Table of Contents

Circular Motion

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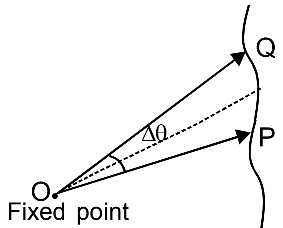
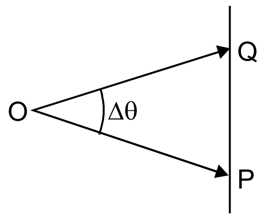
1. ANGULAR DISPLACEMENT

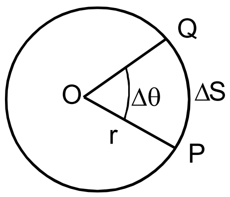
Introduction : Angle subtended by position vector of a particle moving along any arbitrary path w.r.t. some fixed point is called angular displacement.



(a) Particle moving in an arbitrary path



(b) Particle moving in straight line



(c) Particle moving in circular path

(i) Angular displacement is a vector quantity.

(ii) Its direction is perpendicular to plane of rotation and given by right hand screw rule.

Note: Clockwise angular displacement is taken as negative and anticlockwise displacement as positive.

$$\text{angle} = \frac{\text{arc}}{\text{radius}} = \frac{\text{linear displacement}}{\text{radius}}$$

(iii) For circular motion $\Delta S = r \times \Delta\theta$

(iv) Its unit is radian (in M.K.S)

Note : Always change degree into radian, if it occurs in numerical problems.

Note : $1 \text{ radian} = \frac{360^o}{2\pi} \Rightarrow \pi \text{ radian} = 180^o$

(v) It is a dimensionless quantity i.e. dimension $[M^0L^0T^0]$

2. ANGULAR VELOCITY

It is defined as the rate of change of angular displacement of a body or particle moving in circular path.

(i) It is a vector quantity.

(ii) Its direction is same as that of angular displacement i.e. perpendicular to plane of rotation.

Note : If the particle is revolving in the clockwise direction then the direction of angular velocity is perpendicular to the plane downwards. Whereas in case of anticlockwise direction the direction will be upwards.

(iii) Its unit is Radian/sec

(iv) Its dimension is $[M^0L^0T^{-1}]$

Types of Angular Velocity :

2.1 Average Angular Velocity :

$$\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time taken}}$$

2.2 Instantaneous Angular velocity :

The intantaneous angular velocity is defined as the angular velocity at some particular instant.

Instantaneous angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Note: Instantaneous angular velocity can also be called as simply angular velocity.

3. RELATION BETWEEN LINEAR VELOCITYA N D ANGULAR VELOCITY

We have $\omega = \frac{d\theta}{dt} = \frac{d\theta}{ds} \cdot \frac{ds}{dt} = \frac{1}{r} \cdot v$

$[\because \; d\theta = \frac{ds}{dr}, \; \text{angle} = \frac{\text{arc}}{\text{radius}}$

and $v = \frac{ds}{dt} = \text{linear velocity}]$

$\Rightarrow \quad v = r\omega$

In vector form, $\vec{v} = \vec{\omega} \times \vec{r}$

Note :

(i) When a particle moves along a curved path, its linear velocity at a point is along the tangent drawn at that point

(ii) When a particle moves along curved path, its velocity has two components. One along the radius, which increases or decreases the radius and another one perpendicular to the radius, which makes the particle to revolve about the point of observation.

(iii) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{v\sin\theta}{r}$

4. ANGULAR ACCELERATION

The rate of change of angular velocity is defined as angular acceleration.
If $\Delta\omega$ be change in angular velocity in time Δt, then angular acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \vec{\omega}}{\Delta t} = \frac{d\vec{\omega}}{dt}$$

(i) It is a vector quantity
(ii) Its direction is that of change in angular velocity
(iii) Unit : rad/sec^2
(iv) Dimension : $M^0L^0T^{-2}$

5. RELATION BETWEEN ANGULAR ACCELERATION AND LINEAR ACCELERATION

Linear acceleration = Rate of change of linear velocity

$\Rightarrow \quad a = \frac{dv}{dt}$ ....(i)

Angular acceleration = Rate of change of angular velocity

$\Rightarrow \quad \alpha = \frac{d\omega}{dt}$ ....(ii)

From (i) & (ii)

$$\frac{a}{\alpha} = \frac{dv}{d\omega} = \frac{d(r\omega)}{d\omega}$$

$$= r\frac{d\omega}{d\omega} \text{ [r is constant] } = r$$

$\Rightarrow \quad a = \alpha\, r$

In vector form $\quad \vec{a} = \vec{\alpha} \times \vec{r}$

6. EQUATION OF LINEAR MOTION AND ROTATIONAL MOTION

(i)	With constant velocity	$a = 0,\ s = ut$	$\alpha = 0\ ,\ \theta = \omega t$
(ii)	With constant acceleration	(i) Average velocity $v_{av} = \frac{v+u}{2}$ (ii) Average acceleration $a_{av} = \frac{v-u}{t}$ (iii) $s = v_{av}\ t = \frac{v+u}{2}t$ (iv) $v = u + at$ (v) $s = ut + \frac{1}{2}at^2$ (vi) $s = vt - \frac{1}{2}at^2$ (vii) $v^2 = u^2 + 2as$ (viii) $S_n = u + \frac{1}{2}(2n-1)a$ displacement in n^{th} sec	(i) Average angular velocity $\omega_{av} = \frac{\omega_1+\omega_2}{2}$ (ii) Average angular acceleration $a_{av} = \frac{\omega_2-\omega_1}{t}$ (iii) $\theta = \omega_{av}.\ t = \frac{\omega_1+\omega_2}{2}t$ (iv) $\omega_2 = \omega_1 + \alpha t$ (v) $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$ (vi) $\theta = \omega_2 t - \frac{1}{2}\alpha t^2$ (vi) $\omega_2^2 = \omega_1^2 + 2\alpha\theta$ (viii) $\theta_n = \omega_1 + \frac{1}{2}(2n-1)\alpha$ Angular displacement in n^{th} sec
(iii)	With variable acceleration	(i) $v = \frac{ds}{dt}$ (ii) $\int ds = \int v\ dt$ (iii) $a = \frac{dv}{dt} = v\frac{dv}{ds}$ (iv) $\int dv = \int a\ dt$ (v) $\int v\ dv = \int a\ ds$	(i) $\omega = d\theta/dt$ (ii) $\int d\theta = \int \omega\ dt$ (iii) $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$ (iv) $\int d\omega = \int \alpha\ dt$ (v) $\int \omega\ d\omega = \int \alpha\ d\theta$

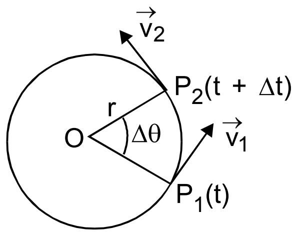
7. CENTRIPETAL ACCELERATION AND CENTRIPETAL FORCE

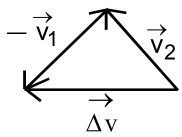
(i) A body or particle moving in a curved path always moves effectively in a circle at any instant.

(ii) The velocity of the particle changes moving on the curved path, this change in velocity is brought by a force known as centripetal force and the acceleration so produced in the body is known as centripetal acceleration.

(iii) The direction of centripetal force or acceleration is always towards the centre of circular path.

7.1 Expression for Centripetal Acceleration





(a) Particle moving in circular path of radius r

(b) Vector diagram of velocities

The triangle OP_1P_2 and the velocity triangle are similar

$$\therefore \quad \frac{P_1P_2}{P_1O} = \frac{AB}{AQ}$$

$$\Rightarrow \quad \frac{\Delta s}{r} = \frac{\Delta v}{v} \quad [|\vec{v}_1| = |\vec{v}_2| = v]$$

$$\Rightarrow \quad \Delta v = \frac{v}{r}\Delta s$$

$$\Rightarrow \quad \frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t}$$

$$\Rightarrow \quad \lim_{\Delta t \to 0}\frac{\Delta v}{\Delta t} = \frac{v}{r}\lim_{\Delta t \to 0}\left(\frac{\Delta s}{\Delta t}\right)$$

$$\Rightarrow \quad a_c = \frac{v}{r}v = \frac{v^2}{r} = r\omega^2$$

This is the magnitude of centripetal acceleration of particle

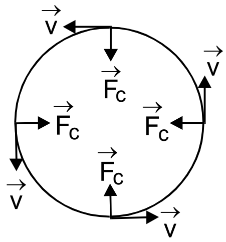
(i) It is a vector quantity. In vector form

$$\vec{a}_c = \vec{\omega} \times \vec{v}$$

(ii) The direction of $\vec{a}_c$ would be the same as that of $\Delta\vec{v}$

(iii) Because velocity vector at any point is tangential to the circular path at that point, the acceleration vector acts along radius of the circle at that point and is directed towards the centre. This is the reason that it is called centripetal acceleration.

7.2 Expression for Centripetal force



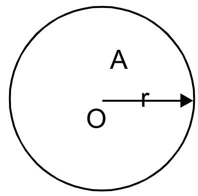
If v = velocity of particle,
r = radius of path

Then necessary centripetal force

F_c = mass × acceleration

$$F_c = m\frac{v^2}{r}$$

This is the expression for centripetal force



(i) It is a vector quantity

(ii) In vector form

$$\vec{F}_c = -\frac{mv^2}{r}.\hat{r} = -\frac{mv^2}{r^2}\vec{r}$$

$$= -m\omega^2 r\,\hat{r} = -m\omega^2\vec{r} = -m(\vec{v} \times \vec{\omega})$$

negative sign indicates direction only

$$|\vec{F}_c| = m(\vec{v} \times \vec{\omega})$$

(iii) For circular motion :

$$|\vec{F}_c| = m(v\,\omega \sin 90^\circ) = mv\omega$$

Note :

1. Centripetal force is not a real force. It is only the requirement for circular motion.
2. It is not a new kind of force. Any of the forces found in nature such as gravitational force, electric friction force, tension in string reaction force may act as centripetal force.

8. TYPE OF CIRCULAR MOTION

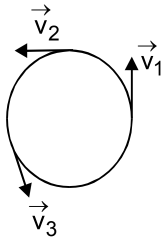
8.1 Uniform circular motion

8.2 Non Uniform Circular Motion :

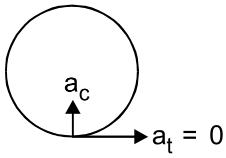
8.1 Uniform Circular Motion :

If m = mass of body,
r = radius of circular orbit,
v = magnitude of velocity

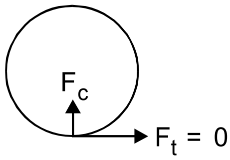
a_c = centripetal acceleration,
a_t = tangential acceleration
In uniform circular motion :



(i) $|\vec{v}_1| = |\vec{v}_2| = |\vec{v}_3|$ = constant
i.e. speed is constant



(ii) As $|\vec{v}|$ is constant
so tangential acceleration
$a_t = 0$



(iii) Tangential force $F_t = 0$

(iv) Total acceleration

$$a = \sqrt{a_c^2 + a_t^2} = a_c = \frac{v^2}{r} \text{ (towards the centre)}$$

Note:

(i) Because F_c is always perpendicular to velocity or displacement, hence the work done by this force will always be zero.

(ii) Circular motion in horizontal plane is usually uniform circular motion.

(iii) There is an important difference between the projectile motion and circular motion.

In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in circular motion the magnitude remains constant but the direction continuously changes.

Hence equations of motion are not applicable for circular motion.

Remember that equations of motion remain valid only when both the magnitude & direction of acceleration are constant.

8.1.1 Hint to solve numerical problem :

(i) Write down the required centripetal force

(ii) Draw the free body diagram of each component of system.

(iii) Resolve the forces acting on the rotating particle along radius and perpendicular to radius

(iv) Calculate net radial force acting towards centre of circular path.

(v) Make it equal to required centripetal force.

(vi) For remaining components see according to question.

Note:

When a system of particles rotates about an axis, the angular velocity of all the particles will be same, but their linear velocity will be different, because of different distances from axis of rotation i.e. $v = r\omega$.

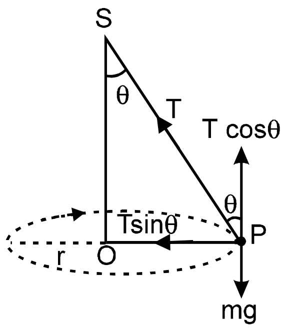
8.1.2 Motion In Horizontal Circle : Conical pendulum

This is the best example of uniform circular motion

A conical pendulum consists of a body attached to a string, such that it can revolve in a horizontal circle with uniform speed. The string traces out a cone in the space.

(i) The force acting on the bob are

(a) Tension T (b) weight mg



(ii) The horizontal component T sin θ of the tension T provides the centripetal force and the vertical component T cos θ balances the weight of bob

$$\therefore \quad T \sin\theta = \frac{mv^2}{r}$$

and $\quad T \cos\theta = mg$

From these equation

$$T = mg\sqrt{1 + \frac{v^4}{r^2g^2}} \quad \text{....(i)}$$

and $$\tan\theta = \frac{v^2}{rg} \quad \text{....(ii)}$$

Also if h = height of conical pendulum

$$\tan\theta = \frac{OP}{OS} = \frac{r}{h} \quad \text{....(iii)}$$

From (ii) & (iii),

$$\omega^2 = \frac{v^2}{r^2} = \frac{g}{h}$$

The time period of revolution

$$T = 2\pi\sqrt{\frac{h}{g}} = 2\pi\sqrt{\frac{\ell\cos\theta}{g}}$$

[where OS = ℓ]

8.2 Non-uniform Circular Motion :

(i) In non-uniform circular motion :
$|\vec{v}| \neq$ constant $\omega \neq$ constant
i.e. speed $\neq$ constant
i.e. angular velocity $\neq$ constant

(ii) If at any instant
v = magnitude of velocity of particle
r = radius of circular path
ω = angular velocity of particle,
then $v = r\omega$

(iii) Tangential acceleration :

$$a_t = \frac{dv}{dt}$$

where $v = \frac{ds}{dt}$ and s = arc - length

(iv) Tangential force :

$$F_t = ma_t$$

(v) Centripetal force :

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

(vi) Net force on the particle :

$$\vec{F} = \vec{F}_c + \vec{F}_t$$

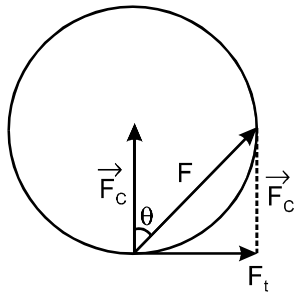
$$\Rightarrow \quad F = \sqrt{F_c^2 + F_t^2}$$

If θ is the angle made by [Note angle between F_c and F_t is 90º] F with F_c, then

$$\tan\theta = \frac{F_t}{F_c}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left[\frac{F_t}{F_c}\right]$$

Angle between F & F_t is (90º – θ)



(vii) Net acceleration towards the centre
= centripetal acceleration

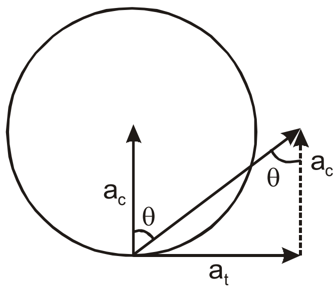
$$\Rightarrow \quad a_c = \frac{v^2}{r} = \omega^2 r = \frac{F_c}{m}$$

(viii) Net acceleration,

$$a = \sqrt{a_c^2 + a_t^2} = \frac{F_{net}}{m}$$

The angle made by 'a' with a_c,

$$\tan\theta = \frac{a_t}{a_c} = \frac{F_t}{F_c}$$



Special Note :

(i) In both uniform & non-uniform circular motion F_c is perpendicular to velocity ; so work done by centripetal force will be zero in both the cases.

(ii) In uniform circular motion $F_t = 0$, as $a_t = 0$, so work done will be zero by tangential force.

But in non-uniform circular motion $F_t \neq 0$, thus there will be work done by tangential force in this case.

Rate of work done by net force in non-uniform circular motion = Rate of work done by tangential force

$$\Rightarrow \quad P = \frac{dW}{dt} = \vec{F}_t \cdot \vec{v} = \vec{F}_t \cdot \frac{d\vec{x}}{dt}$$

Motion in Vertical Circle : Motion of a body suspended by string :

This is the best example of non-uniform circular motion.

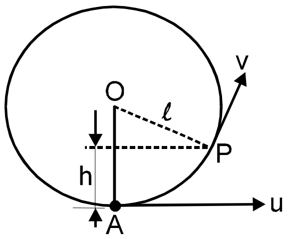
When the body rises from the bottom to the height h apart of its kinetic energy converts into potential energy

Total mechanical energy remains conserved

Total (P.E. + K.E.) at A = Total (P.E. + K.E.) at P

$$\Rightarrow \quad 0 + \frac{1}{2}mu^2 = mgh + \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{u^2 - 2gh} = \sqrt{u^2 - 2g\ell(1-\cos\theta)}$$



[Where ℓ is length of the string]

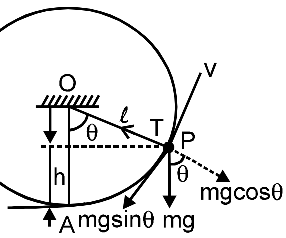
Tension at a point P :

(i) At point P **required centripetal force**

$$= \frac{mv^2}{\ell}$$

(a) Net force towards the centre :

T – mg cos θ, which provides required centripetal force.



$$\therefore \quad T - mg\cos\theta = \frac{mv^2}{\ell}$$

$$T = m\left[g\cos\theta + \frac{v^2}{\ell}\right]$$

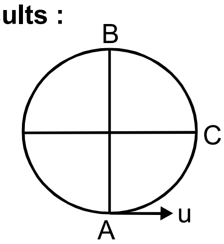
$$= \frac{m}{\ell}\left[u^2 - g\ell\,(2 - 3\cos\theta)\right]$$

(b) Tangential force for the motion

$$F_t = mg\sin\theta$$

This force retards the motion

(ii) Results :



(a) Tension at the lowest point A :

$$T_A = \frac{mv_A^2}{\ell} + mg$$

(Here θ = 0º)

$$T_A = \frac{mu^2}{\ell} + mg$$

(b) Tension at point B :

$$T_B = \frac{mv_B^2}{\ell} - mg$$

$$T_B = \frac{mu^2}{\ell} - 5mg$$

(∵ θ = 180º)

(c) Tension at point C :

$$T_C = \frac{mv_C^2}{\ell}$$

$$T_C = \frac{mu^2}{\ell} - 2mg$$

(Here θ = 90º)

Thus we conclude that

$$T_A > T_C > T_B$$

and also $T_A - T_B = 6\ mg$

$T_A - T_C = 3\ mg$

$T_C - T_B = 3\ mg$

(iii) Cases :

(a) If $u > \sqrt{5g\ell}$

In this case tension in the string will not be zero at any of the point, which implies that the particle will continue the circular motion.

(b) If $u = \sqrt{5g\ell}$

In this case the tension at the top most point (B) will be zero, which implies that the particle will just complete the circular motion.

(c) Critical Velocity : The minimum velocity at which the circular motion is possible

The critical velocity at A = $\sqrt{5g\ell}$

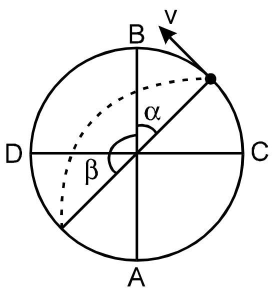
The critical velocity at B = $\sqrt{g\ell}$

The critical velocity at C = $\sqrt{3g\ell}$

Also $T_A = 6$ mg, $T_B = 0$, $T_C = 3$ mg

(d) If $\sqrt{2g\ell} < u < \sqrt{5g\ell}$

In this case particle will not follow circular motion. Tension in string becomes zero somewhere between points C & B whereas velocity remain positive. Particle leaves circular path and follow parabolic trajectory



(e) If u = $\sqrt{2g\ell}$

In this case both velocity and tension in the string becomes zero between A and C and particle will oscillate along semi-circular path.

(f) If u < $\sqrt{2g\ell}$

The velocity of particle remains zero between A and C but tension will not be zero and the particle will oscillate about the point A.

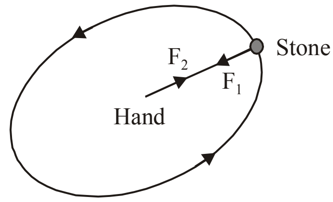
9. CENTRIFUGAL FORCE

The natural tendency of a body is to move uniformly along a straight line. When we apply centripetal force on the body, it is forced to move along a circle. While moving actually along a circle, the body has a constant tendency to regain its natural straight line path. This tendency gives rise to a force called centrifugal force. Hence

Centrifugal force is a force that arises when a body is moving actually along a circular path, by virtue of tendency of the body to regain its natural straight line path.

Centrifugal forces can be regarded as the reaction of centripetal force. As forces of action and reaction are always equal and opposite, therefore, magnitude of centrifugal force = $m v^2/r$, which is same as that of centripetal force. However, direction of centrifugal force is opposite to the direction of centripetal force i.e. **centrifugal force acts along the radius and away from the centre of the circle.**

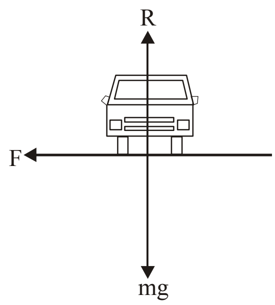
Note that centripetal and centrifugal forces, being the forces of action and reaction act always on different bodies. For example, when a piece of stone tied to one end of a string is rotated in a circle, centripetal force F_1 is applied on the stone by the hand. In turn, the hand is pulled outwards by centrifugal force F_2 acting on it, due to tendency of the stone to regain its natural straight line path. The centripetal and centrifugal forces are shown in Fig.



10. ROUNDING A LEVEL CURVED ROAD

When a vehicle goes round a curved road, it requires some centripetal force. While rounding the curve, the wheels of the vehicle have a tendency to leave the curved path and regain the straight line path. Force of friction between the wheels and the road opposes this tendency of the wheels. This force (of friction) therefore, acts, towards the centre of the circular track and provides the necessary centripetal force.

Three forces are acting on the car, fig.



(i) The weight of the car, mg, acting vertically downwards,

(ii) Normal reaction R of the road on the car, acting vertically upwards,

(iii) Frictional Force F, along the surface of the road, towards the centre of the turn, as explained already.

As there is no acceleration in the vertical direction,

$$R - mg = 0 \text{ or } R = mg \qquad ...(1)$$

The centripetal force required for circular motion is along the surface of the road, towards the centre of the turn.

As explained above, it is the static friction that provides the necessary centripetal force. Clearly,

$$\frac{mv^2}{r} \le F \qquad ...(2)$$

where v is velocity of car while turning and r is radius of circular track.

As $F = \mu_s R = \mu_s mg$, [using (1)]

where μ_s is coefficient of static friction between the tyres and the road. Therefore, from (2),

$$\frac{mv^2}{r} \le \mu_s mg \quad \text{or } v \le \sqrt{\mu_s rg} \quad \therefore \boxed{v_{max} = \sqrt{\mu_s rg}}$$

...(3)

Hence the maximum velocity with which a vehicle can go round a level curve ; without skidding is

$$\boxed{v = \sqrt{\mu_s rg}}.$$

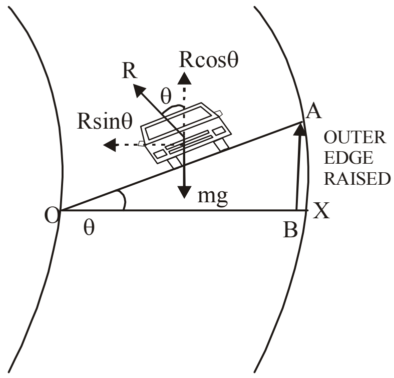
The value of depends on radius r of the curve and on coefficient of static friction (μ_s) between the tyres and the road. Clearly, v is independent of mass of the car.

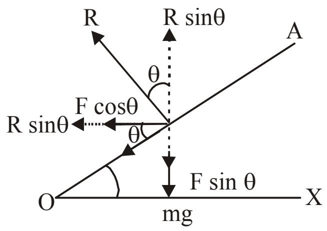
11. BANKING OF ROADS

The maximum permissible velocity with which a vehicle can go round a level curved road without skidding depends on μ, the coefficient of friction between the tyres and the road. The value of μ decreases when road is smooth or tyres of the vehicle are worn out or the road is wet and so on. Thus force of friction is not a reliable source for providing the required centripetal force to the vehicle.

A safer course of action would be to raise outer edge of the curved road above the inner edge. By doing so, a componenet of normal reaction of the road shall be spared to provide the centripetal force. ***The phenomenon of raising outer edge of the curved road above the inner edge is called banking of roads.*** We can calculate the angle of banking θ, as detailed below:

In Fig., OX is a horizontal line. OA is the level of banked curved road whose outer edge has been raised. $\angle XOA = \theta$ = angle of banking.





Three forces are acting on the vehicle as shown in Fig.

(i) Weight mg of the vehicle acting vertically downwards.

(ii) Normal reaction R of the banked road acting upwards in a direction perpendicular to OA.

(iii) Force of friction F between the banked road and the tyres, acting along AO.

R can be resolved into two rectangular components :-

(i) R cos θ, along vertically upward direction

(ii) R sin θ, along the horizontal, towards the centre of the curved road.

F can also be resolved into two rectangular components :

(i) F cos θ, along the horizontal, towards the centre of curved road

(ii) F sin θ, along vertically downward direction.

As there is no acceleration along the vertical direction, the net force along this direction must be zero. Therefore,

$$R\cos\theta = mg + F\sin\theta \qquad ...(1)$$

If v is velocity of the vehicle over the banked circular road of radius r, then centripetal force required = mv^2/r. This is provided by the horizontal components of R and F as shown in Fig.

$$\therefore \quad R\sin\theta + F\cos\theta = \frac{mv^2}{r} \qquad ...(2)$$

But $F \le \mu_s R$, where μ_s is coefficient of static friction between the banked road and the tyres. To obtain v_{max}, we put $F = \mu_s R$ in (1) and (2).

$$R\cos\theta = mg + \mu_s R\sin\theta \qquad ...(3)$$

and $$R\sin\theta + \mu_s R\cos\theta = \frac{mv^2}{r} \qquad ...(4)$$

From (3), $R(\cos\theta - \mu_s \sin\theta) = mg$

$$R = \frac{mg}{\cos\theta - \mu_s \sin\theta} \qquad ...(5)$$

From (4), $R(\sin\theta + \mu_s\cos\theta) = \frac{mv^2}{r}$

Using (5), $$\frac{mg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{mv^2}{r}$$

$$\therefore \quad v^2 = \frac{rg(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} = \frac{rg\cos\theta(\tan\theta + \mu_s)}{\cos\theta(1 - \mu_s\tan\theta)}$$

$$\boxed{v = \left[\frac{rg(\mu_s + \tan\theta)}{(1 - \mu_s\tan\theta)}\right]^{1/2}} \qquad ...(6)$$

This is the max. velocity of vehicle on a banked road.

Discussion

1. If $\mu_s = 0$, i.e., if banked road is perfectly smooth, then from eqn. (51),

$$\boxed{v_0 = (rg\tan\theta)^{1/2}} \qquad ...(7)$$

This is the speed at which a banked road can be rounded even when there is no friction. Driving at this speed on a banked road will cause almost no wear and tear of the tyres.

From (7), $v_0^2 = rg\tan\theta$ or $\boxed{\tan\theta = v_0^2/rg}$ ...(8)

2. If speed of vehicle is less than v_0, frictional force will be up the slope. Therefore, the vehicle can be parked only if $\tan\theta \le \mu_s$.

Roads are usually banked for the average speed of vehicles passing over them. However, if the speed of a vehicle is somewhat less or more than this, the self adjusting static friction will operate between the tyres and the road, and the vehicle will not skid.

The speed limit at which the curve can be negotiated safely is clearly indicated on the sign boards erected along the curved roads.

Note that curved railway tracks are also banked for the same reason. The level of outer rail is raised a little above the level of inner rail, while laying a curved railway track.

12. BENDING OF A CYCLIST

When a cyclist takes a turn, he also requires some centripetal force. If he keeps himself vertical while turning, his weight is balanced by the normal reaction of the ground. In that event, he has to depend upon force of friction between the tyres and the road for obtaining the necessary centripetal force. As force of friction is small and uncertain, dependence on it is not safe.

To avoid dependence on force of friction for obtaining centripetal force, the cyclist has to bend a little inwards from his vertical position, while turning. By doing so, a component of normal reaction in the horizontal direction provides the necessary centripetal force. To calculate the angle of bending with vertical, suppose

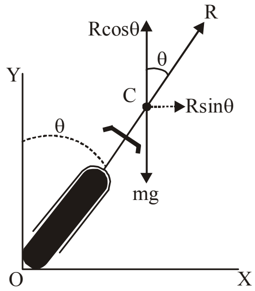
m = mass of the cyclist,

v = velocity of the cyclist while turning,

r = radius of the circular path,

θ = angle of bending with vertical.

In Fig., we have shown weight of the cyclist (mg) acting vertically downwards at the centre of gravity C. R is force of reaction of the ground on the cyclist. It acts at an angle θ with the vertical.



R can be resolved into two rectangular components:

$R\cos\theta$, along the vertical upward direction,

$R\sin\theta$, along the horizontal, towards the centre of the circular track.

In equilibrium, $R\cos\theta$ balances the weight of the cyclist i.e.

$$R\cos\theta = mg \qquad ...(1)$$

and $R\sin\theta$ provides the necessary centripetal force $(m\,v^2/r)$

$$\therefore \qquad R\sin\theta = \frac{mv^2}{r} \qquad ...(2)$$

Dividing (2) by (1), we get $\frac{R\sin\theta}{R\cos\theta} = \frac{mv^2}{rmg}$

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

Clearly, θ would depend on v and r.

For a safe turn, θ should be small, for which v should be small and r should be large i.e. turning should be at a slow speed and along a track of larger radius. This means, a safe turn should neither be fast nor sharp.

TIPS AND TRICKS

1. Centripetal force does not increase the kinetic energy of the particle moving in circular path, hence the work done by the force is zero.
2. Centrifuges are the apparatuses used to separate small and big particles from a liquid.
3. The physical quantities which remain constant for a particle moving in circular path are speed, kinetic energy and angular momentum.
4. If a body is moving on a curved road with speed greater than the speed limit, the reaction at the inner wheel disappears and it will leave the ground first.
5. On unbanked curved roads the minimum radius of curvature of the curve for safe driving is $r = v^2/\mu g$, where v is the speed of the vehicle and μ is small.
6. If r is the radius of curvature of the speed breaker, then the maximum speed with which the vehicle can run on it without leaving contact with the ground is $v = \sqrt{(gr)}$
7. While taking a turn on the level road sometimes vehicles overturn due to centrifugal force.
8. If h is the height of centre of gravity above the road, a is half the wheel base then for road safety $\frac{mv^2}{r}.h < mg.a$, ∴ Minimum safe speed for no overturning is $v = \sqrt{(gar/h)}$.
9. On a rotating platform, to avoid the skidding of an object placed at a distance r from axis of rotation, the maximum angular velocity of the platform, $\omega = \sqrt{(\mu g/r)}$, where μ is the coefficient of friction between the object and the platform.
10. If an inclined plane ends into a circular loop of radius r, then the height from which a body should slide from the inclined plane in order to complete the motion in circular track is h = 5r/2.
11. Minimum velocity that should be imparted to a pendulum to complete the vertical circle is $\sqrt{(5g\ell)}$, where ℓ is the length of the pendulum.
12. While describing a vertical circle when the stone is in its lowest position, the tension in the string is six times the weight of the stone.
13. The total energy of the stone while revolving in vertical circle is $(5/2)\, mg\ell$.
14. When the stone is in horizontal position then the tension in the string is 3mg and the velocity of the stone is $\sqrt{(3g\ell)}$.
15. If the velocity of the stone at the highest point is X mg, then the tension at the lowest point will be (X + 6)mg.
16. If a body of mass m is tied to a string of length ℓ and is projected with a horizontal velocity u such that it does not complete the motion in the vertical circle, then

(a) the height at which the velocity vanishes is

$$h = \frac{u^2}{2g}$$

(b) the height at which the tension vanishes is

$$h = \frac{u^2 + g\ell}{3g}$$

17. K.E. of a body moving in horizontal circle is same throughout the path but the K.E. of the body moving in vertical circle is different at different places.

SOLVED EXAMPLES

Ex.1 The magnitude of the linear acceleration, the particle moving in a circle of radius of 10 cm with uniform speed completing the circle in 4 s, will be -

(A) 5π cm/s^2 (B) 2.5π cm/s^2

(C) $5\pi^2$ cm/s^2 (D) $2.5\pi^2$ cm/s^2

Sol.(D) The distance covered in completing the circle is

$2\pi r = 2\pi \times 10$ cm

The linear speed is

$$v = \frac{2\pi r}{t} = \frac{2\pi \times 10}{4} = 5\pi \text{ cm/s}$$

The linear acceleration is,

$$a = \frac{v^2}{r} = \frac{(5\pi)^2}{10} = 2.5\ \pi^2 \text{ cm/s}^2$$

This acceleration is directed towards the centre of the circle

Hence correct answer is (D)

Ex.2 A cane filled with water is revolved in a vertical circle of radius 4 m and water just does not fall down. The time period of revolution will be -

(A) 1 s (B) 10 s

(C) 8 s (D) 4 s

Sol.(D) We know that

$$\text{Time period} = \frac{\text{Circumference}}{\text{Critical speed}} = \frac{2\pi r}{\sqrt{gr}}$$

$$= \frac{2 \times 22 \times 4}{7 \times \sqrt{10 \times 4}} = 4 \text{ sec}$$

Hence correct answer is (D)

Ex.3 The length of second's hand in a watch is 1 cm. The change in velocity of its tip in 15 seconds is -

(A) 0 (B) $\frac{\pi}{30\sqrt{2}}$ cm/s

(C) $\frac{\pi}{30}$ cm/s (D) $\frac{\pi\sqrt{2}}{30}$ cm/s

Sol.(B) Velocity $= \frac{\text{Circumference}}{\text{Time of revolution}} = \frac{2\pi r}{60}$

$$= \frac{2\pi \times 1}{60} = \frac{\pi}{30} \text{ cm/s}$$

Change in velocity $\Delta v = \sqrt{\left(\frac{\pi}{30}\right)^2 + \left(\frac{\pi}{30}\right)^2}$

$$= \frac{\pi}{30}\sqrt{2} \text{ cm/s}$$

Hence correct answer is (B)

Ex.4 An electron is moving in a circular orbit of radius 5.3×10^{-11} metre around the atomic nucleus at a rate of 6.6×10^{15} revolutions per second. The acceleration of the electron and centripetal force acting on it will be - (The mass of the electron is 9.1×10^{-31}kg)

(A) 8.3×10^{-8}N (B) 3.8×10^{-8}N

(C) 4.15×10^{-8}N (D) 2.07×10^{-8}N

Sol.(A) Let the radius of the orbit be r and the number of revolutions per second be n. Then the velocity of electron is given by

$v = 2\pi nr$,

$$\therefore \text{Acceleration } a = \frac{v^2}{r} = \frac{4\pi^2 r^2 n^2}{r} = 4\ \pi^2\ r\ n^2$$

Substituting the given values, we have

$a = 4 \times (3.14)^2 \times (5.3 \times 10^{-11})\ (6.6 \times 10^{15})^2$

$= 9.1 \times 10^{22}$ m/s^2 towards the nucleus.

The centripetal force is

$F_C = ma = (9.1 \times 10^{-31})\ (9.1 \times 10^{22})$

$= 8.3 \times 10^{-8}$ N towards the nucleus.

Hence correct answer is (A)

Ex.5 An air craft executes a horizontal loop of radius 1 km with a steady speed of 900 km/h. The ratio of centripetal acceleration to that gravitational acceleration will be-

(A) 1 : 6.38 (B) 6. 38 : 1

(C) 2.25 : 9.8 (D) 2.5 : 9.8

Sol.(B) Given that radius of horizontal loop

r = 1 km = 1000 m

$$\text{Speed } v = 900 \text{ km/h} = \frac{9000 \times 5}{18} = 250 \text{ m/s}$$

$$\text{Centripetal acceleration } a_c = \frac{v^2}{r} = \frac{250 \times 250}{1000} = 62.5 \text{ m/s}^2$$

$\therefore \quad \frac{\text{Centripetal acceleration}}{\text{Gravitational acceleration}} = \frac{a_c}{g}$

$= \frac{62.5}{9.8} = 6.38 : 1$

Hence correct answer is (B)

Ex.6 A car driver is negotiating a curve of radius 100 m with a speed of 18 km/hr. The angle through which he has to lean from the vertical will be -

(A) $\tan^{-1}\frac{1}{4}$ (B) $\tan^{-1}\frac{1}{40}$

(C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) $\tan^{-1}\left(\frac{1}{20}\right)$

Sol.(B) We know that, $\tan\theta = \frac{v^2}{rg} = \frac{\left(18\times\frac{5}{18}\right)^2}{100\times10}$

$= \frac{1}{40} \Rightarrow \theta = \tan^{-1}\frac{1}{40}$

Hence correct answer is (B)

Ex.7 Write an expression for the position vector **r** for a particle describing uniform circular motion, using rectangular co-ordinates and the unit vectors **i** and **j**. The vector expressions for the velocity **v** and acceleration a will be-

(A) ωr^2 (B) $-\omega^2 r/2$

(C) $-2\omega r^2$ (D) $-\omega^2 r$

Sol.(D) $r = \hat{i}\ x + \hat{j}\ y,\ x = r\cos\theta,$

$y = r\sin\theta$ where $\theta = \omega t$

$r = \hat{i}\ (r\cos\omega t) + \hat{j}\ (r\sin\omega t)$

$v = dr/dt = -\hat{i}\ (\omega r\sin\omega t) - \hat{j}\ (\omega r\cos\omega t)$

$a = d^2 r/dt^2 = -\omega^2 r$

Hence correct answer is (D)

Ex.8 The vertical section of a road over a canal bridge in the direction of its length is in the form of circle of radius 8.9 metre. Find the greatest speed at which the car can cross this bridge without losing contact with the road at its highest point, the center of gravity of the car being at a height h = 1.1 metre from the ground. (Take $g = 10$ m/sec^2)

(A) 5 m/s (B) 7 m/s

(C) 10 m/s (D) 13 m/s

Sol.(C) Let R be the normal reaction exerted by the road on the car. At the highest point, we have

$\frac{mv^2}{(r+h)} = mg - R$, R should not be negative.

Therefore $v^2 \le (r + a)g = (8.9 + 1.1) \times 10$

or $v^2 \le 10 \times 10 v \le 10$ m/sec

$\therefore \quad v_{max} = 10$ m/sec

Hence correct answer is (C)

Ex.9 The maximum speed at which a car can turn round a curve of 30 metre radius on a level road if the coefficient of friction between the tyres and the road is 0.4, will be -

(A) 10.84 m/s (B) 17.84 m/s

(C) 11.76 m/s (D) 9.02 m/s

Sol.(A) Let W = Mg be the weight of the car. Friction force = 0.4 W

Centripetal force $= \frac{Mv^2}{r} = \frac{Wv^2}{gr}$

$0.4\ W = \frac{Wv^2}{gr}$

$\Rightarrow v^2 = 0.4 \times g \times r = 0.4 \times 9.8 \times 30 = 117.6$

$\Rightarrow v = 10.84$ m/sec

Hence correct answer is (A)

Ex.10 The angular speed with which the earth would have to rotate on it axis so that a person on the equator would weight $(3/5)^{th}$ as much as present will be: (Take the equatorial radius as 6400 km)

(A) 8.7×10^4 rad/sec (B) 8.7×10^3 rad/sec

(C) 7.8×10^4 rad/sec (D) 7.8×10^3 rad/sec

Sol.(C) Let v be the speed of earth's rotation.

We know that W = mg

Hence $\frac{3}{5}W = mg - \frac{mv^2}{r}$

or $\frac{3}{5}mg = mg - \frac{mv^2}{r}$

$\therefore \frac{2}{3}\ mg = \frac{mv^2}{r}$ or $v^2 = \frac{2gr}{5}$

Now $v^2 = \frac{2\times9.8\times(6400\times10^3)}{5}$

Solving, we get $v = 5 \times 10^9$ m/sec,

$\omega = \sqrt{\left(\frac{2g}{5r}\right)} = 7.8 \times 10^4$ radian/sec.

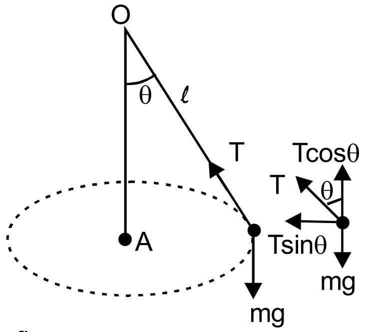
Hence correct answer is (C)

Ex.11 A man whirls a stone round his head on the end of a string 4.0 metre long. Can the string be in a horizontal, plane? If the stone has a mass of 0.4 kg and the string will break, if the tension in it exceeds 8 N. The smallest angle the string can make with the horizontal and the speed of the stone will respectively be (Take g = 10 m/sec^2)

(A) 30°, 7.7 m/s (B) 60°, 7.7 m/s

(C) 45°, 8.2 m/s (D) 60°, 8.7 m/s

Sol.(A)



Form figure

$T \cos \theta = mg$ ...... (1)

$T \sin \theta = \frac{mv^2}{r} = \frac{mv^2}{\ell \sin\theta}$ ...... (2)

Form eq. (1) $T = \frac{mg}{\cos\theta}$

When the string is horizontal, θ must be 90° i.e.,cos 90° = 0

$\therefore \quad T = \frac{mg}{0} = \infty$

Thus the tension must be infinite which is impossible, so the string can not be in horizontal plane.

The maximum angle θ is given by the breaking tension of the string in the equation T cos θ = m.g

Here T (Maximum) = 8 N and m = 0.4 kg

$\therefore \quad 8 \cos \theta = 0.4 \times g = 0.4 \times 10 = 4$

$\cos \theta = (4/8) = \frac{1}{2}, \theta = 60°$

The angle with horizontal = 90° – 60° = 30°

From equation (2), $8 \sin 60° = \frac{0.4 \times v^2}{4 \sin 60°}$

$v^2 = \frac{32 \sin^2 60°}{0.4} = 80 \sin^2 60°$

$\Rightarrow \quad v = \sqrt{80} \sin 60° = 7.7$ m/sec

Hence correct answer is (A)

Ex.12 A smooth table is placed horizontally and a spring of unstreched length ℓ_0 and force constant k has one end fixed to its centre. To the other end of the spring is attached a mass m which is making n revolutions per second around the centre. Tension in the spring will be

(A) $4\pi^2 m \; k \; \ell_0 n^2/ (k - 4\pi^2 m n^2)$

(B) $4\pi^2 m \; k \; \ell_0 n^2/ (k + 4\pi^2 m n^2)$

(C) $2\pi^2 m \; k \; \ell_0 n^2/ (k - 4\pi^2 m n^2)$

(D) $2\pi m \; k \; \ell_0 n^2/ (k - 4\pi^2 m n^2)$

Sol.(A) Let T be the tension produced in the stretched string. The centripetal force required for the mass m to move in a circle is provided by the tension T. The stretched length of the spring is r (radius of the circle). Now,

Elongation produced in the spring = $(r - \ell_0)$

Tension produced in the spring,

$T = k (r - \ell_0)$ .... (1)

Where k is the force constant

Linear velocity of the motion $v = 2\pi r n$

$\therefore$ Centripetal force $= \frac{mv^2}{r} = \frac{m(2\pi rn)^2}{r}$

$= 4\pi^2 r n^2 m$ .... (2)

Equating equation. (1) and (2), we get

$k (r - \ell_0) = 4\pi^2 r n^2 m$

$(\therefore T = mv^2/r)$

$\Rightarrow kr - k \ell_0 = 4 \pi^2 r n^2 m$

$r (k - 4\pi^2 n^2 m) = k \ell_0$

$\Rightarrow \quad r = \frac{k\ell_0}{(k - 4\pi^2 n^2 m)}$ ....(3)

Substituting the value of r in eqn. (1) we have

$$T = k \left[\frac{k\ell_0}{(k - 4\pi^2 n^2 m)} - \ell_0 \right]$$

or $T = \frac{4\pi^2 n^2 m \ell_0 k}{(k - 4\pi^2 n^2 m)}$ .... (4)

Hence correct answer is (A)

Ex.13 A motor car is travelling at 30 m/s on a circular road of radius 500 m. It is increasing its speed at the rate of 2 m/s^2. Its net acceleration is (in m/s^2) –

(A) 2 (B) 1. 8

(C) 2.7 (D) 0

Sol.(C) Two types of acceleration are experienced by the car

(i) Radial acceleration due to circular path,

$$a_r = \frac{v^2}{r} = \frac{(30)^2}{500} = 1.8 \text{ m/s}^2$$

(ii) A tangential acceleration due to increase of tangential speed given by

$$a_t = \Delta v/\Delta t = 2 \text{ m/s}^2$$

Radial and tangential acceleration are perpendicular to each other.
Net acceleration of car a

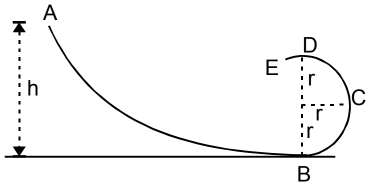
$$= \sqrt{a_r^2 + a_t^2} = \sqrt{(1.8)^2 + (2)^2} = 2.7 \text{ m/s}^2$$

Hence correct answer is (C)

Ex.14 In figure ABCDE is a channel in the vertical plane, part BCDE being circular with radius r. A ball is released from A and slides without friction and without rolling. It will complete the loop path -

(A) if h is greater than 5r/2
(B) if h is less than 5r/2
(C) if h is greater than 2r/5
(D) if h is less than 2r/5

Sol.(A)



Let m be the mass of the ball. When the ball comes down to B, its potential energy mgh which is converted into kinetic energy. Let v_B, be the velocity of the ball at B. Then, mgh = $\frac{1}{2}$ m v_B^2

The ball now rises to a point D, where its potential energy is mg(h – 2r). If v_D be the velocity of the ball at D, then,

$$m\,g\,(h - 2\,r) = \frac{1}{2}\,m\,v_D^2 \quad ......(2)$$

Now to complete the circular path, it is necessary that the centrifugal force acting upward at point D, should be equal or greater than the force mg acting downward at point D should be equal or greater than the force mg acting downward. Therefore

$$\frac{mv_D^2}{r} \geq mg \quad \text{or} \quad v_D^2 \geq r\,g$$

From equation (2) v_D^2 = 2g (h – 2r),

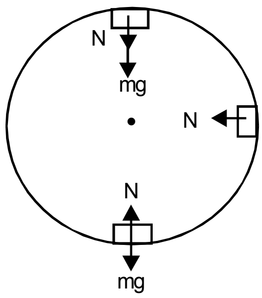
$$\therefore\ 2g\,(h - 2r) \geq r\,g \Rightarrow h \geq \frac{5}{2}r$$

Hence correct answer is (A)

Ex.15 An aircraft loops the loop of radius R = 500 m with a constant velocity v = 360 km/hour. The weight of the flyer of mass m = 70 kg in the lower, upper and middle points of the loop will respectively be-

(A) 210 N, 700 N, 1400 N
(B) 1400 N, 700 N, 2100 N
(C) 700 N, 1400 N, 210 N,
(D) 2100 N, 700 N, 1400 N

Sol.(D) See fig, Here v = 360 km/hr = 100 m/sec



At lower point, $N - mg = \frac{mv^2}{R}$,

$$N = \text{weight of the flyer} = mg + \frac{mv^2}{R}$$

$$N = 70 \times 10 + \frac{70 \times (10000)}{500} = 2100 \text{ N}$$

At upper point, $N + mg = \frac{mv^2}{R}$,

$$N = \frac{mv^2}{R} - mg = 1400 - 700 = 700 \text{ N}$$

At middle point, $N = \frac{mv^2}{R} = 1400$ N

Hence correct answer is (D)

Ex.16 A particle of mass 3 kg is moving under the action of a central force whose potential energy is given by $U(r) = 10\,r^3$ joule. For what energy and angular momentum will the orbit be a circle of radius 10 m-

(A) 2.5×10^4 J, 3000 kgm^2/sec
(B) 3.5×10^4 J, 2000 kgm^2/sec
(C) 2.5×10^3 J, 300 kgm^2/sec
(D) 3.5×10^3 J, 300 kgm^2/sec

Sol.(A) Given that $U(r) = 10r^3$

So the force F acting on the particle is given by,

$$F = -\frac{\partial U}{\partial r} = -\frac{\partial}{\partial r}(10\ r^3)$$

$= -10 \times 3\ r^2 = -30\ r^2$

For circular motion of the particle,

$$F = \frac{mv^2}{r} = 30\ r^2$$

Substituting the given values, we have, $\frac{3 \times v^2}{10}$

$= 30 \times (10)^2$ or $v = 100$ m/s

The total energy in circular motion

$$E = K.E. + P.E. = \frac{1}{2}mv^2 + U(r)$$

$$= \frac{1}{2} \times 3 \times (100)^2 + 10 + (10)^3$$

$= 2.5 \times 10^4$ joule

Angular momentum

$= mvr = 3 \times 100 \times 10 = 3000$ kg–m^2/sec

Also time period $T = \frac{2\pi r}{v} = \frac{2 \times \pi \times 10}{100} = \frac{\pi}{5}$ sec

Hence correct answer is (A)

Ex.17 A particle completes 1.5 revolutions in a circular path of radius 2 cm. The angular displacement of the particle will be - (in radian)

(A) 6π (B) 3π
(C) 2π (D) π

Sol.(D) We have angular displacement

$$= \frac{\text{linear displacement}}{\text{radius of path}}$$

$\Rightarrow$ $\Delta\theta = \frac{\Delta S}{r}$

Here, $\Delta S = n(2\pi r)$

$= 1.5\ (2\pi \times 2 \times 10^{-2})$

$= 6\pi \times 10^{-2}$

$\therefore$ $\Delta\theta = \frac{6\pi \times 10^{-2}}{2 \times 10^{-2}} = 3\pi$ radian

Hence correct answer is (B)

Ex.18 A particle revolving in a circular path completes first one third of circumference in 2 sec, while next one third in 1 sec. The average angular velocity of particle will be : (in rad/sec)

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{3}$

(C) $\frac{4\pi}{3}$ (D) $\frac{5\pi}{3}$

Sol.(A) We have $\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time}}$

For first one third part of circle, angular displacement,

$$\theta_1 = \frac{S_1}{r} = \frac{2\pi r/3}{r}$$

For second one third part of circle,

$$\theta_2 = \frac{2\pi r/3}{r} = \frac{2\pi}{3} \text{ rad}$$

Total angular displacement,

$$\theta = \theta_1 + \theta_2 = 4\pi/3 \text{ rad}$$

Total time $= 2 + 1 = 3$ sec

$$\therefore \quad \vec{\omega}_{av} = \frac{4\pi/3}{3} \text{ rad/s}$$

$$= \frac{4\pi}{6} = \frac{2\pi}{3} \text{ rad/s}$$

Hence correct answer is (A)

Ex.19 The ratio of angular speeds of minute hand and hour hand of a watch is -

(A) 1 : 12 (B) 6 : 1
(C) 12 : 1 (D) 1 : 6

Sol.(C) Angular speed of hour hand,

$$\omega_1 = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{12 \times 60} \text{ rad/sec}$$

angular speed of minute hand,

$$\omega_2 = \frac{2\pi}{60} \text{ rad/sec} \Rightarrow \frac{\omega_2}{\omega_1} = \frac{12}{1}$$

Hence correct answer is (C).

Ex.20 The angular displacement of a particle is given by $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$, where ω_0 and α are constant and $\omega_0 = 1$ rad/sec, $\alpha = 1.5$ rad/sec^2. The angular velocity at time, t = 2 sec will be (in rad/sec) -

(A) 1 (B) 5 (C) 3 (D) 4

Sol.(D) We have $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Rightarrow \frac{d\theta}{dt} = \omega_0 + \alpha t$

This is angular velocity at time t. Now angular velocity at t = 2 sec will be

$\omega = \left(\frac{d\theta}{dt}\right)_{t=2sec} = \omega_0 + 2\alpha$

$= 1 + 2 \times 1.5 = 4$ rad/sec

Hence correct answer is (D)

Ex.21 A particle moves in a circle of radius 20cm with a linear speed of 10m/s. The angular velocity will be -

(A) 50 rad/s (B) 100 rad/s
(C) 25 rad/s (D) 75 rad/s

Sol. The angular velocity is

$\omega = \frac{v}{r}$

Hence $v = 10$ m/s

$r = 20$ cm $= 0.2$ m,

$\therefore \omega = 50$ rad/s

Hence correct answer is (A)

Ex.22 The angular velocity of a particle is given by $\omega = 1.5\,t - 3t^2 + 2$, the time when its angular acceleration decreases to be zero will be -

(A) 25 sec (B) 0.25 sec
(C) 12 sec (D) 1.2 sec

Sol.(B) Given that $\omega = 1.5t - 3t^2 + 2$

$\alpha = \frac{d\omega}{dt} = 1.5 - 6t$

When $\alpha = 0$

$\Rightarrow 1.5 - 6t = 0$

$\Rightarrow t = \frac{1.5}{6} = 0.25$ sec

Hence correct answer is (B)

Ex.23 A particle is moving in a circular path with velocity varying with time as $v = 1.5t^2 + 2t$. If 2 cm the radius of circular path, the angular acceleration at t = 2 sec will be -

(A) 4 rad/sec^2 (B) 40 rad/sec^2
(C) 400 rad/sec^2 (D) 0.4 rad/sec^2

Sol.(C) Given $v = 1.5\,t^2 + 2t$

Linear acceleration a

$= \frac{dv}{dt} = 3t + 2$

This is the linear acceleration at time t

Now angular acceleration at time t

$\alpha = \frac{a}{r} \Rightarrow \alpha = \frac{3t + 2}{2 \times 10^{-2}}$

Angular acceleration at

t = 2 sec

$(\alpha)_{at\ t\ =\ 2sec} = \frac{3 \times 2 + 2}{2 \times 10^{-2}} = \frac{8}{2} \times 10^2$

$= 4 \times 10^2 = 400$ rad/sec^2

Hence correct answer is (C)

Ex.24 A grind stone starts from rest and has a constant-angular acceleration of 4.0 rad/sec^2.The angular displacement and angular velocity, after 4 sec. will respectively be -

(A) 32 rad, 16 rad/sec (B) 16rad, 32 rad/s
(C) 64rad, 32 rad/sec (D) 32 rad, 64rad/sec

Sol. Angular displacement after 4 sec is

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$= \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 4 \times 4^2$

$= 32$ rad

Angular velocity after 4 sec

$\omega = \omega_0 + \alpha t$

$= 0 + 4 \times 4 = 16$ rad/sec

Hence correct answer is (A)

Ex.25 The shaft of an electric motor starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$ during the first 2 seconds after it starts after which $\alpha = 0$. The angular velocity after 6 sec will be -

(A) 10/3 rad/sec (B) 3/10 rad/sec
(C) 30/4 rad/sec (D) 4/30 rad/sec

Sol.(A) Given $\alpha = 3t - t^2$

$\Rightarrow \frac{d\omega}{dt} = 3t - t^2$

$\Rightarrow d\omega = (3t - t^2)dt$

$\Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} + c$

at $t = 0,\ \omega = 0$

$\therefore c = 0,\ \omega = \frac{3t^2}{2} - \frac{t^3}{3}$

Angular velocity at

$t = 2$ sec, $(\omega)_{t\ =\ 2\ sec}$

$= \frac{3}{2}(4) - \frac{8}{3} = \frac{10}{3}$ rad/sec

Since there is no angular acceleration after 2 sec

$\therefore$ The angular velocity after 6 sec remains the same.

Hence correct answer is (A)

Ex.26 A ball is fixed to the end of a string and is rotated in a horizontal circle of radius 5 m with a speed of 10 m/sec. The acceleration of the ball will be -

(A) 20 m/s^2 (B) 10 m/s^2

(C) 30 m/s^2 (D) 40 m/s^2

Sol.(A) We know $a = \frac{v^2}{r}$

Hence $v = 10$ m/s, $r = 5$ m

$\therefore$ $a = \frac{(10)^2}{5} = 20\ m/s^2$

Hence correct answer is (A)

Ex.27 A body of mass 2 kg lying on a smooth surface is attached to a string 3 m long and then whirled round in a horizontal circle making 60 revolution per minute. The centripetal acceleration will be -

(A) 118.4 m/s^2 (B)1.18 m/s^2

(C) 2.368 m/s^2 (D) 23.68 m/s^2

Sol.(A) Given that the mass of the particle,

$m = 2$ kg

radius of circle $= 3$ m

Angular velocity = 60 rev/minute

$= \frac{60 \times 2\pi}{60}$ rad/sec

$= 2\pi$ rad/sec

Because the angle described during 1 revolution is 2π radian

The linear velocity

$v = r\omega$

$= 2\pi \times 3$ m/s $= 6\pi$ m/s

The centripetal acceleration

$= \frac{v^2}{r} = \frac{(6\pi)^2}{3}\ m/s^2 = 118.4\ m/s^2$

Hence correct answer is (A)

Ex.28 A body of mass 0.1 kg is moving on circular path of diameter 1.0 m at the rate of 10 revolutions per 31.4 seconds. The centripetal force acting on the body is -

(A) 0.2 N (B) 0.4 N

(C) 2 N (D) 4 N

Sol.(A) $F = \frac{mv^2}{r} = mr\omega^2$

Here $m = 0.10$ kg,

$r = 0.5$ m

and $\omega = \frac{2\pi n}{t} = \frac{2 \times 3.14 \times 10}{31.4}$

$= 2$ rad/s

$F = 0.10 \times 0.5 \times (2)^2 = 0.2$

Hence correct answer is (A)

Ex.29 A body of mass 4 kg is moving in a horizontal circle of radius 1 m with an angular velocity of 2 rad/s. The required centripetal force, will be -

(A) 16 N (B) 1.6 N

(C) 16 Dyne (D) 1.6 Dyne

Sol.(A) $F = mr\omega^2 = 4 \times 1 \times 2^2 = 16$ N

Hence correct answer is (A)

Ex.30 The safe velocity required for scooterist negotiating a curve of radius 200 m on a road with the angle of repose of $\tan^{-1}(0.2)$ will be-

(A) 20 km/hr (B) 200 m/s

(C) 72 km/hr (D) 72 m/s

Sol.(C) As the centripetal force is supplied by the frictional force, hence

$\mu mg = \frac{mv^2}{r} \Rightarrow 0.2 = \frac{v^2}{200 \times 10}$

$[\theta = \tan^{-1}(0.2) = \tan^{-1}(\mu) \Rightarrow \mu = (0.2)]$

$\Rightarrow v = 20$ m/s

The safe speed is $20 \times \frac{18}{5} = 72$ km/hr

Hence correct answer is (C)

Ex.31 A body of mass 4 kg is tied to one end of a rope of length 40 cm and whirled in a horizontal circle. The maximum number of revolutions per minute it can be whirled so that the rope does not snap as the rope can with stand to a tension of 6.4 Newton, will be -

(A) 1.91 (B) 19.1 (C) 191 (D) 1910

Sol.(B) Tension in the rope $= mr\omega^2 = mr\,4\pi^2n^2$

Maximum tension = 6.4 N

$\therefore$ $6.4 = 4 \times 0.4 \times 4 \times \pi^2n^2$

$\therefore$ Number of revolutions per minutes

$= 60/\pi = 19.1$

Hence correct answer is (B)

Ex.32 A certain string which is 1 m long will break, if the load on it is more than 0.5 kg. A mass of 0.05 kg is attached to one end of it and the particle is whirled round a horizontal circle by holding the free end of the string by one hand. The greatest number of revolutions per minute possible without breaking the string will be-

(A) 9.45 (B) 94.5 (C) 99.5 (D) 9.95

Sol.(B) Mass of the body m = 0.05 kg ,
Radius of circular path = 1 m
The maximum tension in the string can withstand = 0.5 kg wt = 0.5 × 9.8 N = 4.9 N
Hence the centripetal force required to produce the maximum tension in the string is 4.9 N

i.e. $mr\omega^2 = 4.9 \Rightarrow \omega^2 = \frac{4.9}{mr} = \frac{4.9}{0.05 \times 1} = 98$

$\Rightarrow \omega = \sqrt{98} \quad \Rightarrow 2\pi n = \sqrt{98} \Rightarrow n = \frac{\sqrt{98}}{2\pi}$

= 1.1576 rev/sec = 94.5 rev/min
Hence correct answer is (B)

Ex.33 A body of mass m is attached with a string of length ℓ. If it is whirled in a horizontal circular path with velocity v. The tension in the string will be -

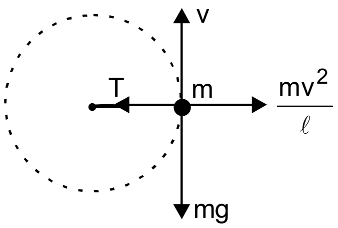
(A) $mv^2\ell$ (B) $\frac{mv^2}{\ell}$

(C) $\frac{m\ell}{v^2}$ (D) $\frac{mv^2}{2\ell}$

Sol.(B) Required centripetal force ,

$$F_c = \frac{mv^2}{\ell}$$

Here centripetal force is provided by the tension in the string



$$T = F_c = \frac{mv^2}{\ell}$$

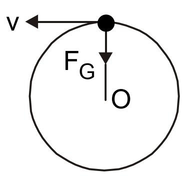
Hence correct answer is (B)

Ex.34 A satellite of mass m is revolving around the earth of mass M in circular orbit of radius r. The orbital velocity of the satellite will be -

(A) $\sqrt{\frac{GM}{r}}$ (B) $\sqrt{\frac{Gm}{r}}$ (C) $\sqrt{\frac{GM}{mr}}$ (D) $\sqrt{\frac{Gm}{Mr}}$

Sol. The required centripetal force,

$F_C = \frac{mv^2}{r}$ (towards the centre)



Net force towards the centre,

$$F_G = \frac{GMm}{r^2}$$

(This force will provide required centripetal force)

Therefore $F_C = F_G$

$$\Rightarrow \frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$\Rightarrow v = \sqrt{\frac{GM}{r}}$$

Hence correct answer is (A)

Note :

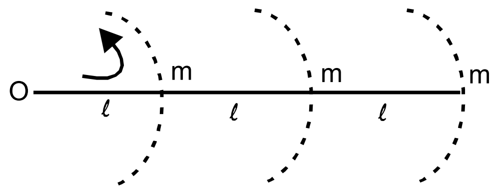
(i) From above example we see that orbital velocity of a body is independent to its mass

(ii) If we are asked to find out time period of above body then time period can be calculated as

$$T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}$$

$T^2 \propto r^3$ this is **Kepler's law.**

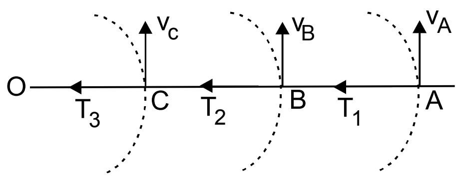
Ex.35 Three identical particles are connected by three strings as shown in fig. These particles are revolving in a horizontal plane. The velocity of outer most particle is v. Then $T_1 : T_2 : T_3$ will be - (Where T_1 is tension in the outer most string etc.)



(A) 3 : 5 : 7 (B) 3 : 5 : 6

(C) 3 : 4 : 5 (D) 7 : 5 : 3

Sol.(B) For A :



Required centripetal force

$$= \frac{mv_A^2}{3l}$$

(net force towards centre = T_1)

This will provide required centripetal force

particle at A,∴ $T_1 = \frac{mv_A^2}{3l}$

For B :

Required centripetal force

$$= \frac{m(v_B^2)}{2\ell}$$

Remember ω i.e. angular velocity, of all the particles is same

$$\therefore \qquad \omega = \frac{v_A}{3\ell} = \frac{v_B}{2\ell} = \frac{v_C}{\ell}$$

Thus for B, centripetal force

$$= \frac{2mv_A^2}{9\ell}$$

Net force towards the centre

$$T_2 - T_1 = \frac{2mv_A^2}{9\ell}$$

$$\Rightarrow \qquad T_2 = \frac{2mv_A^2}{9\ell} + T_1 = \frac{5mv_A^2}{9l}$$

(Putting value of T_1)

For C :

Centripetal force. $\frac{mv_C^2}{3l} = \frac{mv_A^2}{9l}$

Net force towards centre $= T_3 - T_2$

$$\therefore \qquad T_3 - T_2 = \frac{mv_A^2}{9l}$$

$$\Rightarrow \qquad T_3 = \frac{mv_A^2}{9l} + T_2$$

$$T_3 = \frac{6mv_A^2}{9l}$$

(on putting value of T_2)

$$\text{Now } T_1 : T_2 : T_3 = \frac{1}{3} : \frac{5}{9} : \frac{6}{9} = 3 : 5 : 6$$

Note: It is to be pondered from the above example that as the velocity is increased continuously, the innermost string will break first i.e. $T_3 > T_2 > T_1$

Hence correct answer is (B)

Ex.36 A particle describes a horizontal circle on the smooth surface of an inverted cone. The height of the plane of the circle above the vertex is 9.8 cm. The speed of the particle will be -

(A) 9.8 m/s (B) 0.98 m/s
(C) 0.098 m/s (D) 98 m/s

Sol.(B) The force acting on particle are

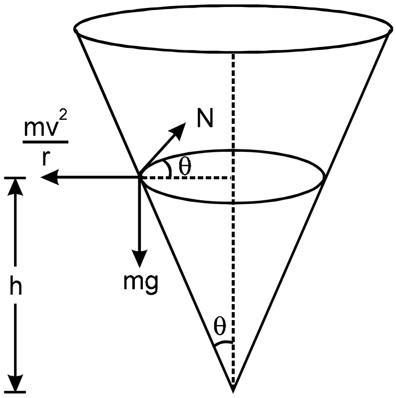
(i) weight mg acting vertically downward

(ii) Normal reaction N of the smooth surface of the cone.

(iii) Reaction of the centripetal force $\frac{mv^2}{r}$ acting radially outwards.

Resolving N into horizontal and vertical components we obtain

$$N\cos\theta = \frac{mv^2}{r} \text{ and } N\sin\theta = mg$$



$$\Rightarrow \qquad \frac{N\sin\theta}{N\cos\theta} = \frac{mg}{mv^2/r}$$

$$\Rightarrow \qquad \tan\theta = \frac{rg}{v^2}$$

$$\text{But} \qquad \tan\theta = \frac{r}{h}$$

$$\therefore \qquad \frac{r}{h} = \frac{rg}{v^2}$$

$$\Rightarrow \; v = \sqrt{hg} = \sqrt{9.8\times9.8\times10^{-2}}$$

$$= 0.98 \text{ m/s}$$

Hence correct answer is (B)

Ex.37 A string of length 1 m is fixed at one end and carries a mass of 100 gm at the other end. The string makes $2/\pi$ revolutions per second about a vertical axis through the fixed end. The angle of inclination of the string with the vertical, and the linear velocity of the mass will respectively be - (in M.K.S. system)

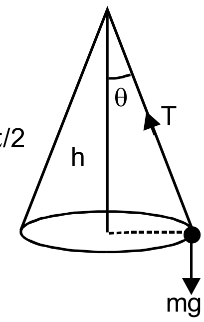
(A) 52°14', 3.16 (B) 50°14', 1.6
(C) 52°14', 1.6 (D) 50°14', 3.16

Sol.(A) Let T be the tension, θ the angle made by the string with the vertical through the point of suspension.

The time period

$$t = 2\pi\sqrt{\frac{h}{g}} = \frac{1}{\text{frequency}} = \pi/2$$

$$\text{Therefore } \omega = \sqrt{\frac{g}{h}} = 4$$



$\Rightarrow \frac{h}{g} = \frac{1}{16}$

$\cos\theta = \frac{h}{\ell} = \frac{g}{16} = 0.6125 \Rightarrow \theta = 52^o\ 14'$

Linear velocity

$= (\ell \sin\theta)\omega = 1 \times \sin 52^o\ 14' \times 4 = 3.16$ m/s

Hence correct answer is (A)

Ex.38 A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it will be -

(A) mk^2t^2r (B) $mk^2r^2t^2$
(C) $m^2k^2t^2r^2$ (D) mk^2r^2t

Sol.(D) Centripetal acceleration,

$$a_c = \frac{v^2}{r} = k^2\ rt^2$$

$\therefore$ Variable velocity

$$v = \sqrt{k^2r^2t^2} = k\ r\ t$$

The force causing the velocity to varies

$$F = m\frac{dv}{dt} = m\ k\ r$$

The power delivered by the force is,

$$P = Fv = mkr \times krt = mk^2r^2t$$

Hence correct answer is (D)

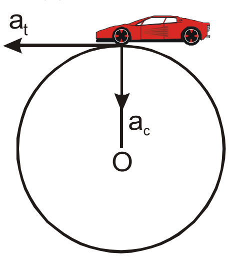
Ex.39 A car is moving in a circular path of radius 100 m with velocity of 200 m/sec such that in each sec its velocity increases by 100 m/s, the net acceleration of car will be -
(in m/sec)

(A) $100\sqrt{17}$ (B) $10\ \sqrt{7}$
(C) $10\sqrt{3}$ (D) $100\ \sqrt{3}$

Sol.(A) We know centripetal acceleration

$$a_c = \frac{(\text{tangential velocity})^2}{\text{radius}}$$

$$= \frac{(200)^2}{100} = 400\ \text{m/sec}^2$$



Tangential acceleration

$a_t = 100$ m/sec^2 (given)

$$\therefore\ a_{net} = \sqrt{a_c^2 + a_t^2 + 2a_ca_t\cos 90^o}$$

$$= \sqrt{a_c{}^2 + a_t{}^2}$$

$$= \sqrt{(400)^2 + (100)^2}$$

$$= 100\sqrt{17}\ \text{m/s}^2$$

[Remember the angle between a_t i.e. the tangential acceleration and a_c i.e. the radial acceleration, is always 90º]

Hence correct answer is (A)

Ex.40 The kinetic energy of a particle moving along a circle of radius R depends on distance covered (s) as T = as^2, where a is constant. The force acting on the particle as a function of s will be -

(A) $2as\left[1+\frac{s^2}{R^2}\right]^{1/2}$ (B) $\frac{2as}{R}$

(C) $2as\sqrt{s^2+R^2}$ (D) $\sqrt{\frac{2as}{R}}$

Sol.(A) The kinetic energy

$$T = as^2$$

$$\Rightarrow \frac{1}{2}mv^2 = as^2$$

$$\therefore \frac{mv^2}{R} = \frac{2as^2}{R}$$

$\therefore$ Centripetal force or Radial force,

$$F_c = \frac{2as^2}{R} \quad .... (1)$$

$\therefore$ Further $mv^2 = 2as^2$

$$\Rightarrow v = \sqrt{\frac{2a}{m}}\ s \quad .... (2)$$

$$\Rightarrow \frac{dv}{dt} = \sqrt{\frac{2a}{m}}\frac{ds}{dt}$$

$$= \sqrt{\frac{2a}{m}}\ v \quad .... (3)$$

Using (2) and (3) gives tangential acceleration,

$$a_t = \frac{dv}{dt} = \sqrt{\frac{2a}{m}}.v$$

$$= \left(\sqrt{\frac{2a}{m}}\right)^2 s = \frac{2a}{m}s$$

$$\Rightarrow m\ a_t = 2as$$

$\therefore$ Tangential force,

$$F_t = ma_t = 2as$$

As centripetal and tangential force are mutually perpendicular, therefore

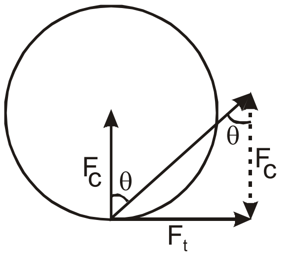
Total Force, $F = \sqrt{F_c^2 + F_t^2}$

$$= \sqrt{\left(\frac{2as^2}{R}\right)^2 + (2as)^2} = 2as\sqrt{\frac{s^2}{R^2}+1}$$

Hence correct answer is (A)

Note:

In the above example the angle made by F from the centripetal acceleration will be θ



$$\tan\theta = \frac{F_t}{F_c} = \frac{2as}{2as^2/R} = \frac{R}{s}$$

Ex.41 A particle of mass m tied with a string of length ℓ is released from horizontal as shown in fig. The velocity at the lowest portion will be -

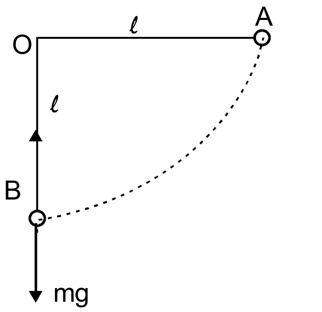
(A) $\sqrt{g\ell}$ (B) $\sqrt{2g\ell}$

(C) $\frac{1}{2}\sqrt{g\ell}$ (D) $\frac{1}{\sqrt{2}}g\ell$

Sol.(B) Suppose v be the velocity of particle at the lowest position B.

According to conservation of energy

(K.E. + P.E.) at A = (K.E. + P.E.) at B



$\Rightarrow \quad 0 + mg\ell = \frac{1}{2}mv^2 + 0$

$\Rightarrow \quad v = \sqrt{2g\ell}$

Hence correct answer is (B)

Ex.42 A 4 kg balls is swing in a vertical circle at the end of a cord 1 m long. The maximum speed at which it can swing if the cord can sustain maximum tension of 163.6 N will be -

(A) 6 m/s (B) 36 m/s

(C) 8 m/s (D) 64 m/s

Sol.(A) Maximum tension $T = \frac{mv^2}{r} + mg$

$\therefore \quad \frac{mv^2}{r} = T - mg$

or $\quad \frac{4v^2}{1} = 163.6 - 4 \times 9.8$

$\Rightarrow \quad v = 6$ m/s

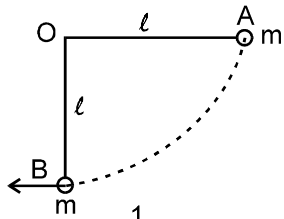
Hence correct answer is (A)

Ex.43 The string of a pendulum is horizontal. The mass of the bob is m. Now the string is released. The tension in the string in the lowest position is -

(1) 1 mg (2) 2 mg

(3) 3 mg (4) 4 mg

Sol.(C) The situation is shown in fig. Let v be the velocity of the bob at the lowest position. In this position the P.E. of bob is converted into K.E. hence -



$$mg\ell = \frac{1}{2}mv^2$$

$\Rightarrow \quad v^2 = 2g\ell \quad ....(1)$

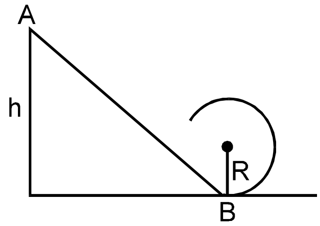
If T be the tension in the string,

then $\quad T - mg = \frac{mv^2}{\ell} \quad ....(2)$

From (1) & (2) T = 3 mg

Hence correct answer is (C)

Ex.44 A ball is released from height h as shown in fig. Which of the following condition hold good for the particle to complete the circular path?



(A) $h \le \frac{5R}{2}$ (B) $h \ge \frac{5R}{2}$

(C) $h < \frac{5R}{2}$ (D) $h > \frac{5R}{2}$

Sol.(B) According to law of conservation of energy (K.E + P.E.) at A = (K.E + P.E) at B

$\Rightarrow \quad 0 + mgh = \frac{1}{2} mv^2 + 0$

$\Rightarrow \quad v = \sqrt{2gh}$

But velocity at the lowest point of circle,

$v \geq \sqrt{5gR} \Rightarrow \sqrt{2gh} \geq \sqrt{5gR} \Rightarrow h \geq \frac{5R}{2}$

Hence correct answer is (B)

Ex.45 The roadway bridge over a canal is the form of an arc of a circle of radius 20 m. What is the minimum speed with which a car can cross the bridge without leaving contact with the ground at the highest point ($g = 9.8\ m/s^2$)

(A) 7 m/s (B) 14 m/s
(C) 289 m/s (D) 5 m/s

Sol.(B) The minimum speed at highest point of a vertical circle is given by

$$v_c = \sqrt{rg} = \sqrt{20 \times 9.8} = 14\ m/s$$

Hence correct answer is (B)

Ex.46 A cane filled with water is revolved in a vertical circle of radius 0.5 m and the water does not fall down. The maximum period of revolution must be -

(A) 1.45 (B) 2.45
(C) 14.15 (D) 4.25

Sol.(A) The speed at highest point must be

$$v > \sqrt{gr}, \quad v = r\omega = r\frac{2\pi}{T}$$

$$\therefore \quad r\frac{2\pi}{T} > \sqrt{rg}$$

$$T < \frac{2\pi r}{\sqrt{rg}} < 2\pi\sqrt{\frac{r}{g}} < 2\pi\sqrt{\frac{0.5}{9.8}} < 1.4 \text{ sec}$$

Maximum period of revolution = 1.4 sec

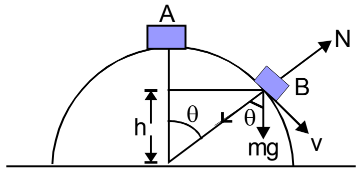
Hence correct answer is (A)

Ex.47 A particle of mass m slides down from the vertex of semi-hemisphere, without any initial velocity. At what height from horizontal will the particle leave the sphere-

(A) $\frac{2}{3}R$ (B) $\frac{3}{2}R$

(C) $\frac{5}{8}R$ (D) $\frac{8}{5}R$

Sol.(A) Let the particles leaves the sphere at height h,



$$\frac{mv^2}{R} = mg\cos\theta - N$$

When the particle leaves the sphere

i.e. $N = 0$

$$\frac{mv^2}{R} = mg\cos\theta$$

$\Rightarrow \quad v^2 = gR\cos\theta \quad ....(1)$

According to law of conservation of energy (K.E. + P.E.) at A = (K.E. + P.E.) at B

$\Rightarrow 0 + mgR = \frac{1}{2} mv^2 + mgh$

$\Rightarrow \quad v^2 = 2g(R - h) \quad ....(2)$

From (1) & (2) $h = \frac{2}{3}R$

Also $\cos\theta = \frac{2}{3}$

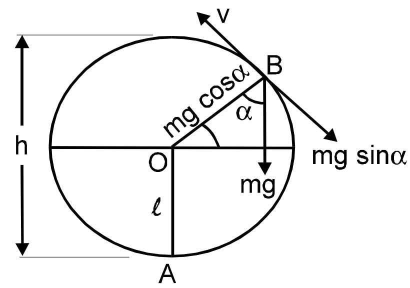
Hence correct answer is (A)

Ex.48 A body of mass m tied at the end of a string of length ℓ is projected with velocity $\sqrt{4lg}$, at what height will it leave the circular path -

(A) $\frac{5}{3}\ell$ (B) $\frac{3}{5}\ell$

(C) $\frac{1}{3}\ell$ (D) $\frac{2}{3}\ell$

Sol.(A) Let the body will have the circular path at height h above the bottom of circle from figure



$$\frac{mv^2}{\ell} = T + mg\cos\alpha$$

On leaving the circular path

$T = 0$

$$\therefore \quad \frac{mv^2}{\ell} = mg\cos\alpha$$

$\Rightarrow \quad v^2 = g\ell\cos\alpha \quad ....(1)$

According to law of conservation of energy
(K.E. + P.E.) at A = (K.E. + P.E.) at B

$\Rightarrow \quad 0 + 2mg\ell = \frac{1}{2}mv^2 + mgh$

$\Rightarrow \quad v^2 = 2g(2\ell - h)$ ....(2)

From (1) & (2) $h = \frac{5}{3}\ell$

Also $\cos\alpha = \frac{h-\ell}{\ell}$

Hence correct answer is (A)

Ex.49 A vehicle of mass 1000 kg is moving along a curved both of length 314 m with a speed of 72 km/hr. If it takes a turn of 90º, the centripetal force needed by the vehicle is -

(A) 20 N (B) 200 N
(C) 2000 N (D) 2 N

Sol. As the vehicle has a turn of 90º, the length of the path is $\frac{1}{4}$ the part of the circle of radius r.

Hence length of the path

$$= 314 = \frac{2\pi r}{4}$$

or $$r = \frac{4\times 314}{2\pi} = 200 \text{ m}$$

Centripetal force, $F_c = \frac{mv^2}{r}$

$$= \frac{1000}{200} \times \left(72\times\frac{5}{18}\right)^2 = 2000 \text{ N}$$

Hence correct answer is (C)

Ex.50 For a heavy vehicle moving on a circular curve of a highway the road bed is banked at an angle θ corresponding to a particular speed. The correct angle of banking of the road for vehicles moving at 60 km/hr will be - (If radius of curve = 0.1 km)

(A) $\tan^{-1}(0.283)$ (B) $\tan^{-1}(2. 83)$
(C) $\tan^{-1}(0.05)$ (D) $\tan^{-1}(0.5)$

Sol.(A) $v = 60$ km/hr $= \frac{50}{3}$ m/s

$r = 0.1$ km $= 100$m

$\therefore \quad \tan\theta = \frac{v^2}{rg} = 0.283$

$\therefore \quad \theta = \tan^{-1}(0.283)$

Hence correct answer is (A)

Ex.51 A train has to negotiate a curve of radius 400 m. By how much should the outer rail be raised with respect to inner rail for a speed of 48 km/hr. The distance between the rail is 1 m.

(A) 12 m (B) 12 cm
(C) 4.5 cm (D) 4.5 m

Sol.(C) We know that $\tan\theta = \frac{v^2}{rg}$ ..... (1)

Let h be the relative raising of outer rail with respect to inner rail. Then

$\tan\theta = \frac{h}{\ell}$ ...... (2)

(ℓ = separation between rails)

From (1) & (2) , $h = \frac{v^2}{rg} \times \ell$

Hence $v = 48$ km/hr $= \frac{120}{9}$ m/s,

($r = 400$ m, $\ell = 1$m),

$\therefore \; h = \frac{(120/9)^2 \times 1}{400\times 9.8} = 0.045 \text{ m} = 4.5 \text{ cm}$

Hence correct answer is (C)

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Equations of Motion

1. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 sec, it rotates through an angle θ_1; in the next 2 sec, it rotates through an additional angle θ_2. The ratio of θ_2 / θ_1 is-

(A) 1 (B) 2
(C) 3 (D) 5

2. In applying the equation for motion with uniform angular acceleration $\omega = \omega_0 + \alpha t$, the radian measure -

(A) must be used for both ω and α
(B) may be used for both ω and α
(C) may be used for ω but not α
(D) cannot be used for both ω and α

3. The linear and angular acceleration of a particle are 10 m/sec^2 and 5 rad/sec^2 respectively it will be at a distance from the axis of rotation -

(A) 50 m (B) 1/2 m
(C) 1 m (D) 2 m

4. A grinding wheel attained a velocity of 20 rad/sec in 5 sec starting from rest. Find the number of revolutions made by the wheel.

(A) $\frac{\pi}{25}$ revolution per sec

(B) $\frac{1}{\pi}$ revolution per sec

(C) $\frac{25}{\pi}$ revolution

(D) None

5. A wheel having a diameter of 3 m starts from rest and accelerates uniformly to an angular velocity of 210 r.p.m. in 5 seconds. Angular acceleration of the wheel is -

(A) $1.4\pi \frac{rad}{s^2}$ (B) $3.3\pi \frac{rad}{s^2}$
(C) $2.2\pi \frac{rad}{s^2}$ (D) $1.1\pi \frac{rad}{s^2}$

6. A wheel starts rotating at 10 rad/sec and attains the angular velocity of 100 rad/sec in 15 seconds. What is the angular acceleration in rad/sec^2?

(A) 10 (B) 110/15
(C) 100/15 (D) 6

Uniform Circular Motion

7. A teachnometer is a device to measure -

(A) gravitational pull (B) speed of rotation
(C) surface tension (D) tension in a spring

8. The ratio of angular speed of hours hand and seconds hand of a clock is-

(A) 1 : 1 (B) 1 : 60
(C) 1 : 720 (D) 3600 : 1

9. The ratio of angular speeds of minutes hand and hour hand of a watch is -

(A) 1 : 12 (B) 6 : 1
(C) 12 : 1 (D) 1:6

10. Two cars of masses m_1 and m_2 are moving along the circular path of radius r_1 and r_2. They take one round in the same time. The ratio of angular velocities of the two cars will be-

(A) $m_1 : m_2$ (B) $r_1 : r_2$
(C) 1 : 1 (D) $m_1 r_1 : m_2 r_2$

11. The angular velocity of earth about its axis of rotation is-

(A) $2\pi / (60 \times 60 \times 24)$ rad / sec
(B) $2\pi / (60 \times 60)$ rad / sec
(C) $2\pi / 60$ rad / sec
(D) $2\pi / (365 \times 24 \times 60 \times 60)$ rad / sec

12. A bottle of soda water is grasped by the neck and swing briskly in a vertical circle. Near which portion of the bottle do the bubbles collect?

(A) near the near bottom
(B) in the middle of the bottle
(C) near the neck
(D) uniformly distributed in the bottle

13. In circular motion, the centripetal acceleration is given by-

(A) $\mathbf{a} \times \mathbf{r}$ (B) $\boldsymbol{\omega} \times \mathbf{v}$
(C) $\mathbf{a} \times \mathbf{v}$ (D) $\boldsymbol{\omega} \times \mathbf{r}$

14. The ratio of angular speeds of minutes hand and hour hand of a watch is -

(A) 1 : 12 (B) 6 : 1
(C) 12 : 1 (D) 1 : 6

15. A particle moves in a circle of radius 25 cm at two revolutions per second. The acceleration of particle in m/s^2 is -

(A) π^2 (B) $8\pi^2$

(C) $4\pi^2$ (D) $2\pi^2$

16. A particle moves in circular path with uniform speed v. The change in its velocity on rotating through 60º is -

(A) $v\sqrt{2}$ (B) $\frac{v}{\sqrt{2}}$

(C) v (D) Zero

17. Two bodies of masses 10 kg and 5 kg moving on concentric orbits of radii R and r such that their period of revolution are same. The ratio of their centripetal acceleration is -

(A) $\frac{R}{r}$ (B) $\frac{r}{R}$

(C) $\frac{R^2}{r^2}$ (D) $\frac{r^2}{R^2}$

18. A particle is moving in a horizontal circle with constant speed. State whether, the-

(A) K.E. is constant

(B) P.E. is constant

(C) Both K.E. and P.E. are constant

(D) Neither K.E. nor P.E. are constant

19. A stone of mass m is tied to a string of length ℓ and rotated in a circle with a constant speed v. If the string is released, the stone flies-

(A) radially outward

(B) radially inward

(C) tangentially outward

(D) with an acceleration mv^2/ℓ

20. If a particle moves in a circle describing equal angles in equal interval of times, its velocity vector –

(A) remains constant

(B) changes in magnitude

(C) changes in direction

(D) changes both in magnitude and direction

21. In uniform circular motion-

(A) both velocity and acceleration are constant

(B) acceleration and speed are constant but velocity changes

(C) both acceleration and velocity change

(D) both acceleration and speed are constant

22. When a body moves with a constant speed along a circle-

(A) no work is done on it

(B) no acceleration is produced in the body

(C) no force acts on the body

(D) its velocity remains constant

23. What happens to the centripetal acceleration of a revolving body if you double the orbital speed v and halve the angular velocity ω ?

(A) the centripetal acceleration remains unchanged

(B) the centripetal acceleration is halved

(C) the centripetal acceleration is doubled

(D) the centripetal acceleration is quadrupled

24. A body of mass m is moving in a circle of radius r with a constant speed v. The force on the body is mv^2/r and u is directed towards the centre. What is the work done by this force in moving the body over half the circumference of the circle?

(A) $mv^2/r \times \pi r$ (B) zero

(C) mv^2/r (D) $\pi r^2/mv^2$

25. Centrifugal force is considerd as pseudo force when

(A) An observer at the centre of circular motion

(B) An outside observe

(C) An observer who is moving with the particle which is experiencing the force

(D) None of the above

26. A stone of mass 0.5 kg tied with a string of length 1 metre is moving in a circular path with a speed of 4 m/sec. The tension acting on the string in Newton is-

(A) 2 (B) 8

(C) 0.2 (D) 0.8

27. The breaking tension of a string is 10 N. A particle of mass 0.1 kg tied to it is rotated along a horizontal circle of radius 0.5 metre. The maximum speed with which the particle can be rotated without breaking the string is-

(A) $\sqrt{5}$ m/sec (B) $\sqrt{(50)}$ m/sec

(C) $\sqrt{(500)}$ m/sec (D) $\sqrt{(1000)}$ m/sec

28. A car of mass m is taking a circular turn of radius 'r' on a frictional level road with a speed v. In order that the car does not skid-

(A) $\frac{mv^2}{r} \geq \mu mg$ (B) $\frac{mv^2}{r} \leq \mu mg$

(C) $\frac{mv^2}{r} = \mu mg$ (D) $\frac{v}{r} = \mu mg$

Where 'μ' is coefficient of friction

29. What happens to centripital force of a revolving body if you double the orbital speed v and halve the angular velocity ω-

(A) Centripetal force remains unchanged

(B) Centripital force is halved

(C) Centripital force is doubled

(D) Centripital force is quadrupled

30. A body is moving with a constant speed v in a circle of radius r. Its angular acceleration is-

(A) Zero (B) $\frac{v}{r}$

(C) $\frac{v^2}{r^2}$ (D) $\frac{v^2}{r}$

31. A body of mass 10 kg is rotated in vertical circle of radius 4 cm at constant angular velocity of 5 rad/ sec. The maximum tension in the string is-

(A) 100 N (B) 600 N

(C) 110 N (D) 1100 N

32. If both the speed and radius of circular path of a revolving body are doubled, the magnitude of centripetal force will be

(A) equal to the former

(B) twice the former

(C) 4 times the former

(D) 8 times the former

33. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. it follows that–

(A) its velocity is constant

(B) its acceleration is constant

(C) its kinetic energy is constant

(D) it moves in circular path

34. A particle is moving along a circular path of radius 6 m with uniform speed of 8 ms^{-1}. The average acceleration when the particle completes one half of the revolution is -

(A) $\frac{16}{3\pi}$ m/s^2 (B) $\frac{32}{3\pi}$ m/s^2

(C) $\frac{64}{3\pi}$ m/s^2 (D) None of these

35. A string of length 1 m is fixed at one end and carries a mass of 100 gm at the other end. The string makes (2/π) revolutions per second around vertical axis through the fixed end. Calculate the tension in the string-

(A) 1.0 N (B) 1.6 N

(C) 2 N (D) 4 N

36. A chain of 125 links is 1.25 m long and has a mass of 2 kg with the ends fastened together it is set rotating at 3000 $\frac{rev}{min}$. Find the centripetal force on each link -

(A) 3.14 N (B) 314 N

(C) $\frac{1}{3.14}$ N (D) $\frac{1}{314}$ N

37. When the road is dry and the coefficient of friction is μ, the maximum speed of a car in a circular path is 10 m/s, if the road becomes wet and μ' = μ/2. What is the maximum speed permitted ?

(A) 5 m/s (B) 10 m/s

(C) $10\sqrt{2}$ m/s (D) $5\sqrt{2}$ m/s

38. A person with a mass of M kg stands in contact against the wall of the cylindrical drum of radius r rotating with an angular velocity ω. The coefficient of friction between the wall and the clothing is μ. The minimum rotational speed of the cylinder which enables the person to remain stuck to the wall when the floor is suddenly removed is -

(A) $\omega_{min} = \sqrt{\frac{g}{\mu r}}$ (B) $\omega_{min} = \sqrt{\frac{\mu r}{g}}$

(C) $\omega_{min} = \sqrt{\frac{2g}{\mu r}}$ (D) $\omega_{min} = \sqrt{\frac{gr}{\mu}}$

39. A body is revolving with a uniform speed V in a circle of radius r. The angular acceleration of the body is -

(A) $\frac{V}{r}$

(B) Zero

(C) $\frac{V^2}{r}$ along the radius and towards the centre

(D) $\frac{V^2}{r}$ along the radius and away from the centre

40. A particle completes 3 revolutions per second on a circular path of radius 8 cm. Find the values of angular velocity and centripetal acceleration of the particle -

(A) $6\pi \frac{rad}{s}$; $288\pi^2 \frac{cm}{s^2}$

(B) $\pi \frac{rad}{s}$; $275\pi^2 \frac{cm}{s^2}$

(C) $6\pi \frac{rad}{s}$; $288 \frac{cm}{s^2}$

(D) None

41. A car of mass 1000 kg moves on a circular track of radius 20 m. if the coefficient of friction is 0.64, what is the maximum velocity with which the car can be moved?

(A) 1.12 m/s (B) 11.2 m/s

(C) $\frac{0.64 \times 20}{1000}$ m/s (D) $\frac{1000}{0.64 \times 20}$ m/s

42. The earth, radius 6400 km, makes one revolution about its own axis in 24 hours. The centripetal acceleration of a point on its equator is nearly -

(A) $340 \frac{cm}{sec^2}$ (B) $3.4 \frac{cm}{sec^2}$

(C) $34 \frac{cm}{sec^2}$ (D) $0.34 \frac{cm}{sec^2}$

43. A stone of mass 0.1 kg tied to one end of a string 1.0 m long is revolved in a horizontal circle at the rate of $10/\pi$ revolution per second. Calculate the tension of the string ?

(A) 30 N (B) 40 N

(C) 50 N (D) 60 N

44. A coin placed on a rotating turn table just slips if it is at a distance of 40 cm from the centre if the angular velocity of the turntable is doubled, it will just slip at a distance of

(A) 10 cm (B) 20 cm

(C) 40 cm (D) 80 cm

45. A stone of mass 0.5 kg tied with a string of length 1 m is moving in a circular path with a speed of 4 m/sec. The tension acting on the string in Newton is

(A) 2 (B) 8

(C) 0.2 (D) 0.8

46. A particle is acted upon by a constant force always normal to the direction of motion of the particle. It is therefore inferred that-

(a) Its velocity is constant
(b) It moves in a straight line
(c) Its speed is constant
(d) It moves in circular path

(A) a, d (B) c, d

(C) a, b (D) a, b, c

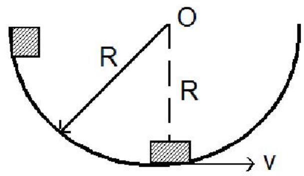
Non uniform Circular motion

47. A particle is projected so as to just move along a vertical circle of radius r. The ratio of the tension in the string when the particle is at the lowest and highest point on the circle is -

(A) 1 (B) finite but large

(C) zero (D) Infinite

48. A block of mass m slides down along the surface of the bowl from the rim to the bottom as shown in fig. The velocity of the block at the bottom will be-



(A) $\sqrt{\pi Rg}$ (B) $2\sqrt{\pi Rg}$

(C) $\sqrt{2Rg}$ (D) $\sqrt{gR}$

49. A sphere is suspended by a thread of length ℓ. What minimum horizontal velocity is to be imparted to the sphere for it to reach the height of suspension?

(A) $\sqrt{gl}$ (B) $g\ell$

(C) $\sqrt{2gl}$ (D) $\sqrt{l/g}$

50. A body of mass 2 kg is moving in a vertical of radius 2 m. The work done when it moves from the lowest point to the highest point is-

(A) 80 J (B) 40 J
(C) 20 J (D) 0

51. A particle rests on the top of the hemisphere of radius R. The small horizontal velocity that must be imparted to the particle if it is to leave the hemisphere without sliding down. is-

(A) $v = (2gR)^{1/2}$ (B) $v = (gR/2)^{1/2}$
(C) $v = (gR)^{1/2}$ (D) $v = (2g/R)^{1/2}$

52. A mass m is revolving in a vertical circle at the end of a string of length 20 cm. By how much does the tension of the string at the lowest point exceed the tension at the top most point?

(A) 2 m g (B) 4 m g
(C) 6 m g (D) 8 m g

53. A car is travelling with linear velocity v on a circular road of radius r. If it is increasing it speed at the rate of 'a' metre/sec^2, then the resultant acceleration will be-

(A) $\sqrt{\left(\frac{v^2}{r^2} - a^2\right)}$ (B) $\sqrt{\left(\frac{v^4}{r^2} + a^2\right)}$

(C) $\sqrt{\left(\frac{v^4}{r^2} - a^2\right)}$ (D) $\sqrt{\left(\frac{v^2}{r^2} + a^2\right)}$

54. On an unbanked road, a cyclist negotiating a bend of radius r at velocity v must lean inwards by an angle θ equal to -

(A) $\tan^{-1}(v^2/g)$ (B) $\tan^{-1}(g/v)$
(C) $\tan^{-1}(v^2/gr)$ (D) $\tan^{-1}(rg/v^2)$

55. A particle of mass m is rotating by means of a string in a vertical circle. The difference in the tension at the bottom and top would be-

(A) 6 mg (B) 4 mg
(C) 3 mg (D) 2 mg

56. A body of mass m crosses the top most point of a vertical circle with critical speed. What will be tension in string when it is horizontal-

(A) mg (B) 2 mg
(C) 3 mg (D) 6 mg

57. A motor-cycle is moving in a vertical circular path. At what stage will the speed of the motor cycle be maximum ?

(A) At the highest point of the path
(B) At the lowest point of the path
(C) At the mid height of the path
(D) At all the points in the path

58. An aeroplane flying at 100 m/sec dives in a vertical plane along the circle of radius 200 m. The mass of the pilot is 75 kg. What will be the force exerted by the pilot on his seat when the aeroplane is at the maximum height

(A) 300 kg wt (B) 200 kg wt
(C) 450 kg wt (D) 100 kg wt

59. **In the above question,** the force exterted when the pilot is at the lowest point is

(A) 450 kg wt (B) 250 kg wt
(C) 300 kg wt (D) 100 kg wt

60. A string can bear a maximum tension of 100 Newton without breaking. A body of mass 1 kg is attached to one end of 1 m length of thin string and it is revolved in a horizontal plane. The maximum linear velocity which can be imparted to the body without breaking the string, will be -

(A) 10 m/s (B) 1 m/s
(C) 100 m/s (D) 1000 m/s

61. A cane filled with water is revolved in a vertical circle of radius 4 metre and the water just does not fall down. The time period of revolution will be -

(A) 1 sec (B) 10 sec
(C) 8 sec (D) 4 sec

62. A 2 kg stone at the end of a string 1 m. long is whirled in a vertical circle at a constant speed. The speed of the stone is 4 m /sec. The tension in the string will be 52 N when the stone is-

(A) at the top of the circle
(B) at the bottom of the circle
(C) half way down
(D) none of the above

63. The roadway of a bridge over a canal is in the form of a circular arc of radius 18 m. What is the greatest speed with which a motor cycle can cross the bridge without leaving ground.

(A) $\sqrt{98}$ m/s (B) $\sqrt{18 \times 9.8}$ m/s
(C) 18×9.8 m/s (D) 18/9.8 m/s

64. The maximum speed with which a car can cross a convex bridge over a river with radius of curvature 9 m is : (given that the centre of gravity of car is 1m above the road)

(A) 50 m/s (B) 30 m/s
(C) 20 m/s (D) 10 m/s

65. A car is moving with speed 30 m/s on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/s^2. The net acceleration of the car is-

(A) 3.7 m/s^2 (B) 2.7 m/s^2
(C) 1.8 m/s^2 (D) 2 m/s^2

Banking of roads

66. A cyclist taking turn bends inwards while a car passenger take the same turn is thrown outwards. The reason is-

(A) car is heavier then cycle
(B) car has four wheels while cycle has only two
(C) difference in the speed of the two
(D) Cyclist has to counteract the centrifugal force while in the case of car only the passenger is thrown by this force

67. A cyclist turns around a curve at 15 miles/hour. If he turns at double the speed, the tendency to overturn is-

(A) doubled (B) quadrupled
(C) halved (D) unchanged

68. A cyclist is moving on a circular track of radius 80 m with a velocity of 72 km/hr. He has to lean from the vertical approximately through an angle

(A) $\tan^{-1}(1/4)$ (B) $\tan^{-1}(1)$
(C) $\tan^{-1}(1/2)$ (D) $\tan^{-1}(2)$

69. Keeping the banking angle same to increase the maximum speed with which a vehicle can travel on a curved road by 10%, the radius of curvature of road has to be changed from 20 m to-

(A) 16 m (B) 18 m
(C) 24.25 m (D) 30.5 m

70. A motor cyclist moving with a velocity of 72 km per hour on a flat road takes a turn on the road at a point where the radius of curvature of the road is 20 metres. The acceleration due to gravity is 10 m/s^2. In order to avoid skidding, he must not bend with respect to the vertical plane by an angle greater than-

(A) $\theta = \tan^{-1} 6$ (B) $\theta = \tan^{-1} 2$
(C) $\theta = \tan^{-1} 25.92$ (D) $\theta = \tan^{-1} 4$

71. A cyclist taking turn bends inwards while a car passenger taking the same turn is thrown outwards. The reason is -

(A) that car is heavier than cycle
(B) that car has four wheels, while cycle has only two
(C) that cyclist has to counteract the centrifugal force, while the passenger is only thrown by it
(D) the difference in the speed of the two

Theta, omega, alpha, equations of motion

72. A particle is moving along a circular path with uniform speed. Through what angle does its angular velocity change when it completes half of the circular path ?

(a) 0° (b) 45°
(c) 180° (d) 360°

73. What is the angular velocity in rad/s of a fly wheel making 300 r.p.m. ?

(a) 600 π (b) 20 π
(c) 10 π (d) 30

74. The angular velocity of a particle moving in a circle of radius 50 cm is increased in 5 min from 100 revolutions per minute to 400 revolutions per minute. Find tangential acceleration of the particle.

(a) 60 m/s^2 (b) $\pi/30$ m/s^2
(c) $\pi/15$ m/s^2 (d) $\pi/60$ m/s^2

75. A particle covers equal distance around a circular path, in equal intervals of time. Which of the following quantities connected with the motion of the particle remains constant with time ?

(a) Displacement (b) Velocity
(c) Speed (d) Acceleration

76. A particle is moving along a circular path of radius 2m with uniform speed of 5 ms^{-1}. What will be the change in velocity when the particle completes half of the revolution?

(a) Zero (b) 10 ms^{-1}
(c) $10\sqrt{2}\ ms^{-1}$ (d) $\frac{10}{\sqrt{2}}\ ms^{-1}$

77. A particle is moving along a circular path of radius 5m with a uniform speed 5 ms^{-1}. What will be the average acceleration when the particle completes half revolution?

(a) Zero (b) 10 ms^{-1}
(c) $10\pi\ ms^{-2}$ (d) $\frac{10}{\pi}\ ms^{-2}$

Vertical Circular Motion

78. What should be the minimum velocity at the highest point of a body tied to a string, so that the string just does not slack ?

(a) $\sqrt{Rg}$ (b) $\sqrt{5Rg}$

(c) $\left(\frac{R}{g}\right)^{3/2}$ (d) $\sqrt{2Rg}$

79. A bead can slide on a smooth circular wire frame of radius r which is fixed in a vertical plane. The bead is displaced slightly from the highest point of the wire frame. The speed of the bead subsequently as a function of the angle θ made by the bead with the vertical line is :

(a) $\sqrt{2gr}$ (b) $\sqrt{2gr(1-\sin\theta)}$

(c) $\sqrt{2gr(1-\cos\theta)}$ (d) $2\sqrt{gr}$

80. A particle moves in a circle of a radius 30 cm. Its linear speed is given by : v=2t, where t in second and v in m/s. Find out its radial and tangential acceleration at t = 3 sec. respectively :

(a) 220 m/sec^2, 50 m/sec^2

(b) 100 m/sec^2, 5 m/sec^2

(c) 120 m/sec^2, 2 m/sec^2

(d) 110 m/sec^2, 10 m/sec^2

EXERCISE - 2 : PREVIOUS YEARS JEE MAINS QUESTIONS

1. If the body is moving in a circle of radius r with a constant speed v, its angular velocity is **[CPMT 1975]**

(a) v^2/r (b) vr

(c) v/r (d) r/v

2. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same duration of time t. The ratio of the angular speed of the first to the second car is **[CBSE PMT 1999]**

(a) $m_1 : m_2$ (b) $r_1 : r_2$

(c) $1:1$ (d) $m_1 r_1 : m_2 r_2$

3. A cyclist turns around a curve at 15 miles/hour. If he turns at double the speed, the tendency to overturn is **[CPMT 1974; AFMC 2003]**

(a) Doubled (b) Quadrupled

(c) Halved (d) Unchanged

4. If a particle moves in a circle describing equal angles in equal times, its velocity vector **[CPMT 1972, 74]**

(a) Remains constant

(b) Changes in magnitude

(c) Changes in direction

(d) Changes both in magnitude and direction

5. When a body moves with a constant speed along a circle **[CBSE PMT 1994]**

(a) No work is done on it

(b) No acceleration is produced in the body

(c) No force acts on the body

(d) Its velocity remains constant

6. A body of mass m moves in a circular path with uniform angular velocity. The motion of the body has constant **[MP PET 2003]**

(a) Acceleration (b) Velocity

(c) Momentum (d) Kinetic energy

7. A cyclist taking turn bends inwards while a car passenger taking same turn is thrown outwards. The reason is **[CPMT 1974]**

(a) Car is heavier than cycle

(b) Car has four wheels while cycle has only two

(c) Difference in the speed of the two

(d) Cyclist has to counteract the centrifugal force while in the case of car only the passenger is thrown by this force

8. A car sometimes overturns while taking a turn. When it overturns, it is **[AFMC 1988]**

(a) The inner wheel which leaves the ground first

(b) The outer wheel which leaves the ground first

(c) Both the wheels leave the ground simultaneously

(d) Either wheel leaves the ground first

9. A tachometer is a device to measure **[DPMT 1999]**

(a) Gravitational pull (b) Speed of rotation

(c) Surface tension (d) Tension in a spring

10. Two bodies of mass 10 kg and 5 kg moving in concentric orbits of radii R and r such that their periods are the same. Then the ratio between their centripetal acceleration is **[CBSE PMT 2001]**

(a) R/r (b) r/R

(c) R^2/r^2 (d) r^2/R^2

11. A particle is moving in a horizontal circle with constant speed. It has constant **[AFMC 1993; CPMT 1997]**

(a) Velocity (b) Acceleration

(c) Kinetic energy (d) Displacement

12. A train is moving towards north. At one place it turns towards north-east, here we observe that **[AIIMS 1980]**

(a) The radius of curvature of outer rail will be greater than that of the inner rail

(b) The radius of the inner rail will be greater than that of the outer rail

(c) The radius of curvature of one of the rails will be greater

(d) The radius of curvature of the outer and inner rails will be the same

13. The angular speed of a fly wheel making 120 *revolutions/minute* is **[CBSE PMT 1995]**

(a) $2\pi\ rad/s$ (b) $4\pi^2\ rad/s$

(c) $\pi\ rad/s$ (d) $4\pi\ rad/s$

14. The length of second's hand in a watch is 1 *cm*. The change in velocity of its tip in 15 seconds is **[MP PMT 1987, 2003]**

(a) Zero (b) $\frac{\pi}{30\sqrt{2}}\ cm/\sec$

(c) $\frac{\pi}{30}\ cm/\sec$ (d) $\frac{\pi\sqrt{2}}{30}\ cm/\sec$

15. A particle moves in a circle of radius 25 *cm* at two revolutions per second. The acceleration of the particle in m/s^2 is **[DPMT 1999]**

(a) π^2 (b) $8\pi^2$

(c) $4\pi^2$ (d) $2\pi^2$

16. An electric fan has blades of length 30 *cm* as measured from the axis of rotation. If the fan is rotating at 1200 *r.p.m.* The acceleration of a point on the tip of the blade is about **[CBSE PMT 1990]**

(a) 1600 $m/\sec^2$ (b) 4740

(c) 2370 (d) 5055

17. The force required to keep a body in uniform circular motion is **[EAMCET 1982; AFMC 2003]**

(a) Centripetal force (b) Centrifugal force

(c) Resistance (d) None of the above

18. A particle moves in a circular orbit under the action of a central attractive force inversely proportional to the distance 'r'. The speed of the particle is **[CBSE PMT 1995]**

(a) Proportional to r^2 (b) Independent of r

(c) Proportional to (d) Proportional to

19. A mass of 2 *kg is* whirled in a horizontal circle by means of a string at an initial speed of 5 *revolutions per minute*. Keeping the radius constant the tension in the string is doubled. The new speed is nearly **[MP PMT/PET 1998; JIPMER 2000]**

(a) 14 *rpm* (b) 10 *rpm*

(c) 2.25 *rpm* (d) 7 *rpm*

20. The magnitude of the centripetal force acting on a body of mass m executing uniform motion in a circle of radius r with speed v is **[AFMC 1998;MP PET 1999]**

(a) mvr (b) mv^2/r

(c) v/r^2m (d) v/rm

21. A 500 *kg* car takes a round turn of radius 50 *m* with a velocity of 36 *km/hr*. The centripetal force is **[CBSE PMT 1999]**

(a) 250 *N* (b) 750 *N*

(c) 1000 *N* (d) 1200 *N*

22. A ball of mass 0.25 *kg* attached to the end of a string of length 1.96 *m* is moving in a horizontal circle. The string will break if the tension is more than 25 *N*. What is the maximum speed with which the ball can be moved **[CBSE PMT 1998]**

(a) 14 *m/s* (b) 3 *m/s*

(c) 3.92 *m/s* (d) 5 *m/s*

23. A body of mass 5 *kg* is moving in a circle of radius 1 *m* with an angular velocity of 2 *radian/sec*. The centripetal force is **[AIIMS 1998]**

(a) 10 *N* (b) 20 *N*

(c) 30 *N* (d) 40 *N*

24. A sphere of mass m is tied to end of a string of length l and rotated through the other end along a horizontal circular path with speed v. The work done in full horizontal circle is **[CPMT 1993]**

(a) 0 (b) $\left(\frac{mv^2}{l}\right).2\pi l$

(c) $mg.2\pi l$ (d) $\left(\frac{mv^2}{l}\right).(l)$

25. A body is whirled in a horizontal circle of radius 20 *cm*. It has angular velocity of 10 *rad/s*. What is its linear velocity at any point on circular path **[CBSE PMT 1996]**

(a) 10 *m/s* (b) 2 *m/s*

(c) 20 *m/s* (d) $\sqrt{2}$ *m/s*

26. Find the maximum velocity for skidding for a car moved on a circular track of radius 100 *m*. The coefficient of friction between the road and tyre is 0.2 **[CPMT 1996]**

(a) 0.14 *m/s* (b) 140 *m/s*

(c) 1.4 *km/s* (d) 14 *m/s*

27. A car when passes through a convex bridge exerts a force on it which is equal to **[AFMC 1997]**

(a) $Mg + \frac{Mv^2}{r}$ (b) $\frac{Mv^2}{r}$

(c) Mg (d) None of these

28. The angular speed of seconds needle in a mechanical watch is **[CPMT 1997]**

(a) $\frac{\pi}{30}\ rad/s$ (b) $2\pi\ rad/s$

(c) $\pi\ rad/s$ (d) $\frac{60}{\pi}\ rad/s$

29. The angular velocity of a particle rotating in a circular orbit 100 times per minute is **[SCRA 1998; DPMT 2000]**

(a) 1.66 *rad/s* (b) 10.47 *rad/s*

(c) 10.47 *deg/s* (d) 60 *deg/s*

30. A body of mass 100 *g* is rotating in a circular path of radius *r* with constant velocity. The work done in one complete revolution is **[AFMC 1998]**

(a) $100\, rJ$ (b) $(r/100)J$

(c) $(100/r)J$ (d) Zero

31. A particle comes round a circle of radius 1 *m* once. The time taken by it is 10 *sec*. The average velocity of motion is **[JIPMER 1999]**

(a) $0.2\ \pi m/s$ (b) $2\ \pi m/s$

(c) $2\ m/s$ (d) Zero

32. An unbanked curve has a radius of $60\ m$. The maximum speed at which a car can make a turn if the coefficient of static friction is 0.75, is **[JIPMER 1999]**

(a) 2.1 (b) 14

(c) 21 (d) 7

33. A particle of mass *M* is moving in a horizontal circle of radius *R* with uniform speed *V*. When it moves from one point to a diametrically opposite point, its **[CBSE PMT 1992]**

(a) Kinetic energy changes by $MV^2/4$

(b) Momentum does not change

(c) Momentum changes by 2*MV*

(d) Kinetic energy changes by MV^2

34. A ball of mass 0.1 *Kg*. is whirled in a horizontal circle of radius 1 *m*. by means of a string at an initial speed of 10 *R.P.M.* Keeping the radius constant, the tension in the string is reduced to one quarter of its initial value. The new speed is **[MP PMT 2001]**

(a) 5 *r.p.m.* (b) 10 *r.p.m.*

(c) 20 *r.p.m.* (d) 14 *r.p.m.*

35. In uniform circular motion, the velocity vector and acceleration vector are **[DCE 2000, 01, 03]**

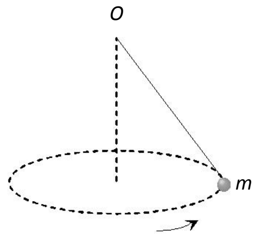
(a) Perpendicular to each other

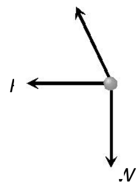
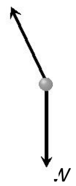
(b) Same direction

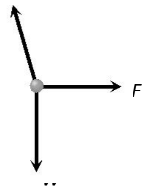
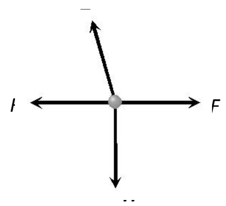
(c) Opposite direction

(d) Not related to each other

36. A point mass *m* is suspended from a light thread of length *l*, fixed at *O*, is whirled in a horizontal circle at constant speed as shown. From your point of view, stationary with respect to the mass, the forces on the mass are **[AMU (Med.) 2001]**



(a)  (b) 

(c)  (d) 

37. If a cyclist moving with a speed of 4.9 *m/s* on a level road can take a sharp circular turn of radius 4 *m*, then coefficient of friction between the cycle tyres and road is **[AIIMS 1999; AFMC 2001]**

(a) 0.41 (b) 0.51

(c) 0.61 (d) 0.71

38. A car moves on a circular road. It describes equal angles about the centre in equal intervals of time. Which of the following statement about the velocity of the car is true **[BHU 2001]**

(a) Magnitude of velocity is not constant

(b) Both magnitude and direction of velocity change

(c) Velocity is directed towards the centre of the circle

(d) Magnitude of velocity is constant but direction changes

39. The maximum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 *m* and coefficient of friction 0.6 to avoid skidding is **[AIEEE 2002]**

(a) 60 (b) 30

(c) 15 (d) 25

40. A car is moving with high velocity when it has a turn. A force acts on it outwardly because of **[AFMC 2002]**

(a) Centripetal force (b) Centrifugal force

(c) Gravitational force (d) All the above

41. A motor cycle driver doubles its velocity when he is having a turn. The force exerted outwardly will be **[AFMC 2002]**

(a) Double (b) Half

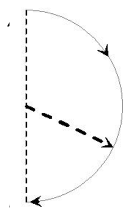
(c) 4 times (d) $\frac{1}{4}$ times

42. If a_r and a_t represent radial and tangential accelerations, the motion of a particle will be uniformly circular if **[CPMT 2004]**

(a) $a_r = 0$ and $a_t = 0$ (b) $a_r = 0$ but $a_t \neq 0$

(c) $a_r \neq 0$ but $a_t = 0$ (d) None of these

43. In 1.0 *s*, a particle goes from point *A* to point *B*, moving in a semicircle of radius 1.0 *m* (see figure). The magnitude of the average velocity is **[IIT-JEE 1999]**



(a) $3.14\, m/s$ (b) $2.0\, m/s$

(c) 1.0 (d) Zero

44. A stone ties to the end of a string $1m$ long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolution in 44 seconds, what is the magnitude and direction of acceleration of the stone **[CBSE PMT 2005]**

(a) $\frac{\pi^2}{4} ms^{-2}$ and direction along the radius towards the centre

(b) $\pi^2 ms^{-2}$ and direction along the radius away from the centre

(c) π^2 ms^{-2} and direction along the radius towards the centre

(d) π^2 ms^{-2} and direction along the tangent to the circle

45. In a circus stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track will be **[CPMT 1979; JIPMER 1997; RPET 1999]**

(a) $\sqrt{2gR}$ (b) $2gR$

(c) $\sqrt{3gR}$ (d) $\sqrt{gR}$

46. A block of mass m at the end of a string is whirled round in a vertical circle of radius . The critical speed of the block at the top of its swing below which the string would slacken before the block reaches the top is **[DCE 1999, 2001]**

(a) Rg (b) $(Rg)^2$

(c) R/g (d) $\sqrt{Rg}$

47. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 *sec*, it rotates through an angle θ_1. In the next 2 *sec*, it rotates through an additional angle θ_2. The ratio of θ_2/θ_1 is **[AIIMS 1985]**

(a) 1 (b) 2

(c) 3 (d) 5

48. A 1 *kg* stone at the end of 1 *m* long string is whirled in a vertical circle at constant speed of 4 *m/sec*. The tension in the string is 6 *N*, when the stone is at ($g = 10\, m/sec^2$) **[AIIMS 1982]**

(a) Top of the circle (b) Bottom of the circle

(c) Half way down (d) None of the above

49. A cane filled with water is revolved in a vertical circle of radius 4 *meter* and the water just does not fall down. The time period of revolution will be **[CPMT 1985]**

(a) 1 *sec* (b) 10 *sec*

(c) 8 *sec* (d) 4 *sec*

50. The string of pendulum of length l is displaced through 90° from the vertical and released. Then the minimum strength of the string in order to withstand the tension, as the pendulum passes through the mean position is **[MP PMT 1986]**

(a) mg (b) 3 mg

(c) 5 mg (d) 6 mg

51. The maximum velocity at the lowest point, so that the string just slack at the highest point in a vertical circle of radius l **[CPMT 1999]**

(a) $\sqrt{gl}$ (b) $\sqrt{3gl}$

(c) $\sqrt{5gl}$ (d) $\sqrt{7gl}$

52. If the equation for the displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then the angular velocity of the particle after 2 *sec* from its start is **[AIIMS 1998]**

(a) 8 *rad/sec* (b) 12 *rad/sec*

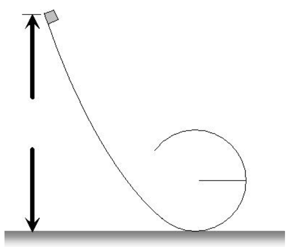
(c) 24 *rad/sec* (d) 36 *rad/sec*

53. The tension in the string revolving in a vertical circle with a mass m at the end which is at the lowest position **[AIIMS 2001]**

(a) $\frac{mv^2}{r}$ (b) $\frac{mv^2}{r} - mg$

(c) $\frac{mv^2}{r} + mg$ (d) mg

54. A block follows the path as shown in the figure from height h. If radius of circular path is r, then relation that holds good to complete full circle is **[RPET 1997]**



(a) $h < 5r/2$ (b) $h > 5r/2$

(c) $h = 5r/2$ (d) $h \geq 5r/2$

55. A pendulum bob on a 2 m string is displaced 60° from the vertical and then released. What is the speed of the bob as it passes through the lowest point in its path **[JIPMER 1999]**

(a) $\sqrt{2}\ m/s$ (b) $\sqrt{9.8}\ m/s$

(c) 4.43 m/s (d) $1/\sqrt{2}\ m/s$

56. A fan is making 600 revolutions per *minute*. If after some time it makes 1200 revolutions per *minute*, then increase in its angular velocity is **[BHU 1999]**

(a) $10\pi\ rad/\sec$ (b) $20\pi\ rad/\sec$

(c) $40\pi\ rad/\sec$ (d) $60\pi\ rad/\sec$

57. A stone tied with a string, is rotated in a vertical circle. The minimum speed with which the string has to be rotated **[CBSE PMT 1999]**

(a) Is independent of the mass of the stone

(b) Is independent of the length of the string

(c) Decreases with increasing mass of the stone

(d) Decreases with increase in length of the string

58. For a particle in a non-uniform accelerated circular motion **[AMU (Med.) 2000]**

(a) Velocity is radial and acceleration is transverse only

(b) Velocity is transverse and acceleration is radial only

(c) Velocity is radial and acceleration has both radial and transverse components

(d) Velocity is transverse and acceleration has both radial and transverse components

59. A ball is moving to and fro about the lowest point *A* of a smooth hemispherical bowl. If it is able to rise up to a height of 20 *cm* on either side of *A*, its speed at *A* must be (Take = 10 m/s^2, mass of the body 5 *g*) **[JIPMER 2000]**

(a) 0.2 m/s (b) 2 m/s

(c) 4 m/s (d) 4.5 ms^{-1}

60. A particle is kept at rest at the top of a sphere of diameter 42 *m*. When disturbed slightly, it slides down. At what height '*h*' from the bottom, the particle will leave the sphere **[BHU 2003]**

(a) 14 m (b) 28 m

(c) 35 m (d) 7 m

61. A small disc is on the top of a hemisphere of radius R. What is the smallest horizontal velocity *v* that should be given to the disc for it to leave the hemisphere and not slide down it ? [There is no friction] **[CPMT 1991]**

(a) $v = \sqrt{2gR}$ (b) $v = \sqrt{gR}$

(c) $v = \frac{g}{R}$ (d) $v = \sqrt{g^2 R}$

62. A body of mass 0.4 *kg* is whirled in a vertical circle making 2 *rev/sec*. If the radius of the circle is 2 *m*, then tension in the string when the body is at the top of the circle, is **[CBSE PMT 1999]**

(a) 41.56 N (b) 89.86 N

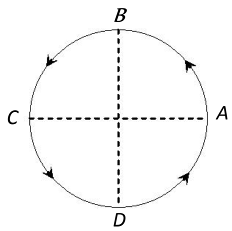
(c) 109.86 N (d) 115.86 N

63. A bucket full of water is revolved in vertical circle of radius 2*m*. What should be the maximum time-period of revolution so that the water doesn't fall off the bucket **[AFMC 2004]**

(a) 1 *sec* (b) 2 *sec*

(c) 3 *sec* (d) 4 *sec*

64. Figure shows a body of mass m moving with a uniform speed v along a circle of radius r. The change in velocity in going from A to B is **[DPMT 2004]**



(a) $v\sqrt{2}$ (b) $v/\sqrt{2}$

(c) v (d) zero

65. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that **(2004)**

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant

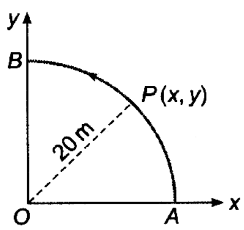
(d) it moves in a straight line

66. For a particle in uniform circular motion the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (here θ is measured from the x–axis **(2010)**

(a) $-\frac{v^2}{R}\cos\theta\,\hat{i}+\frac{v^2}{R}\sin\theta\,\hat{j}$ (b) $-\frac{v^2}{R}\sin\theta\,\hat{i}+\frac{v^2}{R}\cos\theta\,\hat{j}$

(c) $-\frac{v^2}{R}\cos\theta\,\hat{i}-\frac{v^2}{R}\sin\theta\,\hat{j}$ (d) $\frac{v^2}{R}\,\hat{i}+\frac{v^2}{R}\,\hat{j}$

67. A point P moves in counter–clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metre and t is in second. The radius of the path is 20 m. The acceleration of P when t = 2 s is nearly **(2010)**



(a) 13 ms^{-2} (b) 12 ms^{-2}

(c) 7.2 ms^{-2} (d) 14 ms^{-2}

68. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is **(2012)**

(a) $m_1r_1 : m_2r_2$ (b) $m_1 : m_2$

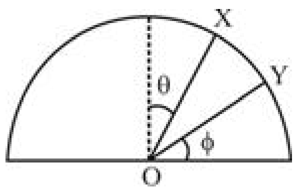
(c) $r_1 : r_2$ (d) 1 : 1

69. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then : **(2018)**

(a) $T \alpha R^{\frac{(n+1)}{2}}$ (b) $T \alpha R^{\frac{n}{2}}$

(c) $T \alpha R^{\frac{3}{2}}$ For any n. (d) $T \alpha R^{\frac{n}{2}+1}$

70. A particle is released on a vertical smooth semicircular track from point X so that OX makes angle θ from the vertical (see figure). The normal reaction of the track on the particle vanishes at point Y where OY makes angle ϕ with the horizontal. Then : **(2014 Online Set-4)**



(a) $\sin\phi = \frac{1}{2}\cos\theta$ (b) $\sin\phi = \cos\theta$

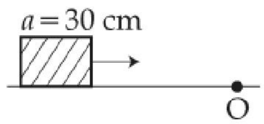
(c) $\sin\phi = \frac{2}{3}\cos\theta$ (d) $\sin\phi = \frac{3}{4}\cos\theta$

71. A particle starts moving on a circle of radius R with initial velocity v_0 such that centripetal L tangential acceleration are equal at all instants. Maximum time for which it can move. **(2015 Online)**

(a) $\frac{R}{v_0}$ (b) $\frac{2R}{v_0}$

(c) $\frac{\pi R}{2v_0}$ (d) Infinite

72. A cubical block of side 30 cm is moving with velocity 2 ms^{-1} on a smooth horizontal surface. The surface has a bump at a point O as shown in figure. The angular velocity (in rad/s) of the block immediately after it hits the bump, is : **(2016 Online Set -1)**



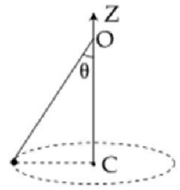
(a) 5.0 (b) 6.7
(c) 9.4 (d) 13.3

73. A civil engineer has to design a circlular banked racing track on which cars can move up to speed of 360 km/hr with coefficient of friction $\mu = \frac{1}{8}$ and radius 4 km. Angle of banking for safe racing should be : **(2016 Online Set-2)**

(a) $\tan^{-1}\left(\frac{7}{33}\right)$ (b) $\tan^{-1}\left(\frac{4}{33}\right)$

(c) $\tan^{-1}\left(\frac{13}{33}\right)$ (d) $\tan^{-1}\left(\frac{7}{29}\right)$

74. A conical pendulum of length 1 m makes an angle $\theta = 45^o$ w.r.t. Z-axis and moves in a circle in the XY plane. The radius of the circle is 0.4m and its center is vertically below O. The speed of the pendulum, in its circular path, will be : (Take $g = 10\ ms^{-2}$) **(2017 Online Set-2)**



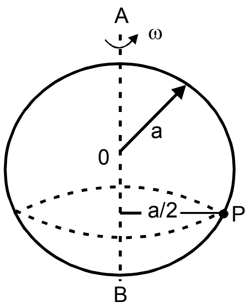
(a) 0.4 m/s (b) 4 m/s
(c) 0.2 m/s (d) 2 m/s

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

Q.1 A rubber band of length ℓ has a stone of mass m tied to its one end. It is whirled with speed v so that the stone describes a horizontal circular path. The tension T in the rubber band is -

(A) zero (B) mv^2/ℓ (C) $> (mv^2)/\ell$ (D) $< mv^2/\ell$

Q.2 A smooth wire is bent into a vertical circle of radius a . A bead P can slide smoothly on the wire. The circle is rotated about diameter AB as axis with a speed ω as shown in figure. The bead P is at rest with respect to the circular ring in the position shown. Then ω^2 is equal to-



(A) $2g/a$ (B) $2g/(a\sqrt{3})$

(C) $g\sqrt{3}\,g/a$ (D) $2a/(g\sqrt{3})$

Q.3 A heavy small sized sphere is suspended by a string of length ℓ. The sphere rotates uniformly in a horizontal circle with the string making an angle θ with the vertical. Then the time period of this conical pendulum is-

(A) $T = 2\pi$ (B) $T=2\pi\sqrt{\left(\frac{l\sin\theta}{g}\right)}$

(C) $T = 2\pi\sqrt{\left(\frac{l\cos\theta}{g}\right)}$ (D) $T=2\pi\sqrt{\left(\frac{l}{g\cos\theta}\right)}$

Q.4 A simple pendulum of length L and mass M is oscillating in a plane about a vertical line between angular limits $-\phi$ and $+\phi$. For an angular displacement θ [$|\theta| < \phi$], the tension in the string and the velocity of the bob are T and v respectively. The following relation holds good under the above conditions-

(A) $T = Mg\cos\theta$

(B) $T\cos\theta = Mg$

(C) $T - Mg\cos\theta = Mv^2/L$

(D) $T + Mg\cos\theta = Mv^2/L$

Q.5 A car is moving with a speed of 30 m/sec on a circular path of radius 500 m. Its is increasing at the rate of 2 m/sec^2. What is the acceleration of the car?

(A) 9.8 m/sec^2 (B) 2.7 m/sec^2

(C) 2.4 m/sec^2 (D) 1.8 m/sec^2

Q.6 The equation of motion of a particle moving on circular path (radius 200 m) is given by $s = 18t + 3t^2 - 2t^3$ where s is the total distance covered from straight point in metres at the end of t seconds. The maximum speed of the particle will be-

(A) 15 m/sec (B) 23 m/sec

(C) 19.5 m/sec (D) 25 m/sec

Q.7 The kinetic energy of a particle moving along a circle of radius R depends on the distance covered s as $T = KS^2$ where K is a constant. Find the force acting on the particle as a function of S -

(A) $\frac{2K}{S}\sqrt{1+\left(\frac{S}{R}\right)^2}$ (B) $2KS\sqrt{1+\left(\frac{R}{S}\right)^2}$

(C) $2KS\sqrt{1+\left(\frac{S}{R}\right)^2}$ (D) $\frac{2S}{K}\sqrt{1+\left(\frac{R}{S}\right)^2}$

Q.8 A point moves along a circle with velocity v = at where a 0.5 m/sec^2. Then the total acceleration of the point at the moment when it covered (1/10) th of the circle after beginning of motion -

(A) 0.5 m/sec^2 (B) 0.6 m/sec^2

(C) 0.7 m/sec^2 (D) 0.8 m/sec^2

Q.9 A solid body rotates about a stationary axis so that its angular velocity depends on the rotation angle φ as $\omega = \omega_0 - k\phi$, where ω_0 and k are positive constants. At the moment t = 0, the angle φ = 0. Find the time dependence of rotation angle -

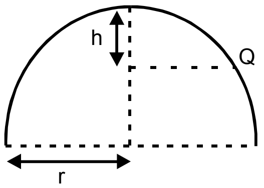
(A) $K.\,\omega_0 e^{-kt}$ (B) $\frac{\omega_0}{K}[e^{-kt}]$

(C) $\frac{\omega_0}{K}[1-e^{-k.t}]$ (D) $\frac{K}{\omega_0}[e^{-kt}-1]$

Q.10 A heavy particle hanging from a fixed point by a light inextensible string of length ℓ is projected horizontally with speed $\sqrt{(gl)}$. Then the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string equal the weight of the particle-

(A) $\sqrt{\frac{3l}{g}}$, $\cos^{-1}(3/2)$ (B) $\sqrt{\frac{lg}{3}}$, $\cos^{-1}(2/3)$
(C) $\sqrt{\frac{3g}{l}}$, $\cos^{-1}(2/3)$ (D) $\sqrt{\frac{gl}{3}}$, $\sin^{-1}(2/3)$

Q.11 A small body of mass m slides without friction from the top of a hemispherical cup of radius r as shown in the following figure. If it leaves the surface of the cup at a vertical distance 'h' below the highest point, then-

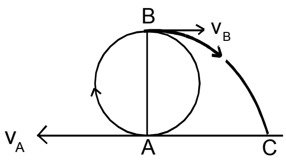


(A) h = r (B) h = r/3
(C) h = r /2 (D) h = 2r /3

Q.12 A body is allowed to slide on a frictionless track from rest position under gravity. The track ends into a circular loop of diameter D. What should be the minimum height of the body in terms of D so that it may complete successfully the loop?

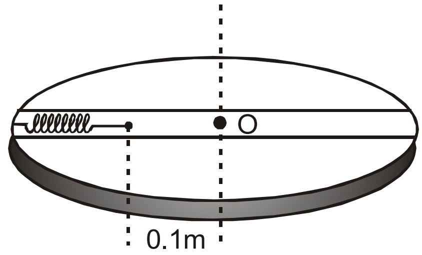
(A) $\frac{4}{5}$D (B) $\frac{5}{4}$D (C) 1 D
(D) 2 D

Q.13 A body is rotated in the vertical plane by means of a thread of length ℓ with minimum possible velocity. When the body up and reaches at the heighest point b of hits path, the thread breaks and the body moves on a parabolic path under the influence the gravitational field as shown in the diagram. The horizontal range AC in the plane of A would be-



(A) x = ℓ (B) x = 2 ℓ
(C) x = $\sqrt{2}\,\ell$ (D) x = 2 $\sqrt{2}\,\ell$

Q.14 A circular turn table of radius 0.5 m has a smooth groove as shown in fig. A ball of mass 90 g is placed inside the groove along with a spring of spring constant 10^2 N/cm. The ball is at a distance of 0.1 m from the centre when the turn table is at rest. On rotating the turn table with a constant angular velocity of 10^2 rad-sec^{-1} the ball moves away from the initial position by a distance nearly equal to-



(A) 10^{-1} m (B) 10^{-2} m
(C) 10^{-3} m (D) 2×10^{-1} m

Q.15 A particle of mass m is attached to one end of a string of length ℓ while the other end is fixed to a point h above the horizontal table, the particle is made to revolve in a circle on the table so as to make p revolutions per second. The maximum value of p if the particle is to be in contact with the table will be-

(A) $2p\sqrt{gh}$ (B) $\sqrt{(g/h)}$
(C) $2p\sqrt{(h/g)}$ (D) $\frac{1}{2\pi}\sqrt{(g/h)}$

Q.16 A gramophone record is revolving with an angular velocity ω. A coin is placed at a distance r from the centre of the record. The static coefficient of friction is μ. The coin will revolve with the record if-

(A) $r > \mu g\, \omega^2$ (B) $r = \mu g/\omega^2$ only
(C) $r < \mu g/\omega^2$ only (D) $r \le \mu g/\omega^2$

Q.17 A car is moving with a speed V on a road inclined at an angle θ in a circular arc of radius r, the minimum coefficent of friction so that the car does not slip away-

(A) $\frac{V^2}{rg} = \mu \tan\theta$ (B) $\mu = V^2/rg$

(C) $\frac{V^2\cos\theta - rg\sin\theta}{rg\cos\theta + V^2\sin\theta}$ (D) $\frac{V^2\cos\theta - rg\sin\theta}{rg\cos\theta - V^2\sin\theta}$

Q.18 A mass of 2.9 kg, is suspended from a string of length 50 cm, and is at rest. Another body of mass 100 gm moving horizontally with a velocity of 150 m/sec, strikes and sticks to it. What is the tension in the string when it makes an angle of 60° with the vertical

(A) 153.3 N (B) 135.3 N
(C) 513.3 N (D) 351.3 N

Q.19 The vertical section of a road over a canal bridge in the direction of its length is in the form of circle of radius 8.9 metre. Then the greatest spped at which the car can cross this bridge wihout losing contact with the road at its hgihest point, the centre of gravity of the car being at a height h = 1.1 metre from the ground. Take g = 10 m/sec^2-

(A) 5 m/sec (B) 10 m/sec
(C) 15 m/sec (D) 20 m/sec

Q.20 A smooth table is placed horizontally and an ideal spring of spring constant k = 1000 N/m and unextended length of 0.5 m has one end fixed to its centre. The other end is attached to a mass of 5 kg which is moving in a circle with constant speed 20 m/s. Then the tension in the spring and the extension of this spring beyond its normal length are-

(A) 500 N, 0.5 m (B) 600 N, 0.6 m (C) 700 N, 0.7 m
(D) 800 N, 0.8 m

Q.21 A body of mass 2 kg is tied at one end of a string 1 m long. The other end is fixed and the body revolves in a horizontal circle. The maximum tension which the string can withstand is 2000 N. Calculate the maximum number of revolutions per minute the body will make and its linear velocity when the string just breaks-

(A) 203 rpm, 13.6 m/sec (B) 32 rpm, 16.3 m/sec
(C) 302 rpm, 61.3 m/sec (D) 302 rpm, 31.6 m/sec

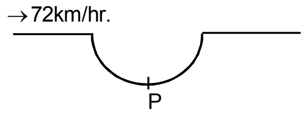
Q.22 A car of mass 1000 kg moves on a circular path with constant speed of 16 m/s. It is turned by 90º after travelling 628 m on the road. The centripetal force acting on the car is-

(A) 160 N (B) 320 N
(C) 640 N (D) 1280 N

Q.23 A car while travelling at a speed of 72 km/hr. Passes through a curved portion of road in the form of an arc of a radius 10 m. If the mass of the car is 500 kg the reaction on the car at the lowest point P is-

(A) 25 KN (B) 50 KN
(C) 75 KN (D) None of these

Q.24 A stone is rotated steadily in a horizontal circle with a time period T by means of a string of length ℓ. If the tension in the string is kept constant and length ℓ increase by 1%, then percentage change in time period T is-

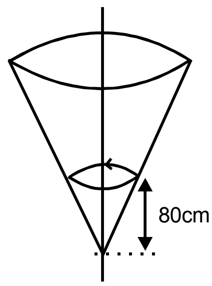


(A) 1 % (B) 0.5 %
(C) 2 % (D) 0.25 %

Q.25 If mass ,speed and radius of rotation of a body moving in a circular path are all increased by 50%, the necessary force required to maintain the body moving in the circular path will have to be increased by-

(A) 225% (B) 125%
(C)150% (D)100 %

Q.26 A particle describing circular motion as shown in figure. The velocity of particle in m/s is-



(A) 2.82
(B) 2.8
(C) 1.42
(D) 1.4

Q.27 A particle P is moving in a circle of radius 'a' with a uniform speed v. C is the centre of the circle and AP is diameter. The angular velocity of P about A and C are in the ratio-

(A) 1 :1 (B) 1 : 2
(C) 2 : 1 (D) 4 : 1

Q.28 A coin placed on a rotating turn table just slips if it is placed at a distace of 4 cm from the centre. If the angular velocity of the turn table is doubled, it will just slip at a distance of-

(A) 1 cm (B) 2 cm
(C) 4 cm (D) 8 cm

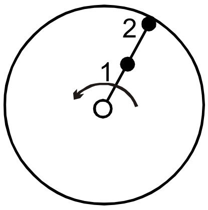
Q.29 In an atom two electrons move round the nucleus in circular orbits of radii r and 4r. The ratio of the time taken by them to complete one revolution is-

(A) 4 :1 (B) 1: 4 (C) 1 : 8
(D) 8 : 1

Q.30 A boy revolves two balls each of mass 100 gm and tied with strings of 1 metre length in horizontal circle as shown in figure. If the speed of outermost ball is 6 m/s , then tension in string-1 is-

(A) 2.4 N (B) 2.7 N
(C) 2 N (D) 1.2 N

Q.31 Three small balls each of mass 100 gm are attached at distance of 1 m, 2 m and 3 m from end D of a 3 m length of string. The string is rotated with uniform angular velocity in a horizontal plane about D. If the outside ball is moving at a speed of 6 m/s, the ratio of tension in the three parts of the string from inside-

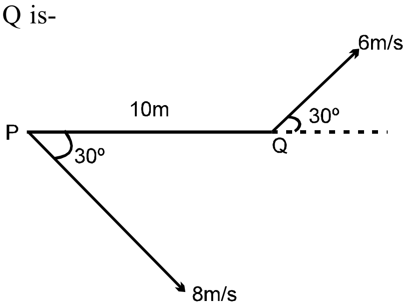


(A) 6 : 5 : 4 (B) 3 : 2 :1
(C) 3 : 5 : 6 (D) 6 : 5 : 3

Q.32 A stone of mass 1 kg tied to a light inextensible string of length 10/3 metre is whirling in a vertical circle. If the ratio of maximum tension to minimum tension in the string is 4, then speed of stone st highest point of the circle is- [g = 10 m/s^2]

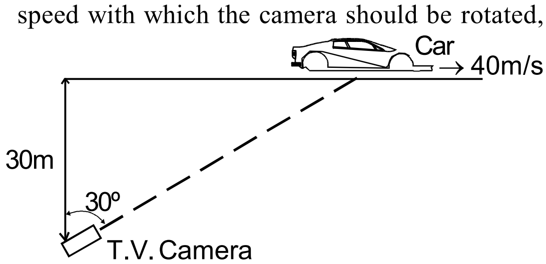
(A) 20 m/s (B) $10\sqrt{3}$ m/s
(C) $5\sqrt{2}$ m/s (D) 10 m/s

Q.33 Tow moving particles P and Q are 10 m apart at a certain instant. The velocity of P is 8 m/s making 30º with the line joining P and Q and that of Q is 6 m/s making an angle 30º with PQ as shown in the firuge .Then angular velocity of P with respect to Q is-



(A) 0 rad/s (B) 0.1 rad/s
(C) 0.4 rad/s (D) 0.7 rad/s

Q.34 A racing car is travelling along a track at a constant speed of 40 m/s. A T.V. camera men is recording the event from a distance of 30 m directly away from the track as shown in figure. In order to keep the car under view in the positio shown, the angular speed with which the camera should be rotated, is-



(A) 4/3 rad/sec (B) 3/4 rad/sec
(C) $8/3\sqrt{3}$ rad/sec (D) 1 rad/sec

Q.35 A particle is moving along a circular path of radiu 3 meter in such a way that the distance travelled measured along the circumference is given by S = $\frac{t^2}{2}+\frac{t^3}{3}$. The accelration of particle when t = 2 sec is-

(A) 1.3 m/s^2 (B) 13 m/s^2
(C) 3 m/s^2 (D) 10 m/s^2

Q.36 A partcile of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2 r t^2$, where k is a constant, the power delivered to the particle by the forces acting on it is-

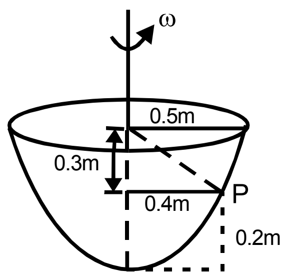
(A) $2 \pi m k^2 r^2 t$ (B) $m k^2 r^2 t$
(C) $(m k^4 r^2 t^5)/3$ (D) 0

Q.37 A partcle rests on the top of a hemisphere of radiu R. Find the smallest horizontal velocity that must be imparted to the particle if it is to leave the hemisphere without sliding down it-

(A) $\sqrt{gR}$ (B) $\sqrt{2gR}$
(C) $\sqrt{3gR}$ (D) $\sqrt{5gR}$

Q.38 A particle P will be in equilibrium inside a hemispherical bowl of radius 0.5 m at a height 0.2 m from the bottom when the bowl is rotated at an angular speed (g = 10 m/sec^2)

(A) $10/\sqrt{3}$ rad/sec
(B) $10\sqrt{3}$ rad/sec
(C) 10 rad/sec
(D $\sqrt{20}$ rad/sec



Theta, omega, alpha, equations o motion

39. A car is moving along a circular path of radius 500 m with a speed of 30 m/s. If at some instant, its speed increases at the rate of 2 m/s^2, then at that instant the magnitude of resultant acceleration will be :

(a) 4.7 m/s^2 (b) 3.8 m/s^2
(c) 3 m/s^2 (d) 2.7 m/s^2

40. A wet, open umbrella is held vertical and is twirled about the handle at a uniform rate of 21 revolutions in 44 second. If the rim of the umbrella is a circle of 1 metre in diameter and the height of the rim above the floor is 4.9 metre, then the angular speed of the umbrella is :

(a) 3 radian/sec (b) 1.5 radian/sec
(c) 1 radian/sec (d) $\sqrt{2.5}$ radian/sec

41. In the above question, the tangential speed of the water drop on leaving the rim of the umbrella is :

(a) 3 m/s (b) 1.5 m/s

(c) 1 m/s (d) $\sqrt{2.5}$ m/s

42. In the above question, the locus of the drops is a circle of radius :

(a) 3 m (b) 1.5 m

(c) 1 m (d) $\sqrt{2.5}$ m

43. If the equation for the displacement of a particle moving on a circular path is given by :

$\theta = 2t^3 + 0.5$

where θ is in radian and t in second, then the angular velocity of the particle at t = 2 sec

(a) 8 rad/sec (b) 12 rad/sec

(c) 24 rad/sec (d) 36 rad/sec

44. A body moves in a circular path of radius r = 500 m with tangential acceleration $a_t = 2\ ms^2$. When its tangential linear velocity is 30 m/s, the total acceleration will be :

(a) 5.4 ms^{-2} (b) 3.9 ms^{-2}

(c) 2.7 ms^{-2} (d) 2.1 ms^{-2}

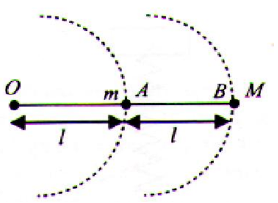
Horizontal Circular Motion

45. A light rigid rod of length L has a bob of mass M attached to one of its end just like a simple pendulum. Speed at the lowest point when it is inverted and relesed is

(a) $\sqrt{gL}$ (b) $\sqrt{2gL}$

(c) $2\sqrt{gL}$ (d) $\sqrt{5gL}$

46. Two identical particles, A and B, are attached to a string of length $2l$, A to middle and B to one of the ends. The string is whirled in a horizontal circle, with the end O fixed. If the kinetic energy of B relative to A is E, then the absolute kinetic energies of A and B are



(a) E and E (b) E and 4E

(c) 4E and E (d) E and 3E

47. A particle moves in a uniform circular motion. Choose the wrong statement :

(a) The particle moves with constant speed

(b) The acceleration is always normal to the velocity

(c) The particle moves with uniform acceleration

(d) The particle moves with variable velocity

48. A particle is moving on a circular track of radius 30 cm with a constant speed of 6 m/s. It acceleration is :

(a) zero (b) 120 m/s^2

(c) 1.2 m/s^2 (d) 36 m/s^2

49. Let a_r and a_t represent radial and tangential acceleration. The motion of a particle may be circular if :

(a) $a_r = 0, a_t = 0$ (b) $a_r = 0, a_t \neq 0$

(c) $a_r \neq 0, a_t = 0$ (d) none of these

50. Two particle of equal masses are revolving in circular paths of radii r_1 and r_2 respectively with the same speed. The ratio of their centripetal forces is :

(a) $\frac{r_2}{r_1}$ (b) $\sqrt{\frac{r_2}{r_1}}$

(c) $\left(\frac{r_1}{r_2}\right)^2$ (d) $\left(\frac{r_2}{r_1}\right)^2$

51. An unbanked curve has a radius of 60 m. The maximum speed at which a car can make a turn if the coefficient of static friction is 0.75 is :

(a) 2.1 m/s (b) 14 m/s

(c) 21 m/s (d) 7 m/s

52. If the banking angle of curved road is given by $\tan^{-1}(3/5)$ and the radius of curvature of the road is 6 m, then the safe driving speed should not exceed : ($g = 10\ m/s^2$)

(a) 86.4 km/h (b) 43.2 km/h

(c) 21.6 km/h (d) 30.4 km/h

53. A circular road of radius 1000 m has banking angle 45°. The maximum safe speed of a car having mass 200 kg will be, if the coefficient of friction between tyres and road is 0.5 :

(a) 172 m/s (b) 124 m/s

(c) 99 m/s (d) 86 m/s

54. A motorcyclist wants to drive on the vertical surface of wooden 'well' or radius 5 m, with a minimum speed of $5\sqrt{5}$ m/s. The minimum value of coefficient of friction between the tyres and the wall of the well must be :

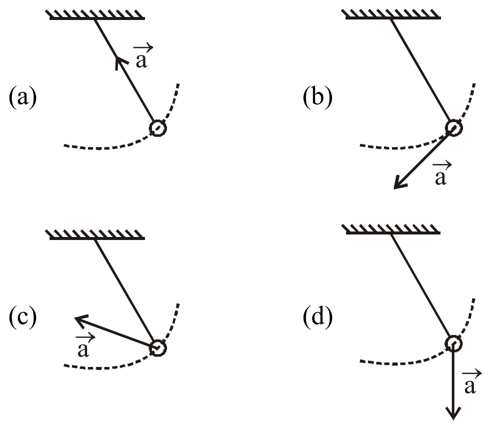
(take $g = 10\ m/s^2$)

(a) 0.10 (b) 0.20

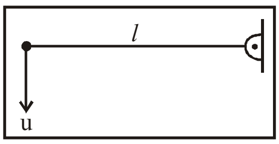
(c) 0.30 (d) 0.40

Vertical Circular Motion

55. A simple pendulum is oscillating without damping. When the displacement of the bob is less than maximum, its acceleration vector $\vec{a}$ is correctly shown in :



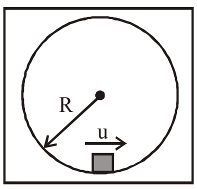
56. A ball of mass m is attached to one end of a light rod of length *l*, the other end of which is hinged. What minimum velocity v should be imparted to the ball downwards, so that it can complete the circle.



(a) $\sqrt{g\ell}$ (b) $\sqrt{5g\ell}$

(c) $\sqrt{3g\ell}$ (d) $\sqrt{2g\ell}$

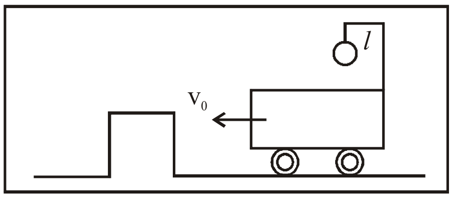
57. A particle is given an initial speed u inside a smooth spherical shell of radius R = 1 m that it is just able to complete the circle. Acceleration of the particle when its velocity is vertical is



(a) $g\sqrt{10}$ (b) g

(c) $g\sqrt{2}$ (d) 3g

58. A bob is suspended from a crane by a cable of length $\ell = 5$ m. The crane and load are moving at a constant speed v_0. The crane is stopped by a bumper and the bob on the cable swings out an angle of 60°. The initial speed v_0 is ($g = 9.8\ m/s^2$)



(a) 10 m/s (b) 7 m/s

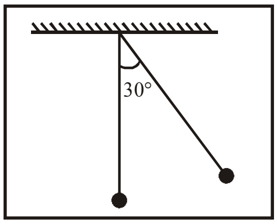
(c) 4 m/s (d) 2 m/s

59. A particle suspended from a fixed point, by a light inextensible thread of length L is projected horizontally from its lowest position with velocity $\sqrt{7gL/2}$. The string will slack after swinging through an angle θ, such that θ equals :

(a) 30° (b) 135°

(c) 120° (d) 150°

60. A simple pendulum is released from rest from the point A at an angle 30° with vertical. Then :



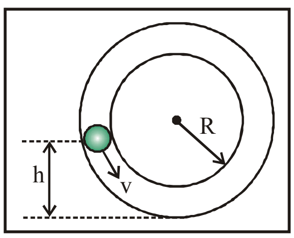
(a) vertical component of velocity of the bob is always less than its, horizontal component.

(b) vertical component of velocity is less than, equal to or more than the horizontal component on different position.

(c) vertical component of velocity is always more than the horizontal component.

(d) acceleration of the bob is constant throughout.

61. With what minimum speed v must a small ball should be pushed inside a smooth vertical tube from a height h so that it may reach the top of the tube ? Radius of the tube is R.



(a) $\sqrt{2g(h+2R)}$ (b) $\frac{5}{2}R$

(c) $\sqrt{g(5R-2h)}$ (d) $\sqrt{2g(2R-h)}$

62. A body of mass 1 kg is moving in a verticle circular path of radius 1 m. The difference between the kinetic energies at its highest and lowest position is :

(a) 20 J (b) 10 J

(c) $4\sqrt{5}$ J (d) $10(\sqrt{5}-1)$ J

63. A block of mass m, slides down along the surface of a bowl (radius R) from the rim to the bottom. The velocity of the block at the bottom will be :

(a) $\sqrt{\pi R g}$ (b) $2\sqrt{\pi R g}$

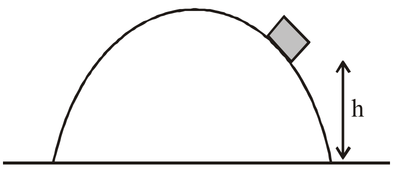
(c) $\sqrt{2Rg}$ (d) $\sqrt{gR}$

64. A simple pendulum 1 metre long has a bob of 10 kg. If the pendulum swings from a horizontal position, the K.E. of the bob, at the instant it passes through the lowest position of its path is

(a) 89 joule (b) 95 joule

(c) 98 joule (d) 85 joule

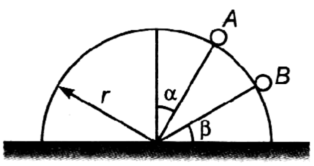
65. A small body of mass m slides without friction fron the top of a hemisphere of radius r. At what height will the body be detached from the centre of hemisphere ?



(a) $h=\frac{r}{2}$ (b) $h=\frac{r}{3}$

(c) $h=\frac{2r}{3}$ (d) $h=\frac{r}{4}$

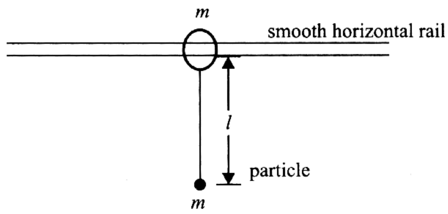
66. A particle moves from rest at A on the surface of a smooth circular cylinder of radius r as shown. At B it leaves the cylinder. The equation relation α and β is :



(a) 3 sin α = 2 cos β (b) 2 sin α = 3 cos β

(c) 3 sin β = 2 cos α (d) 2 sin β = 3 cos α

67. The ring shown in the figure is given a constant horizontal acceleration ($a_0 = g/\sqrt{3}$). Maximum deflection of the string from the vertical is θ_0, then



(a) $\theta_0 = 30°$ (b) $\theta_0 = 60°$

(c) at maximum deflection, tension in string is equal to mg.

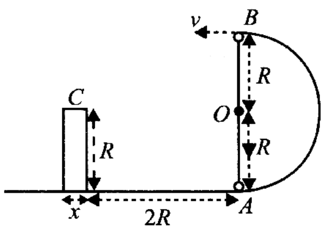
(d) At maximum deflection, tension in string is equal to $\frac{2mg}{\sqrt{3}}$.

Passage - 3

Using the following passage, sovle Q. 68 to 70

A small ball is given some velocity at point A towards right so that it moves on teh semicirculare track and does not leave contact upto the highest point B. After leaving the highest point B. it falls at the top of a building of height R and width x(x << 2R). (All the surface are frictionless.)

68. The velocity given to the ball at point A so that it may hit the top of the building is



(a) $\sqrt{4gR}$ (b) $\sqrt{2gR}$

(c) $\sqrt{gR}$ (d) $\sqrt{6gR}$

69. If the collision of ball with the building is elastic, then the angle with the horizontal at which the ball will rebound from the top of the building is

(a) 60° (b) 45°

(c) 30° (d) none

70. The horizontal distance of the ball from the foot of building where the ball strikes the horizontal ground will be

(a) $\sqrt{2}R$ (b) $(1+\sqrt{2})R$

(c) $2(1+\sqrt{2})R$ (d) 12R

EXERCISE - 4 : PREVIOUS YEARS JEE ADVANCED QUESTIONS

SECTION - A

1. The coordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by – **[AIEEE-2003]**

(A) $3t^2\sqrt{\alpha^2+\beta^2}$ (B) $t^2\sqrt{\alpha^2+\beta^2}$

(C) $\sqrt{\alpha^2+\beta^2}$ (D) $3t\sqrt{\alpha^2+\beta^2}$

2. Which of the following statements is FALSE for a particle moving in a circle with a constant angular speed? **[AIEEE-2004]**

(A) The velocity vector is tangent to the circle
(B) The acceleration vector is tangent to the circle
(C) The acceleration vector ponits to the centre of the circle
(D) The velocity and acceleration vectors and perpendicular to each other

3. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle, the motion of the particle takes place in a plane. It follows that – **[AIEEE-2004]**

(A) Its velocity is constant
(B) Its acceleration is constant
(C) Its kinetic energy is constant
(D) It moves in a straight line

SECTION - B

1. A rod of length L is hinged from one end. It is brought to a horizontal position and released. The angular velocity of the rod when it is in vertical position is – **[IIT- 1990]**

(A) $\sqrt{2g/L}$ (B) $\sqrt{3g/L}$

(C) $\sqrt{g/2L}$ (D) $\sqrt{g/L}$

2. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/s. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1.0 m. The angle made by the rod with the track is– **[IIT- 1992]**

(A) Zero (B) 30º
(C) 45º (D) 60º

3. A tube of length L is filled completely with an incompressible liquid of mass M and closed at both ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω.The force exerted by the liquid at the other end is – **[IIT -1992]**

(A) ML ω^2 /2 (B) ML ω^2
(C) ML ω^2 /4 (D) $ML^2\omega^2$ /2

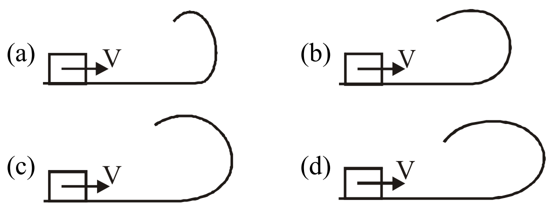
4. A stone of mass m, tied to the end of a string, is whirled around in a horizontal circle (neglect the force due to gravity). The length of the string is reduced gradually keeping the angular momentum of the stone about the centre of the circle constant. Then, the tension in the string is given by $T = Ar^n$, where A is a constant, r is the instantaneous radius of the circle, and n = ... **[IIT– 1993]**

(A) –3 (B) –5 (C) 3 (D) 5

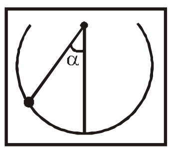
5. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2 rt^2$, where k is a constant . The power delivered to the particle by the forces acting on it is – **[IIT - 1994]**

(A) $2\pi mk^2 r^2 t$ (B) mk^2r^2t
(C) $(mk^4r^2t^5)$ /3 (D) zero

6. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in – **[IIT - 2001]**

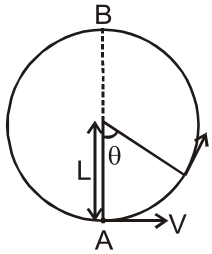


7. An insect crawls up a hemispherical surface very slowly (see the figure). The coefficient of friction between the insect and the surface is 1/3. If the line joining the centre of the hemispherical surface to the insect makes an angle α with the vertical, the maximum possible value of α is given by – **[IIT-2001]**



(A) cot α = 3 (B) tan α = 3
(C) sec = 3 (D) cosec α = 3

8. A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A satisfies – **[IIT-2008]**



(A) $\theta = \frac{\pi}{4}$ (B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

(C) $\frac{\pi}{2} < \theta < \pi$ (D) $\frac{3\pi}{4} < \theta < \pi$

9. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is : **(1998)**

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

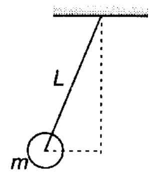
(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

10. The work done on a particle of mass m by a force, $K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is **(2013)**

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$

(c) $\frac{K\pi}{2a}$ (d) 0

11. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizonal circuit path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in rad/s) is : **2011**



(a) 9 (b) 18

(c) 27 (d) 36

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (c)	**2.** (b)	**3.** (d)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (b)	**8.** (c)	**9.** (c)	**10.** (c)
11. (a)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (c)	**16.** (c)	**17.** (a)	**18.** (c)	**19.** (c)	**20.** (c)
21. (c)	**22.** (a)	**23.** (a)	**24.** (b)	**25.** (c)	**26.** (b)	**27.** (b)	**28.** (b)	**29.** (a)	**30.** (a)
31. (c)	**32.** (b)	**33.** (c)	**34.** (c)	**35.** (b)	**36.** (b)	**37.** (d)	**38.** (a)	**39.** (b)	**40.** (a)
41. (b)	**42.** (b)	**43.** (b)	**44.** (a)	**45.** (b)	**46.** (b)	**47.** (d)	**48.** (c)	**49.** (c)	**50.** (a)
51. (c)	**52.** (c)	**53.** (b)	**54.** (c)	**55.** (a)	**56.** (c)	**57.** (b)	**58.** (a)	**59.** (a)	**60.** (a)
61. (d)	**62.** (b)	**63.** (b)	**64.** (d)	**65.** (b)	**66.** (d)	**67.** (b)	**68.** (c)	**69.** (c)	**70.** (b)
71. (c)	**72.** (a)	**73.** (c)	**74.** (d)	**75.** (c)	**76.** (b)	**77.** (d)	**78.** (a)	**79.** (c)	**80.** (c)

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (c)	**2.** (c)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (d)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (a)
11. (c)	**12.** (a, c)	**13.** (d)	**14.** (d)	**15.** (c)	**16.** (b)	**17.** (a)	**18.** (b)	**19.** (d)	**20.** (b)
21. (c)	**22.** (a)	**23.** (b)	**24.** (a)	**25.** (b)	**26.** (d)	**27.** (d)	**28.** (a)	**29.** (b)	**30.** (d)
31. (d)	**32.** (c)	**33.** (c)	**34.** (a)	**35.** (a)	**36.** (c)	**37.** (c)	**38.** (d)	**39.** (b)	**40.** (b)
41. (c)	**42.** (c)	**43.** (b)	**44.** (c)	**45.** (d)	**46.** (d)	**47.** (c)	**48.** (a)	**49.** (d)	**50.** (b)
51. (c)	**52.** (c)	**53.** (c)	**54.** (d)	**55.** (c)	**56.** (b)	**57.** (a)	**58.** (d)	**59.** (b)	**60.** (c)
61. (b)	**62.** (d)	**63.** (c)	**64.** (a)	**65.** (c)	**66.** (c)	**67.** (d)	**68.** (c)	**69.** (a)	**70.** (c)
71. (a)	**72.** (a)	**73.** (b)	**74.** (d)						

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (c)	**2.** (b)	**3.** (c)	**4.** (c)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (d)	**9.** (c)	**10.** (b)
11. (b)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (d)	**16.** (d)	**17.** (c)	**18.** (b)	**19.** (b)	**20.** (a)
21. (d)	**22.** (c)	**23.** (a)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (b)	**28.** (a)	**29.** (c)	**30.** (b)
31. (d)	**32.** (d)	**33.** (d)	**34.** (d)	**35.** (b)	**36.** (b)	**37.** (a)	**38.** (a)	**39.** (d)	**40.** (a)
41. (b)	**42.** (d)	**43.** (c)	**44.** (c)	**45.** (c)	**46.** (b)	**47.** (c)	**48.** (b)	**49.** (c)	**50.** (a)
51. (c)	**52.** (c)	**53.** (a)	**54.** (d)	**55.** (c)	**56.** (d)	**57.** (a)	**58.** (b)	**59.** (c)	**60.** (a)
61. (d)	**62.** (a)	**63.** (c)	**64.** (c)	**65.** (c)	**66.** (c)	**67.** (b, c)	**68.** (d)	**69.** (b)	**70.** (c)

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Section - A

1. (a)	**2.** (b)	**3.** (c)

Section - B

1. (b)	**2.** (c)	**3.** (a)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (a)	**8.** (d)	**9.** (d)	**10.** (d)
11. (d)									

Dream on !!