HYPERBOLA

1. Standard Equation:

Standard equation of the hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $b^2 = a^2 (e^2 - 1)$.

Focii : $S \equiv (\pm ae, 0)$ **Directrices :** $x = \pm \frac{a}{e}$

Vertices : $A \equiv (\pm a, 0)$

Latus Rectum (ℓ) : $\ell = \frac{2b^2}{a} = 2a (e^2 - 1)$.

2. Conjugate Hyperbola :

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ & $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are conjugate hyperbolas of each.

3. Auxiliary Circle : $x^2 + y^2 = a^2$.

4. Parametric Representation : $x = a \sec \theta$ & $y = b \tan \theta$

5. **A Point 'P' w.r.t. A Hyperbola :**

$S_1 \equiv \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$ >, = or < 0 according as the point (x_1, y_1) lies inside, on or outside the curve.

6. **Tangents :**

(i) **Slope Form :** $y = mx \pm \sqrt{a^2m^2 - b^2}$

(ii) **Point Form :** at the point (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

(iii) **Parametric Form :** $\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$.

7. **Normals :**

(a) at the point P (x_1, y_1) is $\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2 = a^2e^2$.

(b) at the point P (a sec θ, b tan θ) is $\frac{ax}{\sec\theta} + \frac{by}{\tan\theta} = a^2 + b^2 = a^2e^2$.

(c) Equation of normals in terms of its slope 'm' are y $= mx \pm \frac{(a^2+b^2)m}{\sqrt{a^2 - b^2m^2}}$.

8. **Asymptotes :** $\frac{x}{a} + \frac{y}{b} = 0$ and $\frac{x}{a} - \frac{y}{b} = 0$.

Pair of asymptotes : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$.

9. **Rectangular Or Equilateral Hyperbola :** $xy = c^2$, eccentricity is $\sqrt{2}$.

Vertices : (± c, ±c) ; Focii : $(\pm\sqrt{2}c, \pm\sqrt{2}c)$. Directrices : $x + y = \pm\sqrt{2}c$

Latus Rectum (*l*) : $\ell = 2\sqrt{2}\,c$ = T.A. = C.A.

Parametric equation $x = ct$, $y = c/t$, $t \in R - \{0\}$

Equation of the tangent at P (x_1, y_1) is $\frac{x}{x_1} + \frac{y}{y_1} = 2$ & at P (t) is $\frac{x}{t} + ty = 2c$.

Equation of the normal at P (t) is $xt^3 - yt = c(t^4 - 1)$.

Chord with a given middle point as (h, k) is $kx + hy = 2hk$.