Q1: NTA Test 01 (Single Choice)

If the general solution of the differential equation $y' = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$, for some function ϕ, is given by $y\ln|cx| = x$, where c is an arbitrary constant, then $\phi(2)$ is equal to $\left(\text{here, } y' = \frac{dy}{dx}\right)$

(A) -4

(B) $-\frac{1}{4}$

(C) $\frac{1}{4}$

(D) 4

Q2: NTA Test 02 (Single Choice)

At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is

(A) 3500

(B) 4500

(C) 2500

(D) 3000

Q3: NTA Test 03 (Single Choice)

Tangent to a curve intersects the y-axis at a point P. A line perpendicular to this tangent through P passes through the point $(1, 0)$. The differential equation of the curve is

(A) $y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$

(B) $x\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 1$

(C) $y\frac{dx}{dy} + x = 1$

(D) None of these

Q4: NTA Test 05 (Numerical)

Let $f[1,\infty) \to [2,\infty)$ be a differentiable function such that $f(1) = \frac{1}{3}$. If $6\int_1^x f(t)dt = 3xf(x) - x^3$ for all $x \geq 1$, then the value of $3f(2)$ is

Q5: NTA Test 06 (Single Choice)

The order of the differential equation whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4e^{x+c_5}$ where c_1, c_2, c_3, c_4 & c_5 are arbitrary constants, is

(A) 5

(B) 4

(C) 3

(D) 2

Q6: NTA Test 07 (Single Choice)

The differential equation obtained by eliminating the arbitrary constants a and b from $xy = ae^x + be^{-x}$ is

(A) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$

(B) $\frac{d^2y}{dx^2} + 2y\frac{dy}{dx} - xy = 0$

(C) $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + xy = 0$

(D) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - xy = 0$

Q7: NTA Test 08 (Single Choice)

The order and degree of the differential equation $\frac{d^2y}{dx^2} = \left[y + \left(\frac{dy}{dx}\right)^6\right]^{1/4}$ are

(A) 2, 4

(B) 3, 4

(C) 2, 5

(D) 2, 6

Q8: NTA Test 10 (Single Choice)

The solution of $dy = \cos x\,(2 - y\,\text{cosec}\,x)dx$, where $y = \sqrt{2}$ when $x = \pi/4$, is

(A) $y = \sin x + \frac{1}{2}\,\text{cosec}\,x$

(B) $y = \tan(x/2) + \cot(x/2)$

(C) $y = (1/\sqrt{2})\sec(x/2) + \sqrt{2}\cos(x/2)$

(D) None of the above

Q9: NTA Test 11 (Single Choice)

The general solution of the differential equation $\left[2\sqrt{xy}-x\right]dy+y\,dx=0$ is (Here x, y > 0)

(A) $\log x+\sqrt{\frac{y}{x}}=c$

(B) $\log y-\sqrt{\frac{x}{y}}=c$

(C) $\log y+\sqrt{\frac{x}{y}}=c$

(D) None of these

Q10: NTA Test 12 (Single Choice)

The solution of the differential equation $\frac{dy}{dx}+x\left(2x+y\right)=x^3(2x+y)^3-2$ is (C being an arbitrary constant)

(A) $\frac{1}{2x+xy}=x^2+1+Ce^x$

(B) $\frac{1}{(2x+y)^2}=x^2+1+Ce^{x^2}$

(C) $\frac{1}{2x+y}=x+1+Ce^{-x^2}$

(D) $\frac{1}{(2x+y)^2}=x^2+1+C$

Q11: NTA Test 13 (Single Choice)

The general solution of the differential equation $(2x-y+1)dx+(2y-x+1)dy=0$ is

(A) $x^2+y^2+xy-x+y=c$

(B) $x^2+y^2-xy+x+y=c$

(C) $x^2-y^2+2xy-x+y=c$

(D) $x^2-y^2-2xy+x-y=c$

Q12: NTA Test 14 (Numerical)

Let $f:R\rightarrow R$ be a differentiable function with $f\left(0\right)=1$ and satisfying the equation

$f\left(x+y\right)=f\left(x\right)f'\left(y\right)+f'\left(x\right)f\left(y\right)$ for all $x,\ y\in R$.

Then, the value of $\log_e\left(f\left(4\right)\right)$ is

Q13: NTA Test 15 (Single Choice)

The general solution of the differential equation $\frac{dy}{dx}+\sin\left(\frac{x+y}{2}\right)=\sin\left(\frac{x-y}{2}\right)$ is (where c is an arbitrary constant)

(A) $\ln\tan\left(\frac{y}{2}\right)=c-2\sin x$

(B) $\ln\tan\left(\frac{y}{4}\right)=c-2\sin\left(\frac{x}{2}\right)$

(C) $\ln\tan\left(\frac{y}{2}+\frac{\pi}{4}\right)=c-2\sin x$

(D) $\ln\tan\left(\frac{y}{4}+\frac{\pi}{4}\right)=c-2\sin\left(\frac{x}{2}\right)$

Q14: NTA Test 16 (Single Choice)

Let the population of ants surviving at a time t be governed by the differential equation $\frac{d(p(t))}{dt}-p\left(t\right)=-100$. If $p\left(0\right)=50$, then $p\left(-\ln 2\right)$ is equal to

(A) 100

(B) 75

(C) 90

(D) 40

Q15: NTA Test 17 (Single Choice)

The curve satisfying the differential equation $\frac{dx}{dy}=\frac{x+2yx^2}{y-2x^3}$ and passing through $(1,\ 0)$ is given by

(A) $x^2+y^2=1$

(B) $x^2+y^2+\frac{y}{x}=1$

(C) $y^2-\frac{y}{x}-x^2=-1$

(D) $x^2-y^2=1$

Q16: NTA Test 18 (Single Choice)

The solution of the differential equation $xdy+\frac{y}{x}dx=\frac{dx}{x}$ is
(where, c is an arbitrary constant)

(A) $y=1+ce^{1/x}$

(B) $y=ce^{1/x}$

(C) $y=ce^{1/x}-1$

(D) $xy=1-ce^{1/x}$

Q17: NTA Test 19 (Single Choice)

The solution of the differential equation $dy - \frac{ydx}{2x} = \sqrt{x}ydy$ is

(where, c is an arbitrary constant)

(A) $\frac{y}{\sqrt{x}} = y + c$

(B) $\frac{y}{\sqrt{x}} = \frac{y^2}{2} + c$

(C) $y = y\sqrt{x} + c$

(D) $\frac{y}{\sqrt{x}} = -y^2 + c$

Q18: NTA Test 20 (Single Choice)

The solution of the differential equation $\left(3x^2 \sin\frac{1}{x} + y\right)dx = x\cos\left(\frac{1}{x}\right)dx - xdy$ is

(where, c is an arbitrary constant)

(A) $\sin\left(\frac{1}{x}\right) = xy + c$

(B) $x^3 \sin\left(\frac{1}{x}\right) + xy = c$

(C) $x^3 \sin\left(\frac{1}{x}\right) = xy + c$

(D) $\sin(x) = x^3 y + c$

Q19: NTA Test 21 (Single Choice)

The solution of the differential equation $\frac{1}{x^2}\left(\frac{dy}{dx}\right)^2 + 6 = \left(\frac{5}{x}\right)\frac{dy}{dx}$ is $y = \lambda x^2 + c$ (where, c is an arbitrary constant). The sum of all the possible value of λ is

(A) $\frac{3}{2}$

(B) $\frac{5}{2}$

(C) $\frac{2}{5}$

(D) 2

Q20: NTA Test 22 (Single Choice)

If the solution of the differential equation $\frac{dy}{dx} = \frac{x^3 + xy^2}{y^3 - yx^2}$ is $y^k - x^k = 2x^2y^2 + \lambda$ (where, λ is an arbitrary constant), then the value of k is

(A) 2

(B) 4

(C) 1

(D) $\frac{3}{2}$

Q21: NTA Test 23 (Single Choice)

The solution of the differential equation $2ydx + xdy = 2x\sqrt{y}dx$ is (where, C is an arbitrary constant)

(A) $x\sqrt{y} = x + C$

(B) $x\sqrt{y} = \frac{x^2}{2} + C$

(C) $\frac{x}{\sqrt{y}} = x + C$

(D) $xy = C$

Q22: NTA Test 24 (Single Choice)

Let $y = f(x)$ be the solution of the differential equation $\frac{dy}{dx} = -2x(y-1)$ with $f(0) = 1$, then $\lim_{x \to \infty} f(x)$ is equal to

(A) $\frac{1}{2}$

(B) 0

(C) e

(D) 1

Q23: NTA Test 25 (Single Choice)

Let the curve $y = f(x)$ satisfies the equation $\frac{dy}{dx} = 1 - \frac{1}{x^2}$ and passes through the point $\left(2, \frac{7}{2}\right)$, then the value of $f(1)$ is

(A) 3

(B) 2

(C) $\frac{7}{2}$

(D) 1

Q24: NTA Test 26 (Single Choice)

The equation of the curve passing through the point $(1,1)$ and satisfying the differential equation $\frac{dy}{dx} = \frac{x+2y-3}{y-2x+1}$ is

(A) $x^2 - 4xy - y^2 + 6x + 2y - 4 = 0$

(B) $x^2 + 4xy - y^2 - 6x + 2y + 4 = 0$

(C) $x^2 + 4xy - y^2 - 6x - 2y + 4 = 0$

(D) $x^2 + 4xy + y^2 - 6x - 2y - 4 = 0$

Q25: NTA Test 27 (Single Choice)

The solution of the differential equation $\sin(x+y)dy = dx$ is

(A) $y + \tan(x+y) - \sec(x+y) = c$

(B) $y - \tan(x+y) - \sec(x+y) = c$

(C) $y + \tan(x+y) + \sec(x+y) = c$

(D) $y - \tan(x+y) + \sec(x+y) = c$

Q26: NTA Test 29 (Single Choice)

The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{(1+\ln x+\ln y)^2}$ is (where, c is an arbitrary constant)

(A) $xy\left[1 + (\ln(xy))^2\right] = \frac{x^2}{2} + c$

(B) $1 + (\ln(xy))^2 = \frac{x^2}{2} + y + c$

(C) $\mathrm{xy}\ (1 + \ln(xy)) = \frac{x^2}{2} + c$

(D) $\mathrm{xy}\ (1 + \ln(xy)) = \frac{x}{2} + c$

Q27: NTA Test 30 (Single Choice)

The solution of the differential equation $\frac{ydx - xdy}{xy} = xdx + ydy$ is (where, C is an arbitrary constant)

(A) $\frac{x}{y} = x + y + C$

(B) $\frac{x}{y} = \frac{x^2+y^2}{2} + C$

(C) $\ln\left(\frac{x}{y}\right) = x^2 + y^2 + C$

(D) $2\ln\left(\frac{x}{y}\right) = x^2 + y^2 + C$

Q28: NTA Test 32 (Single Choice)

The order of the differential equation of the family of curves $y = a3^{bx+c} + d\sin(x+e)$ is (where, $a,\ b,\ c,\ d,\ e$ are arbitrary constants)

(A) 5

(B) 4

(C) 3

(D) 2

Q29: NTA Test 33 (Single Choice)

The solution of the differential equation $\frac{dy}{dx} + xy\ln y = x^3 y$ is equal to (where, C is the constant of integration)

(A) $\ln y = x^2 + Ce^{-x^2}$

(B) $\ln y = x^2 - 2 + Ce^{-x^2}$

(C) $\ln y = x^2 - 2 + Ce^{-\frac{x^2}{2}}$

(D) $\ln y = x^2 + Ce^{-\frac{x^2}{2}}$

Q30: NTA Test 34 (Single Choice)

The population $\mathrm{p(t)}$ at a time t of a certain mouse species satisfies the differential equation $\frac{\mathrm{dp(t)}}{\mathrm{dt}} = 0.5\mathrm{p}(\mathrm{t}) - 450$. If $\mathrm{p}(0) = 850$, then the time at which the population becomes zero is

(A) $\frac{1}{2}\ln 18$

(B) $\ln 18$

(C) $2\ln 18$

(D) $\ln 9$

Q31: NTA Test 35 (Numerical)

Let $\mathrm{y} = \mathrm{f}(\mathrm{x})$ satisfies $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{x+y}}{\mathrm{x}}$ and $\mathrm{f}(\mathrm{e}) = \mathrm{e}$, then the value of $\mathrm{f}(1)$ is

Q32: NTA Test 36 (Single Choice)

The solution of the differential equation $\mathrm{xcosy}\frac{\mathrm{dy}}{\mathrm{dx}} + \sin\mathrm{y} = 1$ is (Here, $\mathrm{x} > 0$ and λ is an arbitrary constant)

(A) $\mathrm{x} - \mathrm{xcosx} = \lambda$

(B) $\mathrm{x} + \mathrm{xcosx} = \lambda$

(C) $\mathrm{x} - \mathrm{xsiny} = \lambda$

(D) $\mathrm{x} + \mathrm{xcosy} = \lambda$

Q33: NTA Test 37 (Single Choice)

If the differential equation $3\mathrm{x}^{\frac{1}{3}}\mathrm{dy} + \mathrm{x}^{-\frac{2}{3}}\mathrm{ydx} = 3\mathrm{xdx}$ is satisfied by $\mathrm{kx}^{\frac{1}{3}}\mathrm{y} = \mathrm{x}^2 + \mathrm{c}$ (where c is an arbitrary constant), then the value of k is

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) 2

(D) 1

Q34: NTA Test 38 (Single Choice)

The solution of the differential equation $\mathrm{xdy} = \left(\tan \mathrm{y} + \frac{\mathrm{e}^{1/\mathrm{x}^2}}{\mathrm{x}} \sec \mathrm{y}\right) \mathrm{dx}$ is (where C is the constant of integration)

(A) $\sin \mathrm{y} = \mathrm{e}^{\frac{1}{\mathrm{x}^2}} + \mathrm{C}$

(B) $\frac{2 \sin \mathrm{y}}{\mathrm{x}} + \mathrm{e}^{\frac{1}{\mathrm{x}^2}} = \mathrm{C}$

(C) $\frac{\sin \mathrm{y}}{\mathrm{x}} - \mathrm{e}^{\frac{1}{\mathrm{x}^2}} = \mathrm{C}$

(D) $\sin \mathrm{y} - \mathrm{xe}^{\frac{1}{\mathrm{x}^2}} = \mathrm{C}$

Q35: NTA Test 39 (Single Choice)

The solution of the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{\mathrm{y}\cos \mathrm{x} - \mathrm{y}^2}{\sin \mathrm{x}}$ is equal to (where c is an arbitrary constant)

(A) $\sin \mathrm{x} = \mathrm{x} - \mathrm{y} + \mathrm{c}$

(B) $\sin \mathrm{x} = \mathrm{x} + \mathrm{y} + \mathrm{c}$

(C) $\sin \mathrm{x} = \mathrm{xy} + \mathrm{cy}$

(D) $\frac{\sin \mathrm{x}}{\mathrm{x}} = \mathrm{y} + \mathrm{c}$

Q36: NTA Test 40 (Numerical)

If the order of the differential equation of the family of circles touching the x-axis at the origin is k, then $2k$ is equal to

Q37: NTA Test 41 (Numerical)

If the solution of the differential equation $x^2 dy + 2xydx = \sin x dx$ is $x^k y + \cos x = C$ (where C is an arbitrary constant), then the value of k is equal to

Q38: NTA Test 42 (Numerical)

If $\int e^{-\frac{x^2}{2}} dx = f(x)$ and the solution of the differential equation $\frac{dy}{dx} = 1 + xy$ is $y = ke^{\frac{x^2}{2}} f\left(x\right) + Ce^{\frac{x^2}{2}}$, then the value of k is equal to (where C is the constant of integration)

Q39: NTA Test 43 (Single Choice)

If the solution of the differential equation $y^3 x^2 \cos\left(x^3\right) dx + \sin\left(x^3\right) y^2 dy = \frac{x}{3} dx$ is $2 \sin\left(x^3\right) y^k = x^2 + C$ (where C is an arbitrary constant), then the value of k is equal to

(A) 3

(B) 2

(C) 1

(D) 4

Q40: NTA Test 44 (Single Choice)

The solution of the differential equation $\frac{dy}{dx} = \frac{x-y}{x+4y}$ is (where C is the constant of integration)

(A) $xy + y^2 = x + C$

(B) $xy - y^2 = x^2 + C$

(C) $xy + 2y^2 = x^2 + C$

(D) $2xy + 4y^2 = x^2 + C$

Q41: NTA Test 45 (Single Choice)

The solution of the differential equation $y\left(2x^4 + y\right) dy + \left(4xy^2 - 1\right) x^2 dx = 0$ is (where C is an arbitrary constant)

(A) $3x^2 y + x^3 - y^3 = C$

(B) $3x^4 y^2 + y^3 - x^3 = C$

(C) $3x^2 y^4 + x^3 - y^3 = C$

(D) $3x^2 y^4 + y^3 - x^3 = C$

Q42: NTA Test 46 (Single Choice)

The solution of the differential equation $y\left(\sin^2 x\right) dy + (\sin x \cos x) y^2 dx = x dx$ is (where C is the constant of integration)

(A) $\sin^2 x \cdot y = x^2 + C$

(B) $\sin^2 x \cdot y^2 = x^2 + C$

(C) $\sin x \cdot y^2 = x^2 + C$

(D) $\sin^2 x \cdot y^2 = x + C$

Q43: NTA Test 47 (Single Choice)

The order of the differential equation of the family of parabolas symmetric about $y = 1$ and tangent to $x = 2$ is

(A) 2

(B) 1

(C) 3

(D) 4

Q44: NTA Test 48 (Single Choice)

The differential equation of the family of curves whose tangent at any point makes an angle of $\frac{\pi}{4}$ with the ellipse $\frac{x^2}{4} + y^2 = 1$ is

(A) $\frac{dy}{dx} = \frac{x+y}{x-y}$

(B) $\frac{dy}{dx} = \frac{x+4y}{x-4y}$

(C) $\frac{dy}{dx} = \frac{x}{4y}$

(D) $\frac{dy}{dx} = \frac{4y}{x}$

Answer Keys

Q1: (B)	**Q2:** (A)	**Q3:** (A)
Q4: 8	**Q5:** (C)	**Q6:** (A)
Q7: (A)	**Q8:** (A)	**Q9:** (C)
Q10: (B)	**Q11:** (B)	**Q12:** 2
Q13: (B)	**Q14:** (B)	**Q15:** (B)
Q16: (A)	**Q17:** (B)	**Q18:** (B)
Q19: (B)	**Q20:** (B)	**Q21:** (B)
Q22: (D)	**Q23:** (A)	**Q24:** (C)
Q25: (D)	**Q26:** (A)	**Q27:** (D)
Q28: (B)	**Q29:** (C)	**Q30:** (C)
Q31: 0	**Q32:** (C)	**Q33:** (C)
Q34: (B)	**Q35:** (C)	**Q36:** 2
Q37: 2	**Q38:** 1	**Q39:** (A)
Q40: (D)	**Q41:** (B)	**Q42:** (B)
Q43: (B)	**Q44:** (B)	

Solutions

Q1: (B) $-\frac{1}{4}$

given : $y' = \frac{y}{x} + \phi\left(\frac{x}{y}\right)$

As $y \ln\left(cx\right) = x \Rightarrow y' \ln\left(cx\right) + y\frac{1}{cx}c = 1$

$$\Rightarrow y'\left(\frac{x}{y}\right) + \frac{y}{x} = 1$$

$$\Rightarrow \frac{1-\left(\frac{y}{x}\right)}{\left(\frac{x}{y}\right)} = \left(\frac{y}{x}\right) + \phi\left(\frac{x}{y}\right)$$

$$x = 2, y = 1 \Rightarrow \frac{1-\left(\frac{1}{2}\right)}{\left(\frac{2}{1}\right)} = \left(\frac{1}{2}\right) + \phi\left(\frac{2}{1}\right)$$

$$\Rightarrow \phi\left(2\right) = -\frac{1}{4}$$

Q2: (A) 3500

$\frac{dP}{dx} = 100 - 12\sqrt{x}$

$\Rightarrow \int dP = \int \left(100 - 12\sqrt{x}\right)dx$

$\Rightarrow P = 100x - 8x^{3/2} + C$

Now, $x = 0$

$\Rightarrow P = 2000 \Rightarrow C = 2000$

hence, $P = 100x - 8x^{\frac{3}{2}} + 2000$

$\because \; x = 25$

$P = 2500 - 1000 + 2000$
$P = 3500$

Q3: (A) $y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$

Equation of tangent at the point

R $\left(x, \; f(x)\right)$ is, $Y - \; f(x) = f'(x) \; (X - x)$

Coordinates of the point are $P(0, f(x) - xf'(x))$

The slope of the perpendicular line through P is

$\frac{f(x) - xf'(x)}{-1} = -\frac{1}{f'(x)}$

$y\frac{dy}{dx} - x\left(\frac{dy}{dx}\right)^2 = 1$ is the differential equation.

Q4: 8

Given, $f(1) = \frac{1}{3}$ and $6\int_1^x f(t)dt = 3x f(x) - x^3$ for all $x \geq 1$

Using Newton- Leibnitz formula.

Differentiating both sides,

$\Rightarrow 6 f(x) \cdot 1 - 0 = 3f(x) + 3xf'(x) - 3x^2$

$\Rightarrow 3xf'(x) - 3f(x) = 3x^2$

$\Rightarrow f'(x) - \frac{1}{x}f(x) = x$

$\Rightarrow \frac{xf'(x) - f(x)}{x^2} = 1$

$\Rightarrow \frac{d}{dx}\left\{\frac{f(x)}{x}\right\} = 1$

Integrating both sides,

$\Rightarrow \frac{f(\mathrm{x})}{\mathrm{x}} = \mathrm{x} + \mathrm{C} \quad [\because f(1) = \frac{1}{3}]$

$\frac{1}{3} = 1 + \mathrm{C}$

$\Rightarrow \mathrm{C} = -\frac{2}{3}$

$f(\mathrm{x}) = \mathrm{x}^2 - \frac{2}{3}\mathrm{x}$

$\Rightarrow f(2) = 4 - \frac{4}{3} = \frac{8}{3}$

$\therefore 3f(2) = 8$

Q5: (C) 3

The given equation can be written as y $= (\mathrm{C}) \cos(x + c_3) - (c_4 e^{c_5}) e^x$ (where $\mathrm{C} = c_1 + c_2$)
$\therefore$ 3 arbitary constants

Q6: (A) $x \frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$

Given, $xy = ae^x + be^{-x}$ ..... (I)

$\Rightarrow \quad x \frac{dy}{dx} + y = ae^x - be^{-x}$

$\Rightarrow \quad x \frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx} = ae^x + be^{-x}$

$\Rightarrow \quad x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} - xy = 0$ [from eq. (I)]

Q7: (A) 2, 4

$\frac{d^2y}{\mathrm{dx}^2} = \left[y + \left(\frac{dy}{dx}\right)^6\right]^{1/4}$

$\Rightarrow \left(\frac{d^2y}{\mathrm{dx}^2}\right)^4 = \left[y + \left(\frac{dy}{dx}\right)^6\right]$

Hence, order is 2 and degree is 4.

Q8: (A) $y = \sin x + \frac{1}{2} \operatorname{cosec} x$

Given, $\frac{dy}{dx} = 2 \cos x - y \cos x \operatorname{cosec} x$

$\Rightarrow \frac{dy}{dx} + y \cot x = 2 \cos x$

$\therefore \mathrm{IF} = e^{\int \cot x \, dx} = e^{\ln(\sin x)} = \sin x$

$\therefore$ Solution is $y \sin x = \int 2 \cos x \sin x \, dx + c$

$\Rightarrow y \sin x = \int \sin 2x \, dx + c$

$\Rightarrow \quad y \sin x = \frac{-\cos 2x}{2} + c$

At $x=\frac{\pi}{4}, y=\sqrt{2}$

$\therefore \sqrt{2}\sin\frac{\pi}{4}=\frac{-\cos 2(\pi/4)}{2}+c$

$\Rightarrow c=1$

$\therefore y\sin x=-\frac{1}{2}\cos 2x+1$

$\Rightarrow y=-\frac{1}{2}\cdot\frac{\cos 2x}{\sin x}+\operatorname{cosec} x$

$\Rightarrow y=-\frac{1}{2\sin x}\left(1-2\sin^2 x\right)+\operatorname{cosec} x$

$\Rightarrow y=\frac{1}{2}\operatorname{cosec} x+\sin x$

Q9: (C) $\log y+\sqrt{\frac{x}{y}}=c$

We have, $\frac{dy}{dx}=\frac{y}{x-2\sqrt{xy}}$ which is homogeneous.

Put, $y=Vx$ so that $\frac{dy}{dx}=x\frac{dV}{dx}+V$

$\Rightarrow x\frac{dV}{dx}=\frac{V}{1-2\sqrt{V}}-V=\frac{2V^{3/2}}{1-2\sqrt{V}}$

$\Rightarrow \frac{dx}{x}=\frac{1-2\sqrt{V}}{2V^{3/2}}dV=\left(\frac{1}{2V^{3/2}}-\frac{1}{V}\right)dV$

Integrating, we get,

$-c+\log x=-V^{-1/2}-\log V=-\sqrt{\frac{x}{y}}-\log y+\log x$

$\Rightarrow \log y+\sqrt{\frac{x}{y}}=c.$

Q10: (B) $\frac{1}{(2x+y)^2}=x^2+1+Ce^{x^2}$

Let, $2x+y=t\Rightarrow\frac{dy}{dx}+2=\frac{dt}{dx}$

$\frac{dt}{dx}+xt=x^3t^3\Rightarrow\frac{1}{t^3}\frac{dt}{dx}+\frac{1}{t^2}x=x^3$

Let, $\frac{1}{t^2}=u\Rightarrow\frac{-2}{t^3}\frac{dt}{dx}=\frac{du}{dx}$

$\frac{du}{dx}+(-2x)u=-2x^3$

I.F. $=e^{-\int 2xdx}=e^{-x^2}\Rightarrow u.e^{-x^2}=\int e^{-x^2}\left(-2x^3\right)dx$

$\frac{e^{-x^2}}{(2x+y)^2}=-2\int e^{-x^2}.x^3dx$

$\frac{e^{-x^2}}{(2x+y)^2}=\int e^{-x^2}.x^2(-2x)dx$

Let, $-x^2=v$

$-2xdx=dv$

$\Rightarrow\frac{e^{-x^2}}{(2x+y)^2}=-\int e^v vdv$

$\frac{e^{-x^2}}{(2x+y)^2}+v\cdot e^v-e^v=C$

$\Rightarrow\frac{e^{-x^2}}{(2x+y)^2}-x^2e^{-x^2}-e^{-x^2}=C$

$\frac{1}{(2x+y)^2}=\left(x^2+1\right)+Ce^{x^2}$

Q11: (B) $x^2+y^2-xy+x+y=c$
$xdy-2ydy-dy=2xdx-ydx+dx$

$(xdy+ydx)-2ydy-dy-2xdx-dx=0$

$d(xy)-2ydy-dy-2xdx-dx=0$

on integrating, we get,

$xy-y^2-y-x^2-x=c$

Q12: 2

$P(x,y):\ f(x+y)=f(x)f'(y)+f'(x)f(y)\ \forall\ x,\ y\in R$

$P(0{,}0):\ f(0)=f(0)f'(0)+f'(0)f(0)$

$\Rightarrow 1=2f'(0)$

$\Rightarrow f'(0)=\frac{1}{2}$

$P(x,0):\ f(x)=f(x).\ f'(0)+f'(x).\ f(0)$

$\Rightarrow\quad f(x)=\frac{1}{2}f(x)+f'(x)$

$\Rightarrow\quad f'(x)=\frac{1}{2}f(x)$

$\Rightarrow\quad f(x)=e^{\frac{1}{2}x}$

$\Rightarrow\quad \ln(f(4))=2$

Q13: (B) $\ln\tan\left(\frac{y}{4}\right)=c-2\sin\left(\frac{x}{2}\right)$
Given equation

$\frac{dy}{dx}+\sin\left(\frac{x+y}{2}\right)=\sin\left(\frac{x-y}{2}\right)$

$\Rightarrow\ \frac{dy}{dx}=\sin\left(\frac{x-y}{2}\right)-\sin\left(\frac{x+y}{2}\right)$

$\Rightarrow\ \frac{dy}{dx}=-2\sin\left(\frac{y}{2}\right)\cos\left(\frac{x}{2}\right)$

$\Rightarrow \text{cosec}\ \left(\frac{y}{2}\right)dy=-2\cos\left(\frac{x}{2}\right)dx$

On integrating both sides, we get

$\int\text{cosec}\ \left(\frac{y}{2}\right)dy=-\int 2\cos\left(\frac{x}{2}\right)dx$

$\Rightarrow\quad \frac{\ln\left(\tan\frac{y}{4}\right)}{\frac{1}{2}}=-\frac{2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}}+c$

$\Rightarrow \ln\left(\tan\frac{y}{4}\right)=c-2\sin\left(\frac{x}{2}\right)$

Q14: (B) 75
$\frac{d(p(t))}{dt}-p(t)=-100$

I.F. $= e^{-\int dt} = e^{-t}$

$e^{-t}(p(t)) = 100e^{-t} + C$

Now $P(0) = 50 \Rightarrow C = -50$

$P(-\ln 2) = 100 - 50e^{\ln\left(\frac{1}{2}\right)} = 100 - 25 = 75.$

Q15: (B) $x^2 + y^2 + \frac{y}{x} = 1$

Given equation is

$2xdx + 2ydy + d\left(\frac{y}{x}\right) = 0$

Integrating, we get,

$x^2 + y^2 + \frac{y}{x} = c$

As $(1,\ 0)$ lies on it, hence, $c = 1$

Q16: (A) $y = 1 + ce^{1/x}$

The given differential equation can be written as,

$\frac{dy}{dx} + \frac{1}{x^2} \cdot y = \frac{1}{x^2}$

Integrating factor is $e^{-1/x}$

$\Rightarrow y.e^{-1/x} = \int \frac{1}{x^2}.e^{-1/x}\,dx$

Hence, the solution is $y = 1 + ce^{1/x}$

Q17: (B) $\frac{y}{\sqrt{x}} = \frac{y^2}{2} + c$

$\frac{dy}{\sqrt{x}} - \frac{ydx}{2x^{\frac{3}{2}}} = ydy \Rightarrow d\left(\frac{y}{\sqrt{x}}\right) = (ydy) \Rightarrow \frac{y}{\sqrt{x}} = \frac{y^2}{2} + c$

Q18: (B) $x^3\sin\left(\frac{1}{x}\right) + xy = c$

Given equation can be written as $d\left(x^3\sin\left(\frac{1}{x}\right)\right) + d(xy) = 0$

$\therefore$ solution is $x^3\sin\left(\frac{1}{x}\right) + xy = c$

Q19: (B) $\frac{5}{2}$

Given equation is $\left(\frac{dy}{dx}\right)^2 - 5x\left(\frac{dy}{dx}\right) + 6x^2 = 0$ or $\left(\frac{dy}{dx} - 3x\right)\left(\frac{dy}{dx} - 2x\right) = 0$

$\Rightarrow y = \frac{3}{2}x^2 + k$ or $\Rightarrow y = x^2 + \mu$

Q20: (B) 4

$\frac{ydy}{xdx} = \frac{x^2+y^2}{y^2-x^2}$

Let, $y^2 = Y; x^2 = X \Rightarrow \frac{ydy}{xdx} = \frac{dY}{dX}$

Hence, the equation is $\frac{dY}{dX} = \frac{X+Y}{Y-X}$

$\Rightarrow YdY - XdX = XdY + YdX$

On integrating we get $\frac{Y^2}{2} - \frac{X^2}{2} = \int d(XY) = XY + c$

or $Y^2 - X^2 = 2XY + \lambda$

$\Rightarrow y^4 - x^4 = 2x^2y^2 + \lambda$

Q21: (B) $x\sqrt{y} = \frac{x^2}{2} + C$

Given equation can be written as $dx\left(\sqrt{y}\right) + x\left(\frac{1}{2\sqrt{y}}dy\right) = xdx$

or $d\left(x\sqrt{y}\right) = xdx$
$\int d\left(x\sqrt{y}\right) = \int xdx \Rightarrow x\sqrt{y} = \frac{x^2}{2} + C$

Q22: (D) 1

$\frac{dy}{dx} = -2x\left(y-1\right)$
$\frac{dy}{dx} + 2xy = 2x$
$\text{I. }F. = e^{\int 2xdx} = e^{x^2}$
$y \cdot e^{x^2} = \int e^{x^2} 2xdx$
$y \cdot e^{x^2} = e^{x^2} + C$
$\because y\left(0\right) = 1 \Rightarrow 1 = 1 + C \Rightarrow C = 0$
Hence, the solution is $ye^{x^2} = e^{x^2}$
$\Rightarrow y = 1 = f\left(x\right)$
$\lim_{x\to\infty} f\left(x\right) = 1$

Q23: (A) 3

$\frac{dy}{dx} = 1 - \frac{1}{x^2}$
So, $y = x + \frac{1}{x} + C$
Since, it passes through $\left(2, \frac{7}{2}\right)$
$\frac{7}{2} = 2 + \frac{1}{2} + C \Rightarrow C = 1$
$y = x + \frac{1}{x} + 1 = f\left(x\right)$
$f\left(1\right) = 3$

Q24: (C) $x^2 + 4xy - y^2 - 6x - 2y + 4 = 0$

$\frac{dy}{dx} = \frac{x+2y-3}{y-2x+1}$
$ydy - 2xdy + dy = xdx + 2ydx - 3dx$
$ydy - 2\left(xdy + ydx\right) + dy - xdx + 3dx = 0$
$ydy - 2d\left(xy\right) + dy - xdx + 3dx = 0$

On integrating, we get,
$\frac{y^2}{2} - 2xy + y - \frac{x^2}{2} + 3x + C = 0$
Since, the curve passes through $(1,1)$

$\Rightarrow \frac{1}{2} - 2 + 1 - \frac{1}{2} + 3 + C = 0$
$C = -2$
$y^2 - 4xy + 2y - x^2 + 6x - 4 = 0$ or $x^2 + 4xy + y^2 - 6x - 2y + 4 = 0$

Q25: (D) $y - \tan\left(x+y\right) + \sec\left(x+y\right) = c$

Let $x + y = v$

$\frac{dv}{dx} - 1 = \frac{1}{\sin v}$

$dx = \left(1 - \frac{1}{1+\sin v}\right)dv$

$\Rightarrow x = v - \int\left(\sec^2 v - \tan v \cdot \sec v\right)dv$

$\Rightarrow x = v - \tan v + \sec v + c$

$\Rightarrow x = x + y - \tan\left(x+y\right) + \sec\left(x+y\right) + c$

Q26: (A) $xy\left[1+(\ln(xy))^{2}\right]=\frac{x^{2}}{2}+c$

$\frac{dy}{dx}+\frac{y}{x}=\frac{1}{(1+\ln xy)^{2}}$

Let $xy=u$ so that $\frac{du}{dx}=\frac{x}{(1+\ln\ u)^{2}}$

$\therefore\quad \int(1+\ln u)^{2}du=\int x\ dx+c$

$\Rightarrow u\left(1+\ln u\right)^{2}-\int\frac{2(1+\ln\ u)}{u}\cdot udu=\frac{x^{2}}{2}+c$

$\Rightarrow u\left(1+2\ \ln u+2u(\ln u)^{2}\right)-2u\ln u=\frac{x^{2}}{2}+c$

$\therefore xy\left(1+\left(\ln(xy)\right)^{2}\right)=\frac{x^{2}}{2}+c$

Q27: (D) $2\ln\left(\frac{x}{y}\right)=x^{2}+y^{2}+C$

The given equation is $\frac{1}{\left(\frac{x}{y}\right)}\cdot\frac{ydx-xdy}{y^{2}}=xdx+ydy$

or $d\left(\ln\left(\frac{x}{y}\right)\right)=xdx+ydy$
On integrating, we get,

$\int d\left(\ln\left(\frac{x}{y}\right)\right)=\int xdx+\int ydy$
$\Rightarrow\ln\left(\frac{x}{y}\right)=\frac{x^{2}}{2}+\frac{y^{2}}{2}+k$
$\Rightarrow 2\ln\left(\frac{x}{y}\right)=x^{2}+y^{2}+C$

Q28: (B) 4
We can re-write the equation of the curve as
$y=a\cdot 3^{c}\cdot 3^{bx}+d\sin(x+e)$
or $y=k\cdot 3^{bx}+d\sin(x+e)$ (where, k is an arbitrary constant)
As there are '4' such constants, hence the order of the differential equation will be '4'

Q29: (C) $\ln y=x^{2}-2+Ce^{-\frac{x^{2}}{2}}$

$\frac{1}{y}\frac{dy}{dx}+x\ln y=x^{3}$
Let $\ln y=t$
$\frac{1}{y}\frac{dy}{dx}=\frac{dt}{dx}$
$\frac{dt}{dx}+tx=x^{3}$
I.F. $=e^{\int xdx}=e^{\frac{x^{2}}{2}}$
$te^{\frac{x^{2}}{2}}=\int e^{\frac{x^{2}}{2}}x^{3}dx$
$\ln ye^{\frac{x^{2}}{2}}=\int\left(e^{\frac{x^{2}}{2}}\cdot x\right)x^{2}dx$
$\ln ye^{\frac{x^{2}}{2}}=e^{\frac{x^{2}}{2}}\cdot x^{2}-\int(2x)e^{\frac{x^{2}}{2}}dx$
$\ln e^{\frac{x^{2}}{2}}=e^{\frac{x^{2}}{2}}x^{2}-2e^{\frac{x^{2}}{2}}+C$
$\ln y\cdot e^{\frac{x^{2}}{2}}=e^{\frac{x^{2}}{2}}\left(x^{2}-2\right)+C$
$\ln y=\left(x^{2}-2\right)+Ce^{\frac{-x^{2}}{2}}$

Q30: (C) $2\ln 18$

Let, $p = p(t)$

$\frac{dp}{dt} = 0.5p - 450$

$= \frac{p-900}{2}$

$\Rightarrow \int_{850}^{p} \frac{2}{p-900} dp = \int_{0}^{t} dt$

$\Rightarrow 2\left|\log|p-900|\right|_{850}^{p} = t$

$\Rightarrow 2\left|\log|p-900| - \log|-50|\right| = t$

$\Rightarrow 2\log\left|\frac{p-900}{-50}\right| = t$ ...(1)

$\Rightarrow \left|\frac{p-900}{-50}\right| = e^{t/2}$

$\Rightarrow p = 900 - 50\ e^{t/2}$

Let, when $t = T,\ p = 0$ (using (1))

$\Rightarrow T = 2\ln 18$

Q31: 0

$\frac{dy}{dx} = 1 + \frac{y}{x}$
$\frac{dy}{dx} - \frac{y}{x} = 1$
$\text{If} = e^{-\int \frac{1}{x} dx} = \frac{1}{x}$
$\frac{y}{x} = \int \frac{1}{x} dx \Rightarrow y = x\ln x + cx = f(x)$
$f(e) = e + ce = e \Rightarrow c = 0$
$f(1) = 0$

Q32: (C) $x - x\sin y = \lambda$

Let, $\sin y = t \Rightarrow \cos y \frac{dy}{dx} = \frac{dt}{dx}$
$\therefore$ the equation becomes
$x\frac{dt}{dx} + t = 1$ or $x\frac{dt}{dx} = 1 - t$
$\Rightarrow \frac{dt}{1-t} = \frac{dx}{x}$
On integrating, we get,
$-\ln|1-t| = \ln x + \ln C$
or $\frac{1}{1-t} = Cx$
i.e. $(1 - \sin y)x = \frac{1}{C} = \lambda$ (say)

Q33: (C) 2

The given equation is $x^{\frac{1}{3}}.dy + \frac{1}{3}x^{-\frac{2}{3}}dx.y = xdx$
or $d\left(x^{\frac{1}{3}}.y\right) = xdx$
Integrating, we get,

$x^{\frac{1}{3}}.y = \frac{x^2}{2} + \lambda$

Or $2x^{\frac{1}{3}}y = x^2 + C$

$\Rightarrow k = 2$

Q34: (B) $\frac{2\sin y}{x} + e^{\frac{1}{x^2}} = C$

$\frac{dy}{dx} = \frac{\tan y}{x} + \frac{e^{\frac{1}{x^2}}}{x^2}\sec y$

$\cos y\frac{dy}{dx} - \frac{\sin y}{x} = \frac{e^{\frac{1}{x^2}}}{x^2}$

Let, $\sin y = t$

$\cos y\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} - \frac{t}{x} = \frac{e^{\frac{1}{x^2}}}{x^2}$

$\text{I.F.} = e^{-\int\frac{1}{x}dx} = e^{\ln\left(\frac{1}{x}\right)} = \frac{1}{x}$

$\Rightarrow \frac{t}{x} = \int\frac{e^{\frac{1}{x^2}}}{x^3}dx$

$\Rightarrow \frac{t}{x} = \frac{-1}{2}\int\left(\frac{-2}{x^3}\right)e^{\frac{1}{x^2}}dx$

$\Rightarrow \frac{\sin y}{x} = \left(\frac{-1}{2}\right)e^{\frac{1}{x^2}} + C$

Q35: (C) $\sin x = xy + cy$

$\frac{dy}{dx} = \frac{y\cos x - y^2}{\sin x}$

$y\cos x dx - \sin x dy = y^2 dx$

$\frac{y\cos x.dx - \sin x.dy}{y^2} = dx$

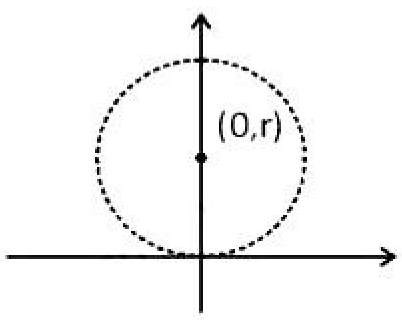
$d\left(\frac{\sin x}{y}\right) = dx$

On integrating, we get,

$\frac{\sin x}{y} = x + c$

$\sin x = xy + cy$

Q36: 2



The equation of the family is $x^2 + (y - r)^2 = r^2$

i.e. only one arbitrary constant.

Hence, the order of its differential equation will be '1'

$\therefore 2k = 2$

Q37: 2

The given equation is

$d\left(x^2y\right) = \sin x dx$

On integrating, we get,

$x^2y = -\cos x + C$

or $x^2y + \cos x = C$

$\Rightarrow k = 2$

Q38: 1

$\frac{dy}{dx} - xy = 1$
Here, I.F. $= e^{-\int x dx} = e^{-\frac{x^2}{2}}$
So, solution is $ye^{-\frac{x^2}{2}} = \int e^{-\frac{x^2}{2}} dx + C$
$y = e^{\frac{x^2}{2}} f(x) + Ce^{\frac{x^2}{2}}$
Hence, $k = 1$

Q39: (A) 3

The given equation is
$y^3 \cdot (3x^2 \cos(x^3) dx) + \sin(x^3)(3y^2 dy) = x dx$
$\Rightarrow d(\sin(x^3) \cdot y^3) = x dx$
On integrating, we get,
$\sin(x^3) \cdot y^3 = \frac{x^2}{2} + C$
$\Rightarrow k = 3.$

Q40: (D) $2xy + 4y^2 = x^2 + C$

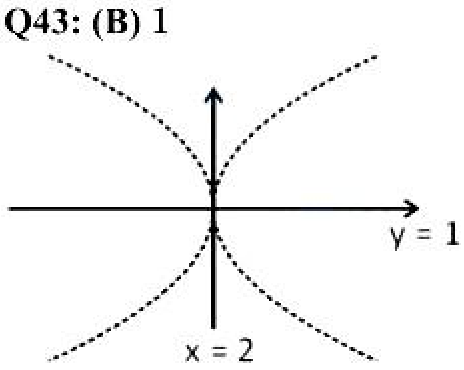
Given equation is $x dy + 4y dy = x dx - y dx$
or $x dy + y dx + 4y dy = x dx$
or $d(xy) + 4y dy = x dx$
Integrating, we get
$xy + 2y^2 = \frac{x^2}{2} + C$

Q41: (B) $3x^4y^2 + y^3 - x^3 = C$
$2x^4 y dy + y^2 dy + 4x^3 y^2 dx - x^2 dx = 0$
$2x^3 y (x dy + 2y dx) + y^2 dy - x^2 dx = 0$
$2x^2 y (x^2 dy + 2xy dx) + y^2 dy - x^2 dx = 0$
$2(x^2 y) d(x^2 y) + y^2 dy - x^2 dx = 0$

On integrating, we get,
$(x^2 y)^2 + \frac{y^3}{3} - \frac{x^3}{3} = C_1$
$3x^4 y^2 + y^3 - x^3 = C$

Q42: (B) $\sin^2 x \cdot y^2 = x^2 + C$
The given equation is $(\sin^2 x)(2y dy) + (2 \sin x \cos x dx) y^2 = 2x dx$
or $d(\sin^2 x \cdot y^2) = 2x dx$
On integrating, we get
$\sin^2 x \cdot y^2 = x^2 + C$

Q43: (B) 1



The equation of the family is which has one arbitrary constant.
Hence, the order of the differential equation will be 1

Q44: (B) $\frac{dy}{dx} = \frac{x+4y}{x-4y}$
Given, $\frac{x^2}{4} + y^2 = 1$

On differentiating w.r.t. x,

$\frac{x}{2} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-x}{4y} = m_1 \text{ (let)}$

Let, the slope of the tangent at a point on ellipse where the tangents meet is m_2

Given, $\tan\left(\frac{\pi}{4}\right) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$1 = \left|\frac{-\frac{x}{4y} - m_2}{1 + \frac{x}{4y} m_2}\right| \Rightarrow |4y - xm_2| = |x + 4ym_2|$

$4y - xm_2 = x + 4ym_2$ or $4y - xm_2 = -x - 4ym_2$

$m_2 = \frac{4y-x}{x+4y}$ or $m_2 = \frac{4y+x}{x-4y}$

$\frac{dy}{dx} = \frac{4y-x}{x+4y}$ or $\frac{dy}{dx} = \frac{4y+x}{x-4y}$