DIFFERENTIAL EQUATION

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are -

(A) 1 , $\frac{2}{3}$ (B) 3 , 1 (C) 1, 2 (D) 3, 3

2. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis are respectively

(A) 2 , 1 (B) 1 , 2 (C) 3 , 2 (D) 2 , 3

3. The order and degree of the differential equation $\sqrt[3]{\frac{dy}{dx}} - 4\frac{d^2y}{dx^2} - 7x = 0$ are a and b, then a + b is -

(A) 3 (B) 4 (C) 5 (D) 6

4. The order of the differential equation whose general solution is given by $y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x+c_5}$ where C_1, C_2, C_3, C_4, C_5 are arbitrary constants, is - **[JEE 98]**

(A) 5 (B) 4 (C) 3 (D) 2

5. The differential equation of the family of curves represented by y = a + bx + ce^{-x} (where a, b, c are arbitrary constants) is -

(A) y''' = y' (B) y''' + y'' = 0 (C) y''' – y'' + y' = 0 (D) y''' + y'' – y' = 0

6. The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant is -

(A) $(x^2 - y^2)\, y' = 2xy$ (B) $2(x^2 + y^2)\, y' = xy$

(C) $2(x^2 - y^2)\, y' = xy$ (D) $(x^2 + y^2)y' = 2xy$

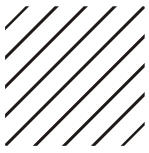
7. Number of values of $m \in N$ for which $y = e^{mx}$ is a solution of the differential equation $D^3y - 3D^2y - 4Dy + 12y = 0$ is -

(A) 0 (B) 1 (C) 2 (D) more than 2

8. If $y = e^{(K+1)x}$ is a solution of differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$, then k =

(A) –1 (B) 0 (C) 1 (D) 2

9. The general solution of the differential equation $\frac{dy}{dx} = \frac{1-x}{y}$ is a family of curves which looks most like which of the following ?

(A)  (B)  (C)  (D) 

10. The solution to the differential equation $y\ell ny + xy' = 0$, where y(1) = e, is -

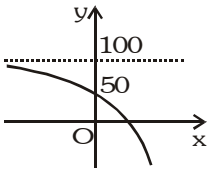
(A) $x(\ell ny) = 1$ (B) $xy(\ell ny) = 1$ (C) $(\ell ny)^2 = 2$ (D) $\ell ny + \left(\frac{x^2}{2}\right) y = 1$

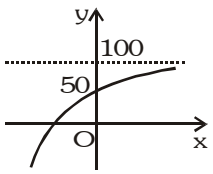
11. The equation of the curve passing through origin and satisfying the differential equation $\frac{dy}{dx}$ = sin (10x + 6y) is -

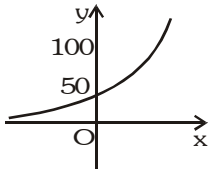
(A) $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4 - 3\tan 4x}\right) - \frac{5x}{3}$ (B) $y = \frac{1}{3}\tan^{-1}\left(\frac{5\tan 4x}{4 + 3\tan 4x}\right) - \frac{5x}{3}$

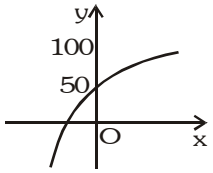
(C) $y = \frac{1}{3}\tan^{-1}\left(\frac{3 + \tan 4x}{4 - 3\tan 4x}\right) - \frac{5x}{3}$ (D) none of these

12. Which one of the following curves represents the solution of the initial value problem Dy = 100 – y, where y(0) = 50

(A) 

(B) 

(C) 

(D) 

13. A curve passing through (2, 3) and satisfying the differential equation $\int_0^x ty(t)dt = x^2y(x), (x > 0)$ is -

(A) $x^2 + y^2 = 13$ (B) $y^2 = \frac{9}{2}x$ (C) $\frac{x^2}{8} + \frac{y^2}{18} = 1$ (D) $xy = 6$

14. A curve passes through the point $\left(1, \frac{\pi}{4}\right)$ & its slope at any point is given by $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$. Then the curve has the equation -

(A) $y = x\tan^{-1}\left(\ell n\frac{e}{x}\right)$ (B) $y = x\tan^{-1}(\ell n + 2)$ (C) $y = \frac{1}{x}\tan^{-1}\left(\ell n\frac{e}{x}\right)$ (D) none

15. The solution of the differential equation $(2x - 10y^3)\frac{dy}{dx} + y = 0$ is -

(A) $x + y = ce^{2x}$ (B) $y^2 = 2x^3 + c$ (C) $xy^2 = 2y^5 + c$ (D) $x(y^2 + xy) = 0$

16. Solution of differential equation $\left(1 + y^2\right)dx + \left(x - e^{\tan^{-1}y}\right)dy = 0$ is -

(A) $ye^{\tan^{-1}x} = \tan^{-1}x + c$ (B) $xe^{\tan^{-1}y} = \frac{1}{2}e^{2\tan^{-1}y} + c$

(C) $2x = e^{\tan^{-1}y} + c$ (D) $y = xe^{-\tan^{-1}x} + c$

17. The general solution of the differential equation, $y' + y\phi'(x) - \phi(x) \cdot \phi'(x) = 0$ where $\phi(x)$ is a known function is -

(A) $y = ce^{-\phi(x)} + \phi(x) - 1$ (B) $y = ce^{\phi(x)} + \phi(x) + K$ (C) $y = ce^{-\phi(x)} - \phi(x) + 1$ (D) $y = ce^{-\phi(x)} + \phi(x) + K$

18. The solution of the differential equation, $e^x(x + 1)dx + (ye^y - xe^x)dy = 0$ with initial condition f(0) = 0, is -

(A) $xe^x + 2y^2e^y = 0$ (B) $2xe^x + y^2e^y = 0$ (C) $xe^x - 2y^2e^y = 0$ (D) $2xe^x - y^2e^y = 0$

19. The solution of the differential equation $ydx + (x + x^2y)dy = 0$ is -

(A) $\frac{1}{xy} + \log y = c$ (B) $\log y = cx$ (C) $-\frac{1}{xy} = c$ (D) $-\frac{1}{xy} + \log y = c$

20. The solution of $y^5x + y - x\frac{dy}{dx} = 0$ is -

(A) $x^4/4 + 1/5\,(x/y)^5 = C$ (B) $x^5/5 + (1/4)\,(x/y)^4 = C$

(C) $(x/y)^5 + x^4/4 = C$ (D) $(xy)^4 + x^5/5 = C$

21. The solution of $\frac{xdy}{x^2 + y^2} = \left(\frac{y}{x^2 + y^2} - 1\right)dx$ is -

(A) $y = x\cot(c - x)$ (B) $\cos^{-1}y/x = -x + c$

(C) $y = x\tan(c - x)$ (D) $y^2/x^2 = x\tan(c - x)$

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

22. The value of the constant 'm' and 'c' for which y = mx + c is a solution of the differential equation $D^2y - 3Dy - 4y = -4x$

(A) is m = –1 (B) is c = 3/4 (C) is m = 1 (D) is c = –3/4

23. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then the solution of the equation is -

(A) $\log\left(\frac{x}{y}\right) = cy$ (B) $\log\left(\frac{y}{x}\right) = cx$ (C) $y = xe^{cx}$ (D) $x = ye^{cx}$

24. Solutions of the differential equation $x^2\left(\frac{dy}{dx}\right)^2 + xy\left(\frac{dy}{dx}\right) - 6y^2 = 0$ -

(A) $y = cx^2$ (B) $x^3 y = c$ (C) $xy^3 = c$ (D) $y = cx$

25. A solution of the differential equation, $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} + y = 0$ is -

(A) $y = 2$ (B) $y = 2x$ (C) $y = 2x - 4$ (D) $y = 2x^2 - 4$

26. The solution the differential equation $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx}\left(e^x + e^{-x}\right) + 1 = 0$ is are -

(A) $y + e^{-x} = c$ (B) $y - e^{-x} = c$ (C) $y + e^x = c$ (D) $y - e^x = c$

27. The solution of $\frac{dy}{dx} = \frac{ax + h}{by + k}$ represent a parabola if -

(A) $a = -2, b = 0$ (B) $a = -2, b = 2$ (C) $a = 0, b = 2$ (D) $a = 0, b = 0$

28. A normal is drawn at a point P(x, y) of a curve. It meets the x-axis and the y-axis in point A and B, respectively, such that $\frac{1}{OA} + \frac{1}{OB} = 1$, where O is the origin, the equation of such a curve is a circle which passes through (5, 4) and has -

(A) centre (1, 1) (B) centre (2, 1) (C) radius 5 (D) radius 4

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	D	B	C	C	B	A	C	C	B	A
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A	B	D	A	C	B	A	B	D	B
Que.	21	22	23	24	25	26	27	28		
Ans.	C	C,D	B,C,D	A,B	C	A,D	A,C	A,C		

EXTRA PRACTICE QUESTIONS ON DIFFERENTIAL EQUATION

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. Which one of the following is homogeneous function ?

(A) $f(x, y) = \dfrac{x-y}{x^2+y^2}$ (B) $f(x, y)= x^{\frac{1}{3}}.y^{-\frac{2}{3}} \tan^{-1}\dfrac{x}{y}$

(C) $f(x, y) = x(\ell n\sqrt{x^2+y^2} - \ell n y) + ye^{x/y}$ (D) $f(x, y) = x\left[\ell n\dfrac{2x^2+y^2}{x} - \ell n(x+y)\right] + y^2\tan\dfrac{x+2y}{3x-y}$

2. The graph of the function $y = f(x)$ passing through the point $(0, 1)$ and satisfying the differential equation $\dfrac{dy}{dx} + y\cos x = \cos x$ is such that -

(A) it is a constant function (B) it is periodic

(C) it is neither an even nor an odd function (D) it is continuous & differentiable for all x.

3. Water is drained from a vertical cylindrical tank by opening a valve at the base of the tank. It is known that the rate at which the water level drops is proportional to the square root of water depth y, where the constant of proportionality $k > 0$ depends on the acceleration due to gravity and the geometry of the hole. If t is measured in minutes and $k = 1/15$ then the time to drain the tank if the water is 4 meter deep to start with is -

(A) 30 min (B) 45 min (C) 60 min (D) 80 min

4. The solution of the differential equation, $x^2\dfrac{dy}{dx}.\cos\dfrac{1}{x} - y\sin\dfrac{1}{x} = -1$, where $y \to -1$ as $x \to \infty$ is -

(A) $y = \sin\dfrac{1}{x} - \cos\dfrac{1}{x}$ (B) $y = \dfrac{x+1}{x\sin\dfrac{1}{x}}$ (C) $y = \cos\dfrac{1}{x} + \sin\dfrac{1}{x}$ (D) $y = \dfrac{x+1}{x\cos\dfrac{1}{x}}$

5. If $y = \dfrac{x}{\ell n|cx|}$ (where c is an arbitrary constant) is the general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{y}{x} + \phi\left(\dfrac{x}{y}\right)$ then the function $\phi\left(\dfrac{x}{y}\right)$ is -

(A) $\dfrac{x^2}{y^2}$ (B) $-\dfrac{x^2}{y^2}$ (C) $\dfrac{y^2}{x^2}$ (D) $-\dfrac{y^2}{x^2}$

6. If $\int_a^x ty(t)dt = x^2 + y(x)$ then y as a function of x is -

(A) $y = 2 - (2+a^2)e^{\frac{x^2-a^2}{2}}$ (B) $y = 1 - (2+a^2)e^{\frac{x^2-a^2}{2}}$ (C) $y = 2 - (1+a^2)e^{\frac{x^2-a^2}{2}}$ (D) none

7. A function f(x) satisfying $\int_0^1 f(tx)dt = nf(x)$, where $x > 0$, is -

(A) $f(x) = c.x^{\frac{1-n}{n}}$ (B) $f(x) = c.x^{\frac{n}{n-1}}$ (C) $f(x) = c.x^{\frac{1}{n}}$ (D) $f(x) = c.x^{(1-n)}$

8. The differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}+\sin y+x^2=0$ is of the following type -

(A) linear (B) homogeneous (C) order two (D) degree one

9. A curve C passes through origin and has the property that at each point (x, y) on it the normal line at that point passes through (1, 0). The equation of a common tangent to the curve C and the parabola $y^2 = 4x$ is -

(A) x = 0 (B) y = 0 (C) y = x + 1 (D) x + y + 1 = 0

10. The function f(x) satisfying the equation, $f^2(x) + 4f'(x) \cdot f(x) + [f'(x)]^2 = 0$ is -

(A) $f(x) = c \cdot e^{(2-\sqrt{3})x}$ (B) $f(x) = c \cdot e^{(2+\sqrt{3})x}$ (C) $f(x) = c \cdot e^{(\sqrt{3}-2)x}$ (D) $f(x) = c \cdot e^{-(2+\sqrt{3})x}$

11. The equation of the curve passing through (3, 4) & satisfying the differential equation,

$y\left(\frac{dy}{dx}\right)^2+(x-y)\frac{dy}{dx}-x=0$ can be -

(A) x – y + 1 = 0 (B) $x^2 + y^2 = 25$ (C) $x^2 + y^2 - 5x - 10 = 0$ (D) x + y – 7 = 0

12. Number of straight lines which satisfy the differential equation $\frac{dy}{dx}+x\left(\frac{dy}{dx}\right)^2-y=0$ is -

(A) 1 (B) 2 (C) 3 (D) 4

13. Let $y = (A + Bx)e^{3x}$ be a solution of the differential equation $\frac{d^2y}{dx^2}+m\frac{dy}{dx}+ny=0$, m, n ∈ I, then -

(A) m + n = 3 (B) $n^2 - m^2 = 64$ (C) m = –6 (D) n = 9

14. The differential equation $2xy\,dy = (x^2 + y^2 + 1)\,dx$ determines -

(A) A family of circles with centre on x-axis

(B) A family of circles with centre on y-axis

(C) A family of rectangular hyperbola with centre on x-axis

(D) A family of rectangular hyperbola with centre on y-axis

15. If $f''(x) + f'(x) + f^2(x) = x^2$ be the differential equation of a curve and let P be the point of maxima then number of tangents which can be drawn from point P to $x^2 - y^2 = a^2$, $a \neq 0$ is -

(A) 2 (B) 1 (C) 0 (D) either 1 or 2

16. The solution of $x^2dy - y^2dx + xy^2(x - y)dy = 0$ is -

(A) $\ell n\left|\frac{x-y}{xy}\right|=\frac{y^2}{2}+c$ (B) $\ell n\left|\frac{xy}{x-y}\right|=\frac{x^2}{2}+c$ (C) $\ell n\left|\frac{x-y}{xy}\right|=\frac{x^2}{2}+c$ (D) $\ell n\left|\frac{x-y}{xy}\right|=x+c$

17. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2=\frac{4}{x}$ are -

(A) $9(y + c)^2 = x^3$ (B) $y + c = \frac{-x^{3/2}}{3}$ (C) $y + c = \frac{x^{3/2}}{3}$ (D) all of these

ANSWER KEY

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Ans.	A,B,C	A,B,D	C	A	D	A	A	C,D	A	C,D
Que.	11	12	13	14	15	16	17			
Ans.	A,B	B	A,C,D	C	A	A	A,B,C,D			