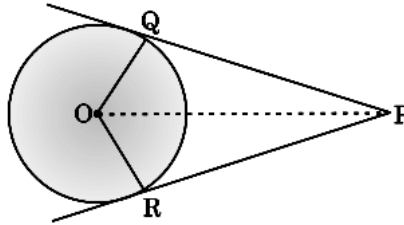# CIRCLES

THEOREMS
1) The tangent to a circle is perpendicular to the radius through the point of contact.
2) The lengths of tangents drawn from an external point to a circle are equal.

Given : A circle $C(O, r)$ and two tangents say PQ and PR from an external point P.

To prove : $PQ = PR$.

Construction : Join OQ, OR and OP.

Proof : In $\triangle OQP$ and $\triangle ORP$

$OQ = OR$	(radii of the same circle)
$OP = OP$	(Common)

$\angle Q = \angle R$ = each $90°$ (The tangent at any point of a circle is perpendicular to the radius through the point of contact)

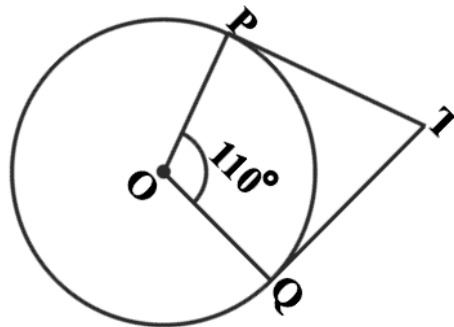Hence $\triangle OQP \cong \triangle ORP$ (By RHS Criterion)

$\therefore \quad PQ = PR$

(By CPCT)

Hence Proved.

IMPORTANT QUESTIONS

1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle

2. In the below figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then find $\angle PTQ$.


3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then find $\angle POA$

4. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

5. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

6. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

7. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

8. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC

9. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

10. Prove that the parallelogram circumscribing a circle is a rhombus.

11. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

12. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

13. XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.
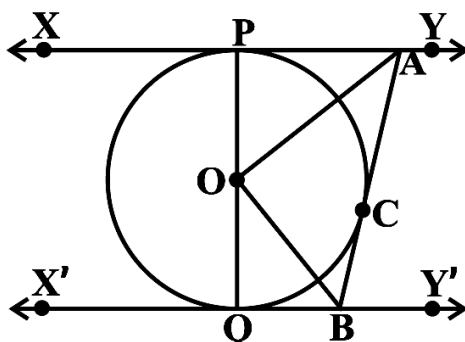

14. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.
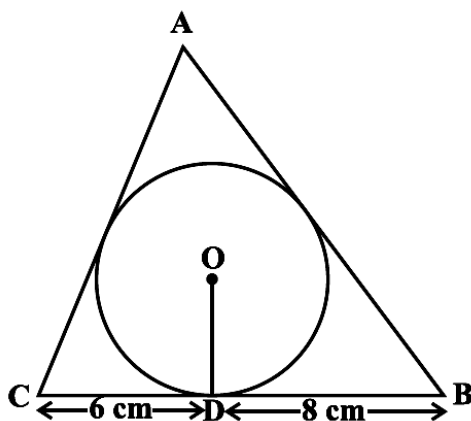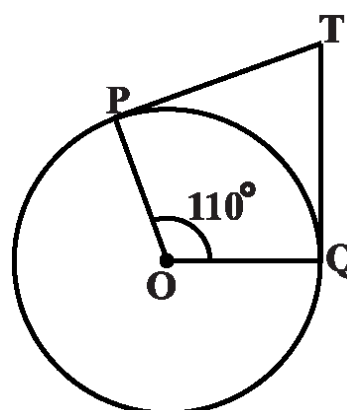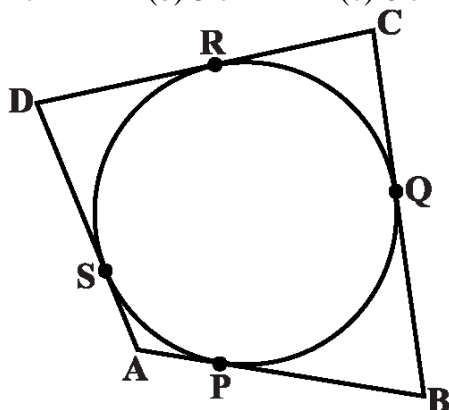

15. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2 \angle OPQ$.

16. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.

17. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

18. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120°$, prove that BC + BD = BO, i.e., BO = 2BC.

19. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

20. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

21. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the the perimeter of the triangle PCD.

22. In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.

MCQ (1 MARK)

1. Find the length of tangent drawn to a circle with radius 7 cm from a point 25 cm away from the centre.
   - (a) 24 cm
   - (b) 27 cm
   - (c) 26 cm
   - (d) 25 cm

2. A point P is 26 cm away from the centre of a circle and the length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.
   - (a) 11 cm
   - (b) 10 cm
   - (c) 16 cm
   - (d) 15 cm

3. From an external point P, tangents PA and PB are drawn to a circle with centre O. If CD is the tangent to the circle at a point E and PA = 14 cm, find the perimeter of the ΔPCD.
   - (a) 28 cm
   - (b) 27 cm
   - (c) 26 cm
   - (d) 25 cm
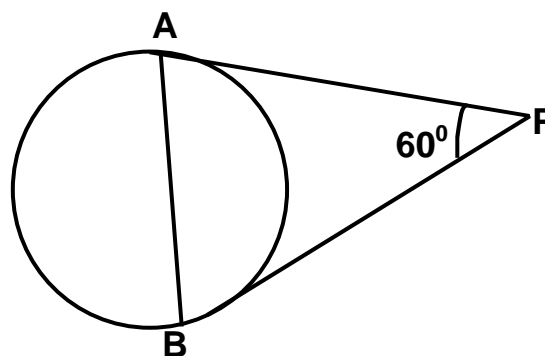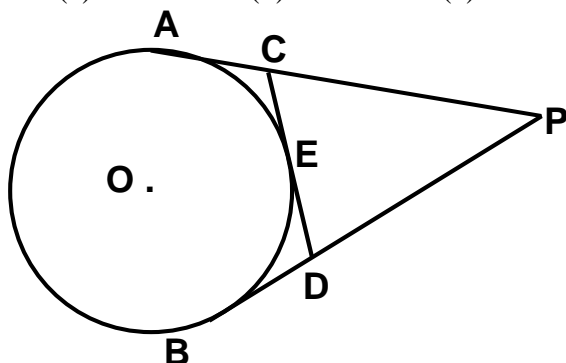
4. In the above sided figure, PA and PB are tangents such that PA = 9cm and ∠APB = 60^0. Find the length of the chord AB.
   - (a) 4 cm
   - (b) 7 cm
   - (c) 6 cm
   - (d) 9 cm

5. In the below figure the circle touches all the sides of a quadrilateral ABCD whose three sides are AB = 6 cm, BC = 7 cm, CD = 4 cm. Find AD.
   - (a) 4 cm
   - (b) 3 cm
   - (c) 6 cm
   - (d) 9 cm

6. In the above sided Fig., if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to
   - (a) 60^0
   - (b) 70^0
   - (c) 80^0
   - (d) 90^0

7. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then ∠POA is equal to
   - (a) 60^0
   - (b) 70^0
   - (c) 80^0
   - (d) 50^0

8. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.
   - (a) 4 cm
   - (b) 3 cm
   - (c) 6 cm
   - (d) 5 cm

9. From a point P, 10 cm away from the centre of a circle, a tangent PT of length 8 cm is drawn. Find the radius of the circle.

    (a) 4 cm   (b) 7 cm   (c) 6 cm   (d) 5 cm

10. PT is tangent to a circle with centre O, OT = 56 cm, TP = 90 cm, find OP

    (a) 104 cm   (b) 107 cm   (c) 106 cm   (d) 105 cm

11. TP and TQ are the two tangents to a circle with center O so that angle $\angle POQ = 130^0$. Find $\angle PTQ$.

    (a) 50^0 (b) 70^0 (c) 80^0 (d) none of these

12. From a point Q, the length of the tangent to a circle is 40 cm and the distance of Q from the centre is 41 cm. Find the radius of the circle.

    (a) 4 cm   (b) 3 cm   (c) 6 cm   (d) 9 cm

13. The common point of a tangent to a circle with the circle is called _____

    (a) centre   (b) point of contact   (c) end point   (d) none of these.