08

Mechanical Properties of Solids

TOPIC 1

Stress, Strain and Hooke's Laws

01 A uniform heavy rod of weight 10 kg ms^{-2}, cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young's modulus of the material of the rod is 2×10^{11} Nm^{-2}. Neglecting the lateral contraction, find the elongation of rod due to its own weight. **[2021, 31 Aug Shift-I]**

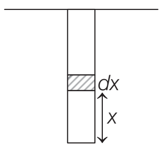
(a) 2×10^{-9} m (b) 5×10^{-8} m
(c) 4×10^{-8} m (d) 5×10^{-10} m

Ans. ***(d)***

Given, weight of rod, $w = 10$ kg ms^{-2}

Area of cross- section, $A = 100$ cm^2
$= 100 \times 10^{-4} m^2$

Length of rod, $l = 20$ cm $= 20 \times 10^{-2}$ m



Young's modulus $(Y) = 2 \times 10^{11}$ Nm^{-2}

Let, elongation $= \Delta l$

Since, Young's modulus, $Y = \dfrac{dmg\, x}{A\, \Delta l}$

Mass of element dx at distance x,

$$dm = \frac{m}{l} dx$$

$$\Rightarrow \quad \Delta l = \frac{\left(\frac{mg}{l}\right)}{AY} x\, dx$$

$$\Rightarrow \quad \Delta l = \frac{mg}{AlY} \left. \frac{x^2}{2} \right|_0^l$$

$$\Rightarrow \quad \Delta l = \frac{mg}{AlY}\frac{l^2}{2} = \frac{mgl}{2AY} = \frac{wl}{2AY}$$

$$= \frac{10 \times 20 \times 10^{-2}}{2 \times 100 \times 10^{-4} \times 2 \times 10^{11}}$$

$$= 5 \times 10^{-10} \text{ m}$$

02 When a rubber ball is taken to a depth of ...... m in deep sea, its volume decreases by 0.5%. (The bulk modulus of rubber $= 9.8 \times 10^8$ Nm^{-2}. Density of sea water $= 10^3$ kg m^{-3}, $g = 9.8$ m/s^2) **[2021, 31 Aug Shift-I]**

Ans. ***(500)***

Given, decrease in volume

$$(\Delta V / V) = -\frac{0.5}{100}$$

Bulk modulus of rubber,

$$B = 9.8 \times 10^8 \text{ Nm}^{-2}$$

Density of sea water, $\rho = 10^3 kgm^{-3}$

Acceleration due to gravity, $g = 9.8 ms^{-2}$

Let h be the depth at which ball is dipped.

Since, $B = \dfrac{\Delta p}{-\Delta V / V}$

where, Δp is change in pressure $= \rho g h$

$$\therefore \quad \rho g h = -B \frac{\Delta V}{V}$$

$$\Rightarrow \quad h = -\frac{1}{\rho g} B \frac{\Delta V}{V}$$

$$\Rightarrow h = -\frac{1}{10^3 \times 9.8} \times 9.8 \times 10^8 \times \left(-\frac{0.5}{100}\right)$$

$$= 0.5 \times 10^{8-3-2} = 0.5 \times 10^3 = 500 \text{ m}$$

03 Four identical hollow cylindrical columns of mild steel support a big structure of mass 50×10^3 kg. The inner and outer radii of each column are 50 cm and 100 cm, respectively. Assuming, uniform local distribution, calculate the compression strain of each column. [Use, $Y = 2.0 \times 10^{11}$ Pa, $g = 9.8$ m/s^2]. **[2021, 31 Aug Shift-II]**

(a) 3.60×10^{-8} (b) 2.60×10^{-7}
(c) 1.87×10^{-3} (d) 7.07×10^{-4}

Ans. ***(b)***

Let inner and outer radii of hollow cylindrical column are r and R, respectively.

Given that, $r = 50$ cm, $R = 100$ cm

Mass supported on four columns,

$$M = 50 \times 10^3 \text{ kg}$$

Mass supported on each column, $m = \dfrac{M}{4}$

$$\Rightarrow \quad m = \frac{50 \times 10^3}{4} = 12.5 \times 10^3 \text{ kg}$$

Now, weight, $w = mg = 12.5 \times 9.8 \times 10^3$ N
$= 1.225 \times 10^5$ N

Area of cross-section of each column

$$A = \pi(R^2 - r^2)$$
$$= 3.14\{(100)^2 - (50)^2\} \times 10^{-4}$$
$$m^2 = 2.35 \text{ m}^2$$

Young's modulus, $Y = 2.0 \times 10^{11}$ Pa

By using Hooke's law,

$$\text{Stress} = Y \times \text{Strain}$$

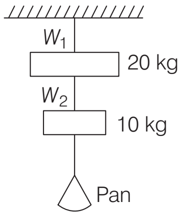
$$\therefore \text{Compressive strain} = \frac{\text{Stress}}{Y} = \frac{w}{AY}$$

Substituting the values, we get

$$\text{Compressive strain} = \frac{1.225 \times 10^5}{2.35 \times 2.0 \times 10^{11}}$$
$$= 2.60 \times 10^{-7}$$

04 Wires W_1 and W_2 are made of same material having the breaking stress of 1.25×10^9 N/m^2. W_1 and W_2 have cross-sectional area of 8×10^{-7} m^2 and $4 \times 10^{-7} m^2$, respectively. Masses of 20 kg and 10 kg hang

from them as shown in the figure. The maximum mass that can be placed in the pan without breaking the wires is .... kg.
(Use, $g = 10\ m/s^2$)
[2021, 27 Aug Shift-II]



Ans. *(40)*

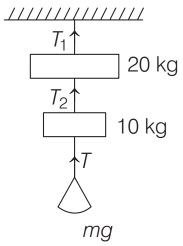
Given, breaking stress of wires 1 and 2 are $\sigma_1 = \sigma_2 = 1.25 \times 10^9\ N/m^2$

Cross-sectional area of wire 1,
$$A_1 = 8 \times 10^{-7}\ m^2$$
Cross-sectional area of wire 2,
$$A_2 = 4 \times 10^{-7}\ m^2$$
Mass hanging from first wire, $m_1 = 20$ kg

Mass hanging from second wire,
$$m_2 = 10\ kg$$
Acceleration due to gravity, $g = 10\ ms^{-2}$

Let m be the maximum mass placed in pan without breaking the wire.

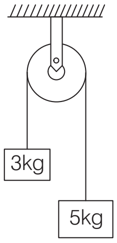
According to free body diagram of given figure,



Since, stress $(\sigma) = \dfrac{\text{Tension } (T)}{\text{Area } (A)}$

$$\therefore \quad T_1 = \sigma A_1$$
$$= 1.25 \times 10^9 \times 8 \times 10^{-7}$$
$$= 10.00 \times 10^2 = 1000\ N$$
and
$$T_2 = \sigma A_2$$
$$= 1.25 \times 10^9 \times 4 \times 10^{-7}$$
$$= 5.00 \times 10^2 = 500\ N$$
By using concept of tension in string
$$T_2 = (10 + m)\ g$$
$$\Rightarrow \quad 500 = (10 + m)\ 10$$
$$\Rightarrow \quad m = 50 - 10 = 40\ kg$$

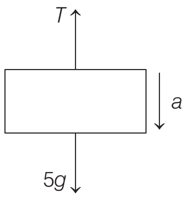
05 Two blocks of masses 3 kg and 5 kg are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $(24/\pi) \times 10^2\ Nm^{-2}$. What is the minimum radius of the wire?
(Take, $g = 10\ ms^{-2}$)
[2021, 26 Aug Shift-II]



(a) 125 cm (b) 1250 cm
(c) 12.5 cm (d) 1.25 cm

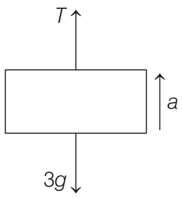
Ans. *(c)*

Given, breaking stress of wire,
$$\sigma = \frac{24}{\pi} \times 10^2\ Nm^{-2}$$
Free body diagram of 5kg block is given as



where, a is common acceleration.

Value of acceleration due to gravity,
$$g = 10\ ms^{-2}$$
From free body diagram of block of mass 5 kg
$$5g - T = 5a$$
$$\Rightarrow \quad 5 \times 10 - T = 5a$$
$$\Rightarrow \quad 50 - T = 5a \quad \ldots (i)$$
Free body diagram of 3 kg block is given as



From free body diagram of block of mass 3 kg,
$$T - 3g = 3a$$
$$\Rightarrow \quad T - 3 \times 10 = 3a$$
$$\Rightarrow \quad T - 30 = 3a \quad \ldots (ii)$$
Add Eqs. (i) and (ii), we get
$$50 - T + T - 30 = 5a + 3a$$
$$\Rightarrow \quad 20 = 8a \Rightarrow a = 2.5\ ms^{-2}$$
Substituting the value of a in Eq. (i), we get
$$50 - T = 5 \times 2.5 \Rightarrow T = 37.5\ N$$

Let us assume the minimum radius of wire is r.

The breaking stress is expressed as
$$\sigma = \frac{T}{\pi r^2}$$
$$\frac{24}{\pi} \times 10^2 = \frac{37.5}{\pi r^2}$$
$$\Rightarrow \quad r^2 = \frac{37.5}{24 \times 10^2}$$
$$= \frac{1}{64}$$
$$\Rightarrow \quad r = \frac{1}{8} m$$
$$= \frac{100}{8} cm$$
$$= 12.5\ cm$$
Thus, the minimum radius of wire should be 12.5 cm.

06 Two wires of same length and radius are joined end-to-end and loaded. The Young's moduli of the materials of the two wires are Y_1 and Y_2. The combination behaves as a single wire, then its Young's modulus is **[2021, 25 July Shift-I]**

(a) $Y = \dfrac{2Y_1Y_2}{3(Y_1 + Y_2)}$ (b) $Y = \dfrac{2Y_1Y_2}{Y_1 + Y_2}$

(c) $Y = \dfrac{Y_1Y_2}{2(Y_1 + Y_2)}$ (d) $Y = \dfrac{Y_1Y_2}{Y_1 + Y_2}$

Ans. *(b)*

Given, length of two wires, $l_1 = l_2 = l$

Radius of two wires, $r_1 = r_2 = r$

Since, Young's moduli are different.

$\therefore$ Change in length of wire 1 and 2 will be Δl_1 and Δl_2.

As we know that,
$$Y = \frac{Fl}{A \Delta l}$$
$$\Rightarrow \quad \Delta l = \frac{Fl}{YA}$$
where, F = force

and A = area of cross-section.

As the wires are in series,

$\therefore$ Net change in length,
$$\Delta l = \Delta l_1 + \Delta l_2$$
$$\frac{F2l}{Y\pi r^2} = \frac{Fl}{Y_1 \pi r^2} + \frac{Fl}{Y_2 \pi r^2}$$
$$\Rightarrow \quad \frac{2}{Y} = \frac{1}{Y_1} + \frac{1}{Y_2} \Rightarrow \frac{2}{Y} = \frac{Y_2 + Y_1}{Y_1Y_2}$$
$$\Rightarrow \quad Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$$
This is the Young's modulus for the combination of curves.

07 The length of a metal wire is l_1, when the tension in it is T_1 and is l_2 when the tension is T_2. The natural length of the wire is **[2021, 20 July Shift-II]**

(a) $\sqrt{l_1 l_2}$ (b) $\dfrac{l_1T_2 - l_2T_1}{T_2 - T_1}$

(c) $\dfrac{l_1T_2 + l_2T_1}{T_2 + T_1}$ (d) $\dfrac{l_1 + l_2}{2}$

Ans. *(b)*

According to question, length of metal wire is l_1 when the tension in it is T_1 and l_2 when the tension is T_2.

Let initial length of metal wire be l_0 and area of cross-section A.

We know that,

$$\frac{T}{A} = \frac{Y\Delta l}{l} \quad \text{...(i)}$$

where, T = tension in the metal wire,

A = area of cross-section of wire,

Y = Young's modulus of the material of wire

and Δl = change in length of wire.

Considering Eq. (i), we can write

$$\frac{T_1}{A} = \frac{Y(l_1 - l_0)}{l_0} \quad \text{...(ii)}$$

and

$$\frac{T_2}{A} = \frac{Y(l_2 - l_0)}{l_0} \quad \text{...(iii)}$$

On dividing Eq. (ii) by Eq. (iii), we get

$$\frac{\frac{T_1}{A}}{\frac{T_2}{A}} = \frac{\frac{Y(l_1 - l_0)}{l_0}}{\frac{Y(l_2 - l_0)}{l_0}}$$

$$\Rightarrow \quad \frac{T_1}{T_2} = \frac{l_1 - l_0}{l_2 - l_0}$$

$$\Rightarrow \quad T_1l_2 - T_1l_0 = T_2l_1 - T_2l_0$$

$$\Rightarrow \quad T_1l_0 - T_2l_0 = T_1l_2 - T_2l_1$$

$$\Rightarrow \quad l_0 = \frac{T_1l_2 - T_2l_1}{T_1 - T_2}$$

$$\text{or} \quad l_0 = \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$$

08 The value of tension in a long thin metal wire has been changed from T_1 to T_2. The lengths of the metal wire at two different values of tension T_1 and T_2 are l_1 and l_2, respectively. The actual length of the metal wire is **[2021, 20 July Shift-I]**

(a) $\dfrac{T_1l_2 - T_2l_1}{T_1 - T_2}$ (b) $\dfrac{T_1l_1 - T_2l_2}{T_1 - T_2}$

(c) $\dfrac{l_1 + l_2}{2}$ (d) $\sqrt{T_1T_2l_1l_2}$

Ans. *(a)*

Suppose, l_0 be the actual length of metal wire and Y be its Young's modulus.

From Hooke's law,

$$Y = \frac{Tl_0}{A\Delta l}$$

where, $\Delta l = l - l_0$

$$\Rightarrow \quad Y = \frac{Tl_0}{A(l - l_0)}$$

$$\text{or} \quad l - l = \frac{Tl_0}{AY}$$

$$\therefore \quad \frac{l_1 - l_0}{l_2 - l_0} = \frac{T_1l_0}{AY} \times \frac{AY}{T_2l_0} = \frac{T_1}{T_2}$$

$$\Rightarrow l_1T_2 - l_0T_2 = l_2T_1 - l_0T_1$$

$$\Rightarrow \quad l_0 = \frac{l_1T_2 - l_2T_1}{T_2 - T_1} = \frac{T_1l_2 - T_2l_1}{(T_1 - T_2)}$$

09 If Y, K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material, respectively. Choose the correct relation for these parameters. **[2021, 24 Feb Shift-I]**

(a) $Y = \dfrac{9K\eta}{2\eta + 3K}$ N/m^2

(b) $Y = \dfrac{9K\eta}{3K - \eta}$ N/m^2

(c) $K = \dfrac{Y\eta}{9\eta - 3Y}$ N/m^2

(d) $\eta = \dfrac{3YK}{9K + Y}$ N/m^2

Ans. *(c)*

We know that,

$$Y = 3K(1 - 2\sigma)$$

$$\Rightarrow \quad \sigma = \frac{1}{2}\left(1 - \frac{Y}{3K}\right) \quad \text{...(i)}$$

$$\text{Also,} \quad Y = 2\eta(1 + \sigma)$$

$$\Rightarrow \quad \sigma = \frac{Y}{2\eta} - 1 \quad \text{...(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$\left(1 - \frac{Y}{3K}\right)\frac{1}{2} = \frac{Y}{2\eta} - 1$$

On solving, we get

$$K = \frac{\eta Y}{9\eta - 3Y} \text{ N/m}^2$$

10 A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be .... cm. **[2021, 24 Feb Shift-II]**

Ans. *(2)*

Let initial length and diameter be l_1 and d_1, whereas final length and diameter be l_2 and d_2.

Given, $l_2 = 2l_1$, $d_2 = 2d_1$, $\Delta l_1 = 0.04$ m

By using formula of Young's modulus of elasticity,

$$Y = \frac{F \cdot l}{A\Delta l}$$

$$\because \quad Y_1 = Y_2$$

$$\Rightarrow \quad \frac{Fl_1}{A_1 \times \Delta l_1} = \frac{Fl_2}{A_2 \times \Delta l_2}$$

$$\Rightarrow \frac{Fl_1}{\pi(d_1/2)^2 \times 0.04} = \frac{F2l_1}{\pi(2d_1/2)^2 \Delta l_2}$$

$$\Rightarrow \quad \frac{1}{1/4 \times 0.04} = \frac{2}{\Delta l_2}$$

$$\Rightarrow \quad \Delta l_2 = 0.02 \text{ m} = 2 \text{ cm}$$

11 The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure p is applied uniformly on all sides, will be **[2021, 26 Feb Shift-I]**

(a) $\dfrac{\rho K}{p}$ (b) $\dfrac{\rho p}{K}$ (c) $\dfrac{K}{\rho p}$ (d) $\dfrac{pK}{\rho}$

Ans. *(b)*

Given, density of material $= \rho$

Bulk modulus of elasticity $= K$

and applied pressure $= p$

Let change in volume and density be ΔV and $\Delta\rho$ respectively and initial volume and density be V and ρ.

$$\text{Since,} \quad K = \frac{p}{-\frac{\Delta V}{V}} \quad \text{...(i)}$$

$$\text{and density } (\rho) = \frac{\text{mass } (m)}{\text{volume } (V)}$$

$$\therefore \quad \frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V}$$

Substituting it in Eq. (i), we get

$$\frac{-\Delta V}{V} = \frac{p}{K} = \frac{\Delta\rho}{\rho}$$

$$\therefore \quad \Delta\rho = \frac{p\rho}{K}$$

12 The length of metallic wire is l_1 when tension in it is T_1. It is l_2 when the tension is T_2. The original length of the wire will be **[2021, 26 Feb Shift-II]**

(a) $\dfrac{l_1 + l_2}{2}$ (b) $\dfrac{T_2l_1 + T_1l_2}{T_1 + T_2}$

(c) $\dfrac{T_2l_1 - T_1l_2}{T_2 - T_1}$ (d) $\dfrac{T_1l_1 - T_2l_2}{T_2 - T_1}$

Ans. *(c)*

Let l_0 be the original length, A be the area of cross-section, α be the coefficient of linear expansion,

Δl be the change in length and Y be the Young's modulus of elasticity.

As, $$l_1 = l_0(1+\alpha\Delta T)$$

$$\Rightarrow \quad l_1 - l_0 = l_0\alpha\Delta T \Rightarrow \Delta l = l_0\alpha\Delta T$$

Initially, $Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{T/A}{\Delta l / l_0}$

$$\Rightarrow \quad Y = \frac{T_1/A}{(l_1 - l_0)/l_0} \quad \text{...(i)}$$

Finally, $$Y = \frac{T_2/A}{(l_2 - l_0)/l_0} \quad \text{...(ii)}$$

Now, from Eqs. (i) and (ii), we get

$$\frac{T_1/A}{(l_1-l_0)/l_0} = \frac{T_2/A}{(l_2-l_0)/l_0}$$

$$\Rightarrow \quad \frac{T_1}{l_1 - l_0} = \frac{T_2}{l_2 - l_0}$$

$$\Rightarrow \quad T_1 l_2 - T_1 l_0 = T_2 l_1 - T_2 l_0$$

$$\Rightarrow \quad T_1 l_2 - T_2 l_1 = (T_1 - T_2) l_0$$

$$\Rightarrow \quad l_0 = \frac{T_1 l_2 - T_2 l_1}{T_1 - T_2} \text{ or } l_0 = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$$

13 An object is located at 2 km beneath the surface of the water. If the fractional compression $\Delta V / V$ is 1.36% , the ratio of hydraulic stress to the corresponding hydraulic strain will be ....

(Take, density of water is 1000 kg m^{-3} and $g = 9.81 \text{ ms}^{-2}$)

[2021, 17 March Shift-II]

(a) $1.96 \times 10^7 \text{ Nm}^{-2}$ (b) $1.44 \times 10^7 \text{ Nm}^{-2}$
(c) $2.26 \times 10^9 \text{ Nm}^{-2}$ (d) $1.44 \times 10^9 \text{ Nm}^{-2}$

Ans. *(d)*

Given, The volumetric strain is $\dfrac{\Delta V}{V} = 1.36\%$

The depth beneath the water surface, $h = 2$ km

The pressure inside the water surface up to 2 km, $p = \rho g h$

Substituting the values in the above equation, we get

$$p = 1000 \times 9.81 \times 2000$$
$$p = 19.62 \times 10^6 \text{ Pa}$$

The bulk modulus of the object, $\beta = \dfrac{p}{\dfrac{\Delta V}{V}}$

Substituting the values in the above equation, we get

$$\beta = \frac{19.62 \times 10^6}{\dfrac{1.36}{100}} = 1.44 \times 10^9 \text{ N/m}^2$$

Hence, the ratio of the hydraulic stress to the corresponding hydraulic strain will be $1.44 \times 10^9 \text{ N/m}^2$.

14 Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2 N. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of $a : b$. Then, a/b can be expressed as $1/x$, where x is ....

[2021, 18 March Shift-I]

Ans. *(32)*

Given,

The change in the length of the wire A, $\Delta L_A = 2 \text{ mm} = 0.002 \text{ m}$

The change in the length of the wire B, $\Delta L_B = 4 \text{ mm} = 0.004 \text{ m}$

The force subjected to the wire, $F = 2$ N

The radius of the wire B is 4 times the radius of the wire A, i.e., $\dfrac{r_B}{r_A} = \dfrac{4}{1}$

Since, the wire is made of the same material, so the Young's modulus of the elasticity of the wire is same.

$$\Rightarrow \quad Y_A = Y_B$$

Using Hooke's law,

Stress = Y (Strain)

$$\frac{F}{A} = Y\left(\frac{\Delta L}{L}\right)$$

$$\Rightarrow \quad L = \frac{Y \Delta L A}{F}$$

$$\Rightarrow \quad \frac{L_A}{L_B} = \frac{Y_A}{Y_B} \times \frac{\Delta L_A}{\Delta L_B} \times \frac{A_A}{A_B} \times \frac{F_B}{F_A}$$

$$\Rightarrow \quad \frac{L_A}{L_B} = \frac{Y_A}{Y_B} \times \frac{0.002}{0.004} \times \frac{\pi r_A^2}{\pi r_B^2} \times \frac{2}{2}$$

$$\Rightarrow \quad \frac{L_A}{L_B} = \frac{0.002}{0.004} \times \frac{r_A^2}{16 r_A^2}$$

$$\Rightarrow \quad \frac{L_A}{L_B} = \frac{a}{b} = \frac{1}{32}$$

Comparing this equation with $1/x$, we get the value of the x is 32.

15 A cube of metal is subjected to a hydrostatic pressure of 4 GPa. The percentage change in the length of the side of the cube is close to

(Take bulk modulus of metal, $B = 8 \times 10^{10}$ Pa) **[2020, 4 Sep Shift-II]**

(a) 1.67 (b) 0.6 (c) 20 (d) 5

Ans. *(a)*

Bulk modulus, $B = \dfrac{\Delta p}{\left(-\dfrac{\Delta V}{V}\right)}$

$$\Rightarrow \quad \frac{\Delta V}{V} = -\frac{\Delta p}{B}$$

$$\frac{\Delta V}{V} \times 100\% = -\frac{\Delta p}{B} \times 100\%$$

$$= \frac{-4 \times 10^9}{8 \times 10^{10}} \times 100\%$$

$$= -\frac{1}{2} \times 10\%$$

$$= -0.5 \times 10\% = -5\%$$

Now, $V = l^3$ (for cube)

$$\Rightarrow \quad l = (V)^{1/3}$$

$$\Rightarrow \quad \frac{\Delta l}{l} \times 100\% = \frac{1}{3}\left(\frac{\Delta V}{V} \times 100\%\right)$$

$$= \frac{1}{3}(-5\%) = -1.67\%$$

So, length of cube will be decreased by 1.67%.

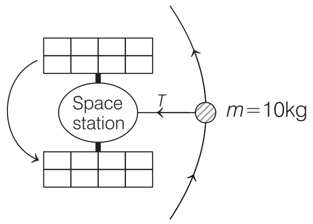
Hence, option (a) is correct.

16 A body of mass $m = 10$ kg is attached to one end of a wire of length 0.3 m. The maximum angular speed (in rad s^{-1}) with which it can be rotated about its other end in space station is (breaking stress of wire $= 4.8 \times 10^7$ Nm^{-2} and area of cross-section of the wire $= 10^{-2} \text{ cm}^2$) is

[2020, 9 Jan Shift-I]

Ans. *(4)*

Centripetal force is provided by the tension in wire.



So, $T = m\omega^2 l$

Stress in wire, $$\sigma = \frac{T}{A} = \frac{m\omega^2 l}{A} \quad \text{...(i)}$$

Here, $\sigma_{max} = 4.8 \times 10^7 \text{ Nm}^{-2}$,

$A = 10^{-2} \text{ cm}^2 = 10^{-2} \times 10^{-4} \text{ m}^2 = 10^{-6} \text{ m}^2$,

$m = 10$ kg and $l = 0.3$ m

If maximum angular speed of rotation is ω_{max}, then from Eq. (i), we have

$$\omega_{max}^2 = \frac{\sigma_{max} A}{ml} = \frac{4.8 \times 10^7 \times 10^{-6}}{10 \times 0.3} = 16$$

or $\omega_{max} = 4 \text{ rad s}^{-1}$

17 A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi\ ms^{-2}$, what will be the tensile stress that would be developed in the wire? **[2019, 8 April Shift-I]**

(a) $6.2\times10^6 Nm^{-2}$ (b) $5.2\times10^6 Nm^{-2}$
(c) $3.1\times10^6 Nm^{-2}$ (d) $4.8\times10^6 Nm^{-2}$

Ans. *(c)*

Given, radius of wire, $r = 2\ mm = 2\times10^{-3}$ m Weight of load, $m = 4$ kg, $g = 3.1\ \pi ms^{-2}$

$\therefore$ Tensile stress $= \dfrac{\text{Force}(F)}{\text{Area}(A)} = \dfrac{mg}{\pi r^2}$

$$= \frac{4\times3.1\times\pi}{\pi\times(2\times10^{-3})^2} = 3.1\times10^6 Nm^{-2}$$

18 Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to **[2019, 8 April Shift-II]**

(a) 1.3 mm (b) 1.5 mm
(c) 1.9 mm (d) 1.7 mm

Ans. *(d)*

When a wire is stretched, then change in length of wire is

$\Delta l = \dfrac{Fl}{\pi r^2 Y}$, where Y is its Young's modulus.

Here, for wires A and B,

$$l_A = 2\ m, l_B = 1.5\ m,$$

$\dfrac{Y_A}{Y_B} = \dfrac{7}{4}$, $r_B = 2\ mm = 2\times10^{-3}$ m and $\dfrac{F_A}{F_B} = 1$

As, it is given that $\Delta l_A = \Delta l_B$

$$\Rightarrow \quad \frac{F_A l_A}{\pi r_A^2 Y_A} = \frac{F_B l_B}{\pi r_B^2 Y_B}$$

$$\Rightarrow \quad r_A^2 = \frac{F_A}{F_B}\cdot\frac{l_A}{l_B}\cdot\frac{Y_B}{Y_A}\cdot r_B^2$$

$$= 1\times\frac{2}{1.5}\times\frac{4}{7}\times4\times10^{-6}\ m$$

$$= 3.04\times10^{-6}\ m$$

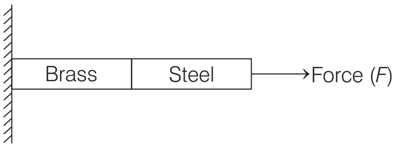
or $\quad r_A = 1.7\times10^{-3}$ m or $r_A = 1.7$ mm

19 In an experiment, brass and steel wires of length 1 m each with areas of cross-section $1 mm^2$ are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress requires to produce a net elongation of 0.2 mm is [Take, the Young's modulus for steel and brass are respectively $120\times10^9\ N/m^2$ and $60\times10^9\ N/m^2$] **[2019, 10 April Shift-II]**

(a) $1.2\times10^6\ N/m^2$ (b) $0.2\times10^6\ N/m^2$
(c) $1.8\times10^6\ N/m^2$ (d) $4.0\times10^6\ N/m^2$

Ans. *(*)*

In given experiment, a composite wire is stretched by a force F.



Net elongation in the wire = elongation in brass wire + elongation in steel wire ...(i)

Now, Young's modulus of a wire of cross-section (A) when some force (F) is applied,

$$Y = \frac{Fl}{A\Delta l}$$

We have,

$$\Delta l = \text{elongation} = \frac{Fl}{AY}$$

So, from relation (i), we have

$$\Delta l_{net} = \Delta l_{brass} + \Delta l_{steel}$$

$$\Rightarrow \quad \Delta l_{net} = \left(\frac{Fl}{AY}\right)_{brass} + \left(\frac{Fl}{AY}\right)_{steel}$$

As wires are connected in series and they are of same area of cross-section, length and subjected to same force, so

$$\Delta l_{net} = \frac{F}{A}\left(\frac{l}{Y_{brass}} + \frac{l}{Y_{steel}}\right)$$

Here, $\Delta l_{net} = 0.2\ mm = 0.2\times10^{-3}$ m and $l = 1$ m

$$Y_{brass} = 60\times10^9 Nm^{-2},$$
$$Y_{steel} = 120\times10^9 Nm^{-2}$$

On putting the values, we have

$$0.2\times10^{-3} = \frac{F}{A}\left(\frac{1}{60\times10^9} + \frac{1}{120\times10^9}\right)$$

$$\Rightarrow \text{Stress} = \frac{F}{A} = 8\times10^6 Nm^{-2}$$

20 A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\dfrac{dr}{r}\right)$ is **[JEE Main 2018]**

(a) $\dfrac{Ka}{mg}$ (b) $\dfrac{Ka}{3mg}$ (c) $\dfrac{mg}{3Ka}$ (d) $\dfrac{mg}{Ka}$

Ans. *(c)*

$\because$ Bulk modulus, $K = \dfrac{\text{Volumetric stress}}{\text{Volumetric strain}}$

$$= \frac{\Delta p}{\dfrac{\Delta V}{V}}$$

$$\Rightarrow K = \frac{mg}{a\left(\dfrac{3\Delta r}{r}\right)} \left[\because V = \frac{4}{3}\pi r^3, \text{ so } \frac{\Delta V}{V} = \frac{3\Delta r}{r}\right]$$

$$\Rightarrow \frac{\Delta r}{r} = \frac{mg}{3aK}$$

21 A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of **[JEE Main 2017 (Offline)]**

(a) $\dfrac{1}{9}$ (b) 81 (c) $\dfrac{1}{81}$ (d) 9

Ans. *(d)*

$\because$ Stress $= \dfrac{\text{Weight}}{\text{Area}}$

Volume will become (9^3) times.

So weight = volume × density × g will also become $(9)^3$ times.

Area of cross-section will become $(9)^2$ times.

$$= \frac{9^3\times W_0}{9^2\times A_0} = 9\left(\frac{W_0}{A_0}\right)$$

Hence, the stress increases by a factor of 9.

22 A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by t°C, the linear stress developed in it is **[AIEEE 2011]**

(a) $\dfrac{\alpha t}{Y}$ (b) $\dfrac{Y}{\alpha t}$
(c) $Y\alpha t$ (d) $\dfrac{1}{Y\alpha t}$

Ans. *(c)*

As change in length,

$$\Delta L = \alpha L\,\Delta T = \frac{FL}{AY}$$

$$\Rightarrow \text{Stress} = \frac{F}{A} = Y\alpha\Delta T$$

23. Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount? **[AIEEE 2009]**

(a) F (b) $4F$
(c) $6F$ (d) $9F$

Ans. *(d)*

As volume is same.

$\therefore \quad A_1 l_1 = A_2 l_2$

$\Rightarrow \quad l_2 = \frac{A_2 l_1}{A_1} = \frac{A \times l_1}{3A} = \frac{l_1}{3}$

$\Rightarrow \quad \frac{l_1}{l_2} = 3$

$\because \quad \Delta x_1 = \frac{F_1}{A\gamma} \times l_1 \quad$ ...(i)

and $\quad \Delta x_2 = \frac{F_2}{3A\gamma} l_2 \quad$ ...(ii)

Here, $\Delta x_1 = \Delta x_2$

$\Rightarrow \quad \frac{F_2}{3A\gamma} l_2 = \frac{F_1}{A\gamma} l_1$

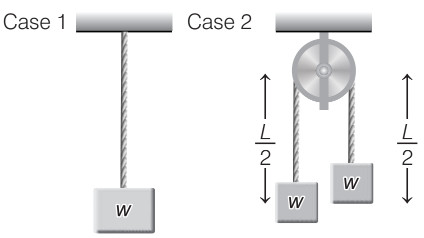
$\Rightarrow \quad F_2 = 3F_1 \times \frac{l_1}{l_2} = 3F_1 \times 3 = 9F$

24 A wire elongates by l mm when a load w is hanged from it. If the wire goes over a pulley and two weights w each are hung at the two ends, the elongation of the wire will be (in mm) **[AIEEE 2006]**

(a) l (b) $2l$
(c) zero (d) $\frac{l}{2}$

Ans. *(a)*

Let us consider the length of wire as L and cross-sectional area A, the material of wire has Young's modulus as Y.



Then, for 1st case, $Y = \frac{w/A}{l/L}$

For 2nd case, $\quad Y = \frac{w/A}{2l'/L} \Rightarrow l' = \frac{l}{2}$

So, total elongation of both sides $= 2l' = l$

TOPIC 2

Stress-Strain Curve, Thermal Stress and Elastic PE

25 A steel rod with $Y = 2.0 \times 10^{11}$ Nm^{-2} and $\alpha = 10^5$ °C^{-1} of length 4 m and area of cross-section 10 cm^2 is heated from 0°C to 400°C without being allowed to extend. The tension produced in the rod is $x \times 10^5$ N, where the value of x is .... **[2021, 1 Sep Shift-II]**

Ans. *(8)*

Given, the Young's modulus of the steel rod, $Y = 2 \times 10^{11}$ Pa

Thermal coefficient of the steel rod, $\alpha = 10^{-5}$ °C

The length of the steel rod, $l = 4$ m

The area of the cross-section, $A = 10$ cm^2

The temperature difference,

$$\Delta T = 400° \text{C}$$

As we know that,

Thermal strain $= \alpha \, \Delta T$

Using the Hooke's law

Young's modulus

$$(Y) = \frac{\text{Thermal stress}}{\text{Thermal strain}} = \frac{F/A}{\alpha \, \Delta T}$$

Thermal stress, $F = YA\alpha\,\Delta T$

Substitute the values in the above equation, we get

$$F = 2 \times 10^{11} \times 10 \times 10^{-4} \times 10^{-5} \times (400)$$

$$= 8 \times 10^5 \text{N}$$

Comparing with, $F = x \times 10^5$ N

The value of the $x = 8$.

26 A stone of mass 20 g is projected from a rubber catapult of length 0.1 m and area of cross-section 10^{-6} m^2 stretched by an amount 0.04 m. The velocity of the projected stone is .... m/s.

(Take, Young's modulus of rubber $= 0.5 \times 10^9$ N/m^2) **[27 July 2021 Shift-I]**

Ans. *(20)*

Given, mass of stone, $m = 20$ g $= 0.20$ kg

Length of catapult, $l = 0.1$ m

Area of cross-section, $A = 10^{-6}$ m^2

Young's modulus of rubber,

$$Y = 0.5 \times 10^9 \text{ N/m}^2$$

During the projection of stone from catapult,

Work done in stretching a catapult = Kinetic energy of stone after releasing from catapult

$\Rightarrow \frac{1}{2} \times Y \times (\text{Strain})^2 \times \text{Volume} = \frac{1}{2} mv^2$

$\Rightarrow \frac{1}{2} \times 0.5 \times 10^9 \times 16 \times 10^{-2} \times 10^{-7}$

$= \frac{1}{2} \times 20 \times 10^{-3} \times v^2$

$\Rightarrow \quad v^2 = 400$

$\Rightarrow \quad v = 20$ ms^{-1}

27 The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of material of track is 10^{-5}/°C. The energy stored per metre in the track is ...... J/m.

(Take, Young's modulus of material of track is 10^{11} Nm^{-2}) **[2021, 22 July Shift-II]**

Ans. *(5)*

Given, area of cross-section, $A = 0.01$ m^2

Change of temperature, $\Delta T = 10$°C

Coefficient of linear expansion, $\alpha = 10^{-5}$ / °C

Young's modulus, $Y = 10^{11}$ Nm^{-2}

Energy stored per unit length

U = Energy stored per unit volume × Area

$= \frac{1}{2} Y (\text{strain})^2 \times A$

$U = \frac{1}{2} Y (\alpha \Delta T)^2 \times A \quad \left[\because \frac{\Delta l}{l} = \alpha \Delta T\right]$

$= \frac{1}{2} \times 10^{11} (10^{-5} \times 10)^2 \times 0.01$

$= \frac{1}{2} \times 10^{11} \times 10^{-8} \times 0.01 = 5$ J/m

28 Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A) When a rod lying freely is heated, no thermal stress is developed in it.

Reason (R) On heating, the length of the rod increases.

In the light of the above statements, choose the correct answer from the options given below **[2021, 25 Feb Shift-I]**

(a) Both A and R are true but R is not the correct explanation of A.
(b) A is false but R is true.
(c) A is true but R is false.
(d) Both A and R are true and R is the correct explanation of A.

Ans. *(a)*

Thermal stress is defined as the stress, experienced by any rod on heating between two fixed rigid supports. On heating, the size of the rod increases but, if the two ends are free, rod will not experience any stress. i.e, there is no thermal stress will be produced in it.

Hence, option (a) is the correct.

29 Two steel wires having same length are suspended from a ceiling under the same load. If the ratio of their energy stored per unit volume is 1 : 4, the ratio of their diameters is **[2020, 9 Jan Shift-II]**

(a) $\sqrt{2}:1$ (b) $1:\sqrt{2}$
(c) 2 : 1 (d) 1 : 2

Ans. *(a)*

Elastic potential energy stored in a loaded wire,

$$U = \frac{1}{2}(\text{Stress} \times \text{Strain} \times \text{Volume})$$

$\therefore$ Energy stored per unit volume,

$$u = \frac{U}{\text{Volume}} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2}\left(\frac{F}{A}\right)^2 \times \frac{1}{Y}$$

Here, both wires are of same material and under same load, so the ratio of stored energies per unit volume, for both the wires will be

$$\frac{u_A}{u_B} = \frac{\frac{1}{2Y} \cdot \frac{F^2}{A_A^2}}{\frac{1}{2Y} \cdot \frac{F^2}{A_B^2}} = \frac{A_B^2}{A_A^2}$$

$$\Rightarrow \quad \frac{u_A}{u_B} = \frac{d_B^4}{d_A^4} \qquad \left(\because A = \pi\frac{d^2}{4}\right)$$

Here, $\frac{u_A}{u_B} = \frac{1}{4}$

So, $\frac{d_B^4}{d_A^4} = \frac{1}{4}$ or $\frac{d_B}{d_A} = \frac{1}{\sqrt{2}}$

$$\Rightarrow \quad \frac{d_A}{d_B} = \sqrt{2}:1$$

30 A rod of length L at room temperature and uniform area of cross-section A, is made of a metal having coefficient of linear expansion α /°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y for this metal is **[2019, 9 Jan Shift-I]**

(a) $\frac{F}{2A\alpha\,\Delta T}$ (b) $\frac{F}{A\alpha(\Delta T - 273)}$
(c) $\frac{2F}{A\alpha\Delta T}$ (d) $\frac{F}{A\alpha\Delta T}$

Ans. *(d)*

If a rod of length L and coefficient of linear expansion α/°C, then with the rise in temperature by ΔT K, its change in length is given as,

$$\Delta L = L\,\alpha\,\Delta T$$

$$\Rightarrow \quad \frac{\Delta L}{L} = \alpha\,\Delta T \qquad \text{...(i)}$$

Also, when a rod is subjected to some compressive force (F), then its' Young's modulus is given as $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$

$$\frac{\Delta L}{L} = \frac{F}{YA} \qquad \text{...(ii)}$$

Since, it is given that the length of the rod does not change. So, from Eqs. (i) and (ii), we get

$$\alpha\,\Delta T = \frac{F}{YA} \Rightarrow Y = \frac{F}{A\alpha\,\Delta T}$$

31 The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod, if it is to support a 400 N load without exceeding its elastic limit? **[2019, 10 April Shift-II]**

(a) 0.90 mm (b) 1.00 mm
(c) 1.16 mm (d) 1.36 mm

Ans. *(c)*

Let d_{min} = minimum diameter of brass.

Then, stress in brass rod is given by

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_{min}^2} \qquad \left[\because A = \frac{\pi d^2}{4}\right]$$

For stress not to exceed elastic limit, we have

$$\sigma \le 379 \text{ MPa}$$

$$\Rightarrow \frac{4F}{\pi d^2_{min}} \le 379 \times 10^6$$

Here, $F = 400$ N

$$\therefore \quad d_{min}^2 = \frac{1600}{\pi \times 379 \times 10^6}$$

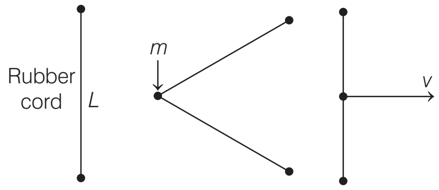
$$\Rightarrow \quad d_{min} = 1.16 \times 10^{-3} \text{ m} = 1.16 \text{ mm}$$

32 A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to **[2019, 8 April Shift-I]**

(a) 10^6 Nm^{-2}
(b) 10^4 Nm^{-2}
(c) 10^8 Nm^{-2}
(d) 10^3 Nm^{-2}

Ans. *(a)*

When rubber cord is stretched, then it stores potential energy and when released, this potential energy is given to the stone as kinetic energy.



So, potential energy of stretched cord

= kinetic energy of stone

$$\Rightarrow \frac{1}{2}Y\left(\frac{\Delta L}{L}\right)^2 A \cdot L = \frac{1}{2}mv^2$$

Here, $\Delta L = 20 \text{ cm} = 0.2 \text{ m}$,
$L = 42 \text{ cm} = 0.42 \text{ m}$,
$v = 20 \text{ ms}^{-1}$,
$m = 0.02 \text{ kg}$,
$d = 6 \text{ mm}$
$= 6 \times 10^{-3}$ m

$$\therefore \quad A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2$$

$$= \pi\left(\frac{6 \times 10^{-3}}{2}\right)^2$$

$$= \pi(3 \times 10^{-3})^2$$

$$= 9\pi \times 10^{-6} \text{ m}^2$$

On substituting values, we get

$$Y = \frac{mv^2L}{A(\Delta L)^2}$$

$$= \frac{0.02 \times (20)^2 \times 0.42}{9\pi \times 10^{-6} \times (0.2)^2}$$

$$\approx 3.0 \times 10^6 \text{ Nm}^{-2}$$

So, the closest value of Young's modulus is 10^6 Nm^{-2}.

33 The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is (For steel, Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1}) **[JEE Main 2014]**

(a) 2.2×10^{8} Pa
(b) 2.2×10^{9} Pa
(c) 2.2×10^{7} Pa
(d) 2.2×10^{6} Pa

Key Idea If the deformation is small, then the stress in a body is directly proportional to the corresponding strain.

Ans. *(a)*

By Hooke's law,

$$\text{Young's modulus}(Y) = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

So, $$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

If the rod is compressed, then compressive stress and strain appear. Their ratio Y is same as that for tensile case.

Given, length of a steel wire $(CL) = 10$ cm

Temperture $(t) = 100°C$

As length is constant.

$\therefore$ $$\text{Strain} = \frac{\Delta L}{L} \alpha\,\Delta\theta \quad [\text{as } \Delta L = (L\Delta\theta)]$$

Now, pressure = stress = $Y\times$ strain

$[\text{given}, Y = 2\times10^{11}\text{ N/m}^2$ and $\alpha = 1.1\times10^{-5}\text{ K}^{-1}]$

$$= 2\times10^{11}\times1.1\times10^{-5}\times100$$
$$= 2.2\times10^{8}\text{ Pa}$$

34 If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is **[AIEEE 2005]**

(a) $2S^2Y$ (b) $\frac{S^2}{2Y}$
(c) $\frac{2Y}{S^2}$ (d) $\frac{S}{2Y}$

Ans. (*b*)

Energy stored in wire

$$= \frac{1}{2}\times\text{Stress}\times\text{Strain}\times\text{Volume}$$

and $$\text{Young's modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$\Rightarrow \text{Strain} = \frac{S}{Y}$$

$$\therefore \frac{\text{Energy stored in wire}}{\text{Volume}} = \frac{1}{2}\times\text{Stress}\times\text{Strain}$$
$$= \frac{1}{2}\times S\times\frac{S}{Y} = \frac{S^2}{2Y}$$

35 A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is **[AIEEE 2004]**

(a) $\frac{F}{2l}$ (b) Fl
(c) $2Fl$ (d) $\frac{Fl}{2}$

Ans. *(d)*

As work done in stretching the wire

$$= \text{Potential energy stored}$$
$$= \frac{1}{2}\times\text{Stress}\times\text{Strain}\times\text{Volume}$$
$$= \frac{1}{2}\times\frac{F}{A}\times\frac{l}{L}\times AL = \frac{1}{2}Fl$$

36 A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then, the elastic energy stored in the wire is **[AIEEE 2003]**

(a) 0.2 J (b) 10 J
(c) 20 J (d) 0.1 J

Ans. *(d)*

Elastic energy stored in the wire is

$$U = \frac{1}{2}\times\text{Stress}\times\text{Strain}\times\text{Volume}$$
$$= \frac{1}{2}\times\frac{F}{A}\times\frac{\Delta l}{L}\times AL = \frac{1}{2}F\,\Delta l$$
$$= \frac{1}{2}\times200\times1\times10^{-3} = 0.1\text{ J}$$