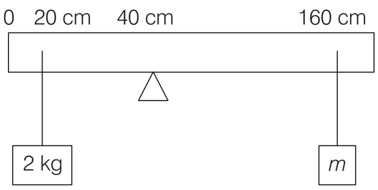
06

System of Particles and Rotational Motion

TOPIC 1
Centre of Mass, Torque and Angular Momentum

01 A uniform rod of length 200 cm and mass 500 g is balanced on a wedge placed at 40 cm mark. A mass of 2 kg is suspended from the rod at 20 cm and another unknown mass m is suspended from the rod at 160 cm mark as shown in the figure. Find the value of m such that the rod is in equilibrium. ($g = 10\ \text{m/s}^2$) **[NEET 2021]**



(a) $\frac{1}{2}$ kg (b) $\frac{1}{3}$ kg

(c) $\frac{1}{6}$ kg (d) $\frac{1}{12}$ kg

Ans. *(d)*

Given, the length of a uniform rod, $L = 200$ cm

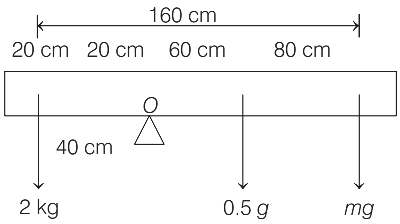
The wedge is placed at the mark = 40 cm

The balanced mass placed at 100 cm mark on wedge = 500 g = 0.5 kg

The mass suspended from the rod at 20 cm distance from the end, $M = 2$ kg

Another unknown mass suspended from the rod at 160 cm distance from the end = m

Let's draw the diagram of the uniform rod suspended with mass.



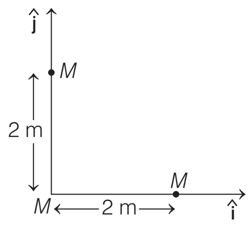
As we know in equilibrium net moment of force is equals to zero.

$\Rightarrow 0.5\,g\,(0.60) + mg\,(1.20) - 2g\,(0.20) = 0$

$\Rightarrow 0.3 + 1.20\,m - 0.4 = 0$

$\Rightarrow 1.20\,m = 0.1 \quad \Rightarrow \quad m = \frac{1}{12}$ kg

02 Three identical spheres, each of mass M, are placed at the corners of a right angle triangle with the mutually perpendicular sides equal to 2 m (see figure). Taking the point of intersection of the two mutually perpendicular sides as the origin, find the position vector of centre of mass. **[NEET (Oct.) 2020]**

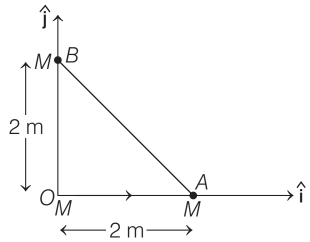


(a) $2(\hat{i} + \hat{j})$ (b) $(\hat{i} + \hat{j})$

(c) $\frac{2}{3}(\hat{i} + \hat{j})$ (d) $\frac{4}{3}(\hat{i} + \hat{j})$

Ans. *(b)*

The given situation is shown in the figure.



$\mathbf{OA} = 2\hat{\mathbf{i}}$

$\mathbf{OB} = 2\hat{\mathbf{j}}$

Position vector of centre of mass,

$$R_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{M_1 + M_2} = \frac{M\mathbf{OA} + M\mathbf{OB}}{M + M} = \frac{M \times 2\hat{\mathbf{i}} + M \times 2\hat{\mathbf{j}}}{2M} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$$

03 Find the torque about the origin when a force of $3\hat{\mathbf{j}}$ N acts on the particle whose position vector is $2\hat{\mathbf{k}}$ m. **[NEET (Sep.) 2020]**

(a) $6\,\hat{\mathbf{j}}$ N-m (b) $-6\,\hat{i}$ N-m

(c) $6\,\hat{\mathbf{k}}$ N-m (d) $6\,\hat{\mathbf{i}}$ N-m

Ans. *(b)*

Given, position vector, $\mathbf{r} = 2\hat{\mathbf{k}}$ m

Force, $\mathbf{F} = 3\hat{\mathbf{j}}$ N

As, torque, $\tau = \mathbf{r} \times \mathbf{F} = 2\hat{\mathbf{k}} \times 3\hat{\mathbf{j}} = 6(-\hat{\mathbf{i}})$

$= -6\hat{\mathbf{i}}$ N-m

Hence, correct option is (b).

04 Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass.

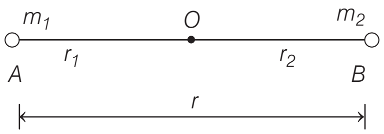
The centre of mass of the system from the 5 kg particle is nearly at a distance of **[NEET (Sep.) 2020]**

(a) 50 cm (b) 67 cm
(c) 80 cm (d) 33 cm

Ans. *(b)*

Given, $m_1 = 5\,\text{kg}$, $m_2 = 10\,\text{kg}$ and
$r = 1\,\text{m} = 100\,\text{cm}$

Let the centre of mass lies at origin O.



$$\therefore \quad \frac{m_1 r_1 - m_2 r_2}{m_1 + m_2} = 0$$

$$\Rightarrow \quad 5r_1 - 10r_2 = 0$$

$$\Rightarrow \quad r_2 = \frac{r_1}{2}$$

Also, $\quad r_1 + r_2 = 100$

$$\Rightarrow \quad r_1 + \frac{r_1}{2} = 100$$

$$\Rightarrow \quad 3r_1 = 200$$

$$\Rightarrow \quad r_1 = \frac{200}{3} \simeq 67\,\text{cm}$$

Hence, correct option is (b).

05 The moment of the force, $\mathbf{F} = 4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$ at $(2, 0, -3)$, about the point $(2, -2, -2)$, is given by **[NEET 2018]**

(a) $-7\hat{\mathbf{i}} - 8\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ (b) $-4\hat{\mathbf{i}} - \hat{\mathbf{j}} - 8\hat{\mathbf{k}}$
(c) $-8\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 7\hat{\mathbf{k}}$ (d) $-7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$

Ans. *(d)*

Key Concept *Moment of force is defined as the cross product of the force and the force arm.*

Given, $\quad \mathbf{F} = 4\hat{\mathbf{j}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}}$

$$\mathbf{r}_1 = 2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$$

$$\mathbf{r}_2 = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$$

Moment of force $= \mathbf{r} \times \mathbf{F}$

$$= (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$$

$$= [-(2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}) + (2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - 3\hat{\mathbf{k}})] \times [4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}}]$$

$$= [0\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 1\hat{\mathbf{k}}] \times [4\hat{\mathbf{i}} + 5\hat{\mathbf{j}} - 6\hat{\mathbf{k}}]$$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 0 & 2 & -1 \\ 4 & 5 & -6 \end{vmatrix}$$

$$= \hat{\mathbf{i}}[(-6 \times 2) - (-1 \times 5)] - \hat{\mathbf{j}}[(-6 \times 0) - (-1 \times 4)] + \hat{\mathbf{k}}[(0 \times 5) - 2 \times 4]$$

$$= -7\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$$

06 A rope is wound around a hollow cylinder of mass 3 kg and radius 40 cm. What is the angular acceleration of the cylinder, if the rope is pulled with a force of 30 N? **[NEET 2017]**

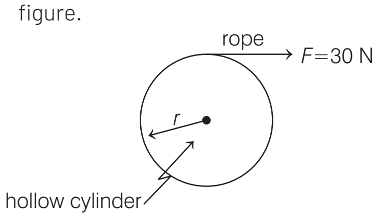
(a) $25\,\text{m/s}^2$ (b) $0.25\,\text{rad/s}^2$
(c) $25\,\text{rad/s}^2$ (d) $5\,\text{m/s}^2$

Ans. *(c)*

Thinking Process Torque (τ) acting on a body and angular acceleration (α) produced in it are related as

$$\tau = I\alpha$$

Consider a hollow cylinder, around which a rope is wounded as shown in the figure.



Torque acting on the cylinder due to the force F is

$$\tau = Fr$$

Now, we have $\quad \tau = I\alpha$

where, I = moment of inertia of the cylinder about the axis through the centre $= mr^2$

α = angular acceleration

$$\Rightarrow \quad \alpha = \frac{\tau}{I} = \frac{Fr}{mr^2} = \frac{F}{mr} = \frac{30}{3 \times 40 \times 10^{-2}}$$

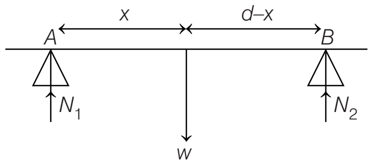
$$= \frac{100}{4} = 25\,\text{rad/s}^2$$

07 A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at distance x from A. The normal reaction on A is **[CBSE AIPMT 2015]**

(a) $\frac{wx}{d}$ (b) $\frac{wd}{x}$
(c) $\frac{w(d-x)}{x}$ (d) $\frac{w(d-x)}{d}$

Ans. *(d)*

As the weight w balances the normal reactions.



So, $\quad w = N_1 + N_2 \quad$ ...(i)

Now balancing torque about the COM, i.e. anti-clockwise momentum = clockwise momentum

$$\Rightarrow \quad N_1 x = N_2 (d - x)$$

Putting the value of N_2 from Eq. (i), we get

$$N_1 x = (w - N_1)(d - x)$$

$$\Rightarrow \quad N_1 x = wd - wx - N_1 d + N_1 x$$

$$\Rightarrow \quad N_1 d = w(d - x)$$

$$\Rightarrow \quad N_1 = \frac{w(d - x)}{d}$$

08 An automobile moves on a road with a speed of $54\,\text{kmh}^{-1}$. The radius of its wheels is 0.45 m and the moment of inertia of the wheel about its axis of rotation is $3\,\text{kg m}^2$. If the vehicle is brought to rest in 15 s, the magnitude of average torque transmitted by its brakes to the wheel is **[CBSE AIPMT 2015]**

(a) $6.66\,\text{kg m}^2\,\text{s}^{-2}$ (b) $8.58\,\text{kg m}^2\,\text{s}^{-2}$
(c) $10.86\,\text{kg m}^2\,\text{s}^{-2}$ (d) $2.86\,\text{kg m}^2\,\text{s}^{-2}$

Ans. *(a)*

As velocity of an automobile vehicle,

$$v = 54\,\text{km/h} = 54 \times \frac{5}{18} = 15\,\text{m/s}$$

Angular velocity of a vehicle, $v = \omega_0 r$

$$\Rightarrow \quad \omega_0 = \frac{v}{R} = \frac{15}{0.45} = \frac{100}{3}\,\text{rad/s}$$

So, angular acceleration of an automobile,

$$\alpha = \frac{\Delta\omega}{t} = \frac{\omega_f - \omega_0}{t} = \frac{0 - \frac{100}{3}}{15}$$

$$= \frac{-100}{45}\,\text{rad/s}^2$$

Thus, average torque transmitted by its brakes to wheel

$$\tau = I\alpha \quad \Rightarrow \quad 3 \times \frac{100}{45} = 6.66\,\text{kgm}^2\text{s}^{-2}$$

09 A force $\mathbf{F} = \alpha\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$ is acting at a point $\mathbf{F} = 2\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - 12\hat{\mathbf{k}}$. The value of α for which angular momentum about origin is conserved is **[CBSE AIPMT 2014]**

(a) – 1 (b) 2 (c) zero (d) 1

Ans. *(a)*

Key Concept When the resultant external torque acting on a system is zero, the total angular momentum of a system remains constant. This is the principle of the conservation of angular momentum.

Given, force $\mathbf{F} = \alpha\,\hat{\mathbf{i}} + 3\,\hat{\mathbf{j}} + 6\,\hat{\mathbf{k}}$ is acting at a point

$$\mathbf{r} = 2\,\hat{\mathbf{i}} - 6\,\hat{\mathbf{j}} - 12\hat{\mathbf{k}}$$

As, angular momentum about origin is conserved.

i.e. $\tau = \text{constant}$

$\Rightarrow$ Torque, $\tau = 0 \Rightarrow \mathbf{r} \times \mathbf{F} = 0$

$$\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -6 & -12 \\ \alpha & 3 & 6 \end{vmatrix} = 0$$

$\Rightarrow$

$(-36 + 36)\hat{\mathbf{i}} - (12 + 12\alpha)\hat{\mathbf{j}} + (6 + 6\alpha)\hat{\mathbf{k}} = 0$

$\Rightarrow \quad 0\,\hat{\mathbf{i}} - 12(1 + \alpha)\hat{\mathbf{i}} + 6(1 + \alpha)\hat{\mathbf{k}} = 0$

$\Rightarrow \quad 6(1 + \alpha) = 0$

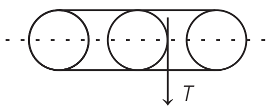
$\Rightarrow \quad \alpha = -1$

So, value of α for angular momentum about origin is conserved, $\alpha = -1$

10 A solid cylinder of mass 50 kg and radius 0.5 m is free to rotate about the horizontal axis. A massless string is wound round the cylinder with one end attached to it and other hanging freely. Tension in the string required to produce an angular acceleration of 2 rev/s^2 is **[CBSE AIPMT 2014]**

(a) 25 N (b) 50 N (c) 78.5 N (d) 157 N

Ans. *(d)*



Fixed horizontal axis

Given, $m = 50$ kg, $r = 0.5$ m,

$$\alpha = 2 \text{ rev / s}^2$$

$\Rightarrow$ Torque produced by the tension in the string

$$= T \times r = T \times 0.5 = \frac{T}{2}\ \text{N-m} \quad \text{...(i)}$$

We know $\tau = I\alpha$ ...(ii)

From Eqs. (i) and (ii), $\dfrac{T}{2} = I\alpha$

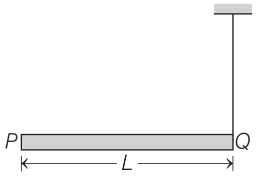
$$= \left(\frac{MR^2}{2}\right) \times (2 \times 2\pi)\ \text{rad / s}^2$$

$$\left[\text{because } I_{\text{Solid cylinder}} = \frac{MR^2}{2}\right]$$

$$\frac{T}{2} = \frac{50 \times (0.5)^2}{2} \times 4\pi$$

$$T = 50 \times \frac{1}{4} \times 4\pi = 50\ \pi = 157\ \text{N}$$

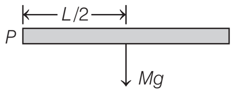
11 A rod PQ of mass M and length L is hinged at end P. The rod is kept horizontal by a massless string tied to a point Q as shown in figure. When string is cut, the initial angular acceleration of the rod is **[NEET 2013]**



(a) $\dfrac{3g}{2L}$ (b) $\dfrac{g}{L}$ (c) $\dfrac{2g}{L}$ (d) $\dfrac{2g}{3L}$

Ans. *(a)*

Concept Torque on the rod is equal to moment of weight of rod about P.



Torque on the rod = Moment of weight of the rod about P

$$\tau = Mg\frac{L}{2} \quad \text{...(i)}$$

$\because$ Moment of inertia of rod

$$\text{about } P = \frac{ML^2}{3} \quad \text{...(ii)}$$

As $\tau = I\alpha$

From Eqs. (i) and (ii), we get

$$Mg\frac{L}{2} = \frac{ML^2}{3}\alpha \Rightarrow \alpha = \frac{3g}{2L}$$

12 When a mass is rotating in a plane about a fixed point, its angular momentum is directed along **[CBSE AIPMT 2012]**

(a) a line perpendicular to the plane of rotation

(b) the line making an angle of 45° to the plane of rotation

(c) the radius

(d) the tangent to the orbit

Ans. *(a)*

As we know that

Angular momentum $\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$

So, here angular momentum is directed along a line perpendicular to the plane of rotation.

13 Two persons of masses 55 kg and 65 kg respectively, are at the opposite ends of a boat. The length of the boat is 3 m and weighs 100 kg. The 55 kg man walks upto the 65 kg man and sits with him. If the boat is in still water the centre of mass of the system shifts by **[CBSE AIPMT 2012]**

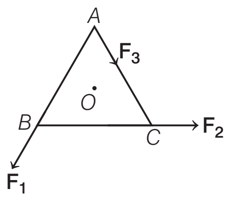
(a) 3 m (b) 2.3 m

(c) zero (d) 0.75 m

Ans. *(c)*

Here on the entire system net external force on the system is zero hence centre of mass remains unchanged.

14 ABC is an equilateral triangle with O as its centre. $\mathbf{F_1}$, $\mathbf{F}_2$ and $\mathbf{F}_3$ represent three forces acting along the sides AB, BC and AC, respectively. If the total torque about O is zero, then the magnitude of $\mathbf{F}_3$ is **[CBSE AIPMT 2012]**



(a) $F_1 + F_2$ (b) $F_1 - F_2$

(c) $\dfrac{F_1 + F_2}{2}$ (d) $2(F_1 + F_2)$

Ans. *(a)*

If we take clockwise torque

$$\tau_{\text{net}} = \tau_{F_1} + \tau_{F_2} + \tau_{F_3}$$

$$0 = F_1 r + F_2 r + F_3 r$$

$$\Rightarrow \quad F_3 = F_1 + F_2$$

15 The instantaneous angular position of a point on a rotating wheel is given by the equation $Q(t) = 2t^3 - 6t^2$. The torque on the wheel becomes zero at **[CBSE AIPMT 2011]**

(a) $t = 0.5$ s (b) $t = 0.25$ s

(c) $t = 2$ s (d) $t = 1$ s

Ans. *(d)*

According to question, torque $\tau = 0$

It means that, $\alpha = 0$

$$\alpha = \frac{d^2\theta}{dt^2}$$

Given, $\theta(t) = 2t^3 - 6t^2$

So, $$\frac{d\theta}{dt} = 6t^2 - 12t$$

$$\alpha = \frac{d^2\theta}{dt^2} = 12t - 12$$

$$12t - 12 = 0 \Rightarrow t = 1\,\text{s}$$

16 Two particles which are initially at rest, move towards each other under the action of their internal attraction. If their speeds are v and $2v$ at any instant, then the speed of centre of mass of the system will be **[CBSE AIPMT 2010]**

(a) $2v$ (b) 0
(c) $1.5v$ (d) v

Ans. *(b)*

As initially both the particles were at rest therefore velocity of centre of mass was zero and there is no external force on the system so speed of centre of mass remains constant i.e it should be equal to zero.

17 Two bodies of masres 1 kg and 3 kg have position vectors $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, respectively. The centre of mass of this system has a position vector **[CBSE AIPMT 2009]**

(a) $-2\hat{\mathbf{i}} + 2\hat{\mathbf{k}}$ (b) $-2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$
(c) $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$ (d) $-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Ans. *(b)*

The position vector of centre of mass

$$\mathbf{r} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

$$= \frac{1(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) + 3(-3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}})}{1+3}$$

$$= \frac{1}{4}(-8\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + 4\hat{\mathbf{k}}) = -2\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}}$$

18 A thin circular ring of mass M and radius R is rotating in a horizontal plane about an axis vertical to its plane with a constant angular velocity ω. If two objects each of mass m be attached gently to the opposite ends of a diameter of the ring, the ring will then rotate with an angular velocity **[CBSE AIPMT 2009, 1998]**

(a) $\frac{\omega(M-2m)}{M+2m}$ (b) $\frac{\omega M}{M+2m}$
(c) $\frac{\omega(M+2m)}{M}$ (d) $\frac{\omega M}{M+m}$

Ans. *(b)*

Concept Apply parallel axes theorem of moment of inertia.

According to question by applying conservation of angular momentum

$$I_1\omega_1 = I_2\omega_2$$

In the given case

$$I_1 = MR^2$$
$$I_2 = MR^2 + 2mR^2$$
$$\omega_1 = \omega$$

Then, $$\omega_2 = \frac{I_1}{I_2}\omega = \frac{M}{M+2m}\omega$$

19 If $\mathbf{F}$ is the force acting on a particle having position vector $\mathbf{r}$ and τ be the torque of this force about the origin, then **[CBSE AIPMT 2009]**

(a) $\mathrm{r}\cdot\tau \neq 0$ and $\mathbf{F}\cdot\tau = 0$
(b) $\mathrm{r}\cdot\tau > 0$ and $\mathbf{F}\cdot\tau < 0$
(c) $\mathrm{r}\cdot\tau = 0$ and $\mathbf{F}\cdot\tau = 0$
(d) $\mathrm{r}\cdot\tau = 0$ and $\mathbf{F}\cdot\tau \neq 0$

Ans. *(c)*

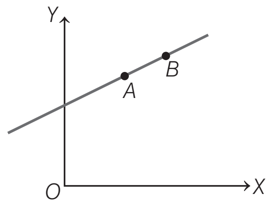
$\tau = \mathbf{r}\times\mathbf{F}$, where $\mathbf{r}$ = position vector

$\mathbf{F}$ = force $\Rightarrow \tau = |\mathbf{r}|\cdot|\mathbf{F}|\sin\theta$

Torque is perpendicular to both $\mathbf{r}$ and $\mathbf{F}$. So, dot product of two vectors will be zero.

$$\tau\cdot\mathbf{r} = 0 \Rightarrow \mathbf{F}\cdot\tau = 0$$

20 A particle of mass m in the XY-plane with a velocity v along the straight line AB. If the angular momentum of the particle with respect to origin O is L_A when it is at A and L_B when it is at B, then **[CBSE AIPMT 2007]**



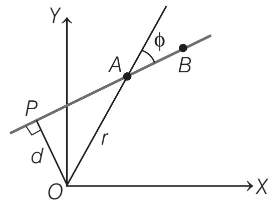
(a) $L_A > L_B$
(b) $L_A = L_B$
(c) the relationship between L_A and L_B depends upon the slope of the line AB
(d) $L_A < L_B$

Ans. *(b)*

From the definition of angular momentum,

$$\mathbf{L} = \mathbf{r}\times\mathbf{p} = rmv\sin\phi(-\mathrm{k})$$

$$\begin{bmatrix} \mathbf{r} = \text{position vector} \\ \mathbf{p} = \text{momentum} \end{bmatrix}$$

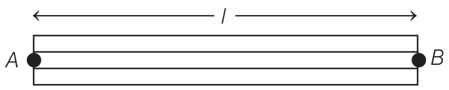


Therefore, the magnitude of L is

$$L = mvr\sin\phi = mvd$$

where, $d = r\sin\phi$ is the distance of closest approach of the particle to the origin. As d is same for both the particles, hence $L_A = L_B$.

21 A uniform rod of length l and mass m is free to rotate in a vertical plane about A. The rod initially in horizontal position is released. The initial angular acceleration of the rod is (moment of inertia of rod about A is $\frac{ml^2}{3}$) **[CBSE AIPMT 2007]**



(a) $\frac{3g}{2l}$ (b) $\frac{2l}{3g}$ (c) $\frac{3g}{2l^2}$ (d) $mg\frac{l}{2}$

Ans. *(a)*

The moment of inertia of the uniform rod about an axis through one end and perpendicular to length is

$$I = \frac{ml^2}{3}$$

where, m is mass of rod and l its length.

Torque ($\tau = I\alpha$) acting on centre of gravity of rod is given by

$$\tau = mg\frac{l}{2}$$

As we know that $\tau = I\alpha$

So $I\alpha = mg\frac{l}{2}$ or $\frac{ml^2}{3}\alpha = mg\frac{l}{2}$

$\therefore$ $$\alpha = \frac{3g}{2l}$$

22 A tube of length L is filled completely with an incompressible liquid of mass M and closed at both the ends. The tube is then rotated in a horizontal plane about one of its ends with a uniform angular velocity ω. The force exerted by the liquid at the other end is **[CBSE AIPMT 2006]**

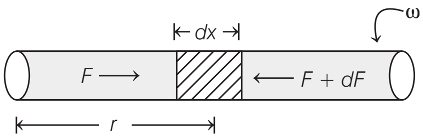
(a) $\frac{ML\omega^2}{2}$ (b) $\frac{ML^2\omega}{2}$ (c) $ML\omega^2$ (d) $\frac{ML^2\omega^2}{2}$

Ans. *(a)*

Let the length of a small element of tube be dx.

Mass of this element

$$dm = \frac{M}{L}dx$$



where, M is mass of filled liquid and L is length of tube. Force on this element

$$dF = dm \times x\omega^2$$

$$\int_0^F dF = \frac{M}{L}\omega^2 \int_0^L x\,dx$$

or $$F = \frac{M}{L}\omega^2\left[\frac{L^2}{2}\right] = \frac{ML\omega^2}{2}$$

or $$F = \frac{1}{2}ML\omega^2$$

23 Consider a system of two particles having masses m_1 and m_2. If the particle of mass m_1 is pushed towards the centre of mass of particles through a distance d, by what distance would the particle of mass m_2 move so as to keep the mass centre of particles at the original position?

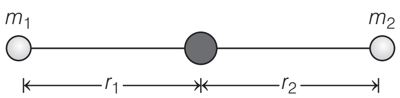
[CBSE AIPMT 2004]

(a) $\frac{m_1}{m_1 + m_2}d$ (b) $\frac{m_1}{m_2}d$

(c) d (d) $\frac{m_2}{m_1}d$

Ans. *(b)*

The system of two given particles of masses m_1 and m_2 are shown in figure.



Initially the centre of mass

$$r_{CM} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} \quad \text{...(i)}$$

When mass m_1 moves towards centre of mass by a distance d, then let mass m_2 moves a distance d' away from CM to keep the CM in its initial position.

So, $$r_{CM} = \frac{m_1(r_1 - d) + m_2(r_2 + d')}{m_1 + m_2} \quad \text{...(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} = \frac{m_1(r_1 - d) + m_2(r_2 + d')}{m_1 + m_2}$$

$$\Rightarrow \quad -m_1 d + m_2 d' = 0$$

$$\Rightarrow \quad d' = \frac{m_1}{m_2}d.$$

If both the masses are equal i.e., $m_1 = m_2$, then second mass will move a distance equal to the distance at which first mass is being displaced.

24 A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is **[CBSE AIPMT 2004]**

(a) $\frac{I_2\omega}{I_1 + I_2}$ (b) ω

(c) $\frac{I_1\omega}{I_1 + I_2}$ (d) $\frac{(I_1 + I_2)\omega}{I_1}$

Ans. *(c)*

Concept Apply conservation of angular momentum

The angular momentum of a disc of moment of inertia I_1 and rotating about its axis with angular velocity ω is

$$L_1 = I_1\omega$$

When a round disc of moment of inertia I_2 is placed on first disc, then angular momentum of the combination is

$$L_2 = (I_1 + I_2)\omega'$$

In the absence of any external torque, angular momentum remains conserved i.e.,

$$L_1 = L_2$$

$$I_1\omega = (I_1 + I_2)\omega'$$

$$\Rightarrow \quad \omega' = \frac{I_1\omega}{I_1 + I_2}$$

25 A thin circular ring of mass M and radius r is rotating about its axis with a constant angular velocity ω. Four objects each of mass m, are kept gently to the opposite ends of two perpendicular diameters of the ring. The angular velocity of the ring will be **[CBSE AIPMT 2003]**

(a) $\frac{(M + 4m)\omega}{M}$ (b) $\frac{(M - 4m)\omega}{M + 4m}$

(c) $\frac{M\omega}{4m}$ (d) $\frac{M\omega}{M + 4m}$

Ans. *(d)*

External torque $\tau_{ext} = 0$

So, $$\frac{dL}{dt} = 0$$

Angular momentum, L = constant

or $I\omega$ = constant

$$\therefore \quad I_1\omega_1 = I_2\omega_2 \quad \text{...(i)}$$

So, for two different cases

Here, $I_1 = Mr^2, \omega_1 = \omega, I_2 = Mr^2 + 4mr^2$

Hence, Eq. (i) can be written as

$$Mr^2\omega = (Mr^2 + 4mr^2)\omega_2$$

$$\therefore \quad \omega_2 = \frac{M\omega}{M + 4m}$$

26 A rod is of length 3 m and its mass acting per unit length is directly proportional to distance x from its one end. The centre of gravity of the rod from that end will be at

[CBSE AIPMT 2002]

(a) 1.5 m (b) 2 m (c) 2.5 m (d) 3 m

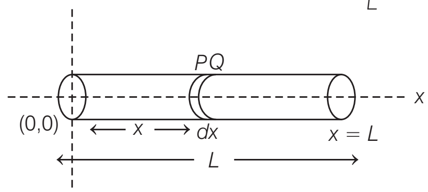
Ans. *(a)*

A rod lying along any of coordinate axes serves for us as continuous body.

Suppose a rod of mass M and length L is lying along the x-axis with its one end at $x = 0$ and the other at $x = L$.

Mass per unit length of the rod $= \frac{M}{L}$

Hence, the mass of the element PQ of length dx situated at $x = x$ is $dm = \frac{M}{L}dx$



The coordinates of the element PQ are $(x, 0, 0)$. Therefore, x-coordinate of centre of gravity of the rod will be

$$x_{CG} = \frac{\int_0^L x\,dm}{\int dm} = \frac{\int_0^L (x)\left(\frac{M}{L}\right)dx}{M} = \frac{1}{L}\int_0^L x\,dx$$

$$= \frac{L}{2}$$

but as given, $L = 3\text{ m}$

$$\therefore \quad x_{CG} = \frac{3}{2} = 1.5\text{ m}$$

The y-coordinate of centre of gravity

$$y_{CG} = \frac{\int y\,dm}{\int dm} = 0 \quad (\text{as } y = 0)$$

Similarly, $z_{CG} = 0$

i.e., the coordinates of centre of gravity of the rod are $(1.5, 0, 0)$ or it lies at the distance 1.5 m from one end.

27 A solid sphere of radius R is placed on a smooth horizontal surface. A horizontal force F is applied at height h from the lowest point. For the maximum acceleration of the centre of mass **[CBSE AIPMT 2002]**

(a) $h = R$
(b) $h = 2R$
(c) $h = 0$
(d) the acceleration will be same whatever h may be

Ans. ***(d)***

The linear acceleration of centre of mass will be $a = \frac{F}{m}$, wherever the force is applied. Hence, the acceleration will be same whatever the value of h may be.

28 A disc is rotating with angular velocity ω. If a child sits on it, what is conserved ? **[CBSE AIPMT 2002]**

(a) Linear momentum
(b) Angular momentum
(c) Kinetic energy
(d) Moment of inertia

Ans. ***(b)***

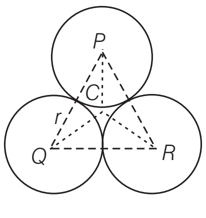
If no external torque is applied on the system, then angular momentum of the system remains constant. When a child sits on rotating disc, then no torque is applied (weight of child acts downward), so angular momentum will remain conserved.

29 Three identical metal balls each of radius r are placed touching each other on a horizontal surface such that an equilateral triangle is formed with centres of three balls joined. The centre of mass of the system is located at **[CBSE AIPMT 1999]**

(a) horizontal surface
(b) centre of one of the balls
(c) line joining the centres of any two balls
(d) point of intersection of the medians

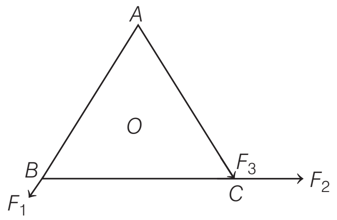
Ans. ***(d)***

The whole mass of the ball will be concentrated at the centre of the ball. All the three balls are identical, i.e., the balls have same mass. On each vertex of equilateral ΔPQR, same mass is kept.



Therefore, centre of mass of the triangle is the centre of mass of the system which is point of intersection of the medians of the triangle.

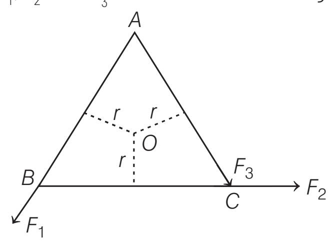
30 O is the centre of an equilateral ΔABC. F_1, F_2 and F_3 are three forces acting along the sides AB, BC and AC as shown in figure. What should be the magnitude of F_3, so that the total torque about O is zero? **[CBSE AIPMT 1998]**



(a) $\frac{(F_1 + F_2)}{2}$ (b) $(F_1 - F_2)$
(c) $(F_1 + F_2)$ (d) $2(F_1 + F_2)$

Ans. ***(c)***

Let r be the perpendicular distance of F_1, F_2 and F_3 from O as shown in figure.



The torque of force F_3 about O is clockwise, while torque due to F_1 and F_2 are anticlockwise.

For total torque to be zero about O, we must have

$$F_1 r + F_2 r - F_3 r = 0$$

$$\Rightarrow \quad F_3 = F_1 + F_2$$

31 A couple produces **[CBSE AIPMT 1997]**

(a) no motion
(b) linear and rotational motion
(c) purely rotational motion
(d) purely linear motion

Ans. ***(c)***

A couple consists of two equal and opposite forces acting at a separation, so that net force becomes zero. When a couple acts on a body it rotates the body but does not produce any translatory motion. Hence, only rotational motion is produced.

32 A cart of mass M is tied to one end of a massless rope of length 10 m. The other end of the rope is in the hands of a man of mass M. The entire system is on a smooth horizontal surface. The man is at $x = 0$ and the cart at $x = 10$ m. If the man pulls the cart by the rope, the man and the cart will meet at the point **[CBSE AIPMT 1997]**

(a) they will never meet
(b) $x = 10$ m
(c) $x = 5$ m
(d) $x = 0$

Ans. ***(c)***

If the man pulls the cart by the rope, the man and cart will meet at the centre of mass.

$$\therefore \quad x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Taking axis at the point where man is present

$$= \frac{M \times 0 + M \times 10}{M + M} \quad \begin{bmatrix} x_1 = 0, x_2 = 10 \\ m_1 = m_2 = M \end{bmatrix}$$

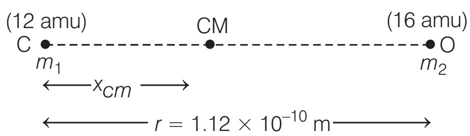
$$= \frac{10M}{2M} = 5 \text{ m}$$

33 In a carbon monoxide molecule, the carbon and the oxygen atoms are separated by a distance 1.12×10^{-10} m. The distance of the centre of mass from the carbon atom is **[CBSE AIPMT 1997]**

(a) 0.64×10^{-10} m (b) 0.56×10^{-10} m
(c) 0.51×10^{-10} m (d) 0.48×10^{-10} m

Ans. ***(a)***

Let the distance of the centre of mass from the carbon atom be x_{cm}.
The mass of carbon, $m_1 = 12$ amu
The mass of oxygen, $m_2 = 16$ amu
[atomic mass unit]



From definition of centre of mass

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{(12 \text{ amu}) \times 0 + (16 \text{ amu}) \times r}{12 \text{ amu} + 16 \text{ amu}}$$

$$= \frac{16}{28} r = \frac{16}{28} \times 1.12 \times 10^{-10} \text{ m}$$

$$= 0.64 \times 10^{-10} \text{ m}$$

34 Find the torque of a force $\mathbf{F} = -3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ acting at the point $\mathbf{r} = 7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$. **[CBSE AIPMT 1997]**

(a) $-21\hat{i} + 3\hat{j} + 5\hat{k}$ (b) $-14\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 16\hat{\mathbf{k}}$
(c) $4\hat{i} + 4\hat{j} + 6\hat{k}$ (d) $14\hat{\mathbf{i}} - 38\hat{\mathbf{j}} + 16\hat{\mathbf{k}}$

Ans. *(d)*

Given, $\mathbf{r} = 7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{F} = -3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}}$

$\therefore \quad \tau = \mathbf{r} \times \mathbf{F} = |\mathbf{r}||\mathbf{F}|\sin\theta$

where, θ is the angle between $\mathbf{r}$ and $\mathbf{F}$

$= (7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (-3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}})$

$$= \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$$

$= \hat{\mathbf{i}}(15 - 1) - \hat{\mathbf{j}}(35 + 3) + \hat{\mathbf{k}}(7 + 9)$

$= 14\hat{\mathbf{i}} - 38\hat{\mathbf{j}} + 16\hat{\mathbf{k}}$

Alternative

$\therefore \quad \tau = \mathbf{r} \times \mathbf{F}$

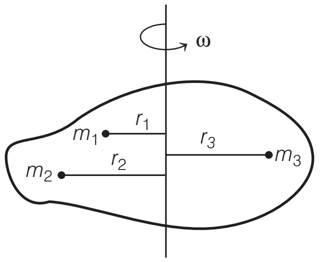
$$\begin{aligned} &= (7\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (-3\hat{\mathbf{i}} + \hat{\mathbf{j}} + 5\hat{\mathbf{k}}) \\ &= -21(\hat{\mathbf{i}} \times \hat{\mathbf{i}}) + 7(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) + 35(\hat{\mathbf{i}} \times \hat{\mathbf{k}}) \\ &\quad - 9(\hat{\mathbf{j}} \times \hat{\mathbf{i}}) + 3(\hat{\mathbf{j}} \times \hat{\mathbf{j}}) + 15(\hat{\mathbf{j}} \times \hat{\mathbf{k}}) \\ &\quad - 3(\hat{\mathbf{k}} \times \hat{\mathbf{i}}) + (\hat{\mathbf{k}} \times \hat{\mathbf{j}}) + 5(\hat{\mathbf{k}} \times \hat{\mathbf{k}}) \\ &= 0 + 7\hat{\mathbf{k}} - 35\hat{\mathbf{j}} + 9\hat{\mathbf{k}} + 0 + 15\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{i}} + 0 \\ &= 14\hat{\mathbf{i}} - 38\hat{\mathbf{j}} + 16\hat{\mathbf{k}} \end{aligned}$$

35 The angular momentum of a body with mass (m) moment of inertia (I) and angular velocity (ω) rad/s is equal to **[CBSE AIPMT 1996]**

(a) $I\omega$ (b) $I\omega^2$
(c) $\frac{I}{\omega}$ (d) $\frac{I}{\omega^2}$

Ans. *(a)*

Consider a rigid body rotating about a given axis with a uniform angular velocity ω. Let the body consists of n particles of masses $m_1, m_2, m_3, \ldots m_n$ at perpendicular distances $r_1, r_2, r_3, \ldots r_n$ respectively from the axis of rotation.



As the body is rigid, angular velocity ω of all the particles is the same. However, as the distances of the particles from the axis of rotation are different, their linear velocities are different. If $v_1, v_2, v_3, \ldots v_n$ are the linear velocities of the particles respectively, then

$$v_1 = r_1\omega$$
$$v_2 = r_2\omega$$
$$v_3 = r_3\omega, \ldots$$

The linear momentum of this particle of mass m_1 is

$$p_1 = m_1 v_1 = m_1(r_1\omega)$$

The angular momentum of this particle about the given axis

$$= p_1 \times r_1 = (m_1 r_1 \omega) \times r_1$$
$$= m_1 r_1^2 \omega$$

Similarly, angular momenta of other particles of the body about the given axis are

$$m_2 r_2^2 \omega, m_3 r_3^2 \omega, \ldots m_n r_n^2 \omega$$

$\therefore$ Angular momentum of the body about the given axis

$$L = m_1 r_1^2 \omega + m_2 r_2^2 \omega + m_2 r_3^2 \omega + \ldots + m_n r_n^2 \omega$$
$$= (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \ldots + m_n r_n^2)\,\omega$$

or $\quad L = \left(\sum_{i=1}^{n} m_i r_i^2 \right) \omega$

or $\quad L = I\omega$

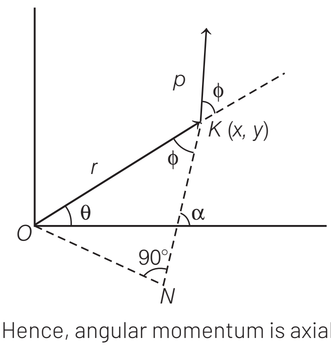
where, $I = \sum_{i=1}^{n} m_i r_i^2$ is moment of inertia of the body about the given axis.

36 Angular momentum is **[CBSE AIPMT 1994]**

(a) vector (axial)
(b) vector (polar)
(c) scalar
(d) None of these

Ans. *(a)*

The radial component of linear momentum does not contribute to angular momentum of the particle. It is only the transverse component of linear momentum (perpendicular to position vector **r**), which when multiplied by distance from the axis of rotation gives us angular momentum.



Hence, angular momentum is axial vector.

37 A particle of mass $m = 5$ kg is moving with a uniform speed $v = 3\sqrt{2}$ in the XOY plane along the line $Y = X + 4$. The magnitude of the angular momentum of the particle about the origin is **[CBSE AIPMT 1991]**

(a) 60 unit
(b) $40\sqrt{2}$ unit
(c) zero
(d) 7.5 unit

Ans. *(a)*

The equation of the line is

$$Y = X + 4$$

or $\quad X - Y + 4 = 0$

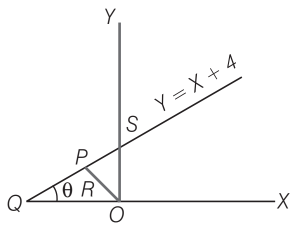
Length of perpendicular from origin on this line is

$$R = \frac{0 - 0 + 4}{\sqrt{1^2 + 1^2}} = \frac{4}{\sqrt{2}}$$

$\therefore$ Angular momentum

$$L = mvR = 5 \times 3\sqrt{2} \times \frac{4}{\sqrt{2}} = 60 \text{ unit}$$

Alternative



$Y = X + 4$ line is shown in the figure.

When $\quad X = 0, \; Y = 4,$

so $\quad OS = 4$

To find slope of this line comparing this with equation of line

$$y = m'x + c$$

$\therefore$ Slope, $\quad m' = \tan\theta = 1$

$\Rightarrow \quad \theta = 45°$

Length of perpendicular $= OP$

In ΔPSO, $\quad \frac{OP}{OS} = \sin 45°$

$\therefore \quad OP = OS \sin 45°$

$$= 4 \times \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

$\therefore$ Angular momentum of particle going along this line

$$= mvR$$
$$= 5 \times 3\sqrt{2} \times \frac{4}{\sqrt{2}}$$
$$= 60 \text{ unit}$$

TOPIC 2
Moment of Inertia

38 From a circular ring of mass M and radius R, an arc corresponding to a 90° sector is removed. The moment of inertia of the remaining part of the ring about an axis passing through the centre of the ring and perpendicular to the plane of the ring is K times MR^2. Then, the value of K is **[NEET 2021]**

(a) $\frac{3}{4}$ (b) $\frac{7}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Ans. *(a)*

Given, the mass of the circular ring $= M$

The radius of the circular ring $= R$

We know that, the moment of inertia of the circular ring, $I = MR^2$

An arc corresponding to a 90° sector in the circular ring is removed, means one-fourth part of circular ring is removed. Then, the remaining mass of the circular ring,

$$M' = \frac{M}{4}$$

The new moment of inertia,

$$I' = M'R^2 = \frac{MR^2}{4}$$

Now, the moment of inertia of the remaining part,

$$I'' = I - I' = MR^2 - \frac{MR^2}{4} \Rightarrow I'' = \frac{3MR^2}{4}$$

Comparing with $I'' = KMR^2$

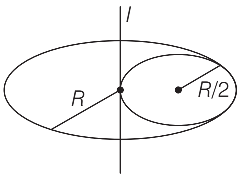
The value of $K = 3/4$.

39 From a disc of radius R and mass M, a circular hole of diameter R, whose rim passes through the centre is cut. What is the moment of inertia of the remaining part of the disc about a perpendicular axis, passing through the centre ? **[NEET 2016]**

(a) $13\,MR^2/32$ (b) $11\,MR^2/32$
(c) $9\,MR^2/32$ (d) $15\,MR^2/32$

Ans. *(a)*

Considering the information given in the question, let us draw the figure



If the above figure is considered, then moment of inertia of disc will be given as

$$I = I_{\text{remain}} + I_{(R/2)} \Rightarrow I_{\text{remain}} = I - I_{(R/2)}$$

Putting the values, we get

$$= \frac{MR^2}{2} - \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2\right]$$

$$= \frac{MR^2}{2} - \left[\frac{MR^2}{32} + \frac{MR^2}{16}\right]$$

$$= \frac{MR^2}{2} - \left[\frac{MR^2 + 2MR^2}{32}\right]$$

$$= \frac{MR^2}{2} - \frac{3MR^2}{32}$$

$$= \frac{16MR^2 - 3MR^2}{32}$$

$$I_{\text{remain}} = \frac{13MR^2}{32}$$

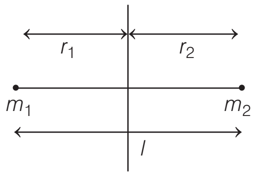
40 A light rod of length l has two masses m_1 and m_2 attached to its two ends. The moment of inertia of the system about an axis perpendicular to the rod and passing through the centre of mass is **[NEET 2016]**

(a) $\frac{m_1 m_2}{m_1 + m_2} l^2$ (b) $\frac{m_1 + m_2}{m_1 m_2} l^2$
(c) $(m_1 + m_2) l^2$ (d) $\sqrt{m_1 m_2}\, l^2$

Ans. *(a)*

COM of m_1 and m_2 masses lies at

$$r = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}$$



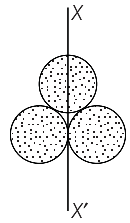
$\therefore$ Moment of inertia of the point masses about the given axis is

$$I = \Sigma m_i r_i^2 \Rightarrow I = m_1 r_1^2 + m_2 r_2^2$$

$$= m_1 \left(\frac{m_2 l}{m_1 + m_2}\right)^2 + m_2 \left(\frac{m_1 l}{m_1 + m_2}\right)^2$$

$$= \frac{m_1 m_2 l^2}{(m_1 + m_2)^2}(m_2 + m_1) = \frac{m_1 m_2 l^2}{(m_1 + m_2)}$$

41 Three identical spherical shells, each of mass m and radius r are placed as shown in figure. Consider an axis XX', which is touching to two shells and passing through diameter of third shell.

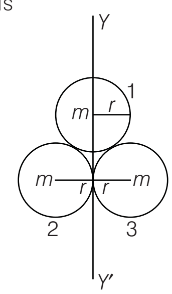


Moment of inertia of the system consisting of these three spherical shells about XX' axis is **[CBSE AIPMT 2015]**

(a) $\frac{11}{5}\text{mr}^2$ (b) $3\,\text{mr}^2$
(c) $\frac{16}{5}\text{mr}^2$ (d) $4\,\text{mr}^2$

Ans. *(d)*

The total moment of inertia of the system is



$$I = I_1 + I_2 + I_3 \quad \text{...(i)}$$

Here, $I_1 = \frac{2}{3}mr^2$

$$I_2 = I_3 = \frac{2}{3}mr^2 + mr^2$$

[From parallel axis theorem]

$$= \frac{5}{3}mr^2$$

From Eq. (i), $I = \frac{2}{3}mr^2 + 2 \times \frac{5}{3}mr^2$

$$= mr^2\left(\frac{2}{3} + \frac{10}{3}\right)$$

$$I = 4mr^2$$

42 The moment of inertia of a thin uniform rod of mass M and length L about an axis passing through its mid-point and perpendicular to its length is I_0. Its moment of inertia about an axis passing through one of its ends and perpendicular to its length is **[CBSE AIPMT 2011]**

(a) $I_0 + ML^2/4$ (b) $I_0 + 2ML^2$
(c) $I_0 + ML^2$ (d) $I_0 + ML^2/2$

Ans. *(a)*

Concept Apply parallel axes theorem of moment of inertia.

According to parallel axes theorem of moment of inertia,

$$I = I_{CM} + Mh^2$$

So, $$I = I_0 + M\left(\frac{L}{2}\right)^2$$

$$\Rightarrow \quad I = I_0 + \frac{ML^2}{4}$$

43 Four identical thin rods each of mass M and length l, form a square frame. Moment of inertia of this frame about an axis through the centre of the square and perpendicular to its plane is **[CBSE AIPMT 2009]**

(a) $\frac{4}{3}Ml^2$ (b) $\frac{2}{3}Ml^2$

(c) $\frac{13}{3}Ml^2$ (d) $\frac{1}{3}Ml^2$

Ans. *(a)*

Moment of inertia of rod about an axis through its centre of mass and perpendicular to rod = (mass of rod) × (perpendicular distance between two axes)

$$= \frac{Ml^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{3}$$

Moment of inertia of the system

$$= \frac{Ml^2}{3} \times 4$$

$$= \frac{4}{3}Ml^2$$

44 A thin rod of length L and mass M is bent at its mid-point into two halves so that the angle between them is 90°. The moment of inertia of the bent rod about an axis passing through the bending point and perpendicular to the plane defined by the two halves of the rod is **[CBSE AIPMT 2008]**

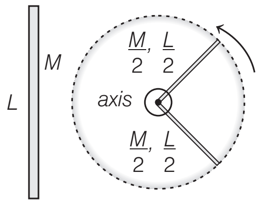
(a) $\frac{ML^2}{24}$ (b) $\frac{ML^2}{12}$

(c) $\frac{ML^2}{6}$ (d) $\frac{\sqrt{2}ML^2}{24}$

Ans. *(b)*

As the rod is bent into two equal halves, the mass and length of each half is $\frac{M}{2}$ and $\frac{L}{2}$ respectively.

The moment of inertia about an axis passing through its edge and perpendicular to the rod



$$= \frac{\text{Mass} \times (\text{length})^2}{3}$$

$$\therefore \quad I = 2 \times \frac{1}{3} \times \frac{M}{2}\left(\frac{L}{2}\right)^2 = \frac{ML^2}{12}$$

45 The ratio of the radii of gyration of a circular disc to that of a circular ring, each of same mass and radius, around their respective axes is **[CBSE AIPMT 2008]**

(a) $\sqrt{3} : \sqrt{2}$ (b) $1 : \sqrt{2}$

(c) $\sqrt{2} : 1$ (d) $\sqrt{2} : \sqrt{3}$

Ans. *(b)*

As we know that radius of gyration

$$k = \sqrt{\frac{I}{m}}$$

So, for two different cases

$$\frac{k_{\text{ring}}}{k_{\text{disc}}} = \sqrt{\frac{I_{\text{ring}}}{I_{\text{disc}}}} = \sqrt{\frac{MR^2}{\frac{1}{2}MR^2}}$$

$$\therefore \quad \frac{k_{\text{ring}}}{k_{\text{disc}}} = \sqrt{2} \Rightarrow \frac{k_{\text{disc}}}{k_{\text{ring}}} = \frac{1}{\sqrt{2}}$$

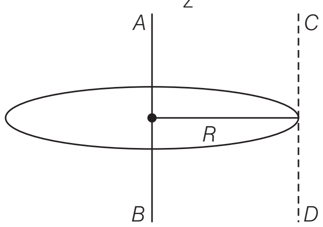
46 The moment of inertia of a uniform circular disc of radius R and mass M about an axis passing from the edge of the disc and normal to the disc is **[CBSE AIPMT 2006]**

(a) $\frac{1}{2}MR^2$ (b) MR^2 (c) $\frac{7}{2}MR^2$ (d) $\frac{3}{2}MR^2$

Ans. *(d)*

Moment of inertia of disc passing through its centre of gravity and perpendicular to its plane is

$$I_{AB} = \frac{1}{2}MR^2$$

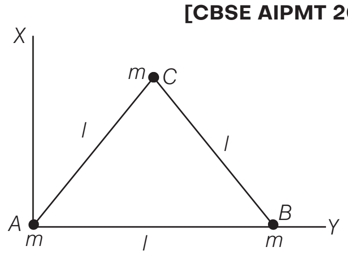


Using theorem of parallel axes, we have,

$$I_{CD} = I_{AB} + MR^2$$

$$= \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

47 Three particles, each of mass m grams situated at the vertices of an equilateral ΔABC of side l cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC in g-cm^2 units will be **[CBSE AIPMT 2004]**

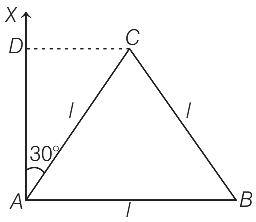


(a) $\left(\frac{3}{4}\right)ml^2$ (b) $2\,ml^2$

(c) $\left(\frac{5}{4}\right)ml^2$ (d) $\left(\frac{3}{2}\right)ml^2$

Ans. *(c)*

Moment of inertia of the system about AX is given by



$$\text{Moment of inertia} = m_A r_A^2 + m_B r_B^2 + m_C r_C^2$$

$$\begin{bmatrix} r_A = 0 \\ r_B = l \\ r_c = l \sin 30° \end{bmatrix}$$

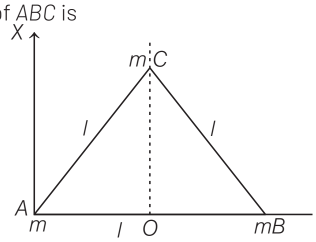
Moment of inertia

$$= m(0)^2 + m(l)^2 + m(l \sin 30°)^2$$

$$= ml^2 + \frac{ml^2}{4} = \frac{5}{4}ml^2$$

Alternative

Moment of inertia of a system about a line OC perpendicular to AB in the plane of ABC is



$$I_{CO} = m \times 0 + m \times \left(\frac{l}{2}\right)^2 + m \times \left(\frac{l}{2}\right)^2$$

$$\therefore \quad I_{CO} = \frac{ml^2}{4} + \frac{ml^2}{4} = \frac{ml^2}{2}$$

Now, by applying parallel axes theorem

$$I_{AX} = I_{CO} + Mx^2$$

where, x = distance of AX from CO

M = total mass of system

$$I_{AX} = \frac{ml^2}{2} + 3m \times \left(\frac{l}{2}\right)^2$$

$$I_{AX} = \frac{ml^2}{2} + \frac{3ml^2}{4} = \frac{5}{4}ml^2$$

48 The ratio of the radii of gyration of a circular disc about a tangential axis in the plane of the disc and of a circular ring of the same radius about a tangential axis in the plane of the ring is **[CBSE AIPMT 2004]**

(a) 2 : 3 (b) 2 : 1
(c) $\sqrt{5} : \sqrt{6}$ (d) $1 : \sqrt{2}$

Ans. *(c)*

Moment of inertia of a disc and circular ring about a tangential axis in their planes are respectively. Momentum inertia of disc about tangential axis

$$I_d = \frac{5}{4} M_d R^2$$

Moment of inertia of ring about a tangential axis

$$I_r = \frac{3}{2} M_r R^2$$

but $I = Mk^2 \Rightarrow k = \sqrt{\frac{I}{M}}$

$$\therefore \quad \frac{k_d}{k_r} = \sqrt{\frac{I_d}{I_r} \times \frac{M_r}{M_d}}$$

or $$\frac{k_d}{k_r} = \sqrt{\frac{(5/4) M_d R^2}{(3/2) M_r R^2} \times \frac{M_r}{M_d}} = \sqrt{\frac{5}{6}}$$

$$\therefore \quad k_d : k_r = \sqrt{5} : \sqrt{6}$$

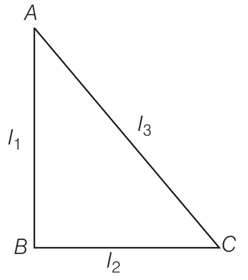
49 A circular disc is to be made using iron and aluminium. To keep its moment of inertia maximum about a geometrical axis, it should be so prepared that **[CBSE AIPMT 2002]**

(a) aluminium is at the interior and iron surrounds it
(b) iron is at the interior and aluminium surrounds it
(c) aluminium and iron layers are in alternate order
(d) sheet of iron is used at both external surfaces and aluminium sheet as inner material

Ans. *(a)*

Moment of inertia depends on distribution of mass and about axis of rotation. Density of iron is more than that of aluminium, therefore for moment of inertia to be maximum, the iron should be far away from the axis. Thus, aluminium should be at interior and iron surrounds it.

50 ABC is a right angled triangular plate of uniform thickness. The sides are such that $AB > BC$ as shown in figure. I_1, I_2, I_3 are moments of inertia about AB, BC and AC respectively. Then, which of the following relations is correct? **[CBSE AIPMT 2000]**



(a) $I_1 = I_2 = I_3$
(b) $I_2 > I_1 > I_3$
(c) $I_3 < I_2 < I_1$
(d) $I_3 > I_1 > I_2$

Ans. *(b)*

The moment of inertia of a body about an axis depends not only on the mass of the body, but also on the distribution of mass from the axis. For a given body, mass is same, so it will depend only on the distribution of mass from the axis.

The mass is farthest from axis BC, so I_2 is maximum. Mass is nearest to axis AC, so I_3 is minimum.

Hence, the correct sequence will be

$$I_2 > I_1 > I_3$$

NOTE

In a rotational motion, moment of inertia is also known as rotational inertia.

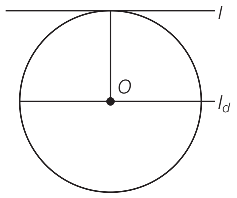
51 The moment of inertia of a disc of mass M and radius R about a tangent to its rim in its plane is **[CBSE AIPMT 1999]**

(a) $\frac{2}{3}MR^2$ (b) $\frac{3}{2}MR^2$
(c) $\frac{4}{5}MR^2$ (d) $\frac{5}{4}MR^2$

Ans. *(d)*

Moment of inertia of a disc about its diameter is

$$I_d = \frac{1}{4}MR^2$$



Now, according to perpendicular axis theorem, moment of inertia of disc about a tangent passing through rim and in the plane of disc is

$$I = I_d + MR^2 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$$

52 ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB}, I_{BC} and I_{CA} are the moments of inertia of the plate about AB, BC and CA as axes respectively. Which one of the following relations is correct? **[CBSE AIPMT 1995]**

(a) $I_{AB} > I_{BC}$ (b) $I_{BC} > I_{AC}$
(c) $I_{AB} + I_{BC} = I_{CA}$ (d) I_{CA} is maximum

Ans. *(b)*

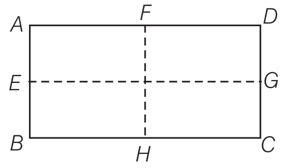
Moment of inertia of the triangular plate is maximum about the shortest side because effective distance of mass distribution about this side is maximum. Since, distances of centre of mass from the sides are related as

$$x_{BC} < x_{AB} < x_{AC}$$

Therefore

$$I_{BC} > I_{AB} > I_{AC} \quad \text{or} \quad I_{BC} > I_{AC}$$

53 In a rectangle $ABCD$ $(BC = 2AB)$. The moment of inertia is minimum along axis through **[CBSE AIPMT 1993]**



(a) BC (b) BD
(c) HF (d) EG

Ans. *(d)*

Concept Magnitude of moment of inertia depends on the distribution of mass taken from the axis.

From the axis EG, the distribution of masses is at minimum distance while from the axis BD the distribution of masses is at maximum distance. Hence, the moment of inertia is minimum along axis through EG.

TOPIC 3
Kinematics and Dynamics of Rotational Motion

54 The angular speed of the wheel of a vehicle is increased from 360 rpm to 1200 rpm in 14 s. Its angular acceleration is **[NEET (Oct.)2020]**

(a) 2π rad/s^2 (b) 28π rad/s^2
(c) 120π rad/s^2 (d) 1 rad/s^2

Ans. ***(a)***

Initial angular speed of wheel,

$$\omega_0 = 2\pi f_0 = 2\pi \times \frac{360}{60} \text{ rad/s}$$
$$= 12\pi \text{ rad/s}$$

Final angular speed of wheel,

$$\omega = 2\pi f$$
$$= 2\pi \times \frac{1200}{60} \text{ rad/s} = 40\pi \text{ rad/s}$$
$$t = 14 \text{ s}$$

From the equation of rotational motion,

$$\omega = \omega_0 + \alpha t$$
$$\Rightarrow \quad \alpha = \frac{\omega - \omega_0}{t}$$
$$= \frac{40\pi - 12\pi}{14} = \frac{28\pi}{14} = 2\pi \text{ rad/s}^2$$

55 A particle starting from rest, moves in a circle of radius 'r'. It attains a velocity of v_0 m/s in the n^{th} round. Its angular acceleration will be **[NEET (Odisha) 2019]**

(a) $\frac{v_0}{n}$ rad/s^2 (b) $\frac{v_0^2}{2\pi n r^2}$ rad/s^2
(c) $\frac{v_0^2}{4\pi n r^2}$ rad/s^2 (d) $\frac{v_0^2}{4\pi n r}$ rad/s^2

Ans. ***(c)***

From third equation of motion for circular motion

$$\omega^2 - \omega_0^2 = 2\alpha\theta \quad \text{...(i)}$$

where, ω = final angular velocity of particle
ω_0 = initial angular velocity
α = angular acceleration and
θ = angular displacement

Here, $\omega = \frac{v_0}{r}$ rad/s (where, r radius of the circle)

$\omega_0 = 0$ (initially particle is at rest)
$\theta = 2\pi n$ (for n rounds)

Substituting these values in Eq. (i), we get

$$\left(\frac{v_0}{r}\right)^2 - 0 = 2\alpha(2\pi n)$$
$$\Rightarrow \quad \alpha = \frac{v_0^2}{4\pi n r^2} \text{ rad/s}^2$$

56 A solid cylinder of mass 2 kg and radius 50 cm rolls up an inclined plane of angle inclination 30°. The centre of mass of cylinder has speed of 4 m/s. The distance travelled by the cylinder on the inclined surface will be :
(Take $g = 10 \text{ m/s}^2$) **[NEET (Odisha) 2019]**

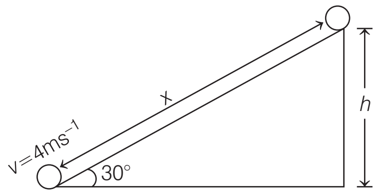
(a) 2.2 m (b) 1.6 m
(c) 1.2 m (d) 2.4 m

Ans. ***(d)***

When a body rolls i.e. have rotational motion, the total kinetic energy of the system will be

$$KE = \frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right)$$

where, m = mass of body, v = velocity and
k = radius of gyration



Given, $m = 2$ kg, $\theta = 30°$, $v = 4 \text{ ms}^{-1}$

Let h be the height of the inclined plane, then from law of conservation of energy,

$$KE = PE$$
$$\frac{1}{2}mv^2\left(1 + \frac{k^2}{R^2}\right) = mgh$$

Substituting the given values in the above equation, we get

$$\frac{1}{2} \times 2 \times 16\left(1 + \frac{1}{2}\right) = 2 \times 10 \times h$$
$$\left[\because \text{For cylinder } \frac{k^2}{R^2} = \frac{1}{2}\right]$$
$$\Rightarrow \quad 8 \times \frac{3}{2} = 10 \text{ h} \Rightarrow h = 1.2 \text{ m}$$

From the above diagram

$$\sin\theta = \frac{h}{x}$$
$$\Rightarrow x = \frac{h}{\sin\theta} = \frac{1.2}{\sin 30°} = 1.2 \times 2 = 2.4 \text{ m}$$
$$\left[\because \sin 30° = \frac{1}{2}\right]$$

57 A disc of radius 2 m and mass 100 kg rolls on a horizontal floor. Its centre of mass has speed of 20 cm/s. How much work is needed to stop it? **[NEET (National) 2019]**

(a) 30 kJ (b) 2 J
(c) 1 J (d) 3 J

Ans. ***(d)***

Given, radius $R = 2$ m
mass, $m = 100$ kg and
v_{CM} (velocity centre of mass) $= v = 20$ cm/s $= 20 \times 10^{-2}$ m/s.

Then, according to work energy theorem, the work done in stopping the disc is equal to the change in its kinetic energy, i.e.

$$W = \text{KE}_f - \text{KE}_i$$

As, the disc stops at the end, so final velocity is zero. Thus, $\text{KE}_f = 0$

Since, the disc is rolling so, its initial kinetic energy would have both rotational and translational kinetic energy component.

$$\text{KE}_i = \text{KE}_R + \text{KE}_T = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$
$$= \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega^2 + \frac{1}{2}mv^2$$
$$\left[\because \text{for disc, } I = \frac{1}{2}mR^2\right]$$
$$= \frac{1}{4}mR^2\omega^2 + \frac{1}{2}mv^2$$
$$= \frac{1}{4}mR^2\frac{v^2}{R^2} + \frac{1}{2}mv^2 \quad [\because v = r\omega]$$
$$= \frac{3}{4}mv^2$$
$$\therefore W = |\text{KE}_i| = \frac{3}{4}mv^2$$

Substituting the given values, we get

$$W = \frac{3}{4} \times 100 \times (20 \times 10^{-2})^2$$
$$= \frac{3}{4} \times 400 \times 100 \times 10^{-4} = 3 \text{ J}$$

58 A solid cylinder of mass 2 kg and radius 4 cm is rotating about its axis at the rate of 3 rpm. The torque required to stop after 2π revolutions is **[NEET (National) 2019]**

(a) 2×10^{-3} N-m (b) 12×10^{-4} N-m
(c) 2×10^{6} N-m (d) 2×10^{-6} N-m

Ans. ***(d)***

Key Idea *According to work-energy theorem, the change in kinetic energy of a particle is the amount of work done on the particle to move, i.e.*

$$W = -\Delta\text{KE} = \text{KE}_f - \text{KE}_i$$

Given, mass of cylinder, $m = 2$ kg
radius of cylinder, $r = 4 \text{ cm} = 4 \times 10^{-2}$ m
rotational velocity, $\omega = 3$ rpm
$= 3 \times \frac{2\pi}{60} = \frac{\pi}{10}$ rad/s and $\theta = 2\pi$ revolution
$= 2\pi \times 2\pi = 4\pi^2$ rad.

The work done in rotating an object by an angle θ from rest is given by $W = \tau\theta$

As the cylinder is brought to rest, so the work done will be negative.

According to work-energy theorem,

Work done = Change in rotational kinetic energy

$$-\tau\theta = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = \frac{1}{2}I(\omega_f^2 - \omega_i^2)$$

$$\Rightarrow \quad \tau = \frac{I(-\omega_i^2)}{2\theta} \qquad [\because \omega_f = 0]$$

$$= \frac{1}{2}\left(\frac{1}{2}mr^2\right)\frac{\omega_i^2}{\theta}$$

$$\left[I = \frac{1}{2}MR^2 \text{ (for cylinder)}\right]$$

$$= \frac{1}{4}mr^2\,\frac{\omega^2}{\theta} \qquad [\because \omega_i = \omega]$$

$$= \frac{1}{4}\times 2\times(4\times 10^{-2})^2\times\left(\frac{\pi}{10}\right)^2\times\frac{1}{4\pi^2}$$

$$= \frac{1}{4}\times 2\times 16\times 10^{-4}\times\frac{\pi^2}{100}\times\frac{1}{4\pi^2}$$

$$= \frac{2}{100}\times 10^{-4} = 2\times 10^{-6}\text{ N-m}$$

59 Three objects, A : (a solid sphere), B : (a thin circular disk) and C : (a circular ring), each have the same mass M and radius R. They all spin with the same angular speed ω about their own symmetry axes. The amounts of work (W) required to bring them to rest, would satisfy the relation **[NEET 2018]**

(a) $W_B > W_A > W_C$ (b) $W_A > W_B > W_C$
(c) $W_C > W_B > W_A$ (d) $W_A > W_C > W_B$

Ans. ***(c)***

Work done required to bring an object to rest is given as

$$W = \frac{1}{2}I\omega^2$$

where, I is the moment of inertia and ω is the angular velocity.

Since, here all the objects spin with the same ω, this means,

$$W \propto I$$

As, I_A (for a solid sphere) $= \frac{2}{5}MR^2$

I_B (for a thin circular disk) $= \frac{1}{2}MR^2$

I_C (for a circular ring) $= MR^2$

$$\therefore \quad W_A : W_B : W_C = I_A : I_B : I_C$$

$$= \frac{2}{5}MR^2 : \frac{1}{2}MR^2 : MR^2$$

$$= \frac{2}{5} : \frac{1}{2} : 1$$

$$= 4 : 5 : 10$$

$$\Rightarrow \quad W_A < W_B < W_C$$

60 A solid sphere is rotating freely about its symmetry axis in free space. The radius of the sphere is increased keeping its mass same. Which of the following physical quantities would remain constant for the sphere? **[NEET 2018]**

(a) Rotational kinetic energy
(b) Moment of inertia
(c) Angular velocity
(d) Angular momentum

Ans. ***(d)***

Moment of inertia of a rotating solid sphere about its symmetrical (diametric) axis is given as, $I = \frac{2}{5}mR^2$

Rotational kinetic energy of solid sphere is

$$K_r = \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}\times\frac{2}{5}mR^2\omega^2 = \frac{1}{5}mR^2\omega^2$$

Angular velocity, $\omega = V_{cm} R$

As, we know that external torque,

$$\tau_{ext} = \frac{dL}{dt}$$

where, L is the angular momentum.

Since, in the given condition, $\tau_{ext} = 0$

$$\Rightarrow \quad \frac{dL}{dt} = 0 \text{ or } L = \text{constant}$$

Hence, when the radius of the sphere is increased keeping its mass same, only the angular momentum remains constant. But other quantities like moment of inertia, rotational kinetic energy and angular velocity changes.

61 A solid sphere is in rolling motion. In rolling motion, a body possesses translational kinetic energy (K_t) as well as rotational kinetic energy (K_r) simultaneously. The ratio $K_t : (K_t + K_r)$ for the sphere is **[NEET 2018]**

(a) 10 : 7 (b) 5 : 7
(c) 7 : 10 (d) 2 : 5

Ans. ***(b)***

Translational kinetic energy of a rolling body is

$$K_t = \frac{1}{2}mv_{CM}^2 \qquad \text{...(i)}$$

Total kinetic energy of a rolling body

$$= K_t + K_r = \text{Rotational KE} + \text{Translational KE}$$

$$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv_{CM}^2 \qquad \text{...(ii)}$$

For a solid sphere, moment of inertia about its diametric axis, $I = \frac{2}{5}MR^2$

Substituting the value of I in Eq. (ii), we get

$$K_t + K_r = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 + \frac{1}{2}mv_{CM}^2$$

$$= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v_{CM}}{R}\right)^2 + \frac{1}{2}mv_{CM}^2$$

$$[\because v_{CM} = R\omega]$$

$$= \frac{1}{5}mv_{CM}^2 + \frac{1}{2}mv_{CM}^2$$

$$= \left(\frac{1}{5}+\frac{1}{2}\right)mv_{CM}^2$$

$$= \frac{7}{10}mv_{CM}^2 \qquad \text{...(iii)}$$

$$\therefore \quad \text{Ratio, } \frac{K_t}{K_t + K_r} = \frac{\frac{1}{2}mv_{CM}^2}{\frac{7}{10}mv_{CM}^2}$$

$$= \frac{1}{2}\times\frac{10}{7} = \frac{5}{7}$$

$$\therefore \quad K_t : K_t + K_r = 5 : 7$$

Alternate Method

Suppose, moment of inertia,

$$I = xMR^2 \qquad \text{...(i)}$$

For solid sphere, moment of inertia,

$$I = \frac{2}{5}MR^2 \qquad \text{...(ii)}$$

Thus, from Eqs. (i) and (ii), we get

$$x = \frac{2}{5}$$

Since, the ratio of translational energy to the total energy can be written as

$$\frac{K_t}{K_t + K_r} = \frac{\frac{1}{2}mv_{CM}^2}{\frac{1}{2}mv_{CM}^2\left(1+\frac{k^2}{R^2}\right)} \qquad \text{...(iii)}$$

where, k is called the radius of gyration.

As, $K = \sqrt{\frac{I}{m}}$ or $K^2 = \frac{I}{m}$

From Eq. (i), we get

$$K^2 = \frac{xmR^2}{m} = xR^2$$

Substituting the value of K^2 in Eq. (iii), we get

$$\frac{K_t}{K_t + K_r} = \frac{1}{\left(1+\frac{xR^2}{R^2}\right)} = \frac{1}{1+x}$$

Here, $x = \frac{2}{5}$

$$\Rightarrow \quad \frac{K_t}{K_t + K_r} = \frac{1}{1+2/5} = \frac{5}{7}$$

62 Two discs of same moment of inertia rotating about their regular axis passing through centre and perpendicular to the plane of disc with angular velocities ω_1 and ω_2. They are brought into contact face to face coinciding the axis of rotation. The expression for loss of energy during this process is **[NEET 2017]**

(a) $\frac{1}{2}I(\omega_1+\omega_2)^2$ (b) $\frac{1}{4}I(\omega_1-\omega_2)^2$

(c) $I(\omega_1-\omega_2)^2$ (d) $\frac{I}{8}(\omega_1-\omega_2)^2$

Ans. ***(b)***

Thinking Process When no external torque acts on system then, angular momentum of system remains constant.

Angular momentum before contact

$$=I_1\omega_1+I_2\omega_2$$

Angular momentum after the discs brought into contact

$$=I_{\text{net}}\omega=(I_1+I_2)\,\omega$$

So, final angular speed of system $=\omega$

$$=\frac{I_1\omega_1+I_2\omega_2}{I_1+I_2}$$

Now, to calculate loss of energy, we subtract initial and final energies of system.

$\Rightarrow$ Loss of energy

$$=\frac{1}{2}I\omega_1^2+\frac{1}{2}I\omega_2^2-\frac{1}{2}(2I)\,\omega^2$$

$$=\frac{1}{4}I(\omega_1-\omega_2)^2$$

63 Two rotating bodies A and B of masses m and $2m$ with moments of inertia I_A and $I_B(I_B>I_A)$ have equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then **[NEET 2016]**

(a) $L_A=\frac{L_B}{2}$

(b) $L_A=2L_B$ (c) $L_B>L_A$

(d) $L_A>L_B$

Ans. ***(c)***

As we know that, the kinetic energy of a rotating body,

$$\text{KE}=\frac{1}{2}I\omega^2=\frac{1}{2}\frac{I^2\omega^2}{I}=\frac{L^2}{2I}$$

Also, angular momentum, $L=I\omega$

Thus, $K_A=K_B$

$$\Rightarrow \quad \frac{1}{2}\frac{L_A^2}{I_A}=\frac{1}{2}\frac{L_B^2}{I_B}$$

$$\Rightarrow \quad \left(\frac{L_A}{L_B}\right)^2=\frac{I_A}{I_B}\Rightarrow\frac{L_A}{L_B}=\sqrt{\frac{I_A}{I_B}}$$

$$L\propto\sqrt{I}$$

$$\therefore \quad L_A<L_B \qquad [\because I_B>I_A]$$

64 A disc and a sphere of same radius but different masses roll off on two inclined planes of the same altitude and length. Which one of the two objects gets to the bottom of the plane first? **[NEET 2016]**

(a) Sphere

(b) Both reach at the same time

(c) Depends on their masses

(d) Disc

Ans. ***(a)***

Acceleration of an object rolling down an inclined plane is given by

$$a=\frac{g\sin\theta}{1+I/mr^2}$$

where, $\theta=$ angle of inclination of the inclined plane

$m=$ mass of the object

$I=$ moment of inertia about the axis through centre of mass

For disc, $\frac{I}{mr^2}=\frac{1/2\,mr^2}{mr^2}=\frac{1}{2}$

For solid sphere, $\frac{I}{mr^2}=\frac{2/5mr^2}{mr^2}=\frac{2}{5}$

For hollow sphere, $\frac{I}{mr^2}=\frac{2/3mr^2}{mr^2}=\frac{2}{3}$

$$\therefore \quad a_{\text{disc}}=\frac{g\sin\theta}{1+\frac{1}{2}}=\frac{2}{3}g\sin\theta=0.66$$

$$g\sin\theta\ a_{\text{solid sphere}}=\frac{g\sin\theta}{1+\frac{2}{5}}=\frac{5}{7}g\sin\theta$$

$$=0.71\,g\sin\theta$$

$$a_{\text{hollow sphere}}=\frac{g\sin\theta}{1+\frac{2}{3}}=\frac{3}{5}g\sin\theta$$

$$=0.6\,g\sin\theta$$

Clearly, $a_{\text{solid sphere}}>a_{\text{disk}}>a_{\text{hollow sphere}}$

Type of sphere is not mentioned in the question. Therefore, we will assume the given sphere as solid sphere.

$$\therefore \quad a_{\text{solid sphere}}=a_{\text{hollow sphere}}>a_{\text{disk}}$$

65 A solid sphere of mass m and radius R is rotating about its diameter. A solid cylinder of the same mass and same radius is also rotating about its geometrical axis with an angular speed twice that of the sphere. The ratio of their kinetic energies of rotation $(E_{\text{sphere}}/E_{\text{cylinder}})$ will be **[NEET 2016]**

(a) 2 : 3 (b) 1 : 5 (c) 1 : 4 (d) 3 : 1

Ans. ***(b)***

Key Idea *KE of a rotating rigid body,*

$$\text{KE}=\frac{1}{2}I\omega^2$$

$$\therefore \text{ KE of sphere, } K_S=\frac{1}{2}I\omega_1^2=\frac{1}{2}\frac{2}{5}mR^2\omega_1^2$$

$$=\frac{1}{5}mR^2\omega_1^2$$

$$\text{KE of cylinder, } K_C=\frac{1}{2}\frac{1}{2}mR^2\omega_2^2$$

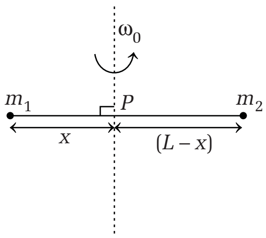
$$=\frac{1}{4}mR^2\omega_2^2$$

$$\therefore \quad \frac{K_S}{K_C}=\frac{\frac{mR^2\omega_1^2}{5}}{\frac{mR^2\omega_2^2}{4}}$$

$$=\frac{4}{5}\frac{\omega_1^2}{\omega_2^2}=\frac{4}{5}\frac{\omega_1^2}{(2\omega_1)^2}$$

$$=\frac{1}{5} \qquad (\text{given}, \omega_2=2\omega_1)$$

66 Point masses m_1 and m_2 are placed at the opposite ends of a rigid rod of length L and negligible mass. The rod is to be set rotating about an axis perpendicular to it. The position of point P on this rod through which the axis should pass, so that the work required to set the rod rotating with angular velocity ω_0 is minimum, is given by **[CBSE AIPMT 2015]**



(a) $x=\frac{m_1L}{m_1+m_2}$ (b) $x=\frac{m_1}{m_2}L$

(c) $x=\frac{m_2}{m_1}L$ (d) $x=\frac{m_2L}{m_1+m_2}$

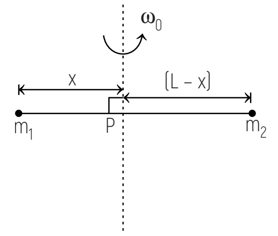
Ans. ***(d)***

As two point masses m_1 and m_2 are placed at opposite ends of a rigid rod of length L and negligible mass as shown in figure.

Total moment of inertia of the rod

$I=m_1x^2+m_2(L-x)^2$

$I=m_1x^2+m_2L^2+m_2x^2-2m_2Lx$



As, I is minimum i.e.

$$\frac{dI}{dx} = 2m_1 x + 0 + 2xm_2 - 2m_2 L = 0$$

$$\Rightarrow \quad x(2m_1 + 2m_2) = 2m_2 L$$

$$\Rightarrow \quad x = \frac{m_2 L}{m_1 + m_2}$$

When I is minimum, then work done on rotating a rod $1/2\, I\omega^2$ with angular velocity ω_0 will be minimum.

Shortcut Way *The position of point P on rod through which the axis should pass, so that the work required to set the rod rotating with minimum angular velocity ω_0 is their centre of mass, we have*

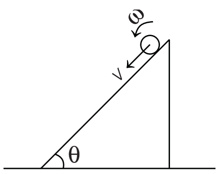
$$m_1 x = m_2 (L - x) \Rightarrow x = \frac{m_2 L}{m_1 + m_2}$$

67 The ratio of the accelerations for a solid sphere (mass m and radius R) rolling down an incline of angle θ without slipping and slipping down the incline without rolling is **[CBSE AIPMT 2014]**

(a) 5 : 7 (b) 2 : 3
(c) 2 : 5 (d) 7 : 5

Ans. *(a)*

A solid sphere rolling without slipping down an inclined plane



In this case, $a_1 = \dfrac{g\sin\theta}{1+\dfrac{k^2}{R^2}} = \dfrac{g\sin\theta}{1+\dfrac{(2/5)R^2}{R^2}}$

$$\left[\because \text{ for solid sphere}, k^2 = \frac{2}{5}R^2\right]$$

$$= \frac{g\sin\theta}{7/5} \Rightarrow a_1 = \frac{5}{7} g\sin\theta$$

For a sphere slipping down an inclined plane

$$\Rightarrow \quad a_2 = g\sin\theta \Rightarrow \frac{a_1}{a_2} = \frac{5/7\, g\sin\theta}{g\sin\theta}$$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{5}{7}$$

68 A small object of uniform density rolls up a curved surface with an initial velocity v'. It reaches upto a maximum height of $\dfrac{3v^2}{4g}$ with respect to the initial position. The object is **[NEET 2013]**

(a) ring (b) solid sphere
(c) hollow sphere (d) disc

Ans. *(d)*

As, $$v = \sqrt{\frac{2gh}{1+\dfrac{k^2}{r^2}}}$$

Given, $$h = \frac{3v^2}{4g}$$

So, $$v^2 = \frac{2gh}{1+\dfrac{k^2}{r^2}}$$

$$= \frac{2g\, 3v^2}{4g\left(1+\dfrac{k^2}{r^2}\right)} = \frac{6\, gv^2}{4\, g\left(1+\dfrac{k^2}{r^2}\right)}$$

$$1 = \frac{3}{2\left(1+\dfrac{k^2}{r^2}\right)}$$

or $$1+\frac{k^2}{r^2} = \frac{3}{2} \quad \text{or} \quad \frac{k^2}{r^2} = \frac{3}{2} - 1 = \frac{1}{2}$$

$$k^2 = \frac{1}{2}r^2 \quad \text{(Equation of disc)}$$

Hence, the object is disc.

69 A circular disc of moment of inertia I_t is rotating in a horizontal plane, about its symmetry axis, with a constant angular speed ω_i. Another disc of moment of inertia I_b is dropped coaxially onto the rotating disc. Initially the second disk has zero angular speed. Eventually both the discs rotate with a constant angular speed ω_f. The energy lost by initially rotating disc due to friction is **[CBSE AIPMT 2010]**

(a) $\dfrac{1}{2}\dfrac{I_b^2}{(I_t + I_b)}\omega_i^2$ (b) $\dfrac{1}{2}\dfrac{I_t^2}{(I_t + I_b)}\omega_i^2$

(c) $\dfrac{1}{2}\dfrac{I_b - I_t}{(I_t + I_b)}\omega_i^2$ (d) $\dfrac{1}{2}\dfrac{I_b\, I_t}{(I_t + I_b)}\omega_i^2$

Ans. *(d)*

Loss of energy is given by

$$\Delta E = \frac{1}{2} I_t\, \omega_i^2 - \frac{1}{2}\frac{I_t^2\, \omega_i^2}{(I_t + I_b)}$$

$$= \frac{1}{2}\frac{I_b\, I_t\, \omega_i^2}{(I_t + I_b)}$$

70 A wheel has angular acceleration of 3 rad / s^2 and an initial angular speed of 2 rad/s. In a time of 2 s, it has rotated through an angle (in radian) of **[CBSE AIPMT 2007]**

(a) 6 (b) 10 (c) 12 (d) 4

Ans. *(b)*

By definition $\alpha = \dfrac{d\omega}{dt}$

i.e. $$d\omega = \alpha\, dt$$

So, if in time t the angular speed of a body changes from ω_0 to ω

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha\, dt$$

If α is constant

$$\omega - \omega_0 = \alpha t \quad \text{or} \quad \omega = \omega_0 + \alpha t \qquad \text{...(i)}$$

Now, as by definition $\omega = \dfrac{d\theta}{dt}$

Eq. (i) becomes $\dfrac{d\theta}{dt} = \omega_0 + \alpha t$

i.e. $$d\theta = (\omega_0 + \alpha t)\, dt$$

So, if in time t angular displacement is θ.

$$\int_0^{\theta} d\theta = \int_0^{1} (\omega_0 + \alpha t)\, dt$$

or $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \text{...(ii)}$$

Given $$\alpha = 3\text{ rad/s}^2$$

$$\omega_0 = 2\text{ rad/s},\ t = 2\text{s}$$

Here $$\theta = 2\times 2 + \frac{1}{2}\times 3\times (2)^2$$

or $$\theta = 4 + 6 = 10\text{ rad}$$

Alternative

As we know that equation of circular motion

$$\theta = \omega t + \frac{1}{2}\alpha t^2$$

$$\left(\begin{array}{l}\text{where symbols have}\\ \text{their usual meaning}\end{array}\right)$$

Putting the value of ω, t, α from question.

So, $$\theta = 2\times 2 + \frac{1}{2}\times 3\times 2\times 2 = 10\text{ rad}$$

71 Two bodies have their moments of inertia I and $2I$ respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio **[CBSE AIPMT 2005]**

(a) 1 : 2 (b) $\sqrt{2}:1$ (c) 2 : 1 (d) $1:\sqrt{2}$

Ans. *(d)*

Concept As for linear motion $KE = \dfrac{p^2}{2m}$

Similarly, for rotational motion $KE_{\text{rot}} = \dfrac{L^2}{2I}$

As said, $(KE)_{rot}$ remains same.

i.e. $\frac{1}{2} I_1 \omega_1^2 = \frac{1}{2} I_2 \omega_2^2$

$\Rightarrow \frac{1}{2I_1}(I_1\omega_1)^2 = \frac{1}{2I_2}(I_2\omega_2)^2$

$\Rightarrow \frac{L_1^2}{I_1} = \frac{L_2^2}{I_2}$

$\Rightarrow \frac{L_1}{L_2} = \sqrt{\frac{I_1}{I_2}}$

but $I_1 = I, I_2 = 2I$

$\therefore \frac{L_1}{L_2} = \sqrt{\frac{I}{2I}} = \frac{1}{\sqrt{2}}$

or $L_1 : L_2 = 1 : \sqrt{2}$

72 A ball rolls without slipping. The radius of gyration of the ball about an axis passing through its centre of mass is k. If radius of the ball be R, then the fraction of total energy associated with its rotational energy will be **[CBSE AIPMT 2003]**

(a) $\frac{k^2}{k^2+R^2}$ (b) $\frac{R^2}{k^2+R^2}$

(c) $\frac{k^2+R^2}{R^2}$ (d) $\frac{k^2}{R^2}$

Ans. *(a)*

Kinetic energy of rotation is

$$K_{rot} = \frac{1}{2} I\omega^2 = \frac{1}{2} Mk^2 \frac{v^2}{R^2} \quad \left[K = \sqrt{\frac{I}{M}}\right]$$

where, k is radius of gyration.

Kinetic energy of translation is

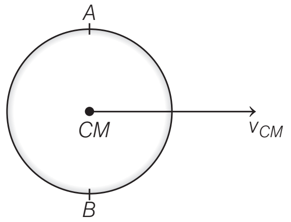
$$K_{trans} = \frac{1}{2} Mv^2$$

Thus, total energy

$$E = K_{rot} + K_{trans}$$
$$= \frac{1}{2} Mk^2 \frac{v^2}{R^2} + \frac{1}{2} Mv^2$$
$$= \frac{1}{2} Mv^2 \left(\frac{k^2}{R^2} + 1\right)$$
$$= \frac{1}{2} \frac{Mv^2}{R^2}(k^2 + R^2)$$

Hence, $$\frac{K_{rot.}}{E} = \frac{\frac{1}{2} Mk^2 \frac{v^2}{R^2}}{\frac{1}{2}\frac{Mv^2}{R^2}(k^2+R^2)}$$
$$= \frac{k^2}{k^2+R^2}$$

73 A wheel of bicycle is rolling without slipping on a level road. The velocity of the centre of mass is v_{CM}, then true statement is **[CBSE AIPMT 2001]**



(a) The velocity of point A is $2v_{CM}$ and velocity of point B is zero

(b) The velocity of point A is zero and velocity of point B is $2v_{CM}$

(c) The velocity of point A is $2v_{CM}$ and velocity of point B is $-v_{CM}$

(d) The velocities of both A and B are v_{CM}

Ans. *(a)*

Similarly, velocity of point A is given by

v_A = velocity of centre of mass (v_{CM}) + Linear velocity of point A $(R\omega)$

$$= v_{CM} + v_{CM} \quad (\because v_{CM} = R\omega)$$
$$= 2v_{CM}$$

Velocity of point B is,

$$v_B = v_{CM} - R\omega = v_{CM} - v_{CM} = 0$$

Thus, the velocity of point A is $2v_{CM}$ and velocity of point B is zero.

74 If a flywheel makes 120 rev/min, then its angular speed will be **[CBSE AIPMT 1996]**

(a) 8π rad/s (b) 6π rad/s

(c) 4π rad/s (d) 2π rad/s

Ans. *(c)*

Angular velocity of flywheel is given by

$$\omega = 2\pi\nu$$

where, ν is number of revolutions per second or frequency of revolution

Here, ν = 120 rev/min

$$\therefore \quad \omega = \frac{2\pi \times 120}{60} = 4\pi \text{ rad/s}$$

75 The angular speed of an engine wheel making 90 rev/min is **[CBSE AIPMT 1995]**

(a) 1.5π rad/s (b) 3π rad/s

(c) 4.5π rad/s (d) 6π rad/s

Ans. *(b)*

Angular velocity of an object in circular motion is defined as the time rate of change of its angular displacement.

$$\therefore \quad \omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi\nu \quad \left(\because T = \frac{1}{\nu}\right)$$

Number of revolutions made by the engine wheel $(\nu) = 90$/min.

$\therefore$ Angular velocity of the engine wheel

$$\omega = \frac{2\pi\nu}{60} = \frac{2\pi \times 90}{60} = 3\pi \text{ rad/s}$$

76 A spherical ball rolls on a table without slipping. Then, the fraction of its total energy associated with rotation is **[CBSE AIPMT 1994]**

(a) $\frac{2}{5}$ (b) $\frac{2}{7}$ (c) $\frac{3}{5}$ (d) $\frac{3}{7}$

Ans. *(b)*

Concept The total kinetic energy of the ball rolling on a table without slipping is equal to its rotational kinetic energy and translational kinetic energy. Total kinetic energy of spherical ball is given by

K = Kinetic energy rotational (K_{rot}) + Kinetic energy translational (K_{trans})

$$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$

For sphere, moment of inertia about its diameter $I = \frac{2}{5} mr^2$

$$\therefore \quad K = \frac{1}{2}\left(\frac{2}{5} mr^2\right)\omega^2 + \frac{1}{2} mv^2$$
$$= \frac{1}{5} mr^2 \omega^2 + \frac{1}{2} mv^2$$
$$= \frac{1}{5} mv^2 + \frac{1}{2} mv^2 \quad (\text{as } v = r\omega)$$
$$= \frac{7}{10} mv^2$$

$$\therefore \quad \frac{K_r}{K} = \frac{\frac{1}{5} mv^2}{\frac{7}{10} mv^2} = \frac{2}{7}$$

77 A thin uniform circular ring is rolling down an inclined plane of inclination 30° without slipping. Its linear acceleration along the inclined plane will be **[CBSE AIPMT 1994]**

(a) $\frac{g}{2}$ (b) $\frac{g}{3}$ (c) $\frac{g}{4}$ (d) $\frac{2g}{3}$

Ans. *(c)*

Acceleration of the centre of mass of the rolling body is given by

$$a = \frac{g\sin\theta}{1 + \left(\frac{I}{MR^2}\right)}$$

Moment of inertia of the ring about an axis perpendicular to the plane of the ring and passing through its centre is given by

$$I = MR^2$$

$$\therefore \quad a = \frac{g\sin\theta}{1 + MR^2/MR^2}$$
$$= \frac{g\sin 30°}{1+1} = \frac{g}{4}$$

78 A solid sphere, disc and solid cylinder all of the same mass and made of the same material are allowed to roll down (from rest) on the inclined plane, then **[CBSE AIPMT 1993]**

(a) solid sphere reaches the bottom first
(b) solid sphere reaches the bottom last
(c) disc will reach the bottom first
(d) all reach the bottom at the same time

Ans. *(a)*

Let us consider that solid sphere, disc and solid cylinder are rolling on an inclined plane. M, I and R be mass, moment of inertia and radius of the rolling section in each case.

(i) **Solid sphere** The moment of inertia of a solid sphere about its diameter is given by

$$I = \frac{2}{5} MR^2 \quad \text{or} \quad K = \frac{I}{MR^2} = \frac{2}{5}$$

As from the concept, acceleration

$$a = \frac{g \sin\theta}{1 + K}$$

So, $$a = \frac{g \sin\theta}{1 + \frac{2}{5}} = \frac{5}{7} g \sin\theta$$

(ii) **Disc** The moment of inertia of disc about an axis perpendicular to the plane of disc and passing through its centre is given by

$$I = \frac{1}{2} MR^2$$

or $$\frac{I}{MR^2} = \frac{1}{2}$$

$\therefore$ $$a = \frac{g \sin\theta}{1 + \frac{1}{2}} = \frac{2}{3} g \sin\theta$$

(iii) **Solid cylinder** The moment of inertia of a cylinder about the axis passing through its centre and perpendicular to its plane is given by

$$I = \frac{1}{2} MR^2$$

or $$\frac{I}{MR^2} = \frac{1}{2}$$

$\therefore$ $$a = \frac{g \sin\theta}{1 + \frac{1}{2}} = \frac{2}{3} g \sin\theta$$

So, acceleration of solid sphere is more. It implies that solid sphere reaches the bottom first.

79 The speed of a homogeneous solid sphere after rolling down an inclined plane of vertical height h from rest without sliding is **[CBSE AIPMT 1992]**

(a) $\sqrt{\frac{10}{7} gh}$ (b) $\sqrt{gh}$
(c) $\sqrt{\frac{6}{5} gh}$ (d) $\sqrt{\frac{4}{3} gh}$

Ans. *(a)*

When solid sphere rolls on inclined plane, then it has both rotational as well as translational kinetic energy

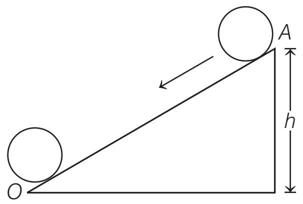
Total kinetic energy

$$K = K_{\text{rot}} + K_{\text{trans}} = \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$

For sphere, moment of inertia about its diameter

$$I = \frac{2}{5} mr^2$$

$\therefore$ $$K = \frac{1}{2}\left(\frac{2}{5} mr^2\right)\omega^2 + \frac{1}{2} mv^2$$
$$= \frac{1}{5} mr^2\omega^2 + \frac{1}{2} mv^2$$
$$= \frac{1}{5} mv^2 + \frac{1}{2} mv^2 \quad (\text{as } v = r\omega)$$
$$= \frac{7}{10} mv^2$$



On reaching sphere at O, it has only kinetic energy

$\therefore$ $$\text{PE} = \text{Total KE}$$
$$mgh = \frac{7}{10} mv^2$$
$\Rightarrow$ $$v = \sqrt{\frac{10 gh}{7}}$$

80 If a sphere is rolling, the ratio of the translational energy to total kinetic energy is given by **[CBSE AIPMT 1991]**

(a) 7 : 10 (b) 2 : 5 (c) 10 : 7 (d) 5 : 7

Ans. *(d)*

When sphere rolls, then it has both translational and rotational kinetic energy

$\therefore$ $$K = K_{\text{rot}} + K_{\text{trans}}$$
$$= \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$$

$\because$ Moment of inertia of the sphere about its diameter is

$$I = \frac{2}{5} mr^2$$

$\therefore$ $$K = \frac{1}{2}\left(\frac{2}{5} mr^2\right)\omega^2 + \frac{1}{2} mv^2$$
(as $v = r\omega$)

$\therefore$ $$K = \frac{1}{5} mv^2 + \frac{1}{2} mv^2 = \frac{7}{10} mv^2$$

$\therefore$ $$\frac{K_t}{K} = \frac{\frac{1}{2} mv^2}{\frac{7}{10} mv^2} = \frac{5}{7}$$

81 The moment of inertia of a body about a given axis is 1.2 kg-m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 J, an angular acceleration of 25 rad/s^2 must be applied about that axis for a duration of **[CBSE AIPMT 1990]**

(a) 4 s (b) 2 s
(c) 8 s (d) 10 s

Ans. *(b)*

Given,

Moment of inertia, $I = 1.2 \text{ kg-m}^2$

Rotational kinetic energy, $K_r = 1500 \text{ J}$

Angular acceleration

$$\alpha = 25 \text{ rad/s}^2, \omega_0 = 0, t = ?$$

Kinetic energy of rotation is given by

$$K_{\text{rot}} = \frac{1}{2} I\omega^2$$

$\therefore$ $$\omega = \sqrt{\frac{2K_r}{I}} = \sqrt{\frac{2 \times 1500}{1.2}}$$
$$= 50 \text{ rad/s}$$

Now, from equation of rotational motion

$$\omega = \omega_0 + \alpha t$$
$$t = \frac{\omega - \omega_0}{\alpha}$$
$$= \frac{50 - 0}{25} = 2 \text{ s}$$

82 Moment of inertia of a uniform circular disc about a diameter is I. Its moment of inertia about an axis perpendicular to its plane and passing through a point on its rim will be **[CBSE AIPMT 1990]**

(a) $5I$ (b) $3I$
(c) $6I$ (d) $4I$

Ans. *(c)*

Problem Solving Strategy

For this type of problem, always apply parallel and perpendicular axes theorem of moment of inertia.

Moment of inertia of uniform circular disc about its diameter $= I$

According to theorem of perpendicular axes,

Moment of inertia of disc about its axis $= 2I$

Applying theorem of parallel axes,

Moment of inertia of disc about the given axis

$$= 2I + mr^2$$
$$= 2I + 4I$$
$$\left(\text{as } 2I = \frac{1}{2}mr^2 \therefore mr^2 = 4I\right)$$
$$\therefore \quad = 6I$$

83 A flywheel rotating about a fixed axis has a kinetic energy of 360 J when its angular speed is 30 rad/s. The moment of inertia of the wheel about the axis of rotation is **[CBSE AIPMT 1990]**

(a) 0.6 kg -m^2 (b) 0.15 kg -m^2
(c) 0.8 kg -m^2 (d) 0.75 kg -m^2

Ans. *(c)*

A flywheel is a large heavy wheel with a long cylindrical axle supported on ball bearings. Its centre of mass lies on its axis of rotation, so that it remains at rest in any position. Rotational kinetic energy of flywheel is given by

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

where, I = moment of inertia of the wheel about the axis of rotation

ω = angular velocity of flywheel

Given, Rotational kinetic energy $K_r = 360$ J

Angular velocity $\omega = 30$ rad/s

$$\therefore \quad I = \frac{2K_r}{\omega^2} = \frac{2 \times 360}{(30)^2}$$
$$= 0.8 \text{ kg-m}^2$$

84 At any instant, a rolling body may be considered to be in pure rotation about an axis through the point of contact. This axis is translating forward with speed **[CBSE AIPMT 1989]**

(a) equal to centre of mass
(b) zero
(c) twice of centre of mass
(d) None of the above

Ans. *(a)*

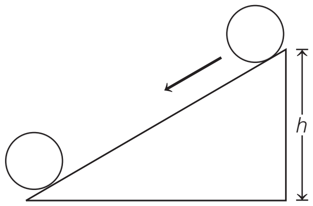
Since, in this case, instantaneous axis of rotation is always below the centre of mass. This is possible only when point of contact moves with a velocity equal to centre of mass.

85 A solid cylinder of mass M and radius R rolls down an inclined plane of height h without slipping. The speed of its centre of mass when it reaches the bottom is **[CBSE AIPMT 1989]**

(a) $\sqrt{2gh}$ (b) $\sqrt{\frac{4gh}{3}}$
(c) $\sqrt{\frac{3gh}{4}}$ (d) $\sqrt{\frac{4g}{h}}$

Ans. *(b)*

When solid cylinder rolls down on an inclined plane, then it has both rotational and translational kinetic energy



Total kinetic energy $K = K_{\text{rot}} + K_{\text{trans}}$

or $\quad K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

where, I = moment of inertia of solid cylinderabout its axis

$$= \frac{1}{2}mr^2$$
$$\therefore \quad K = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\omega^2 + \frac{1}{2}mv^2$$
$$= \frac{1}{4}mv^2 + \frac{1}{2}mv^2 \quad (\text{as } v = r\omega)$$
$$= \frac{3}{4}mv^2$$

Now, gain in KE = loss in PE

$$\therefore \quad \frac{3}{4}mv^2 = mgh$$
$$\Rightarrow \quad v = \sqrt{\left(\frac{4}{3}gh\right)}$$

86 A ring of mass m and radius r rotates about an axis passing through its centre and perpendicular to its plane with angular velocity ω. Its kinetic energy is **[CBSE AIPMT 1988]**

(a) $\frac{1}{2}mr^2\omega^2$ (b) $mr\omega^2$
(c) $mr^2\omega^2$ (d) $\frac{1}{3}mr^2\omega^2$

Ans. *(a)*

Kinetic energy of rotation of a body is the energy possessed by the body on account of its rotation about a given axis. If I is the moment of inertia of the body about the given axis of rotation, ω is angular velocity of the body, then kinetic energy of rotation

$$K_{\text{rot}} = \frac{1}{2}I\omega^2$$

Moment of inertia of the ring about an axis perpendicular to the plane of the ring and passing through its centre is

$$I = mr^2$$

So, $\quad K_{\text{rot}} = \frac{1}{2}mr^2\omega^2$

87 A solid homogeneous sphere of mass M and radius R is moving on a rough horizontal surface, partly rolling and partly sliding. During this kind of motion of the sphere **[CBSE AIPMT 1988]**

(a) total kinetic energy is conserved
(b) he angular momentum of the sphere about the point of contact with the plane is conserved
(c) only the rotational kinetic energy about the centre of mass is conserved
(d) angular momentum about the centre of mass is conserved

Ans. *(b)*

Angular momentum about the point of contact, for solid homogeneous sphere of mass M and radius R is conservd.