes on the screen will be -

(a) Straight line (b) Parabola

(c) Hyperbola (d) Circle

3. In an experiment for interference light of 700 nm wavelength is used. There is third bright fringe at a point on screen. For getting fifth bright fringe at the same point what is the required wavelength of light-

(a) 210 nm (b) 315 nm

(c) 420 nm (d) 490 nm

4. In Young's double slit experiment ratio of widths of slits is 4 : 9than what is the ratio of intensity of maximum and minima -

(a) 196 : 25 (b) 82 : 16

(c) 25 : 1 (d) 9 : 4

5. Light of two different wave lengths is used in young's double slit experiment. Position of third fringe for yellow-orange colour (> = 600 nm) concides with the position of fourth bright fringe for other colour. What is wave length of other colour-

(a) 5000 n m (b) 450 n m

(c) 225 n m (d) 350 n m

6. In Young's double slit experiment maximum intensity of light is I_{max}, what is the intensity when path difference is $\lambda/2$.

(a) I_{max} (b) $I_{\max}/2$

(c) $I_{\max}/4$ (d) Zero

7. Which of the following statement given more correct understanding that in most of the situations possibility of diffraction of sound is more than diffraction of light-

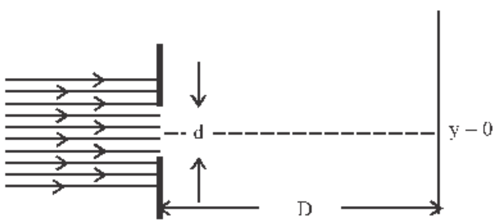
(a) Medium is necessary for second transmission

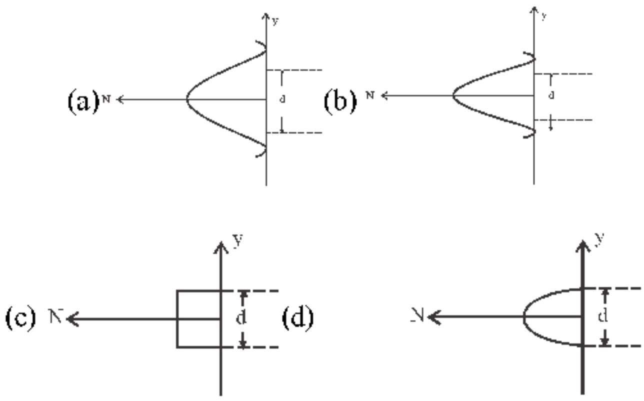
(b) Sound waves are longitudinal but light waves are transverse

(c) Wavelength of light is very short 1 n comparison to second

(d) Velocity of sound is very low than velocity of light

8. According to figure in an experiment electron's passes through a narrow slit of width 'd' which is of the range of its D' Broglie wave length and are detected on the screen at distance D from the slit. The intensity pattern on the screen is -





9. Light of 5000 Å wavelength is incident on a sreen. In diffraction pattern fifth minima is formed at 5 mm from the central maxima if distance between screen and slit is 1 m than width of slit is -

(a) 0.1 mm (b) 0.3 mm

(c) 0.5 mm (d) 0.8 mm

10. A beam of microwaves of wave length 0.052 m is propogating toward a rectangular hole. Resultant diffraction pattern is formed on a wall at a distance 8.0 m from the hole. What is the distance between first and second order outer fringes -

(a) 1.3 m (b) 1.8 m

(c) 2.1 m (d) 2.5 m

11. Aperture of astronomical telescope is large -

(a) To remove defect of spherical aberration

(b) For high resolution

(c) To increase thearea of observation

(d) For low dispersion

12. Two white points are 1 mm apart from each other on a black paper then are seem from pupil of eye of 3 mm diameter what is the maximum distance between them so that they are just resolve by eye. (wave length of light = 500 nm)

(a) 6 m (b) 3 m

(c) 5 m (d) 1 m

13. Electromagnetic waves are transverse in nature. Its proof is -

(a) Polarisation (b) Interference

(c) Reflection (d) Diffraction

14. The angle of incidence when reflection is from air to glass and reflected light is fully polarised is given

by (refraction index n)

(a) $\tan^{-1}(1/n)$ (b) $\sin^{-1}(1/n)$

(c) $\sin^{-1}(n)$ (d) $\tan^{-1}(n)$

15. A beam of unpolarised light is incident on four polarised plates. Plates are arranged so that each plate direction is at an angle 30^0 with its previous one. What is the intensity of light transmitted by each polarisar -

(a) 50% (b) 20 %

(c) 50 % (d) 21 %

16. Two nichol prisms are arranged so that the angle between their principle planes is 60^0. What percentage of incident unpolarised light passes through the system -

(a) 50 % (b) 100 %

(c) 12.5 % (d) 37.5 %

Very Short Answer Type Questions -

1. Line perpendicular to wave front gives direction of which quantity?
2. Which physical quantities affect the width of Young's fringes?
3. Write expression of Huygen's principle for diffraction of light?
4. Which type of wavefront emerges from

   (i) Point source (ii) Far light source
5. Which is the most important condition for interference of two waves?
6. How the angular separation between fringes changes in single slit diffraction experiment when distance between slit and screen doubled?
7. For clear diffraction of waves what should be the range of size of obstacle or hole?
8. Write expression of two physical events which proves wave nature of light?
9. Why light seems to propogate in straight line although it is of wave nature?
10. In an experiment for diffraction through hole, which light waves super imposes?
11. Write down mathematical form of Malus law?

Short Answer Type Questions -

1. Give Huygen's principle for light waves?
2. Define interference of waves.
3. What do you mean by Coherent sources?
4. What you understand by diffraction of light? Compare diffraction of light and sound waves?
5. Define resolving power of microscope. How it is affected -

   (i) When wave length of incident radiations is reduced.

   (ii) When diameter of objective lens is reduced. Give reason for your answer.
6. Frings are formed on screen due to interference of light from two thin slits. If distance between slits rises four times and distance of screen from slits is halws than how many times will be the fringe width?
7. Explain construction of polariod.
8. What do you mean by double refraction?
9. Write the difference between interference and diffraction?
10. Write down main difference between Fresnel and Fraunhofer diffraction.

Essay Type Questions -

1. On the basis of Huygen's secondary wave principle explain refraction of light and derive Snell's law.
2. On the basis of Huygen's wave principle of give intepretation for reflection of light.
3. Write down analytical explanation of interference and write down condition's for construction and destructive interference.
4. What do you mean by diffraction of light? Why diffraction of sound waves is easily observed than diffraction of light waves? Compare Fresnel's and Fraunhofer diffraction.
5. Interpret Fraunhofer diffraction through single slit.
6. What is polarisation? Interpret polarisation with the help of electric vector. Make it clear why it is a property of transverse wave only?
7. Write down name of four methods for producing polarised light? Define double refraction and explain it.
8. By reflection how we can get plane polarised light? What is Brewster's law? Prove that if light is

incident on plane transparent slab at angle of polarisation than reflected and refracted rays are perpendicular to each other.

9. Define plane of vibration and plane of polarisation give explation of Malus law and explain parallel and crossed arrangment?

Answer (Multiple Choice Questions)

1. (a) 2. (b) 3. (c) 4. (a) 5. (d) 6. (d) 7. (b) 8. (b)

9. (a) 10. (c) 11. (a) 12. (b)

Short Answer Type -

1. Direction of rays
2. Wave length of light, distance between sources, medium, distance of screen
4. (i) Spherical (ii) Plane
5. Both sources should be Coharent
6. It changes to half
7. Range of the wave length
8. Interference, diffraction, polarsation
9. Their wave length is very short
10. Between the waves from different sources at mid portion of the hole

Numberical Questions -

1. For two waves of same shape the ratio of amplitude is 2 : 1. Find out maximum and minimum ratios of amplitudes and intensities of vibrations within interference region.

(Ans : 3 : 1 and 9)

2. In an experiment for interference two sources of intensities I and 4 I are used. Find out the intensities an those points where the phase difference between the waves from two sources interfering each other is (a) zero (b) $\pi / 2$ (c) π

(Ans : 9 I, 5 I, I)

3. Find out distance between two holes which forms fringes of 1 mm width on the screen placed at 1 m distance, the wavelength of light is 5000 Å.

(Ans : 0.5 mm)

4. Light of wave length 5500 Å is incident perpendicularly on a linear hole of width 22 x 10^{-5} cm. Find out angular poision of first two minima situated on both sides of central maixma.

(Ans : $\theta_1 = 0.25$ rad , $\theta_2 = 0.50$ rad)

5. Two polaroides are placed in position so that intensity of emergent light is maximum. If one of the polaroide is rotated by 30^0, 90^0 relative to other than under new position the intensity of emergent light is how much part of maximum intensity?

(Ans : 3/4, 0)

6. When sun is at 37^0 with horizon than reflected light from surface of water is totally polarised. Find out refractive index of water.

(Ans : 1.33)

7. The polarising directions of two polariser plates are parallel hence the intensity of emergent light is maximum. What is the minimum rotation of one of the plates so that intensity of emergent light remains one fourth of maximum intensity?

(Ans : I 60^0)