2 Quadratic Equations and Expressions

QUICK LOOK

Identity: $f(x)=\phi(x)$ is an identity if $f(x)$ and $\phi(x)$ have the same value for every real value in R. An equation with arbitrary coefficients may be an identity under certain conditions.

- $ax^2+bx+c=0$ will be an identity (or can have more than two solutions) if coefficient of each power of x is separately zero, i.e., $a=0, b=0, c=0, d=0$.
- $ax^3+bx^2+cx+d=0$ will be an identity if $a=0, b=0, c=0, d=0$.

Polynomial equations and their solutions: If $f(x)$ is a function of x then $f(x)=0$ is an equation in one unknown (or variable) and zeros of $f(x)$ or roots of $f(x)=0$ are the values of x which make $f(x)$ equal to 0.

(i) If $f(x)$ is a polynomial of the first degree in x then the equation $f(x)=0$ is of the first degree in one unknown. $ax+b=0$ is an equation of the first degree in x. Its solution (or root) is found like this: $ax=-b$;

$\therefore\ x=\frac{-b}{a}$. A first degree equation has only one solution.

(ii) If $f(x)$ is a polynomial of the second degree in x then the equation $f(x)=0$ is of the second degree (or quadratic equation) in one unknown.

$ax^2+bx+c=0$ is an equation of the second degree in x where $a\neq 0$.

The roots of $ax^2+bx+c=0$ are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

The roots of $ax^2+bx+c=0$ can also be found by factorizing ax^2+bx+c and equating each factor to 0 separately.

A second degree equation has two solutions (different or equal).

(iii) If $f(x)$ is a polynomial of the degree three (or more) then the equation $f(x)=0$ is cubic (or of higher degree) in one unknown.

Such equations can be solved if $f(x)$ can be factorized in linear or quadratic factors.

An equation of the nth degree has n solutions (different or equal).

Example: Solve $x^3+x^2+x=84$.

Here the equation is $x^3+x^2+x-84=0$

We have to factorize x^3+x^2+x-84.

Trying $x=1,-1,2,-2$, etc., we get,

when $x=4, x^3+x^2+x-84=4^3+4^2+4-84=0$

$\therefore$ $(x-4)$ is a factor of x^3+x^2+x-84.

Dividing x^3+x^2+x-84 by $x-4$. We get $x^2+5x+21$.

$\therefore$ $x^3+x^2+x-84=0$

$\Rightarrow$ $(x-4)(x2+5x+21)=0$

$\therefore$ $x-4=0$ or $x^2+5x+21=0$

$\therefore$ $x=4$ or $x=\frac{-5\pm\sqrt{25-84}}{2}$

Exponential equations and their solutions: If the equation involves terms or factors of the type $a^{f(x)}$ or $\{\phi(x)^{f(x)}$, it will be an exponential equation.

(iv) If the exponential equation is such that it can be put in the form $a^{f(x)}=a\psi(x), a\neq 1$ $a\neq 1$ then $f(x)=\psi(x)$ will give the solution.

Example: Solve $(2\sqrt{2})^{x^2}=8^{3x}$.

Here $(2\sqrt{2})^{x^2}=\{(2\sqrt{2})^2\}^{3x}$

or $(2\sqrt{2})^{x^2}=(2\sqrt{2})^{6x}$

$\therefore$ $x^2=6x$ or $x(x-6)=0$;

$\therefore$ $x=0,6$.

(v) If the exponential equation cannot be put in the above form, select an exponential as y so that the equation changes into a polynomial equation in y.

(vi) In the exponential a^x, a is greater than 0. So, no negative value of a^x is possible.

Logarithmic equations and their solutions: If the equation involves logarithm of some function of the unknown then it will be a logarithmic equation.

- If the logarithmic equation is such that it can be put in the form $\log_a f(x)=\log_a \phi(x)$ then $f(x)=\phi(x)$ will give the solution. Only those values of x from $f(x)=\phi(x)$ will give admissible solutions which make both $f(x)$ and $g(x)$ greater than 0.

- If the logarithmic equation cannot be put in the above form, select a logarithm as y so that the equation changes into a polynomial equation in y.
- If the bases of the logarithms are also functions of x, the admissible solutions must make the values of the bases greater than 0 but not equal to 1.

Equations Involving Modulus, Greatest Integer Function, etc., and Their Solutions

- $f(x) = |x-a|$ is a piecewisely defined function whose definition is $f(x) = x-a,\ x \geq a\ \ -(x-a),\ x < a$
- $f(x) = [x]$ is a piecewisely defined function whose definition is
  $f(x) = n, n \leq x < n+1$ where n is an integer.
- $f(x) = [x+n] = [x]+n,$ where n is an integer.
- If the definition of the function is not uniform over R, the set R of real numbers should be divided into subsets according to the definitions of the function and the equation in the corresponding interval is to be solved. A solution will be admissible if it lies in the interval of definition of that equation.

In-equations and their Solutions

Laws of inequality are as follows

- $a+b > a+c \Rightarrow b > c$
  and $a > b \Rightarrow a+c > b+c.$
- $a > b \Rightarrow ca > bc$ if $c > 0$ $ca < bc$ if $c > 0$
- $ab > ac \Rightarrow b > c$ if $a > 0$ $b < c$ if $a < 0$
- $a > b$ and $c > d \Rightarrow a+c > b+d.$
- $a > 0, b > 0 \Rightarrow a+b > 0$ and $ab > 0$
  $a < 0, b < 0 \Rightarrow a+b < 0$ and $ab > 0$
  $a > 0, b < 0 \Rightarrow ab < 0$
- $ax > ay \Rightarrow x > y$ if $a > 1$ $x < y$ if $0 < a < 1$

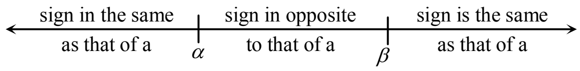
$f(x) > 0,\ f(x) < 0,\ f(x) \geq 0,\ f(x) \leq 0$ are all inequations in one variable if they hold for all $x \in A \subset R.$ But if they hold for all $x \in R$ then they are inequalities.

- $ax+b > 0$ is a linear inequation. For this inequation, $ax > -b,$ we have $x > \frac{-b}{a}$ if $a > 0$ or $x < \frac{-b}{a}$ if $a < 0.$ The solution set is an infinite set
- $ax^2+bx+c > 0$ (or < 0 or ≥ 0 or ≤ 0) is a quadratic inequation. The solution of the inequation is the set of real values of x for which the inequality is true. The set can be obtained conveniently by sign-scheme.

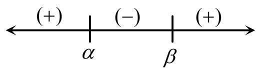
The sign-scheme for $ax^2+bx+c, x \in R$

It is as follows: Let the roots of the corresponding equation $ax^2+bx+c = 0$ be $\alpha, \beta.$

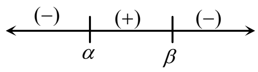
- If α, β are real and unequal $(\alpha < \beta)$ then



$\therefore$ if $a > 0,$]

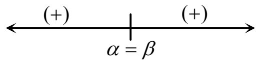


if $a < 0.$

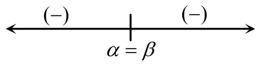


- If α, β are real and equal then

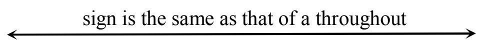
If $a > 0,$



If $a < 0,$



- If α, β are imaginary (non-real complex) then



$\therefore$ if $a > 0,$ the expression is always positive

If $a < 0,$ the expression is always negative.

Note

$|x| < a, (a > 0)$ holds when $-a < x < a$

$|x| > a, (a > 0)$ holds when $x > a$ or $x < -a$

$x^2 > a^2$ holds when $x > a$ or $x < -a$

$x^2 < a^2$ holds when $-a < x < a.$

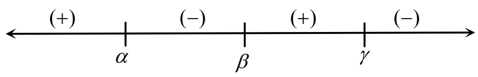
Positive definiteness and negative definiteness of a quadratic polynomial

- $ax^2+bx+c > 0$ holds for all $x \in R,$ i.e. ax^2+bx+c is positive definite, if $D < 0$ and $a > 0$ where $D = b^2-4ac.$
- $ax^2+bx+c \geq 0$ holds for all $x \in R,$ i.e., ax^2+bx+c is non-negative, if $D \leq 0$ and $a > 0.$
- $ax^2+bx+c < 0$ holds for all $x \in R,$ i.e., ax^2+bx+c is negative definite, if $D < 0$ and $a < 0.$

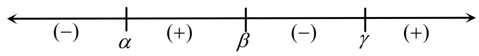
The sign-scheme for $ax^3+bx^2+cx+d, x \in R$

It is as follows: Let the roots of the corresponding equation $ax^3+bx^2+cx+d = 0$ be $\alpha, \beta, \gamma.$

- If α, β, γ are real and unequal $(\alpha < \beta < \gamma)$ then



Where a value between β and γ makes the expression positive;



Where a value between β and γ makes the expression negative.

Note

- The sign-scheme for fourth or higher degree polynomials is also prepared as above by detecting the sign of the value of the polynomial for x belonging to an interval determining by two consecutive roots and then setting alternate signs in the other intervals.
- If two roots of $f(x)=0$ for the in-equation $f(x)>0$ be real and equal to α then $(x-\alpha)^2$ is a factor of $f(x)$, which is positive for all real x except $x=\alpha$ where $f(x)$ is zero.

$\therefore$ The solution of $f(x)>0$ will be the same as $\dfrac{f(x)}{(x-\alpha)^2}>0.$ So the omission of the factor $(x-\alpha)^2$ from $f(x)$ will not affect the solution of the in-equation.

- If two roots of $f(x)=0$ for the in-equation be complex conjugate $\alpha \pm i\beta$ then $(x-\alpha)^2+\beta^2$ is a positive factor of $f(x)$. So the omission of the factor $(x-\alpha)^2+\beta^2$ from $f(x)$ will not affect the solution of the in-equaiton.
- The solution of an in-equation $f(x)>0$ or <0 or ≥ 0 or ≤ 0 is directly dependent on the solution of the corresponding equation $f(x)=0$.

Quadratic equation and its roots: If $ax^2+bx+c=0, (a \neq 0)$ be a quadratic equation whose only two roots are α, β then

- Roots $\alpha, \beta = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$,

where b^2-4ac is the discriminant D.

- The nature of the roots will be as follows :

  $D>0 \Leftrightarrow$ roots are equal and unequal (*a, b, c* being real)

  $D=0 \Leftrightarrow$ roots are real and equal (*a, b, c* being real)

  $D<0 \Leftrightarrow$ roots are non real conjugate complex (*a, b, c* being real)

  D is a perfect square $\Leftrightarrow$ roots are rational (*a, b, c* being real)

  D is a perfect square $\Leftrightarrow$ roots are rational (*a, b, c* being rational)

  D is not a perfect square (but positive) $\Leftrightarrow$ roots are conjugate irrational (*a, b, c* being rational)

Note

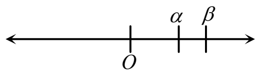
If any of the coefficients *a, b, c* is normal complex and $p+iq$ is a complex root of $ax^2+bx+c=0$ then the other root need not be $p-iq$. If any of the coefficients *a, b, c* is irrational and $p+\sqrt{q}$ be an irrational root of $ax^2+bx+c=0$ then the other root need not be $p-\sqrt{q}$. The above notes hold for equation of higher degrees also. If $a+b+c=0$ then the equation $ax^2+bx+c=0$ has the root $x=1$.

$$\alpha+\beta=\frac{-b}{a}, \alpha\beta=\frac{c}{a}$$

Sign of real roots

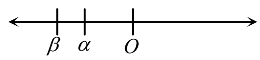
When $D \geq 0$, both roots are positive if

$$\alpha+\beta=\frac{-b}{a}>0,\ \alpha\beta=\frac{c}{a}>0$$



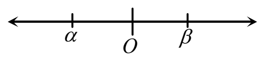
both roots are negative if

$$\alpha+\beta=\frac{-b}{a}<0,\ \alpha\beta=\frac{c}{a}>0$$



When $D>0$, one root is positive and one root is negative if

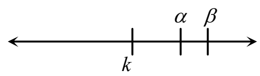
$$\alpha\beta=\frac{c}{a}<0$$



Location of real roots:

When $D \geq 0$,

both roots are greater than k

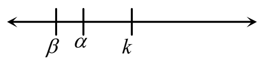


$\Rightarrow \quad \alpha-k>0, \beta-k>0$

$\Rightarrow \quad (\alpha-k)+(\beta-k)>0, (\alpha-k)(\beta-k)>0$

$\Rightarrow \quad \alpha+\beta-2k>0, \alpha\beta-k(\alpha+\beta)+k^2>0$

both roots are less than k



$\Rightarrow \quad \alpha-k<0, \beta-k<0$

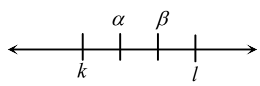
$\Rightarrow \quad (\alpha-k)+(\beta-k)<0, (\alpha-k)(\beta-k)>0$

both roots lie between k and l $(k<l)$

$\Rightarrow \quad \alpha-k>0, \beta-k>0, \alpha-l<0, \beta-l<0$

$\Rightarrow \alpha-k+\beta-k>0, \alpha-l+\beta-l<0$

$(\alpha-k)(\beta-k)>0, (\alpha-l)(\beta-l)>0$

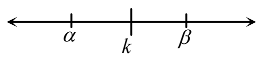


(b) When $D>0$,

(i) one root is less than k and the other greater than k

$\Rightarrow \alpha-k<0, \beta-k>0$

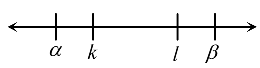
$\Rightarrow (\alpha-k)(\beta-k)<0$



(ii) one root is less than k and the other greater than l (k<1)

$\Rightarrow \alpha-k<0, \beta-k>0, \alpha-l<0, \beta-l>0$

$\Rightarrow (\alpha-k)(\beta-k)<0, (\alpha-l)(\beta-l)<0$



Equations of higher degrees and their roots

$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+....+a_n=0$, when a is are real $(a_0 \neq 0)$, is an nth degree polynomial equation in one variable x. It has n roots (unequal or equal) which are either real or nonreal complex.

MULTIPLE CHOICE QUESTIONS

Identity and Polynomial

1. Both the roots of given equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are always:

a. Positive **b.** Negative
c. Real **d.** Imaginary

2. If 3 is a root of $x^2 + kx - 24 = 0$, it is also a root of:

a. $x^2+5x+k=0$ **b.** $x^2-5x+k=0$
c. $x^2-kx+6=0$ **d.** $x^2+kx+24=0$

3. For what values of k will the equation $x^2-2(1+3k)x+7\ (3+2k)=0$ have equal roots?

a. 1, –10/9 **b.** 2, –10/9
c. 3, –10/9 **d.** 4, –10/9

Equations of Higher Degrees and Their Roots

4. If the difference between the corresponding roots of $x^2+ax+b=0$ and $x^2+bx+a=0$ is same and $a \neq b$, then:

a. $a+b+4=0$ **b.** $a+b-4=0$
c. $a-b-4=0$ **d.** $a-b+4=0$

5. If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $a/c, b/a, c/b$ are in:

a. A.P. **b.** G.P.
c. H.P. **d.** None of these

6. If the roots of the equation $x^2-5x+16=0$ are α, β and the roots of equation $x^2+px+q=0$ are $\alpha^2+\beta^2$, $\alpha\beta/2$, then:

a. $p=1, q=-56$ **b.** $p=-1, q=-56$
c. $p=1, q=56$ **d.** $p=-1, q=56$

7. If one root of the equation $x^2+px+q=0$ is the square of the other, then:

a. $p^3+q^2-q(3p+1)=0$ **b.** $p^3+q^2+q(1+3p)=0$
c. $p^3+q^2+q(3p-1)=0$ **d.** $p^3+q^2+q(1-3p)=0$

8. Let α and β be the roots of the equation $x^2+x+1=0$, the equation whose roots are α^{19}, β^7 is:

a. $x^2-x-1=0$ **b.** $x^2-x+1=0$
c. $x^2+x-1=0$ **d.** $x^2+x+1=0$

9. If one root of a quadratic equation is $\frac{1}{2+\sqrt{5}}$, then the equation is:

a. $x^2+4x+1=0$ **b.** $x^2+4x-1=0$
c. $x^2-4x+1=0$ **d.** None of these

10. If one of the roots of the equation $x^2+ax+b=0$ and $x^2+bx+a=0$ is coincident. Then the numerical value of $(a+b)$ is:

a. 0 **b.** – 1
c. 2 **d.** 5

Properties of Quadratic Equation

11. The value of 'a' for which one root of the quadratic equation $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as large as the other is:

a. 2/3 **b.** – 2/3
c. 1/3 **d.** – 1/3

Quadratic Expression

12. If x be real, then the minimum value of $x^2-8x+17$ is:

a. 0– 1 **b.** 0
c. 1 **d.** 2

Solution of Quadratic Equations and Nature of Roots

13. The roots of the equation $a(x^2+1)-(a^2+1)x=0$ are:

a. $a, \frac{1}{a}$ **b.** $a, 2a$
c. $a, \frac{1}{2a}$ **d.** None of these

14. The roots of the equation $ix^2-4x-4i=0$ are:

a. $-2i$ **b.** $2i$
c. $-2i, -2i$ **d.** $2i, 2i$

15. The number of roots of the quadratic equation $8\sec^2\theta-6\sec\theta+1=0$ is:

a. Infinite **b.** 1
c. 2 **d.** 0

16. The number which exceeds its positive square root by 12 is:

a. 9 **b.** 16
c. 25 **d.** None of these

17. If $x^{2/3}-7x^{1/3}+10=0$, then $x=?$

a. {125} **b.** {8}
c. ϕ **d.** {125, 8}

18. The solution set of the equation $x^{\log_x(1-x)^2}=9$ is:

a. {– 2, 4} **b.** {4}
c. {0, – 2, 4} **d.** None of these

19. The number of real roots of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ are:

a. 1 **b.** 2
c. Infinite **d.** None

20. The solution of the equation $x + \frac{1}{x} = 2$ will be:

a. 2, –1 **b.** 0, –1, $-\frac{1}{5}$
c. $-1, -\frac{1}{5}$ **d.** None of these

21. If $\sqrt{3x^2 - 7x - 30} + \sqrt{2x^2 - 7x - 5} = x + 5$,then x is equal to:

a. 2 **b.** 3
c. 6 **d.** 5

22. If x_1, x_2, x_3 are distinct roots of the equation $ax^2 + bx + c = 0$ then:

a. $a = b = 0, c \in R$ **b.** $a = c = 0, b \in R$
c. $b^2 - 4ac \geq 0$ **d.** $a = b = c = 0$

23. The value of $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + ...}}}$ is:

a. –1 **b.** 1
c. 2 **d.** 3

24. The equation $e^x - x - 1 = 0$ has:

a. Only one real root $x = 0$
b. At least two real roots
c. Exactly two real roots
d. Infinitely many real roots

25. A real root of the equation $\log_4\{\log_2(\sqrt{x+8} - \sqrt{x})\} = 0$ is:

a. 1 **b.** 2
c. 3 **d.** 4

26. If the roots of the equations $px^2 + 2qx + r = 0$ and $qx^2 - 2\sqrt{pr}x + q = 0$ be real, then:

a. $p = q$ **b.** $q^2 = pr$
c. $p^2 = qr$ **d.** $r^2 = pq$

27. If $a > 0, b > 0, c > 0$ then both the roots of the equation $ax^2 + bx + c = 0$?

a. Are real and negative
b. Have negative real parts
c. Are rational numbers
d. None of these

Relation between Roots and Coefficients

28. If one root of $5x^2 + 13x + k = 0$ is reciprocal of the other, then k = ?

a. 0 **b.** 5
c. 1/6 **d.** 6

29. If the product of the roots of the equation $(a+1)x^2 + (2a+3)x + (3a+4) = 0$ be 2, then the sum of roots is:

a. 1 **b.** –1
c. 2 **d.** –2

30. If α, β are the roots of the equation $ax^2 + bx + c = 0$ then the equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is:

a. $acx^2 + (a+c)bx + (a+c)^2 = 0$
b. $abx^2 + (a+c)bx + (a+c)^2 = 0$
c. $acx^2 + (a+b)cx + (a+c)^2 = 0$
d. None of these

31. If α and β be the roots of the equation $2x^2 + 2(a+b)x + a^2 + b^2 = 0$, then the equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$ is:

a. $x^2 - 2abx - (a^2 - b^2)^2 = 0$ **b.** $x^2 - 4abx - (a^2 - b^2)^2 = 0$
c. $x^2 - 4abx + (a^2 - b^2)^2 = 0$ **d.** None of these

32. If the sum of the roots of the equation $\lambda x^2 + 2x + 3\lambda = 0$ be equal to their product, then λ = ?

a. 4 **b.** –4
c. 6 **d.** None of these

33. If α and β are the roots of the equation $2x^2 - 3x + 4 = 0$, then the equation whose roots are α^2 and β^2 is:

a. $4x^2 + 7x + 16 = 0$ **b.** $4x^2 + 7x + 6 = 0$
c. $4x^2 + 7x + 1 = 0$ **d.** $4x^2 - 7x + 16 = 0$

34. If the ratio of the roots of the equation $ax^2 + bx + c = 0$ be $p:q$, then:

a. $pqb^2 + (p+q)^2ac = 0$ **b.** $pqb^2 - (p+q)^2ac = 0$
c. $pqa^2 - (p+q)^2bc = 0$ **d.** None of these

35. If α, β be the roots of the equation $x^2 - 2x + 3 = 0$, then the equation whose roots are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ is:

a. $x^2 + 2x + 1 = 0$ **b.** $9x^2 + 2x + 1 = 0$
c. $9x^2 - 2x + 1 = 0$ **d.** $9x^2 + 2x - 1 = 0$

36. If α,β be the roots of $x^2-px+q=0$ and α',β' be the roots of $x^2-p'x+q'=0$, then the value of $(\alpha-\alpha')^2+(\beta-\alpha')^2+(a-\beta')^2+(\beta-\beta')^2$ is:

a. $2\{p^2-2q+p'^2-2q'-pp'\}$

b. $2\{p^2-2q+p'^2-2q'-qq'\}$

c. $2\{p^2-2q-p'^2-2q'-pp'\}$

d. $2\{p^2-2q-p'^2-2q'-qq'\}$

37. If α,β are the roots of $(x-a)(x-b)=c,\ c\neq 0$, then the roots of $(x-\alpha)(x-\beta)+c=0$ shall be:

a. $a,\ c$ **b.** b,c

c. a,b **d.** $a+c,b+c$

38. If the roots of the equation $\frac{1}{x+p}+\frac{1}{x+q}=\frac{1}{r}$ are equal in magnitude but opposite in sign, then the product of the roots will be:

a. $\frac{p^2+q^2}{2}$ **b.** $-\frac{(p^2+q^2)}{2}$

c. $\frac{p^2-q^2}{2}$ **d.** $-\frac{(p^2-q^2)}{2}$

39. If α,β are roots of $x^2-3x+1=0$, then the equation whose roots are $\frac{1}{\alpha-2},\frac{1}{\beta-2}$ is:

a. $x^2+x-1=0$ **b.** $x^2+x+1=0$

c. $x^2-x-1=0$ **d.** None of these

40. If α and β are the roots of $6x^2-6x+1=0$, then the value of $\frac{1}{2}[a+b\alpha+c\alpha^2+d\alpha^3]+\frac{1}{2}[a+b\beta+c\beta^2+d\beta^3]$ is:

a. $\frac{1}{4}(a+b+c+d)$ **b.** $\frac{a}{1}+\frac{b}{2}+\frac{c}{3}+\frac{d}{4}$

c. $\frac{a}{2}-\frac{b}{2}+\frac{c}{3}-\frac{d}{4}$ **d.** None of these

41. If the roots of the quadratic equation $\frac{x-m}{mx+1}=\frac{x+n}{nx+1}$ are reciprocal to each other, then:

a. $n=0$ **b.** $m=n$

c. $m+n=1$ **d.** $m^2+n^2=1$

42. If a and b are roots of $x^2-px+q=0$, then $\frac{1}{a}+\frac{1}{b}=?$

a. $\frac{1}{p}$ **b.** $\frac{1}{q}$ **c.** $\frac{1}{2p}$ **d.** $\frac{p}{q}$

43. If α,β are the roots of $ax^2+bx+c=0$ and $\alpha+\beta$, $\alpha^2+\beta^2$, $\alpha^3+\beta^3$ are in G.P., where $\Delta=b^2-4ac$, then:

a. $\Delta\neq 0$ **b.** $b\Delta=0$

c. $cb\neq 0$ **d.** $c\Delta=0$

44. If $3p^2=5p+2$ and $3q^2=5q+2$ where $p\neq q$, then the equation whose roots are $3p-2q$ and $3q-2p$ is:

a. $3x^2-5x-100=0$ **b.** $5x^2+3x+100=0$

c. $3x^2-5x+100=0$ **d.** $5x^2-3x-100=0$

Condition for Common Roots, Quadratic Expressions and Position of Roots

45. If the roots of the equation $x^2-2ax+a^2+a-3=0$ are real and less than 3, then:

a. $a<2$ **b.** $2\leq a\leq 3$

c. $3<a\leq 4$ **d.** $a>4$

46. If both the roots of $k(6x^2+3)+rx+2x^2-1=0$ and $6k(2x^2+1)+px+4x^2-2=0$ are common, then $2r-p$ is equal to:

a. –1 **b.** 0

c. 1 **d.** 2

47. If the two equations $x^2-cx+d=0$ and $x^2-ax+b=0$ have one common root and the second has equal roots, then $2(b+d)=?$

a. 0 **b.** $a+c$

c. ac **d.** $-ac$

48. If every pair of the equations $x^2+px+qr=0$, $x^2+qx+rp=0$, $x^2+rx+pq=0$ have a common root, then the sum of three common roots is:

a. $\frac{-(p+q+r)}{2}$ **b.** $\frac{-p+q+r}{2}$

c. $-(p+q+r)$ **d.** $-p+q+r$

49. If the equation $x^2+px+q=0$ and $x^2+qx+p=0$, have a common root, then $p+q+1=?$

a. 0 **b.** 1 **c.** 2 **d.** – 1

50. $x^2-11x+a$ and $x^2-14x+2a$ will have a common factor, if $a=?$

a. 24 **b.** 0, 24 **c.** 3, 24 **d.** 0, 3

51. If x^2-3x+2 be a factor of x^4-px^2+q, then $(p,q)=?$

a. (3, 4) **b.** (4, 5)

c. (4, 3) **d.** (5, 4)

52. If x is real, the expression $\frac{x+2}{2x^2+3x+6}$ takes all value in the interval:

a. $\left(\frac{1}{13},\frac{1}{3}\right)$ **b.** $\left[-\frac{1}{13},\frac{1}{3}\right]$

c. $\left(-\frac{1}{3},\frac{1}{13}\right)$ **d.** None of these

53. If x, y, z are real and distinct, then $u = x^2+4y^2+9z^2-6yz-3zx-zxy$ is always:

a. Non-negative **b.** Non-positive

c. Zero **d.** None of these

54. If x is real, the function $\frac{(x-a)(x-b)}{(x-c)}$ will assume all real values, provided:

a. $a>b>c$ **b.** $a<b<c$

c. $a>c<b$ **d.** $a<c<b$

55. If the roots of the equation $x^2-2ax+a^2+a-3=0$ are real and less than 3, then:

a. $a<2$ **b.** $2\le a\le 3$

c. $3<a\le 4$ **d.** $a>4$

56. If a,b,c are real numbers such that $a+b+c=0$, then the quadratic equation $3ax^2+2bx+c=0$ has:

a. At least one root in [0, 1]

b. At least one root in [1, 2]

c. At least one root in $[-1,0]$

d. None of these

57. If the equation $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x = 0$ $a_1 \ne 0$, $n \ge 2$, has a positive root $x=\alpha$, then the equation $na_n x^{n-1}$ $+(n-1)$ $a_{n-1}x^{n-2}+....+a_1=0$ has a positive root, which is

a. Greater than or equal to α **b.** Equal to α?

c. Greater than α **d.** Smaller than α

Descarte's Rule of Signs

58. The maximum possible number of real roots of equation $x^5-6x^2-4x+5=0$ is:

a. 0 **b.** 3

c. 4 **d.** 5

Calculus in Problems of Equations and Expressions

59. If $\frac{2x}{2x^2+5x+2} > \frac{1}{x+1}$, then:

a. $-2>x>-1$ **b.** $-2\ge x\ge -1$

c. $-2<x<-1$ **d.** $-2<x\le -1$

60. If for real values of x, $x^2-3x+2>0$ and $x^2-3x-4\le 0$, then:

a. $-1\le x<1$ **b.** $-1\le x<4$

c. $-1\le x<1$ or $2<x\le 4$ **d.** $2<x\le 4$

Equation and In-equation Containing Absolute Value

61. The roots of $|x-2|^2+|x-2|-6=0$ are:

a. 0, 4 **b.** –1, 3 **c.** 4, 2 **d.** 5, 1

62. The set of all real numbers x for which $x^2-|x+2|+x>0$, is:

a. $(-\infty,-2)\cup(2,\infty)$ **b.** $(-\infty,-\sqrt{2})\cup(\sqrt{2},\infty)$

c. $(-\infty,-1)\cup(1,\infty)$ **d.** $(\sqrt{2},\infty)$

63. Product of real roots of the equation $t^2x^2+|x|+9=0$ $(t\ne 0)$

a. Is always +*ve* **b.** Is always –*ve*

c. Does not exist **d.** None of these

64. The number of solution of $\log_4(x-1)=\log_2(x-3)$?

a. 3 **b.** 1

c. 2 **d.** 0

NCERT EXEMPLAR PROBLEMS

More than One Answer

65. Let $a\in R$ and $f:R\to R$ be given by $f(x)=x^5-5x+a$. Then:

a. $f(x)$has three real roots, if $a>4$

b. $f(x)$has only one real root, if $a>4$

c. $f(x)$has three real roots, if $a<-4$

d. $f(x)$has three real roots, if -4 $a<4$

66. Let $f(x)$ be a quadratic expression which is positive for all real x. If $g(x)=f(x)-f'(x)+f''(x)$, then for any real x:

a. $g(x)>0$ **b.** $g(x)\ge 0$

c. $g(x)\le 0$ **d.** $g(x)<0$

67. The real values of λ for which the equation, $3x^3+x^2-7x+\lambda=0$, has two distinct real roots in [0, 1] lie in the interval: (s)

a. $(-2,0)$ **b.** $[0,1]$

c. $[0,2]$ **d.** $(-\infty,\infty)$

68. The roots of the equation, $(x^2+1)^2=x(3x^2+4x+3)$, are given by:

a. $2-\sqrt{3}$ **b.** $(-1+i\sqrt{3})/2, i=\sqrt{-1}$

c. $2+\sqrt{3}$ **d.** $(-1-i\sqrt{3})/2, i=\sqrt{-1}$

69. If A, G and H are the Arithmetic mean, Geometric mean and Harmonic mean between two unequal positive integers. Then the equation $Ax^2 - |G|\,x - H = 0$ has:

a. both roots are fraction

b. at least one root which is negative fraction

c. exactly one positive root

d. at least one root which is an integer

70. If $a, b, c \in R$ and the equality $ax^2 - bx + c = 0$ has complex roots which are reciprocal of each:

a. $|b| \le |a|$ **b.** $|b| \le |c|$

c. $a = c$ **d.** $b \ge a$

71. The equation $|x+1||x-1| = a^2 - 2a - 3$ can have real solution for x, if a belongs x to:

a. $(-\infty, -1] \cup [3, \infty)$ **b.** $[1-\sqrt{5}, 1+\sqrt{5}]$

c. $[1-\sqrt{5}, -1] \cup [3, 1+\sqrt{5}]$ **d.** none of these

72. The equation $x^2 + a^2x + b^2 = 0$ has two roots each of which exceeds a number c, then:

a. $a^4 > 4b^2$ **b.** $c^2 + a^2c + b^2 > 0$

c. $-a^2/2 > c$ **d.** none of these

73. A quadratic equation whose difference of roots is 3 and the sum of the squares of the roots is 29 is given by:

a. $x^2 + 9x + 14 = 0$ **b.** $x^2 + 7x + 10 = 0$

c. $x^2 - 7x - 10 = 0$ **d.** $x^2 - 7x + 10 = 0$

74. If a, b, c are distinct number in arithmetic progression, then both the roots of the quadratic equation $(a + 2b - 3c)\ x^2 + (b + 2c - 3a)x\ + (c + 2a - 3b) = 0$ are:

a. real **b.** positive

c. negative **d.** rational

Assertion and Reason

Note: Read the Assertion (A) and Reason (R) carefully to mark the correct option out of the options given below:

a. If both assertion and reason are true and the reason is the correct explanation of the assertion.

b. If both assertion and reason are true but reason is not the correct explanation of the assertion.

c. If assertion is true but reason is false.

d. If the assertion and reason both are false.

e. If assertion is false but reason is true.

75. **Assertion:** If $a,b,c \in R - \{0\}$, then at least one $ax^2 + bx + c = 0$, $bx^2 + cx + a = 0$ and $cx + ax + b = 0$ has imaginary roots.

Reason: If $a,b,c \in R, a \ne 0$, then imaginary roots of the equation $ax^2 + bx + c = 0$ occur in conjugate pair.

76. **Assertion:** The equation $f(x) 1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} = 0$ has two pairs of repeated roots.

Reason: Polynomial equation $P(x) = 0$ has a repeated root α if $P(\alpha) = 0$ and $P'(\alpha) = 0$.

77. **Assertion:** If all the four roots of $x^4 - 4x^3 + ax^2 - bx + 1 = 0$ are positive, then $a = 6$ and $b = 4$.

Reason: If polynomial equation $P(x) = 0$ has four positive roots, then the polynomial equation $P'(x) = 0$ has 3 positive roots.

78. **Assertion:** If $a, b, c \in Q$ & $2^{1/3}$ satisfies $a + bx + cx^2 = 0$, then $a = 0, b = 0, c = 0$.

Reason: A polynomial equation with rational coefficients cannot have irrational roots.

79. Let $a, b, c \in R$, a>0 and function $f : R \to R$ be defined by $f(x) = ax^2 + 2bx + c$.

Assertion: $b^2 < ac \Rightarrow f(x) > 0$ for every value of x

Reason: f is strictly decreasing in the interval $(-\infty, b/a)$ and strictly increasing in the interval $(-b/a, \infty)$

80. **Assertion:** If $a,b,c \in \mathrm{R}$ and $2a + 3b + 6c = 0$, then the equation $ax^2 + bx + c = 0$ has at least one root in [0, 1]

Reason: If a continuous function f defined on R assumes both positive and negative values, then it, vanishes at least once.

81. **Assertion:** Let $f(x) = ax^2 + bx + c, a,b,c \in \mathrm{R}$. If $f(x)$ assumes real values for real values of x and non-real values of for non-real values of x, then $a = 0$

Reason: If a, b, c are complex numbers, $a \ne 0$ then $a + i\beta, \beta \ne 0$ is a root of $ax^2 + bx + c = 0$ if and only if $\alpha - i\beta$ is a root of $ax^2 + bx + c = 0$

82. **Assertion:** If $a \ne 0$ and the equation $ax^2 + bx + c = 0$ has two roots α and β such that $\alpha < -2$ and $\beta > 2$, then $a + |b| + c$ and a have the opposite signs.

Reason: If $a>0$ and $\gamma,\delta(\gamma<\delta)$ are the roots of $ax^2+bx+c=0,$ then graph of the parabola $y=ax^2+bx+c,$ for $\gamma<x<\delta$ lies below the x-axis.

83. Let a, b, c, p, q be the real numbers. Suppose α, β are the roots of the equation $x^2+2px+q=0$ and $\alpha,\frac{1}{\beta}$ are the roots of the equation $ax^2+2bx+c=0,$ where $\beta^2\notin\{-1,0,1\}$.

Assertion: $(p^2-q)(b^2-ac)\geq 0$ and

Reason: $b\notin pa$ or $c\notin qa$.

Comprehension Based

Paragraph-I

Read the following passage and answer the questions. If a continuous f defined on the real line R, assumes positive and negative values in R, then the equation $f(x)=0$ has a root in R. For Illustration:, if it is known that a continuous function f on R is positive at some point and its minimum values is negative, then the equation $f(x)=0$ has a root in R. Consider $f(x)=ke^x-x$ for all real x where k is real constant.

84. The line $y=x$ meets $y=ke^x$ for $k\leq 0$ at:

a. no point **b.** one point
c. two points **d.** more than two points

85. The positive value of k for which $ke^x-x=0$ has only one root is:

a. $\frac{1}{e}$ **b.** 1
c. e **d.** $\log_e 2$

86. For $k>0,$ the set of all values of k for which $ke^x-x=0$ has two distinct roots, is:

a. $\left(0,\frac{1}{e}\right)$ **b.** $\left(\frac{1}{e},1\right)$
c. $\left(\frac{1}{e},\infty\right)$ **d.** (0,1)

Paragraph-II

Consider the polynomial $f(x)=1+2x+3x^2+4x^3.$ Let s be the sum of all distinct real roots of $f(x)$ and let $t=|s|$

87. The real numbers s lies in the interval:

a. $\left(-\frac{1}{4},0\right)$ **b.** $\left(-11,-\frac{3}{4}\right)$ **c.** $\left(-\frac{3}{4},-\frac{1}{2}\right)$ **d.** $\left(0,\frac{1}{4}\right)$

88. The area bounded by the curve $y=f(x)$ and the lines $x=0, y=0$ and $x=t,$ lies in the interval:

a. $\left(\frac{3}{4},3\right)$ **b.** $\left(\frac{21}{64},\frac{11}{16}\right)$
c. (9,10) **d.** $\left(0,\frac{21}{64}\right)$

89. The function $f'(x)$ is:

a. increasing in $\left(-t,-\frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4},t\right)$
b. decreasing in $\left(-t,-\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4},t\right)$
c. increasing in $(-t,t)$
d. decreasing in $(-t,t)$

Match the Column

90. A Observe the following columns:

Column I	Column II
(A) A The number of solution of the system of equations $x+2y=6$ and $\lvert x-3\rvert = y$ is m, then.	**1.** m is the AM of n and p
(B) If x and y are integers and $(x-8)(x-10)=2^y$ the number of solution is n, then.	**2.** n is the GM of m and p
(C) The number of integral solution for the equation $x+2y=2xy$ is p then.	**3.** p is the HM of m and n
	4. $n=\frac{m^p+p^m}{mp}$
	$m=\sqrt{n\sqrt{p\sqrt{n\sqrt{p\sqrt{n....\infty}}}}}$

a. A→1,5; B→2,4; C→3
b. A→2,5; B→5, 1 C→1
c. A→3,4; B→1,3; C→2,4
d. A→2,3; B-3,4; C→1,4

91. Observe the following columns:

Column I	Column II
(A) If $a+b+2c=0$, $c\neq$ then equation $ax^2+bx+c=0$ has	**1.** at least one root in (–2, 0)

(B) Let $a, b, c \in R$ such that $2a - 3b\ 6c = 0$, then equation $ax^2 + bx + c = 0$ has	**2.** at least one root in $(-1, 0)$
(C) Let a, b, c be zero real numbers such that $\int_0^1 (1+\cos^8 x)\ (ax^2 + bx + c)\ dx = \int_0^2 (1+\cos^8 x)\ (ax^2 + bx + c)\ dx,$ then the equation $ax^2 + bx + c = 0$ has	**3.** at least one root in $(-1, 1)$
	4. at least one root in $(0, 1)$
	5. at least one root in $(0, 2)$

a. A→3,4,5; B→1,2,3; C→3,4,5
b. A→1,2,3; B→1,3,5; C→3,4,5
c. A→1,2,5; B→1,5,3; C→3,2,5
d. A→2,5,3; B→1,4,5; C→2,1,5

92. Observe the following columns:

Column I	**Column II**
(A) If a, b, c, d are four non zero numbers such that $(d + a - b)^2 + (d + b - c)^2 = 0$ and roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are real and equal, then	**1.** $a + b + c \neq 0$
(B) If a, b, c are three non zero real numbers such that the roots of the equation	**2.** , b, c are in AP
(C) If the three equations $x^2 + px + 12 = 0$, $x^2 + qx + 15 = 0$ and $x^2(p + q)x + 36 = 0$have a common positive root and a, b, c be their other roots, then	3. a, b, c are in GP
	4. a, b, c are in HP
	5. $a = b = c$

a. A→1,2,3,4,5; B→1,2; C-1
b. A→2,1,4,3,5; B→1,3; C-1
c. A→2,1,3,5,4; B→3,2; C→2
d. A→5,2,3,4,1; B→1,2; C→5

Integer

93. If α,β are the roots of the equation $\lambda(x^2 - x) + x + 5 = 0$. If λ_1 and λ_2 are two values of λ for which the roots α,β are related by $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$, then the value of $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_3}{\lambda_1}$ must be equal to:

94. If α,β,γ are such that $\alpha + \beta + \gamma = 4$, $\alpha^2 + \beta^2 + \gamma^2 = 6$, $\alpha^3 + \beta^3 + \gamma^3 = 8$, then the value of $[\alpha^4 + \beta^4 + \gamma^4]$ must be equal to: (where $[\cdot]$ denotes the greatest integer function)

95. Sum of all roots of the equation

$\underbrace{\sqrt{x + 2\sqrt{x + 2\sqrt{x + \ldots + 2\sqrt{x + 2\sqrt{3x}}}}}}_{n \text{ radical signs}} = x$ must be equal to:

96. In copying a quadratic equation of the form $x^2 + px + q = 0$ then coefficient of x was wrongly written as -10 in place of -11 and the roots were found to be 4 and 6. If α, β are the roots of the correct equation, then the value of $\alpha^4 + \beta^4$ must be equal to:

97. If α and β are the roots of the equation $x^2 + px + q = 0$ and also $x^{3900} + p^{1950}x^{1950}\ \ +q^{1950} = 0$ and if $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $x^n + 1 + (x+1)^n = 0$, then the value of n must be equal to:

98. The smallest value of k, for which both the roots of the equation $x^2 - 8kx + 16(k^2 - k + 1) = 0$ are real, distinct and have values at least 4, is:

99. Let (x, y, z) be points with integer coordinates satisfying the system of homogeneous equations $3x - y - z = 0$, $-3x + z = 0$, $-3x + 2y + z = 0$ Then the number of such points for which $x^2 + y^2 + z^2 \leq 100$ is:

100. Let ω be the complex number $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$. Then the number of district complex number z satisfying $\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega \end{vmatrix} = 0$ is equal to:

ANSWER

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
c	c	b	a	c	b	d	d	b	d
11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
a	c	a	c	d	b	d	a	d	d
21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
c	d	c	a	a	b	b	b	b	a
31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
b	d	a	b	b	a	c	b	c	b
41.	**42.**	**43.**	**44.**	**45.**	**46.**	**47.**	**48.**	**49.**	**50.**
a	d	d	a	a	b	c	a	a	b
51.	**52.**	**53.**	**54.**	**55.**	**56.**	**57.**	**58.**	**59.**	**60.**
d	b	a	d	a	a	d	b	c	c
61.	**62.**	**63.**	**64.**	**65.**	**66.**	**67.**	**68.**	**69.**	**70.**
a	b	c	b	b,d	a,d	All	All	b,c	a,b,c
71.	**72.**	**73.**	**74.**	**75.**	**76.**	**77.**	**78.**	**79.**	**80.**
b,c	a,b,c	b,d	a,b	b	d	b	c	b	b
81.	**82.**	**83.**	**84.**	**85.**	**86.**	**87.**	**88.**	**89.**	**90.**
c	a	b	a	a	a	c	a	b	a
91.	**92.**	**93.**	**94.**	**95.**	**96.**	**97.**	**98.**	**99.**	**100.**
a	a	254	7	3	4177	1950	k =2	7	1

SOLUTION

Multiple Choice Questions

1. **(c)** Given equation $(x-a)(x-b)+(x-b)(x-c)+$ $(x-c)(x-a)=0$ can be re-written as $3x^2-2(a+b+c)x$ $+(ab+bc+ca)=0$

$D=4[(a+b+c)^2-3(ab+bc+ca)]$

$=4[a^2+b^2+c^2-ab-bc-ac]$

$=2[(a-b)^2+(b-c)^2+(c-a)^2]\geq 0$

Hence both roots are always real.

2. **(c)** Equation $x^2+kx-24=0$ has one root as 3,

$\Rightarrow \quad 3^2+3k-24=0$

$\Rightarrow \quad k=5$

Put $x=3$ and $k=5$ in option

Only (c) gives the correct answer *i.e.*

$\Rightarrow \quad 3^2-15+9=0$

$\Rightarrow \quad 0=0$

3. **(b)** Since roots are equal then $[-2(1+3k)]^2$ $=4.1.7(3+2k)$

$\Rightarrow \quad 1+9k^2+6k=21+14k$

$\Rightarrow \quad 9k^2-8k-20=0$

Solving, we get $k=2,-10/9$

4. **(a)** $\alpha+\beta=-a$, $\alpha\beta=b$

$\Rightarrow \quad \alpha-\beta=\sqrt{a^2-4b}$ and $\gamma+\delta=-b$, $\gamma\delta=a$

$\Rightarrow \quad \gamma-\delta=\sqrt{b^2-4a}$

According to question, $\alpha-\beta=\gamma-\delta$

$\Rightarrow \quad \sqrt{a^2-4b}=\sqrt{b^2-4a}$

$\Rightarrow \quad a+b+4=0$

5. **(c)** As given, if α,β be the roots of the quadratic equation, then

$\Rightarrow \quad \alpha+\beta=\dfrac{1}{\alpha^2}+\dfrac{1}{\beta^2}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}$

$\Rightarrow \quad -\dfrac{b}{a}=\dfrac{b^2/a^2-2c/a}{c^2/a^2}=\dfrac{b^2-2ac}{c^2}$

$\Rightarrow \quad \dfrac{2a}{c}=\dfrac{b^2}{c^2}+\dfrac{b}{a}=\dfrac{ab^2+bc^2}{ac^2}$

$\Rightarrow \quad 2a^2c=ab^2+bc^2$

$\Rightarrow \quad \dfrac{2a}{b}=\dfrac{b}{c}+\dfrac{c}{a}\,\dfrac{c}{a},\dfrac{a}{b},\dfrac{b}{c}$ are in A.P.

$\Rightarrow \quad \dfrac{a}{c},\dfrac{b}{a},\dfrac{c}{b}$ are in H.P.

6. **(b)** Since roots of the equation $x^2-5x+16=0$ are α,β.

$\Rightarrow \quad \alpha+\beta=5,\alpha\beta=16$ and $\alpha^2+\beta^2+\dfrac{\alpha\beta}{2}=-p$

$\Rightarrow \quad (\alpha+\beta)^2-2\alpha\beta+\dfrac{\alpha\beta}{2}=-p$

$\Rightarrow \quad 25-2(16)+\dfrac{16}{2}=-p$

$\Rightarrow \quad p=-1$ and $(\alpha^2+\beta^2)\left(\dfrac{\alpha\beta}{2}\right)=q$

$\Rightarrow \quad [(\alpha+\beta)^2-2\alpha\beta]\dfrac{\alpha\beta}{2}=q$

$\Rightarrow \quad (25-32)8=q$

$\Rightarrow \quad q=-56$

7. **(d)** Let α and α^2 be the roots then $\alpha+\alpha^2=-p$, $\alpha.\alpha^2=q$

Now $(\alpha+\alpha^2)^3=\alpha^3+\alpha^6+3\alpha^3(\alpha+\alpha^2)$

$\Rightarrow \quad -p^3=q+q^2-3pq \Rightarrow p^3+q^2+q(1-3p)=0$

8. **(d)** Roots of $x^2+x+1=0$ are

$x=\dfrac{-1\pm\sqrt{1-4}}{2}, \;=\dfrac{-1\pm\sqrt{3}i}{2}=\omega,\omega^2$

Take $\alpha=\omega,\beta=\omega^2$

$\therefore \quad \alpha^{19}=w^{19}=w,\beta^7=(w^2)^7=w^{14}=w^2$

$\therefore \quad$ Required equation is $x^2+x+1=0$

9. **(b)** Given root $=\frac{1}{2+\sqrt{5}}=\frac{2-\sqrt{5}}{-1}=-2+\sqrt{5}$

$\therefore$ Other root $=-2-\sqrt{5}$

Again, sum of roots = – 4 and product of roots = – 1.

The required equation is $x^2+4x-1=0$

10. **(b)** If α is the coincident root, then $x^2+a\,\alpha+b=0$ and

$\alpha^2+b\alpha+a=0 \Rightarrow \frac{\alpha^2}{a^2-b^2}=\frac{\alpha}{b-a}=\frac{1}{b-a}$

$\alpha^2=-(a+b)$, $\alpha=1$

$\Rightarrow$ $-(a+b)=1 \Rightarrow (a+b)=-1$

11. **(a)** Let the roots are α and 2α

Now, $\alpha+2\alpha=\frac{1-3a}{a^2-5a+3}$, $\alpha.2\alpha=\frac{2}{a^2-5a+3}$

$\Rightarrow$ $3\alpha=\frac{1-3a}{a^2-5a+3}$, $2\alpha^2=\frac{2}{a^2-5a+3}$

$\Rightarrow$ $2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2-5a+3)^2}\right]=\frac{2}{a^2-5a+3}$

$\Rightarrow$ $\frac{(1-3a)^2}{a^2-5a+3}=9 \Rightarrow 9a^2-45a+27=1+9a^2-6a$

$\Rightarrow$ $39a=26 \Rightarrow a=2/3$

12. **(c)** Since $a=1>0$ therefore its minimum value is

$=\frac{4ac-b^2}{4a}=\frac{4(1)(17)-64}{4}=\frac{4}{4}=1$

13. **(a)** Equation $a(x^2+1)-(a^2+1)x=0$

$\Rightarrow$ $ax^2-(a^2+1)x+a=0$

$\Rightarrow$ $(ax-1)(x-a)=0 \Rightarrow x=a,\frac{1}{a}$.

14. **(c)** We have $ix^2-4x-4i=0$

$\Rightarrow$ $x^2+4ix-4=0 \Rightarrow x^2+2ix+2ix-4=0$

$\Rightarrow$ $(x+2i)(x+2i)=0 \Rightarrow x=-2i,\ -2i$.

15. **(d)** $8\sec^2\theta-6\sec\theta+1=0 \Rightarrow \sec\theta=\frac{1}{2}$

or $\sec\theta=\frac{1}{4}$, but $\sec\theta\geq 1$ or $\sec\theta\leq -1$.

Hence the given equation has no solution.

16. **(b)** Let the required number is x

So, $x=\sqrt{x}+12 \Rightarrow x-12=\sqrt{x} \Rightarrow x^2-25x+144=0$

$\Rightarrow$ $x^2-16x-9x+144=0 \Rightarrow x=16$

Since $x=9$ does not hold the condition.

By inspection, since 16 exceeds its positive square root i.e.,4 by 12.

17. **(d)** Given that $x^{2/3}-7x^{1/3}+10=0$. Given equation can be written as $(x^{1/3})^2-7(x^{1/3})+10=0$

Let $a=x^{1/3}$, then it reduces to the equation

$a^2-7a+10=0 \Rightarrow (a-5)(a-2)=0 \Rightarrow a=5,2$

Putting these values, we have $a^3=x \Rightarrow x=125$ and 8.

18. **(a)** $x^{\log_x(1-x)2}=9$

$\Rightarrow$ $\log_x(9)=\log_x(1-x)^2 (\because a^x=N \Rightarrow \log_a N=x)$

$\Rightarrow$ $9=(1-x)^2 \Rightarrow 1+x^2-2x-9=0$

$\Rightarrow$ $x^2-2x-8=0 \Rightarrow (x+2)(x-4)=0 \Rightarrow x=-2,4$.

19. **(d)** Given equation $e^{\sin x}-e^{-\sin x}-4=0$

Let $e^{\sin x}=y$, then given equation can be written as

$y^2-4y-1=0 \Rightarrow y=2\pm\sqrt{5}$

But the value of $y=e^{\sin x}$ is always positive, so

$y=2+\sqrt{5}$ $(\because 2<\sqrt{5})$

$\Rightarrow$ $\log_e y=\log_e(2+\sqrt{5}) \Rightarrow \sin x=\log_e(2+\sqrt{5})>1$

Which is impossible, since $\sin x$ cannot be greater than 1. Hence we cannot find any real value of x which satisfies the given equation.

20. **(d)** $x+\frac{1}{x}=2 \Rightarrow x+\frac{1}{x}-2=0$ $(\because x\neq 0)$

$\Rightarrow$ $x^2-2x+1=0 \Rightarrow (x-1)^2=0 \Rightarrow x=1,1$.

21. **(c)** $\sqrt{3x^2-7x-30}+\sqrt{2x^2-7x-5}=x+5$

$\sqrt{3x^2-7x-30}=(x+5)-\sqrt{2x^2-7x-5}$

on squaring, $\sqrt{2x^2-7x-5}=5$

$2x^2-7x-30=0 \Rightarrow x=6$.

22. **(d)** Since quadratic equation $ax^2+bx+c=0$ has three distinct roots so it must be an identity.

So, $a=b=c=0$.

23. **(c)** $x=\sqrt{2+x} \Rightarrow x^2-x-2=0$

$\Rightarrow$ $(x-2)(x+1)=0 \Rightarrow x=2,-1$

But $\sqrt{2+\sqrt{2+.....}}\neq -1$, so it is equal to 2.

24. **(a)** $e^x=x+1 \Rightarrow 1+\frac{x}{1!}+\frac{x^2}{2!}+......=x+1$

$\Rightarrow$ $\frac{x^2}{2!}+\frac{x^3}{3!}+......=0$; $x^2=0, x^3=0,\ ......\ x^n=0$

Hence, $x=0$ only one real root.

Check the equation with options then only option (a) satisfies the equation.

25. **(a)** $\log_4 \log_2(\sqrt{x+8}-\sqrt{x}) = 0$

$\Rightarrow$ $4^0 = \log_2(\sqrt{x+8}-\sqrt{x}) \Rightarrow 2^1 = \sqrt{x+8}-\sqrt{x}$

$\Rightarrow$ $4 = x+8+x-2\sqrt{x^2+8x} \Rightarrow 2\sqrt{x^2+8x} = 2x+4$

$\Rightarrow$ $x^2+8x = x^2+4+4x \Rightarrow 4x = 4 \Rightarrow x = 1$.

26. **(b)** Equations $px^2+2qx+r=0$ and $qx^2-2(\sqrt{pr})x+q=0$ have real roots, then from first

$4q^2-4pr \ge 0 \Rightarrow q^2-pr \ge 0 \Rightarrow q^2 \ge pr$ . . .(i)

and from second $4(pr)-4q^2 \ge 0$ (for real root)

$\Rightarrow$ $pr \ge q^2$ . . . (ii)

From (i) and (ii), we get result $q^2 = pr$.

27. **(b)** The roots of the equations are given by

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ Let $b^2-4ac > 0, b > 0$

Now if $a > 0, c > 0, b^2-4ac < b^2$

$\Rightarrow$ The roots are negative.

Let $b^2-4ac < 0$, then the roots are given by

$x = \dfrac{-b \pm i\sqrt{(4ac-b^2)}}{2a}$, $(i = \sqrt{-1})$

Which are imaginary and have negative real part $(\because b > 0)$

$\therefore$ In each case, the roots have negative real part.

28. **(b)** Let first root $= \alpha$ and second root $= \dfrac{1}{\alpha}$

Then $\alpha . \dfrac{1}{\alpha} = \dfrac{k}{5} \Rightarrow k = 5$.

29. **(b)** It is given that $\alpha\beta = 2 \Rightarrow \dfrac{3a+4}{a+1} = 2$

$\Rightarrow$ $3a+4 = 2a+2 \Rightarrow a = -2$

Also $\alpha+\beta = -\dfrac{2a+3}{a+1}$

Putting this value of a, we get sum of roots

$= -\dfrac{2a+3}{a+1} = -\dfrac{-4+3}{-2+1} = -1$.

30. **(a)** Here $\alpha+\beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

If roots are $\alpha+\dfrac{1}{\beta}, \beta+\dfrac{1}{\alpha}$, then sum of roots are

$= \left(\alpha+\dfrac{1}{\beta}\right)+\left(\beta+\dfrac{1}{\alpha}\right) = (\alpha+\beta)+\dfrac{\alpha+\beta}{\alpha\beta} = -\dfrac{b}{ac}(a+c)$

and product $= \left(\alpha+\dfrac{1}{\beta}\right)\left(\beta+\dfrac{1}{\alpha}\right)$

$= \alpha\beta+1+1+\dfrac{1}{\alpha\beta} = 2+\dfrac{c}{a}+\dfrac{a}{c} = \dfrac{2ac+c^2+a^2}{ac} = \dfrac{(a+c)^2}{ac}$

Hence required equation is given by

$x^2+\dfrac{b}{ac}(a+c)x+\dfrac{(a+c)^2}{ac} = 0$

$\Rightarrow$ $acx^2+(a+c)bx+(a+c)^2 = 0$.

Trick: Let $a = 1$, $b = -3, c = 2$, then $\alpha = 1$, $\beta = 2$

$\therefore$ $\alpha+\dfrac{1}{\beta} = \dfrac{3}{2}$ and $\beta+\dfrac{1}{\alpha} = 3$

Therefore, required equation must be

$(x-3)(2x-3) = 0$ *i.e.* $2x^2-9x+9 = 0$

Here (a) gives this equation on putting $a = 1, b = -3, c = 2$.

31. **(b)** Sum of roots $\alpha+\beta = -(a+b)$ and $\alpha\beta = \dfrac{a^2+b^2}{2}$

$\Rightarrow$ $(\alpha+\beta)^2 = (a+b)^2$ and $(\alpha-\beta)^2 = \alpha^2+\beta^2-2\alpha\beta$

$= 2ab-(a^2+b^2) = -(a-b)^2$

Now the required equation whose roots are $(\alpha+\beta)^2$ and $(\alpha-\beta)^2$

$x^2-\{(\alpha+\beta)^2+(\alpha-\beta)^2\}x+(\alpha+\beta)^2(\alpha-\beta)^2 = 0$

$\Rightarrow$ $x^2-\{(a+b)^2-(a-b)^2\}x-(a+b)^2(a-b)^2 = 0$

$\Rightarrow$ $x^2-4abx-(a^2-b^2)^2 = 0$

32. **(d)** Under condition, $-\dfrac{2}{\lambda} = 3 \Rightarrow \lambda = -\dfrac{2}{3}$

33. **(a)** $\alpha+\beta = \dfrac{3}{2}$ and $\alpha\beta = 2$

$\alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = \dfrac{9}{4}-4 = -\dfrac{7}{4}$

Hence required equation $x^2-(\alpha^2+\beta^2)x+\alpha^2\beta^2 = 0$

$\Rightarrow$ $x^2+\dfrac{7}{4}x+4 = 0$

$\Rightarrow$ $4x^2+7x+16 = 0$

34. **(b)** Let $p\alpha, q\alpha$ be the roots of the given equation $ax^2+bx+c = 0$.

Then $p\alpha+q\alpha = -\dfrac{b}{a}$ and $p\alpha . q\alpha = \dfrac{c}{a}$

From first relation, $\alpha = -\dfrac{b}{a(p+q)}$

Substituting this value of α in second relation, we get

$\dfrac{b^2}{a^2(p+q)^2} \times pq = \dfrac{c}{a}$

$\Rightarrow$ $b^2pq-ac(p+q)^2 = 0$

Students should remember this question as a fact.

35. **(b)** α, β be the roots of $x^2 - 2x + 3 = 0$, then $\alpha + \beta = 2$ and $\alpha\beta = 3$ Now required equation whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ is $x^2 - \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)x + \frac{1}{\alpha^2\beta^2} = 0$

$\Rightarrow \quad x^2 - \left(-\frac{2}{9}\right)x + \frac{1}{9} = 0 \Rightarrow 9x^2 + 2x + 1 = 0$.

36. **(a)** As given, $\alpha + \beta = p,\ \alpha\beta = q, \alpha' + \beta' = p', \alpha'\beta' = q'$

Now, $(\alpha - \alpha')^2 + (\beta - \alpha')^2 + (\alpha - \beta')^2 + (\beta - \beta')^2$

$= 2(\alpha^2 + \beta^2) + 2(\alpha'^2 + \beta'^2) - 2\alpha'(\alpha + \beta) - 2\beta'(a + \beta)$

$= 2\{(\alpha + \beta)^2 - 2\alpha\beta + (\alpha' + \beta')^2 - 2\alpha'\beta' - (\alpha + \beta)(a' + \beta')\}$

$= 2\{p^2 - 2q + p'^2 - 2q' - pp'\}$.

37. **(c)** As given, $\alpha + \beta = a + b$, $\alpha\beta = ab - c$ or $ab = \alpha\beta + c$.

Then the required equation is $x^2 - x(\alpha + \beta) + \alpha\beta + c = 0$

$\Leftrightarrow x^2 - x(a + b) + ab = 0$, whose roots are a, b.

38. **(b)** Given equation can be written as

$x^2 + x(p + q - 2r) + pq - pr - qr = 0$ ... (i)

whose roots are α and $-\alpha$, then the product of roots

$-\alpha^2 = pq - pr - qr = pq - r(p + q)$ ... (ii)

and sum $0 = p + q - 2r \Rightarrow r = \frac{p + q}{2}$ ... (iii)

From (ii) and (iii), we get

$-\alpha^2 = pq - \frac{p + q}{2}(p + q) = -\frac{1}{2}\{(p + q)^2 - 2pq\}$

$= -\frac{(P^2 + q^2)}{2}$.

39. **(c)** α, β are the roots of the equation $x^2 - 3x + 1 = 0$

$\therefore \alpha + \beta = 3$ and $\alpha\beta = 1$

$S = \frac{1}{\alpha - 2} + \frac{1}{\beta - 2} = \frac{\alpha + \beta - 4}{\alpha\beta - 2(\alpha + \beta) + 4} = \frac{3 - 4}{1 - 2.3 + 4} = 1$

and $P = \frac{1}{(\alpha - 2)(\beta - 2)} = \frac{1}{\alpha\beta - 2(\alpha + \beta) + 4} = -1$

Hence the equation whose roots are $\frac{1}{\alpha - 2}$ and $\frac{1}{\beta - 2}$ are $x^2 - Sx + P = 0 \Rightarrow x^2 - x - 1 = 0$.

40. **(b)** α, β are the roots of the equation $6x^2 - 6x + 1 = 0$

$\Rightarrow \quad \alpha + \beta = 1,\ \alpha\beta = 1/6$

$\therefore \quad \frac{1}{2}[a + b\alpha + c\alpha^2 + d\alpha^3] + \frac{1}{2}[a + b\beta + c\beta^2 + d\beta^3]$

$= a + \frac{1}{2}b(\alpha + \beta) + \frac{1}{2}c(\alpha^2 + \beta^2) + \frac{1}{2}d(\alpha^3 + \beta^3)$

$= a + \frac{1}{2}b + \frac{1}{2}c[(\alpha + \beta)^2 - 2\alpha\beta] + \frac{1}{2}d[(\alpha + \beta)^3$

$-3\alpha\beta(\alpha + \beta)]$

$= a + \frac{b}{2} + \frac{1}{2}c\left[(1)^2 - 2.\frac{1}{6}\right] + \frac{1}{2}d\left[(1)^3 - 3.\frac{1}{6}\right]$

$= \frac{a}{1} + \frac{b}{2} + \frac{c}{3} + \frac{d}{4}$.

41. **(a)** Given, $\frac{x - m}{mx + 1} = \frac{x + n}{nx - 1}$

$\Rightarrow \quad x^2(m - n) + 2mnx + (m + n) = 0$

Roots are $\alpha, \frac{1}{\alpha}$ respectively, then $\alpha.\frac{1}{\alpha} = \frac{m + n}{m - n}$

$\Rightarrow \quad m - n = m + n \Rightarrow n = 0$.

42. **(d)** Roots of given equation $x^2 - px + q = 0$ is a and b

i.e., $a + b = p$ ... (i)

and $ab = q$ ... (ii)

Then $\frac{1}{a} + \frac{1}{b} = \frac{a + b}{ab} = \frac{p}{q}$.

43. **(d)** $(\alpha^2 + \beta^2)^2 = (\alpha + \beta)(\alpha^3 + \beta^3)$

$\left(\frac{b^2 - 2ac}{a^2}\right)^2 = \left(\frac{-b}{a}\right)\left(\frac{-b^2 + 3abc}{a^3}\right)$

$\Rightarrow \quad 4a^2c^2 = acb^2 \Rightarrow ac(b^2 - 4ac) = 0$

As $a \neq 0 \Rightarrow c\Delta = 0$

44. **(a)** Given roots are $3p - 2q$ and $3q - 2p$.

Sum of roots $= (3p - 2q) + (3q - 2p) = (p + q) = \frac{5}{3}$

Product of roots $= (3p - 2q)(3q - 2p)$

$= 9pq - 6q^2 - 6p^2 + 4pq = 13pq - 2(3p^2 + 3q^2)$

$= 13\left(\frac{-2}{3}\right) - 2(5p + 2 + 5q + 2)$

$= 13\left(\frac{-2}{3}\right) - 2\left[5\left(\frac{5}{3}\right) + 4\right]$

$= \frac{-26}{3} - 2\left[\frac{25}{3} + 4\right] = \frac{-100}{3}$

Hence, equation is $3x^2 - 5x - 100 = 0$.

45. **(a)** Given equation is $x^2 - 2ax + a^2 + a - 3 = 0$

If roots are real, then $D \geq 0$

$\Rightarrow \quad 4a^2 - 4(a^2 + a - 3) \geq 0$

$\Rightarrow \quad -a + 3 \geq 0$

$\Rightarrow \quad a - 3 \leq 0$

$\Rightarrow \quad a \leq 3$

As roots are less than 3, hence $f(3) > 0$

$9-6a+a^2+a-3>0$

$\Rightarrow \quad a^2-5a+6>0$

$\Rightarrow \quad (a-2)(a-3)>0$

$\Rightarrow \quad a<2, a>3$.

Hence $a<2$ satisfy all the conditions.

46. **(b)** Given equation can be written as

$(6k+2)x^2+rx+3k-1=0$ . . . (i)

and $2(6k+2)x^2+px+2(3k-1)=0$ . . . (ii)

Condition for common roots is

$\frac{12k+4}{6k+2}=\frac{p}{r}=\frac{6k-2}{3k-1}=2$ or $2r-p=0$

47. **(c)** Let roots of $x^2-cx+d=0$ be α,β then roots of $x^2-ax+b=0$ be α,α

$\therefore \quad \alpha+\beta=c, \alpha\beta=d, \alpha+\alpha=a, \alpha^2=b$

Hence $2(b+d)=2(\alpha^2+\alpha\beta)=2\alpha(\alpha+\beta)=ac$

48. **(a)** Let the roots be $\alpha,\beta;\beta,\gamma$ and γ,α respectively.

$\therefore \quad \alpha+\beta=-p, \beta+\gamma=-q, \gamma+\alpha=-r$

Adding all, we get $\Sigma\alpha=-(p+q+r)/2$ etc.

49. **(a)** Let α is the common root, so $\alpha^2+p\alpha+q=0$ . . . (i)

and $\alpha^2+q\alpha+p=0$ . . . (ii)

from (i) – (ii),

$\Rightarrow \quad (p-q)\alpha+(q-p)=0 \Rightarrow \alpha=1$

Put the value of α in (i), $p+q+1=0$.

50. **(b)** Expressions are $x^2-11x+a$ and $x^2-14x+2a$ will have a common factor, then

$\Rightarrow \quad \frac{x^2}{-22a+14a}=\frac{x}{a-2a}=\frac{1}{-14+11}$

$\Rightarrow \quad \frac{x^2}{-8a}=\frac{x}{-a}=\frac{1}{-3} \Rightarrow x^2=\frac{8a}{3}$ and $x=\frac{a}{3}$

$\Rightarrow \quad \left(\frac{a}{3}\right)^2=\frac{8a}{3} \Rightarrow \frac{a^2}{9}=\frac{8a}{3} \Rightarrow a=0, 24$.

Trick: We can check by putting the values of a from the options.

51. **(d)** x^2-3x+2 be factor of $x^4-px^2+q=0$

Hence $(x^2-3x+2)=0 \Rightarrow (x-2)(x-1)=0$

$\Rightarrow \quad x=2,1,$ putting these values in given equation

So, $4p-q-16=0$ . . . (i)

and $p-q-1=0$ . . . (ii)

Solving (i) and (ii), we get $(p, q)=(5, 4)$

52. **(b)** If the given expression be y, then

$y=2x^2y+(3y-1)x+(6y-2)=0$

If $y\neq 0$ then $\Delta\geq 0$ for real x *i.e.* $B^2-4AC\geq 0$

or $-39y^2+10y+1\geq 0$ or $(13y+1)(3y-1)\leq 0$

$\Rightarrow \quad -1/13\leq y\leq 1/3$

If $y=0$ then $x=-2$ which is real and this value of y is included in the above range.

53. **(a)** $x, y, z\in R$ and distinct.

Now, $u=x^2+4y^2+9z^2-6yz-3zx-2xy$

$=\frac{1}{2}(2x^2+8y^2+18z^2-12yz-6zx-4xy)$

$=\frac{1}{2}\{(x^2-4xy+4y^2)+(x^2-6zx+9z^2)+(4y^2-12yz+9z^2)\}$

$=\frac{1}{2}\{(x-2y)^2+(x-3z)^2+(2y-3z)^2\}$

Since it is sum of squares. So u is always non- negative.

54. **(d)** Let $y=\frac{(x-a)(x-b)}{(x-c)}$ or $y(x-c)=x^2-(a+b)x+ab$

or $x^2-(a+b+y)x+ab+cy=0$

$\Delta=(a+b+y)^2-4(ab+cy)$

$=y^2+2y(a+b-2c)+(a-b)^2$

Since x is real and y assumes all real values, we must have $\Delta\geq 0$ for all real values of y. The sign of a quadratic in y is same as of first term provided its discriminant $B^2-4AC<0$

This will be so if $4(a+b-2c)^2-4(a-b)^2<0$

or $4(a+b-2c+a-b)(a+b-2c-a+b)<0$

or $16(a-c)(b-c)<0$ or $16(c-a)(c-b)=-ve$

$\therefore \quad c$ lies between a and b i.e., $a<c<b$ . . . (i)

Where $a<b$, but if $b<a$ then the above condition will be $b<c<a$ or $a>c>b$ . . . (ii)

Hence from (i) and (ii) we observe that (d) is correct answer.

55. **(a)** Given equation is $x^2-2ax+a^2+a-3=0$

If roots are real, then $D\geq 0$

$\Rightarrow \quad 4a^2-4(a^2+a-3)\geq 0 \Rightarrow -a+3\geq 0$

$\Rightarrow \quad a-3\leq 0 \Rightarrow a\leq 3$

As roots are less than 3, hence $f(3)>0$

$9-6a+a^2+a-3>0 \Rightarrow a^2-5a+6>0$

$\Rightarrow \quad (a-2)(a-3)>0 \Rightarrow$ either $a<2$ or $a>3$

Hence $a<2$ satisfy all.

56. **(a)** Let $f'(x)$ denotes the quadratic expression $f'(x) \equiv 3ax^2 + 2bx + c$, whose antiderivative be denoted by $f(x) = ax^3 + bx^2 + cx$

Now $f(x)$ being a polynomial in $R, f(x)$ is continuous and differentiable on R. To apply Rolle's theorem.

We observe that $f(0) = 0$ and $f(1) = a + b + c = 0$, by hypothesis. So there must exist at least one value of x, say $x = \alpha \in (0,1)$ such that $f'(\alpha) = 0 \Leftrightarrow 3a\alpha^2 + 2b\alpha + c = 0$

That is, $f'(x) = 3ax^2 + 2bx + c = 0$ has at least one root in [0, 1].

57. **(d)** Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x$;

$f(0) = 0$; $f(\alpha) = 0$

$\Rightarrow$ $f'(x) = 0$, has atleast one root between $(0, \alpha)$

i.e., equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$

has a positive root smaller than α.

58. **(b)** $f(x) = \underset{+}{x^5} \underset{-}{- 6x^2} \underset{-}{- 4x} \underset{+}{+ 5} = 0$

2 changes of sign $\Rightarrow$ maximum two positive roots.

$f(-x) = \underset{-}{-x^5} \underset{-}{- 6x^2} \underset{+}{+ 4x} \underset{+}{+ 5} = 0$

1 changes of sign $\Rightarrow$ maximum one negative roots.

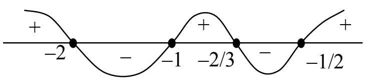
$\Rightarrow$ total maximum possible number of real roots = 2 + 1 = 3.

59. **(c)** Given $\dfrac{2x}{2x^2 + 5x + 2} - \dfrac{1}{x+1} > 0$

$\Rightarrow \dfrac{2x^2 + 2x - 2x^2 - 5x - 2}{(2x+1)(x+2)(x+1)} > 0 \Rightarrow \dfrac{-3x-2}{(2x+1)(x+2)(x+1)} > 0$

$\Rightarrow \dfrac{-3(x+2/3)}{(x+1)(x+2)(2x+1)} > 0 \Rightarrow \dfrac{(x+2/3)}{(x+1)(x+2)(2x+1)} < 0$

Equating each factor equal to 0,



We get $x = -2, -1, -2/3, -1/2$

$\therefore$ $x \in]-2, -1[\cup]-2/3, -1/2$

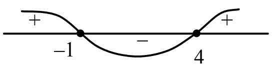
$\Rightarrow$ $-2/3 < x < -1/2$ or $-2 < x < -1$

60. **(c)** $x^2 - 3x + 2 > 0$ or $(x-1)(x-2) > 0$

$\therefore$ $x \in (-\infty, 1) \cup (2, \infty)$ ...(*i*)

Again $x^2 - 3x - 4 \le 0$ or $(x-4)(x+1) \le 0$

$\therefore$ $x \in [-1, 4]$ ...(*ii*)



From eq. (i) and (ii), $x \in [-1, 1) \cup (2\,4]$

$\Rightarrow$ $-1 \le x < 1$ or $2 < x \le 4$

61. **(a)** We have $|x-2|^2 + |x-2| - 6 = 0$

Let $|x-2| = X$; $X^2 + X - 6 = 0$

$\Rightarrow$ $X = \dfrac{-1 \pm \sqrt{1+24}}{2} = 2, -3$

$\Rightarrow$ $X = 2$ and $X = -3$

$\therefore$ $|x-2| = 2$ and $|x-2| = -3$, which is not possible.

$\Rightarrow$ $x - 2 = 2$

or $x - 2 = -2$

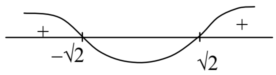
$\therefore$ $x = 4$

or $x = 0$

62. **(b)** Case (i): If $x + 2 \ge 0$

i.e. $x \ge -2$,

We get $x^2 - x - 2 + x > 0$



$\Rightarrow$ $x^2 - 2 > 0$

$\Rightarrow$ $(x - \sqrt{2})(x + \sqrt{2}) > 0$

$\Rightarrow$ $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

But $x \ge -2$

$\therefore$ $x \in [-2, -\sqrt{2}) \cup (\sqrt{2}\ \infty)$ . . . (*i*)

Case (ii): $x + 2 < 0$ *i.e.* $x < -2$,

then $x^2 + x + 2 + x > 0$

$\Rightarrow$ $x^2 + 2x + 2 > 0$

$\Rightarrow$ $(x+1)^2 + 1 > 0$. Which is true for all x

$\therefore$ $x \in (-\infty, -2)$ . . . (*ii*)

From (i) and (ii),

we get, $x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

63. **(c)** Expression is always +*ve*,

so $t^2x^2 + |x| + 9 \ne 0$.

Hence roots of given equation does not exist.

64. **(b)** We have $\log_4(x-1) = \log_2(x-3)$

$(x-1) = (x-3)^2$

$\Rightarrow$ $x - 1 = x^2 + 9 - 6x$

$\Rightarrow$ $x^2 - 7x + 10 = 0$

$\Rightarrow$ $(x-5)(x-2) = 0$

$x = 5$ or $x = 2$

But $x - 3 < 0$, when $x = 2$.

$\therefore$ Only solution is $x = 5$.

Hence number of solution is one.

NCERT Exemplar Problems

More than One Answer

65. (b, d) Plan: Concepts of curve tracing are used in this question.

Number of roots are taken out from the curve traced.

Let $y = x^5 - 5x$

(a) As $x \to \infty, y \to \infty$ and as $x \to -\infty, y \to -\infty$

(b) Also, at $x = 0, y = 0,$ thus the curve passes through the origin.

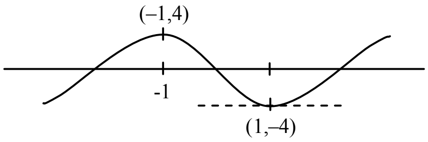
(c) $\frac{dy}{dx} = 5x^4 - 5 = 5(x^4 - 1)$

$= 5(x^2 - 1)(x^2 + 1) = 5(x-1)(x+1)(x^2+1)$

Now, $\frac{dy}{dx} > 0$ in $(-\infty, -1) \cup (1, \infty)$, thus $f(x)$ is increasing in these interval. Also, $\frac{dy}{dx} < 0$ in (–1, 1), thus decreasing in (–1, 1).

(d) Also, at $x = -1, dy/dx$ its sign from +ve to –ve.

$\therefore$ $x = -1$ is point of local minima.



Local maximum value, $y = (-1)^5 - 5(-1) = 4$

Local minimum value, $y = (1)^5 - 5(1) = -4$

Now, let $y = -a$

As, evident from the graph, if $-a \in (-4, 4)$

i.e., $a \in (-4, +4)$

Then, $f(x)$ has three real roots and if $-a > 4$

Or $-a < -4$, then $f(x)$ has one real root. i.e., for $a < -4$ or $a > 4, f(x)$ has one real root.

66. (a, d) Let $f(x) = ax^2 + bx + c > 0. \forall x \in R$

$\Rightarrow$ $b^2 - 4ac < 0$

And $a > 0$ . . .(i)

Now, $g(x) = f(x) - f'(x) + f''(x)$

$= ax^2 + (b-2a)x + (2a - b + c)$

Discriminant $= (b-2a)^2 - a(2a - b + c)$

$= (b^2 - 4ac) - 4a^2 < 0$ (from Eq. (i)]

$\Rightarrow$ $g(x) > 0, \forall x \in R \Rightarrow$ $g(x) \geq 0, \forall x \in R.$

67. (a, b, c, d) Given equation is $3x^3 + x^2 - 7x + \lambda = 0$

Let $f(x) = 3x^3 + x^2 - 7x + \lambda$

$\therefore$ $f'(x) = 9x^2 + 2x - 7$

$= 9(x+1)(x - 7/9)$

For max or min $f'(x) = 0$

$\therefore$ $x = -1, x = 7/9 \ \frac{7}{9} \in [0,1]$

Hence Eq. (i) has two distinct real roots in [0, 1] for all values of λ

68. (a, b, c, d) Given equation is $(x^2+1)^2 = x(3x^2 + 4x + 3)$

$\Rightarrow$ $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$

$\Rightarrow$ $x^2\left(x^2 - 3x - 2 - \frac{3}{x} + \frac{1}{x^2}\right) = 0$

$\Rightarrow$ $x \neq 0$

$\therefore$ $x^2 + \frac{1}{x^2 - 3}\left(x + \frac{1}{x}\right) - 2 = 0$

$\Rightarrow$ $\left(x + \frac{1}{x}\right)^2 - 3\left(x + \frac{1}{x}\right) - 4 = 0$

$\Rightarrow$ $\left(x + \frac{1}{x} - 4\right)\left(x + \frac{1}{x} + 1\right) = 0$

Or $(x^2 - 4x + 1)(x^2 + x + 1) = 0$

Or $\{(x-2)^2 - 3\}\left(\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}\right) = 0$

$\therefore$ $x = 2 \pm \sqrt{3}, \frac{-1 \pm i\sqrt{3}}{2}$

69. (b, c) Given equation is

$Ax^2 - |G|x - H = 0$ . . .(i)

Let α, β are the roots then $\alpha + \beta = \frac{|G|}{A}$ and $\alpha\beta = -\frac{H}{A}$

Since, $A > |G| > H$ or $1 > \frac{|G|}{A} > \frac{H}{A}$

Hence, A is positive

$\therefore$ $\alpha + \beta$ and $\alpha\beta$ has positive and negative fraction respectively.

Also, $|G|^2 = AH$ . . .(ii)

Discriminant of Equation (i) $= (-|G|)^2 - 4 \cdot A \cdot (-H)$

$= |G|^2 + 4AH = 5|G|^2 > 0$ [From Equation (ii)]

Hence roots of Eq. (i) are and distinct.

$\because$ $\alpha + \beta > 0$ and $\alpha\beta < 0$

One root is positive and other is negative and at least one root is a fraction. So, the equation has a negative fraction root.

70. **(a, b, c)** If roots is $\alpha, \frac{1}{\alpha}$

$\therefore \quad \alpha \cdot \frac{1}{\alpha} = \frac{c}{a} \Rightarrow c = a$

Since, $(|a| - |b|)^2 \geq 0$ or $|a| \geq |b|$

But $a = c, \therefore |c| \geq |b|$.

71. **(b, c)** $|x+1||x-1| = a^2 - 2a - 3$

$\Rightarrow \quad |x^2 - 1| = a^2 - 2a - 3$

$\therefore \quad a^2 - 2a - 3 \geq 0 \Rightarrow (a+1)(a-3) \geq 0$

$\therefore \quad a \in (-\infty, -1) \cup [3, \infty)$

72. **(a, b, c)** Roots are real

$\therefore \quad B^2 - 4AC > 0 \Rightarrow a^4 > 4b^2$, (a) is correct.

If $f(x) = x^2 + a^2 x + b^2$ ($\because$ c lie outside the roots)

$\therefore \quad f(c) > 0$, then $c^2 + a^2 c + b^2 > 0$

(b) is correct. Also (x- coordinate of vertex) $> c$

$\Rightarrow \quad -\frac{a^2}{2} > c$

$\therefore \quad$ (c) is correct.

73. **(b, d)** $|\alpha - \beta| = 3$ and $\alpha^2 + \beta^2 = 29$. $|\alpha - \beta|^2 = 9$

$\Rightarrow \quad \alpha^2 + \beta^2 - 2\alpha\beta = 9$

$\therefore \quad \alpha\beta = 10$ ($\because \alpha^2 + \beta^2 = 29$)

Then $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta = 29 + 20 = 49$

$\therefore \quad \alpha + \beta = \pm 7$

$\therefore \quad$ Required equation is $x^2 \pm 7x + 10 = 0$

74. **(a, b)** Let $A = a + 2b - 3c$, $B = b + 2c - 3a$,

$C = c + 2a - 3b$

$\therefore \quad A + B + C = 0$

Hence, roots are 1 and $\frac{C}{A}$.

Assertion and Reason

75. **(b)** If each of the three equation has real roots, then $b^2 - 4ac \geq 0, c^2 - 4ab \geq 0$ and $a^2 - 4ab \geq 0$

$\Rightarrow \quad a^2 b^2 c^2 \geq 64 a^2 b^2 c^2$. A contradiction.

76. **(d)** If α is a repeated root of $f(x) = 0$, then $f(\alpha) = 0$ and $f'(\alpha) = 0$.

$\therefore \quad 1 + \frac{\alpha}{1} + \frac{\alpha^2}{2!} + \frac{\alpha^3}{3!} + \frac{\alpha^4}{4!} = 0$ and $1 + \frac{\alpha}{1} + \frac{\alpha^2}{2!} + \frac{\alpha^3}{3!} = 0$

$\Rightarrow \quad \frac{\alpha^4}{4!} = 0 \Rightarrow \alpha = 0$. But $\alpha = 0$ does not satisfy $f(x) = 0$.

77. **(b)** Let x_1, x_2, x_3, x_4 be four positive roots of $x^4 - 4x^3 + ax^2 - bx + 1 = 0$, then $x_1 + x_2 + x_3 + x_4 = 4$ and $x_1 x_2 x_3 x_4 = 1$.

$\Rightarrow \quad \frac{1}{4}(x_1 + x_2 + x_3 + x_4) = (x_1 x_2 x_3 x_4)^{1/4}$

That is A.M. = G.M. This is possible if and only if $x_1 = x_2 = x_3 = x_4 = 1$. Thus the given equation becomes $(x-1)^2 = 0 \Rightarrow a = 6, b = 4$.

Reason follows immediately from the Rolle's theorem.

78. **(c)** By multiplying a,b,c by an appropriate natural number, we may assume that a, b, c are integers. We may further assume that a, b, c have no factor in common.

Now, $a + b2^{1/3} + c2^{2/3} = 0$

$\Rightarrow \quad a^3 = -2(b + 2^{1/3}c)^3 = -2(b^3 + 2c^3 - 3abc)$

$\Rightarrow \quad 2 | a^2 \Rightarrow 2 | a \Rightarrow a = 2a_1$ for some $a_1 \in$ I.

Thus, $4a_1^3 = -(b^3 + 2c^3 - 6a_1 bc)$

$\Rightarrow \quad b^3 = -2(2a_1^3 + c^3 - 3a_1 bc)$

$\Rightarrow \quad 2 | b^3 \Rightarrow 2 | b \Rightarrow b = 2b_1$ for some $b_1 \in$ I.

Therefore $4b_1^3 = -(2a_1^3 + c^3 - 6a_1 b_1 c)$

$\Rightarrow \quad c^3 = -2(a_1^3 + 2b_1^3 - 3a_1 b_1 c)$

$\Rightarrow \quad 2 | c^3 \Rightarrow 2 | c$.

This is a contradiction. Reason is false as $2^{\frac{1}{3}}$ is irrational but is a root of $x^3 - 2 = 0$.

79. **(b)** We have $f'(x) = 2ax + 2b = 2a\left(x + \frac{b}{a}\right)$

Not that f '(x)=0 for $x = -b/a$ and $f'(x) < 0$ for $x < -b/a$. and $f'(x) > 0$ for $x > -b/a$

This shows that f is strictly decreasing on $(-\infty, -b/a)$ and strictly increasing on $(-b/a, \infty)$.

Also note that f has a local minimum at $x = -b/a$.

We have $\min f(x) = f\left(\frac{-b}{a}\right) = \frac{b^2 - ac}{a}$

If $b^2 - ac > 0$ and $a > 0$, then $f(x) \geq \min f(x) > 0$

Thus, Assertion is true and Reason is true. However, Reason alone is not the complete explanation for Reason.

80. **(b)** Consider f: $R \rightarrow R$ defined by $f(x) = \frac{1}{3}ax^2 + \frac{1}{2}bx^2 + cx$

Then is f continuous and differentiable on R and $f(1)\frac{1}{2}a + \frac{1}{2}b + c = \frac{1}{2}(2a + 3b + 6c) = 0$ and $f(0) = 0$. By the Rolle's theorem there exists $\alpha \in (0,1)$ such that $f'(\alpha) = 0$

But $f'(x)=ax^2+bx+c$

Thus, $ax^2+bx+c=0$ has at least one root in (0, 1)

Reason is true but its not the correct explanation of Assertion.

81. **(c)** If $a\neq 0,$ we rewrite $f(x)$ as follows:

$$f(x)=a\left\{x^2+\frac{b}{a}x+\frac{c}{a}\right\}=a\left\{\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right\}$$

$$f\left(-\frac{b}{2a}+i\right)=a\left\{(i)^2+\frac{4ac-b^2}{4a^2}\right\}=a\left(1+\frac{4a-b^2}{4a^2}\right)$$

Which is real? This contradicts our assumption that is non real for non-real *x*. Therefore, *a* = 0. Reason is false since –*i* is a root of $x^2+x+1+i=0$ but *i* is not a root of $x^2+x+1+i=0.$

82. **(a)** α,β are roots of $ax^2+bx+c=0\Leftrightarrow\alpha,\beta$ are roots of $x^2+\frac{b}{a}x+\frac{c}{a}=0.$ As coefficient of $x^2>0,$ graph of the parabola $y=x^2+\frac{b}{a}x+\frac{c}{a}$ for $\alpha<x<\beta$ lies below the *x*-axis.

As $-1,,\in(\alpha,\beta),\ (-1)^2+\frac{b}{a}(-1)+\frac{c}{a}<0$ and $1^2\frac{b}{a}+\frac{c}{a}<0$

$\Rightarrow\quad 1-\frac{b}{a}+\frac{c}{a}<0$ and $1+\frac{b}{a}+\frac{c}{a}<0$

$\Rightarrow\quad \frac{1}{a}(a+|b|+c)<0$

Thus, $a+|b|+c$ and *a* have the opposite signs.

83. **(b)** Given, $x^2+2px+q=0$

$\therefore\quad \alpha+\beta=-2p\qquad ...(i)$

$\alpha\beta=q\qquad ...(ii)$

and $ax^2+2bx+c=0$

$\therefore\quad \alpha+\frac{1}{\beta}=-\frac{2b}{a}\qquad ...(iii)$

and $\frac{\alpha}{\beta}=\frac{c}{a}\qquad ...(iv)$

Now, $(p^2-q)(b^2-ac)$

$$=\left[\left(\frac{\alpha+\beta}{-2}\right)^2-\alpha\beta\right]\left[\left(\frac{\alpha+\frac{1}{\beta}}{2}\right)^2-\frac{\alpha}{\beta}\right]a^2$$

$$=\frac{(\alpha-\beta)^2}{16}\left(\alpha-\frac{1}{\beta}\right)^2.\ a^2\geq 0$$

$\therefore$ Assertion is true.

Again, now $pa=-\left(\frac{\alpha+\beta}{2}\right)a=-\frac{\alpha}{2}(\alpha+\beta)$

and $b=-\frac{a}{2}\left(\alpha+\frac{1}{\beta}\right)$

Since, $pa\neq b$

$\Rightarrow\quad \alpha+\frac{1}{\beta}\neq\alpha+\beta$

$\Rightarrow\quad \beta^2\neq 1,\beta\neq\{-1,0,1\},$ b which is correct. Similarly, if $c\neq qa$

$\Rightarrow\quad a\frac{\alpha}{\beta}\neq a\alpha\beta$

$\Rightarrow\quad \alpha\left(\beta-\frac{1}{\beta}\right)\neq 0$

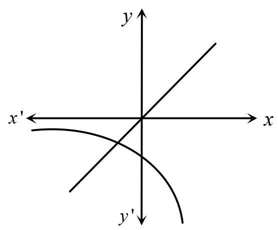
$\Rightarrow\quad \alpha\neq 0$ and $\beta-\frac{1}{\beta}\neq 0$

$\Rightarrow\quad \beta\neq\{-1,0,1\}$

Reason is true. Both Assertion and Reason are true. But Reason does not explain Assertion.

Comprehension Based

84. **(a)**



Let $y=x$ intersect the curve $y=ke^x$ at exactly one point when $k\leq 0.$

85. **(a)** Let $f(x)=ke^x-x$

$f'(x)=ke^x-1=0\Rightarrow x=-\text{In}\,k$

$f''(x)=ke^x$

$\therefore\quad [f''(x)]_{x=-\text{In}\,k}=1>0$

Hence, $f(-\text{In}\,k)=1+\text{In}\,k$

For one root of given equation

$1+\text{In}\,k=0\Rightarrow\ k=\frac{1}{e}$

86. **(a)** For two distinct roots,

$1+\text{In}\,k<0\ (k>0)$

$\text{In}\,k<-1\qquad k<1/e$

Hence, $k\in\left(0,\frac{1}{e}\right)$

87. **(c)** Given, $f(x)=4x^3+3x^2+2x+1$

$f'(x)=2(6x^2+3x+1)$

$D=9-24<0$

Hence, $f(x)=0$ has only one real root.

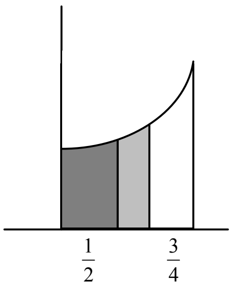
$f\left(-\frac{1}{2}\right)=1-1+\frac{3}{4}-\frac{4}{8}>0$

$f\left(-\frac{3}{4}\right)=1-\frac{6}{4}+\frac{27}{16}-\frac{108}{64}=\frac{64-96+108-108}{64}<0$

$f(x)$ changes its sign in $\left(-\frac{3}{4},-\frac{1}{2}\right)$, hence

$f(x)=0$ has a root in $\left(-\frac{3}{4},-\frac{1}{2}\right)$

88. **(a)**



$\int_0^{1/2} f(x)dx<\int_0^{t} f(x)dx<\int_0^{3/4} f(x)dx$

Now, $\int f(x)dx=\int(1+2x+3x^2+4x^3)dx$

$=x+x^2+x^3+x^4$

$\Rightarrow \int_0^{1/2} f(x)dx=\frac{15}{16}>\frac{3}{4}$

$\int_0^{3/4} f(x)dx=\frac{530}{256}<3$

89. **(b)**

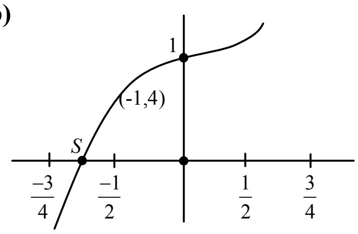


Figure is self explanatory

Match the Column

90. **(a)** **(A)** Case (i): If $x>3$, the $x+2y=6, x-3=y$ We get $x=4, y=1$

Case (ii):If $x<3$, then $\left.\begin{array}{l}x+2y=6\\3-4=y\end{array}\right\}y=3, x=0$

Hence, the only solution are (0, 3) and (4, 1) *i.e*, $m=2$

(B) Since, $2^y$is positive for all values of y, then $(x-8)(x-10)$ should be positive, Therefore, $x>10$ or $x<8$

Since, 2^y is a power of 2, $x-10$ and $x-8$ should be both powers of 2.

$\therefore$ $x=12$ and $x=6$ are only values of x that fit for this condition.

Hence, (12, 3) and (6, 3) are the only solutions.

ie, $n=2$

(C) $x+2y=2xy \Rightarrow \quad x=2y(x-1)$

or $2y=\frac{x}{x-1}=1+\frac{1}{x-1}$

$\therefore$ $x-1=1,-1$ or $x=2,0$

Then $2y=2,0 \Rightarrow y=1,0$

Hence, the only solutions are (2, 1) and (0, 0) *ie*, $p=2$

$P\to m=\frac{n+p}{2}+\frac{2+2}{2}=2, Q\to n=\sqrt{np}=\sqrt{4}=2,$

$R\to p=\frac{2mn}{m+n}=\frac{8}{4}=2, S\to n=\frac{2^2+2^2}{4}=2.$

$T\to\sqrt{n\sqrt{p\sqrt{n\sqrt{p\sqrt{n...\infty}}}}}=\sqrt{2\sqrt{2\sqrt{2\sqrt{2...}}}}=2^{1/2+1/4+1/8+}$

$=2^{\frac{\frac{1}{2}}{1-1/2}}=2=m$

91. **(a)** **(A)**Let $f(x)=ax^2+bx+c$

Then $f(1)=a+b+c=-c$ $(\because a+b+2c=0)$ and $f(0)=c$

$\therefore$ $f(0)f(1)=-c^2<0 \quad (\because c\neq 0)$

$\therefore$ Equation $f(x)=0$ has a root in (0, 1)

$\therefore$ $f(x)=0$ has a root in (0, 2) (*T*) as well as in (–1, 1).

(B) Let $f'(x)=ax^2+bx+c$

$\therefore$ $f(x)=\frac{ax^3}{3}+\frac{bx^2}{2}+cx+d=\frac{(2ax^3+3bx^2+6cx)}{6}+d$

$\therefore$ $f(0)=d$

and $f(-1)=\frac{(-2a+3b-6c)}{6}+d=-\frac{(2a-3b+6c)}{6}+d$

$=0+d$ $(\because 2a-3b+6c=0)$ $=d$

Hence, $f(0)=f(-1)$

Hence, $f'(x)=0$ has at least one root in (–1, 0).

$\therefore$ $f(x)=0$ has a root in (–2, 0) (*P*) as well as in (–1 1).

(C) Let $f(x)=\int(1+\cos^8 x)(ax^2+bx+c)dx$

Given $f(1)-f(0)=f(2)-f(0) \Rightarrow f(1)=f(2)$

$\Rightarrow$ $f'(x)=0$ has at least one root in (0, 1)

$\Rightarrow$ $(1+\cos^8 x)(ax^2+b+c)=0$ has at least one root in (0, 1)

$\Rightarrow$ $ax^2+bx+c=0$ has at least one root in (0, 1)

$\therefore$ $ax^2+bx+c=0$ has a root in (0,2) (*T*) as well as in (–1, 1)

92. **(a) (A)** $a(b-c)+b(c-a)+c(a-b)=0$

$\therefore$ $x=1$ is a root of $a(b-c)x^2+b(c-a)x+c(a-b)=0$

$\because$ Roots are equal then other root is also 1.

$\therefore$ Product of roots $=\dfrac{c(a-b)}{a(b-c)}=1$

$\therefore$ $b=\dfrac{2ac}{a+c}\Rightarrow a,b,c$ an in HP.

Also, $(d+a-b)^2+(d+b-c)^2=0$ which is possible only when

$d+a-b=0, d+b-c=0$

$\therefore$ $b-a=c-b=d$

ie, $2b=a+c$

$\Rightarrow$ a,b,c are in AP

$\because$ a,b,c are in AP and HP then $a=b=c$ then *a*, *b*, *c* are also in GP

$\because$ $a=b=c$

$\therefore$ $a+b+c\neq 0$ (*P*)

(B) $(b-c)+(c-a)+(a-b)=0$

$\therefore$ $x=1$ is a root of $(b-c)x^2+(c-a)x+(a-b)=0$

$\because$ Roots are equal

$\therefore$ Other root is also 1.

Then product of roots $=\dfrac{a-b}{b-c}=1$

$b=\dfrac{a+c}{2}\Rightarrow a,b,c$ are in AP.

and $a+b+c\neq 0$ (*P*)

(C) Let α be a common positive root, then

$\alpha^2+p\alpha+12=0$ . . .(*i*)

$\alpha^2+q\alpha+15=0$ . . .(*ii*)

and $\alpha^2+(p+q)\alpha+36=0$ . . .(*iii*)

Applying Eqs. (*i*) + (*ii*) – (*iii*) , we get

$\alpha^2+27-36=0$

or $\alpha^2=9\Rightarrow\alpha=3$ ($\because\alpha$ is positive)

Let other root of Eq. (*i*) is a (given)

then $a\times 3=12$

$\Rightarrow$ $a=4$

Let other root of Eq. (*ii*) is b (given) then $b\times 3=15$

$\Rightarrow$ $b=5$

and let other root of Eq. (*iii*) is c (given) then $c\times 3=36$

$\Rightarrow$ $c=12$

$\therefore$ $a+b+c=21\neq 0$

Integer

93. **(254)** The given equation can be written as

$\lambda x^2-(\lambda-1)x+5=0$

$\therefore$ $\alpha+\beta=\dfrac{\lambda-1}{\lambda},\ \alpha\beta=\dfrac{5}{\lambda}$ but given $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{4}{5}$

or $5(\alpha^2+\beta^2)=4\alpha\beta$

or $5\{(\alpha+\beta)^2-2\alpha\beta\}=4\alpha\beta$ or $5\left\{\left(\dfrac{\lambda-1}{\lambda}\right)^2-\dfrac{10}{\lambda}\right\}=\dfrac{20}{\lambda}$

or $\lambda^2-16\lambda+1=0$

It is a quadratic equation in λ, let roots be λ_1 and λ_2, then $\lambda_1+\lambda_2=16,\ \lambda_1\lambda_2=1$

$\therefore$ $\dfrac{\lambda_1}{\lambda_2}+\dfrac{\lambda_2}{\lambda_1}=\dfrac{\lambda_1^2+\lambda_2^2}{\lambda_1\lambda_2}=\dfrac{(\lambda_1+\lambda_2)^2-2\lambda_1\lambda_2}{\lambda_1\lambda_2}$

$=\dfrac{(16)^2-2.1}{1}=256-2=254$

94. **(7)** We have

$(\alpha+\beta+\gamma)^2=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$\Rightarrow$ $16=6+2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$\therefore$ $\alpha\beta+\beta\gamma+\gamma\alpha=5$

Also, $\alpha^3+\beta^3+\gamma^3-3\alpha\beta\gamma$

$=(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)$

$\Rightarrow$ $8-3\alpha\beta\gamma=4(6-5)$

or $3\alpha\beta\gamma=4$ or $\alpha\beta\gamma=4/3$

Now, $(\alpha^2+\beta^2+\gamma^2)^2=\Sigma\alpha^4+2\Sigma\beta^2\gamma^2$

$=\Sigma\alpha^4+2\{(\Sigma\beta\gamma^2)-2\alpha\beta\gamma\,\Sigma\alpha\}$

$(6)^2=\Sigma\alpha^4+2\left\{25-2\cdot\dfrac{4}{3}\cdot 4\right\}$

$\Sigma\alpha^4=36-50+\dfrac{64}{3}=\dfrac{64}{3}-14=\dfrac{22}{3}$

$\therefore$ $[\alpha^4+\beta^4+\gamma^4]=\left[\dfrac{22}{3}\right]=7$

95. **(3)** Rewrite the given equation

$$\sqrt{x+2\sqrt{x+2\sqrt{x+\ldots+2\sqrt{x+2\sqrt{x+2x}}}}}=x \quad \ldots(i)$$

On replacing the last latter *x* on the LHS of equation (*i*) by the value of *x* expressed by equation (*i*) we obtain

$$x=\underbrace{\sqrt{x+2\sqrt{x+2\sqrt{x+\ldots+2\sqrt{x+2x}}}}}_{(2n\text{ radical signs})}$$

Further, let us replace the last latter *x* by the same expression; again and again yields

$\therefore \quad x=\underbrace{\sqrt{x+2\sqrt{x+2\sqrt{x+\ldots+2\sqrt{x+2x}}}}}_{(3n \text{ radical signs})}$

$x=\underbrace{\sqrt{x+2\sqrt{x+2\sqrt{x+\ldots+2\sqrt{(x+2x)}}}}}_{(4n \text{ radical signs})}$

We can write $x=\sqrt{x+2\sqrt{x+2\sqrt{x+\ldots}}}$

$=\lim_{N\to\infty}\sqrt{x+2\sqrt{x+2\sqrt{x+\ldots+2\sqrt{x+2x}}}}$ (N radical signs)

If follows that,

$x=\sqrt{x+2\sqrt{x+2\sqrt{x+\ldots}}}=\sqrt{x+2\left(\sqrt{x+2\sqrt{x+\ldots}}\right)}=\sqrt{x+2x}$

Hence, $x^2=x+2x \quad \Rightarrow x^2-3x=0$

Therefore, $x=0,3 \quad \therefore$ Sum of roots $=0+3=3$

96. **(4177)** Correct equation is $x^2-11x+q=0$ . . .(i)

Incorrect equation is $x^2-10x+q=0$ . . .(ii)

Given roots of equation (ii) are 4 and 6

$\therefore$ Product of roots of the incorrect equation is 4×6

i.e., $q=4\times6=24$

From equation (i), correct equation is $x^2-11x+24=0$

$\therefore \quad x=3,8$ i.e., $\alpha=3, \beta=8$

$\therefore \quad \alpha^4+\beta^4=3^4+8^4=81+4096=4177$

97. **(1950)** α,β are the roots of $x^2+px+q=0$

Then $\alpha+\beta=-p,\ \alpha\beta=q$ . . .(i)

Also α,β are roots of $x^{3900}+p^{1950}x^{1950}+q^{1950}=0$

$\therefore \quad \alpha^{1950}+\beta^{1950}=-p^{1950}$ and $\alpha^{1950}\beta^{1950}=q^{1950}$ . . .(ii)

Now, α/β is a root of $x^n+1+(x+1)^n=0$

Then $\left(\frac{\alpha}{\beta}\right)^n+1+\left(\frac{\alpha}{\beta}+1\right)^n=0 \Rightarrow \alpha^n+\beta^n+(\alpha+\beta)^n=0$

$\Rightarrow \quad \alpha^n+\beta^n+(-p)^n=0 \quad (\because\ \alpha+\beta=-p\ )$

or $\quad \alpha^n+\beta^n=-(-p)^n$ . . .(iii)

From equation (ii) and (iii), we get $n=1950$

98. **(k = 2)** Given, $x^2-8kx+16(k^2-k+1)=0$

Now, $D=64\{k^2-(k^2-k+1)\}=64(k-01)>0$

$-\frac{b}{2a}>4 \Rightarrow \frac{8k}{2}>4 \Rightarrow k>1$

$f(4)\geq0 \quad 16-32k+16(k^2-k+1)\geq0$

$\Rightarrow \quad k^2-3k+2\geq0 \Rightarrow \quad (k-2)(k-1)\geq0$

$\Rightarrow \quad k\leq1$ or $k\geq2$ Hence, $k=2$.

99. **(7)** Given, $3x-y-z=0$ . . .(i)

$-3x+2y+z=0$ . . .(ii)

and $\quad -3x+z=0$ . . .(iii)

On adding Equation (i) and (ii), we get $y=0$

So, $\quad 3x=z$

Now, $x^2+y^2+z^2\leq100 \Rightarrow x^2+(3x)^2+0\leq100$

$\Rightarrow \quad 10x^2\leq100 \Rightarrow x^2\leq10 \quad x=-3,-2,-1,0,1,2,3$

So, Number of such 7 points are possible.

100. **(1)** Let $A=\begin{bmatrix}1 & \omega & \omega^2\\ \omega & \omega^2 & 1\\ \omega^2 & 1 & \omega\end{bmatrix}$

Now, $A^2=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$ and Tr $(A)=0$, $|A|=0$

$\therefore \quad A^3=0$

$\Rightarrow \quad \begin{vmatrix}z+1 & \omega & \omega^2\\ \omega & z+\omega^2 & 1\\ \omega^2 & 1 & z+\omega\end{vmatrix}=|A+zI|=0$

$\Rightarrow \quad z^3=0$

$\Rightarrow \quad z=0$, the number of z satisfying the given equation is 1.

* * *