Chapter 3

Motion in a Plane

Solutions (Set-1)

Very Short Answer Type Questions :

1. Which of the following is a scalar quantity? Momentum, Acceleration, Work, Force.

Sol. Work is a scalar quantity, others are vectors.

2. Name two vector quantities.

Sol. Displacement and velocity.

3. Can three vectors of different magnitudes be combined to give a zero resultant?

Sol. If three vectors form the sides of a triangle when taken in the same order, the resultant vector is a null vector.

4. If $|\vec{A}+\vec{B}|=|\vec{A}-\vec{B}|$, what is the angle between $\vec{A}$ and $\vec{B}$?

Sol. Let the angle between $\vec{A}$ and $\vec{B}$ be θ.

$$|\vec{A}+\vec{B}|=\sqrt{A^2+B^2+2AB\cos\theta}$$

$$|\vec{A}-\vec{B}|=\sqrt{A^2+B^2-2AB\cos\theta}$$

$\Rightarrow$ $A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$\Rightarrow$ $4AB\cos\theta = 0$

$\theta = 90°$

5. What is the average value of acceleration of an object in uniform circular motion in one complete revolution?

Sol. $\vec{\bar{a}}=\dfrac{\Delta\vec{v}}{\Delta t}$

In one complete rotation $\Delta\vec{v}=\vec{v}-\vec{v}=0$. On reaching the same point, the object has same velocity.

$\therefore$ $\vec{\bar{a}}=0$

6. Electric current has both magnitude and direction, so is it a vector?

Sol. No, electric current does not obey vector laws of addition.

7. Can the sum of two vectors be a scalar?

Sol. No, sum of two vectors is always a vector.

8. What is the magnitude of $\hat{i}+\hat{j}$?

Sol. The vector is $\hat{i} + \hat{j}$

$\therefore$ Its magnitude = $\sqrt{1^2+1^2}$

= $\sqrt{2}$

9. What is the direction of $\vec{A}+\vec{B}$ for parallel vectors $\vec{A}$ and $\vec{B}$?

Sol. It is along either $\vec{A}$ or $\vec{B}$, both have same direction.

10. When can the sum of two vectors be minimum and maximum?

Sol. Minimum, when two vectors are in opposite directions.

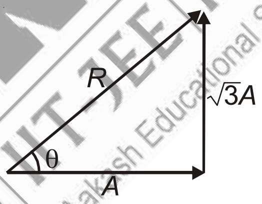
Maximum, when two vectors are parallel *i.e.*, in same direction.

Short Answer Type Questions :

11. Two vectors having magnitudes A and $\sqrt{3}\,A$ are perpendicular to each other. What is the angle between their resultant and $\vec{A}$?

Sol. Let $\vec{R}$ be their resultant.

Then by law of cosines



$|\vec{R}|=\sqrt{A^2+(\sqrt{3}A)^2}$

$|\vec{R}|=2A$

Using law of sines, $\dfrac{|\vec{R}|}{\sin 90^\circ}=\dfrac{\sqrt{3}A}{\sin\theta}$

$\Rightarrow \quad \sin\theta=\dfrac{\sqrt{3}A}{2A}\times\sin 90^\circ$

$\sin\theta=\dfrac{\sqrt{3}}{2}$

$\theta = 60^\circ$

12. Find the angle of the vector $\vec{A}=3\hat{i}+6\hat{j}-\hat{k}$ with y-axis.

Sol. Here $|\vec{A}|=\sqrt{3^2+6^2+(-1)^2}$

= $\sqrt{9+36+1}$

= $\sqrt{46}$

Let $\vec{A}$ make angle θ with y-axis, then

$|\vec{A}|\cos\theta = 6$

$\Rightarrow \quad \cos\theta = \dfrac{6}{\sqrt{46}} = 0.88465$

$\Rightarrow \quad \theta = \cos^{-1}(0.88465)$

$\theta = 27.79°$

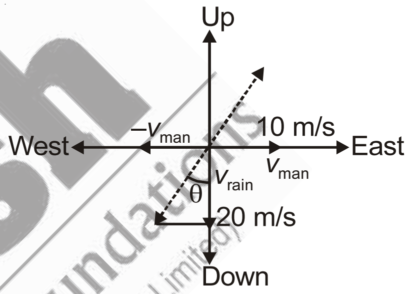
13. A boy sitting in a train moving with constant velocity, throws a ball vertically upwards. How does the ball appear to move to an observer, (i) inside the train, (ii) outside the train?

Sol. (i) To an observer sitting inside the train, the ball will appear to move straight vertically upwards and then downwards.

(ii) To an observer sitting outside the train, the ball will appear to move along the parabolic path.

14. Rain is falling vertically with speed 20 m/s. A man runs with speed of 10 m/s towards east. In which direction should he hold his umbrella?

Sol. Velocity of rain with respect to man



$\vec{v}_{rm} = \vec{v}_{rain} - \vec{v}_{man}$

$\Rightarrow \quad \tan\theta = \dfrac{10}{20}$

$= \dfrac{1}{2}$

Or θ = 26.56° inclined to vertical towards east direction.

15. Are the two vectors (2 kg) (4 m/s, towards east) and 2(4 m/s, towards east) same?

Sol. No, (2 kg) (4 m s^{-1}, towards east) shows the multiplication of a scalar quantity *i.e.*, mass of 2 kg with a velocity vector. So it is a momentum vector of magnitude 8 kg m/s directed towards east.

2 (4 m/s, towards east) is the velocity vector of magnitude 8 m/s. Hence, the two are not same.

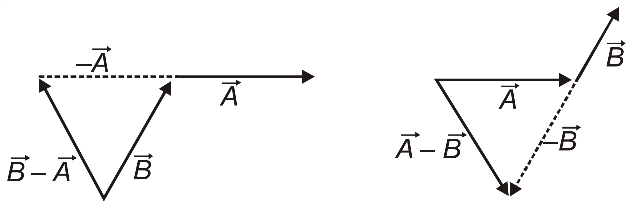
16. If $\vec{A}+\vec{B}=\vec{C}$ and $|\vec{C}|>|\vec{A}|$ and $|\vec{B}|$. Does that mean $|\vec{C}|>|\vec{A}|+|\vec{B}|$?

Sol. No, In above case, $\vec{A}, \vec{B}$ and $\vec{C}$ represent three sides of a triangle. And no side is ever greater than the sum of the other two.

Hence, $|\vec{C}|<|\vec{A}|+|\vec{B}|$

17. Show graphically that subtraction of two vectors is not commutative.

Sol. The two figures given show that $\vec{A}-\vec{B}$ and $\vec{B}-\vec{A}$ are two different vectors.



Hence, subtraction of vectors is not commutative.

18. A projectile is fired with kinetic energy 4 kJ. If its range is maximum, what is its K.E. at the highest point of its path?

Sol. Since the range is maximum, the angle of projection is 45°.

If velocity of projection = v

$\Rightarrow \quad 4\text{ kJ} = \frac{1}{2}mv^2$

Velocity at the highest point = v cos45° = $\frac{v}{\sqrt{2}}$

$\therefore$ K.E. at highest point = $\frac{1}{2}m\left(\frac{v}{\sqrt{2}}\right)^2$

$\Rightarrow \quad \frac{1}{2}\left(\frac{1}{2}mv^2\right)$

$\Rightarrow \quad \frac{4\text{ kJ}}{2} = 2\text{ kJ}$

19. For a given velocity in projectile motion, name the quantities related to a projectile which have maximum values when the maximum height attained by the projectile is the largest.

Sol. (i) Vertical component of initial velocity

(ii) Angle of projection

(iii) Time of flight

20. Give a few examples of motion in two dimensions.

Sol. (i) Projectile motion

(ii) Uniform circular motion

(iii) Non-uniform circular motion : The object moving in a circle has different speeds at different points. For example, A stone tied with a thread whirled in a vertical plane.

21. Find the angle of projection at which the horizontal range and maximum height of a projectile are equal.

Sol. $\therefore$ $H_{max} = R$

$\Rightarrow \quad \frac{v_0^2 \sin^2\theta_0}{2g} = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{\sin\theta_0}{2} = 2\cos\theta_0$

$\Rightarrow$ $\tan\theta_0 = 4$ or $\theta_0 = 75.96°$

22. Why is centripetal acceleration called so? Is it a constant vector?

Sol. Centripetal acceleration always acts towards the centre of the circular path at every point. Hence, it is called centripetal acceleration which in Greek means "center seeking". Since its direction changes continuously, it is not a constant vector.

23. What do you mean by resolution of a vector and components of a vector?

Sol. Splitting a vector into two or more component vectors is called resolution of the vector. A vector can be resolved into infinite number of components. However a vector can have only two rectangular components in a plane.

Components of vector : If vectors $\vec{A}, \vec{B}$ and $\vec{C}$, when added using vectors laws, result into vector $\vec{D}$, then they are said to be component vectors of $\vec{D}$ along directions $\vec{A}, \vec{B}$ and $\vec{C}$.

24. Find components of vector addition of $2\hat{i}+4\hat{j}-\hat{k}$ and $3\hat{i}-2\hat{j}+2\hat{k}$.

Sol. Resultant $\vec{R}=(2\hat{i}+4\hat{j}-\hat{k})+(3\hat{i}-2\hat{j}+2\hat{k})$

$$\vec{R}=(5\hat{i}+2\hat{j}+\hat{k})$$

Hence, rectangular components of $\vec{R}$ are

R_x = 5 along *x*-axis

R_y = 2 along *y*-axis

R_z = 1 along *z*-axis

25. If the position of particle at time *t* is given by $\vec{r}=(2t^2\hat{i}+6t\hat{j}+8\hat{k})$ m. Find the component of velocity along z-axis at time *t* = 4 s.

Sol. $\vec{r}=2t^2\hat{i}+6t\hat{j}+8\hat{k}$

$$\vec{v}=\frac{d\vec{r}}{dt}$$

$$=4t\hat{i}+6\hat{j}+0\hat{k}$$

$$=4t\hat{i}+6\hat{j}$$

$\because$ v_z = 0, here. The particle does not have any velocity along *z*-axis.

26. An object is projected at an angle 30°. If its horizontal velocity is 50 km/h, what is its vertical velocity?

Sol. θ_0 = 30°

$v_0\cos\theta_0$ = 50 km/h

$v_0\sin\theta_0$ = ?

$$v_0\sin\theta_0=\frac{50}{\cos 30^\circ}\times\sin 30^\circ$$

$$=50\times\frac{1}{\sqrt{3}}\text{ km/h}$$

= 28.86 km/h

27. What is the relative velocity of a man swimming downstream with speed 12 km/h (in still water) with respect to a child running towards the river with speed 4 km/h in direction perpendicular to water flow? Speed of water flow = 4 km/h.

Sol. Velocity of man in still water = 12 km/h = $\vec{v}_m$

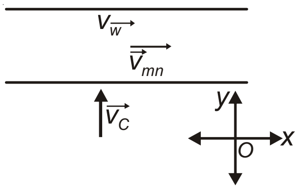
Velocity of water flow $\vec{v}_w=4$ km/h

$\therefore$ Net velocity of man w.r.t. ground = $\vec{v}_{mn}=16$ km/h $\hat{i}$

Velocity of child = $\vec{v}_C=4$ km/h $\hat{j}$

$\therefore$ Velocity of man w.r.t. child

$\vec{v}_{(mn)c} = 16\hat{i} - 4\hat{j}$



$|\vec{v}_{(mn)c}| = \sqrt{(16)^2 + (4)^2} = 4\sqrt{17}$ km/h

$\theta = \tan^{-1}\left(\frac{-1}{4}\right)$

$= -14.04°$ with x-axis (*i.e.*, direction of flow of river).

28. What do you mean by angular speed in uniform circular motion? How is it related to time period and centripetal acceleration?

Sol. Angular speed (ω) : The time rate of change of the angular displacement of an object having uniform circular motion is called angular speed (ω).

$\omega = \frac{2\pi}{T}$ or $a_C = \omega^2 r$

29. A cyclist going round a circular path with constant speed 10 km/h completes 42 revolutions in 30 minutes. Find its centripetal acceleration.

Sol. Frequency $\nu = \frac{42}{30 \times 60} = \frac{7}{300}\ \text{s}^{-1}$ ...(i)

$\therefore$ Time period $T = \frac{1}{\nu} = \frac{300}{7}$ s

Also, $T = \frac{2\pi R}{v}$

$\Rightarrow R = \frac{vT}{2\pi}$

Given that $v = 10$ km h^{-1}

$\Rightarrow R = \frac{10 \times \frac{5}{18} \times \frac{300}{7}}{2 \times \frac{22}{7}}\ \text{m} = \frac{625}{33}\ \text{m}$ ...(ii)

$a_C = 4\pi^2\nu^2 R$

$= 4 \times \frac{22}{7} \times \frac{22}{7} \times \frac{7}{300} \times \frac{7}{300} \times \frac{625}{33}$ [using equation (i) and (ii)]

$= \frac{11}{27}\ \text{m/s}^2$

$= 0.41\ \text{m s}^{-2}$

30. Find time of flight of an object projected at angle 30° with speed 60 m/s. [take $g = 10\ \text{m/s}^2$]

Sol. $T = \frac{2v_0 \sin\theta_0}{g} = \frac{2 \times 60 \times 1}{10 \times 2} = 6$ s

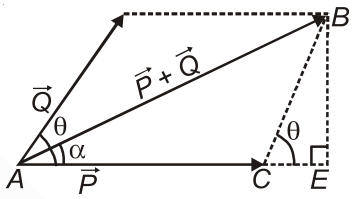
Long Answer Type Questions :

31. State and prove the law of cosines.

Sol. Law of cosines : The resultant $\vec{R}$ of two vectors $\vec{P}$ and $\vec{Q}$ inclined at an angle θ is given by

$$\left|\vec{R}\right| = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}.$$

Let the two vectors $\vec{P}$ and $\vec{Q}$ are inclined at an angle θ with each other. Then, the resultant of their vector addition can be obtained by using parallelogram method as shown below.



Here, $\overrightarrow{AB} = \vec{P} + \vec{Q}$

From B, draw a line perpendicular to AC which meets it at E when extended.

Now in ΔABE

$AB^2 = AE^2 + BE^2$ [Pythagoras theorem]

$\Rightarrow \quad AB^2 = (AC + CE)^2 + BE^2$ ...(i)

From ΔBCE, we can have

$CE = CB\cos\theta = Q\cos\theta$ [$CB = Q$, opposite sides of a parallelogram]

Also $BE = CB\sin\theta = Q\sin\theta$

Substituting these values in equation (i) above,

$AB^2 = (P + Q\cos\theta)^2 + Q^2\sin^2\theta$

$AB^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\Rightarrow \quad AB = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ ← Law of cosines

32. Define a unit vector. Find unit vector along the vector $6\hat{i} + 8\hat{j} + 10\hat{k}$.

Sol. A vector having unit magnitude is called a unit vector. The unit vector along the direction of vector $\vec{A}$ is denoted by $\hat{A}$. $\hat{A}$ is given by $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$.

It does not have any unit.

Let $\vec{A} = 6\hat{i} + 8\hat{j} + 10\hat{k}$

$= |\vec{A}| = \sqrt{6^2 + 8^2 + 10^2}$

$= \sqrt{200}$

$= 10\sqrt{2}$

$\therefore$ Unit vector $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|} = \dfrac{6\hat{i}}{10\sqrt{2}} + \dfrac{8\hat{j}}{10\sqrt{2}} + \dfrac{1\hat{k}}{\sqrt{2}} = \dfrac{3\hat{i}}{5\sqrt{2}} + \dfrac{4\hat{j}}{5\sqrt{2}} + \dfrac{1\hat{k}}{\sqrt{2}}$

33. (i) Derive expression for the horizontal range of a projectile,

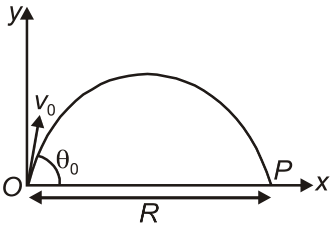
(ii) The maximum range of a projectile is $\dfrac{2}{\sqrt{3}}$ times the actual range for a given velocity of projection. What is the angle of projection for the actual range?

Sol. (i) The maximum horizontal distance travelled by the projectile during its flight is called the horizontal range of the projectile. This is the straight distance OP as shown in figure. It is denoted by R.

R can be calculated by using equation

$x = (v_0 \cos\theta_0)t$

When $x = R$, t = Time of flight, $T_f = \dfrac{2v_0 \sin\theta_0}{g}$



$$\therefore \quad R = (v_0 \cos\theta_0)\frac{(2v_0 \sin\theta_0)}{g} = \frac{2v_0^2(\sin\theta_0)(\cos\theta_0)}{g}$$

$$R = \frac{v_0^2 \sin 2\theta_0}{g} \qquad [\because\ 2\sin\theta_0 \cos\theta_0 = \sin(2\theta_0)]$$

R is maximum when $2\theta_0 = 90°$ *i.e.*, $R_{max} = \dfrac{v_0^2}{g}$

(ii) $R_{max} = \dfrac{v_0^2}{g} = \dfrac{2}{\sqrt{3}}R$ [R = actual Range]

$$\frac{v_0^2}{g} = \frac{2}{\sqrt{3}}\left(\frac{v_0^2 \sin 2\theta_0}{g}\right)$$

$$\sin 2\theta_0 = \frac{\sqrt{3}}{2}$$

$\Rightarrow \quad 2\theta_0 = 60°$

$\Rightarrow \quad \theta_0 = 30°$

34. Can the resultant of three non parallel, coplanar forces of magnitudes 4 N, 8 N and 3 N acting on a particle be zero? Explain.

Sol. A vector having zero magnitude and having any arbitrary direction is called a null vector. It is denoted by $\vec{0}$.

Examples of null vector :

(i) Zero displacement

(ii) Zero change in velocity

(iii) Resultant of two equal and opposite forces acting at a point.

Properties of null vector $\vec{0}$.

$$\vec{A} + \vec{0} = \vec{A}$$

$$\vec{A} - \vec{0} = \vec{A}$$

$$\vec{A}.0 = \vec{0}$$

$$A.\vec{0} = \vec{0}$$

Vector sum of three non-parallel and coplanar vectors will be zero only when the given vectors are represented by three sides of a triangle. From geometry, the sum of any two sides of a triangle must be greater, than the third side. Here in given problem (4 + 3) N < 8 N. The forces can not be represented by the side of a triangle. Hence their resultant can not be zero.

35. (i) Show that a projectile follows a parabolic path.

(ii) Find the horizontal distances travelled by a projectile projected at an angle 30° with horizontal with speed 15 m/s, at the time it is at height 2.5 m above the point of projection. [take g = 10 ms^{-2}]

Sol. (i) Consider a stone projected with velocity v_0 at angle θ with x-axis.

$v_{0x} = v_0 \cos\theta$ [Horizontal speed at the time of release]

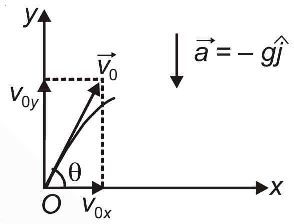
$v_{0y} = v_0 \sin\theta$ [Vertical speed at the time of release]

Along horizontal :

$\Rightarrow \quad \vec{v}_x = \vec{v}_{0x} = v_0 \cos\theta \hat{i}$

If we take the time at which the stone is projected as $t = 0$,

then its horizontal displacement at any time t is



$x\hat{i} = (v_0 \cos\theta)t\hat{i}$

or $x = v_0\cos\theta t$ ...(i)

Along vertical : The stone moves under a constant acceleration $\vec{a} = -g\hat{j}$. The velocity $\vec{v}_y$ of the stone at time t is given by

$v_y \hat{j} = v_{0y}\hat{j} + (-g)t\hat{j} \quad \Rightarrow \quad v_y = v_0\sin\theta - gt$

Its vertical displacement y in this time is given by

$$y = v_0 \sin\theta t - \frac{1}{2}gt^2$$

Substituting $t = \dfrac{x}{v_0 \cos\theta}$ from equation (i)

$$y = v_0 \sin\theta\left(\frac{x}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{x^2}{v_0^2\cos^2\theta}\right)$$

$\Rightarrow \quad y = (\tan\theta)x - \dfrac{1}{2}\dfrac{g}{(v_0^2\cos^2\theta)}x^2$ **Equation of trajectory of the projectile.**

(ii) v_0 = 15 m/s

θ = 30°

$$y = (v_0 \sin\theta)t - \frac{1}{2}gt^2 = 2.5 \text{ m} \ \text{(vertical height)}$$

$$2.5 = (15\sin 30^\circ)t - \frac{1}{2}\times 10t^2$$

$$2.5 = \frac{15}{2}t - 5t^2$$

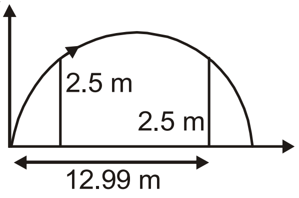
$\Rightarrow \quad 10t^2 - 15t + 5 = 0$

$\Rightarrow \quad t = 0.5$ s, 1 s

Horizontal distance travelled $x = v_0\cos\theta_0 t$

$= 15\cos 30^\circ \times 0.5$ [Taking t = 0.5 s]

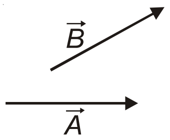
$= 15 \times \frac{\sqrt{3}}{2} \times 0.5 = 6.5$ m



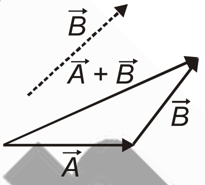
Similarly for t = 1 s, x = 12.99 m

36. Prove the commutative and associative properties of vector addition. How do we specify the position of an object using vectors?

Sol. (i) Commutative Property : To obtain $\vec{A}+\vec{B}$, we shift $\vec{B}$ parallel to itself so that its tail coincides with the head of $\vec{A}$. The line segment joining the tail of $\vec{A}$ to the head of $\vec{A}$ represents $\vec{A}+\vec{B}$ and its direction is as shown in the figure (ii).

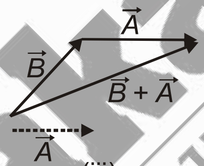


(i)



(ii)

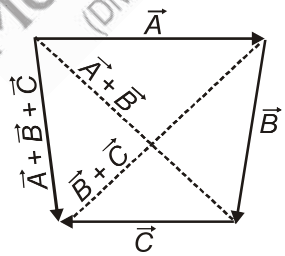
If instead of $\vec{B}$, we shift $\vec{A}$ parallel to itself such that the tail of $\vec{A}$ coincides with the head of $\vec{B}$, the vector obtained by joining the tail of $\vec{B}$ to the head of $\vec{B}$ gives $\vec{B}+\vec{A}$.



(iii)

If we compare $\vec{A}+\vec{B}$ and $\vec{B}+\vec{A}$ from figures (ii) and (iii), we find the two vectors are equal *i.e.*, $\vec{A}+\vec{B}=\vec{B}+\vec{A}$.

(ii) Associative Property : The figure shows that $\vec{A}+\vec{B}+\vec{C}=\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$



We can describe the position of an object using position vector. The position vector of a point P is the vector joining origin O to the point P. Thus it is the line OP having arrow at P.

37. The position of an object is described by the vector $\vec{r} = t^2\hat{i} + 2t\hat{j} - t\hat{k}$ at any time t. Find its position, velocity and acceleration at time t = 6 s.

Sol. $\vec{r} = t^2\hat{i} + 2t\hat{j} - t\hat{k}$

$\vec{v} = \frac{d\vec{r}}{dt} = 2t\hat{i} + 2\hat{j} - \hat{k}$

$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 0\hat{j} + 0\hat{k}$

at $t = 6$ s,

(i) Position : $\vec{r} = 6^2\hat{i} + 12\hat{j} - 6\hat{k} = 36\hat{i} + 12\hat{j} - 6\hat{k}$

(ii) Velocity : $\vec{v} = 2\times 6\hat{i} + 2\hat{j} - 1\hat{k} = 12\hat{i} + 2\hat{j} - \hat{k}$

$|\vec{v}| = \sqrt{12^2 + 2^2 + (-1)^2} = \sqrt{149}$ m s^{-1}

(iii) Acceleration : $\vec{a} = 2\hat{i}$

A constant acceleration of 2 m s^{-2} along positive x-axis.

38. If $\vec{A} = 2\hat{i} - \hat{j}$ and $\vec{B} = \hat{i} - 2\hat{j}$, find the scalar magnitude and directions of

(i) $\vec{A}$

(ii) $\vec{B}$ and

(iii) $\vec{A} + \vec{B}$

Sol. (i) $\vec{A} = 2\hat{i} - \hat{j}$

Magnitude of $\vec{A} = \sqrt{(2)^2 + (-1)^2} = \sqrt{5}$

Direction of $\vec{A}$ with x-axis.

$\theta = \tan^{-1}\left(\frac{-1}{2}\right) = -26.56°$

(ii) $\vec{B} = \hat{i} - 2\hat{j}$

Magnitude of $\vec{B} = \sqrt{(1)^2 + (-2)^2} = \sqrt{5}$

Direction of $\vec{B}$ with x-axis

$\phi = \tan^{-1}\left(\frac{-2}{1}\right) = -63.43°$

(iii) $\vec{A} + \vec{B} = (2\hat{i} - \hat{j}) + (\hat{i} - 2\hat{j}) = (3\hat{i} - 3\hat{j})$

$\therefore$ Magnitude of $\vec{A} + \vec{B} = \sqrt{3^2 + (-3)^2} = 3\sqrt{2}$

Direction of $\vec{A} + \vec{B}$ with x-axis.

$\alpha = \tan^{-1}\left(\frac{-3}{3}\right) = -45°$

39. Show that a two-dimensional uniformly accelerated motion is a combination of two one-dimensional motions along perpendicular directions.

Sol. A body is said to be moving with uniform acceleration if its velocity vector suffers the same change in equal interval of time, however small. **For this case the average acceleration of the object is same as its instantaneous acceleration over a given time interval.**

The motion in a plane with uniform acceleration can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions. This can be shown as follows.

Let a constant acceleration $\vec{a}$ act on an object moving in a plane. This acceleration changes its velocity from $\overrightarrow{v_0}$ at time $t = 0$ to $\vec{v}$ at time $t = t$. Then

$$\vec{a} = \frac{\vec{v} - \overrightarrow{v_0}}{t - 0}$$

$$\Rightarrow \quad \vec{v} = \overrightarrow{v_0} + \vec{a}t \qquad \text{...(i)}$$

As we have studied in the last chapter, that for an object having constant acceleration, average velocity is given as

$$\vec{\bar{v}} = \frac{\overrightarrow{v_0} + \vec{v}}{2} \qquad \text{...(ii)}$$

From the definition of average velocity during the time interval $\Delta t = t - 0$, it can be expressed as $\vec{\bar{v}} = \dfrac{\vec{r} - \overrightarrow{r_0}}{t - 0}$.

Where $\vec{r}$ and $\vec{r}_0$ are the position vectors of the particle at time $t = t$ and $t = 0$ respectively.

$$\Rightarrow \quad \vec{\bar{v}}t = \vec{r} - \overrightarrow{r_0}$$

or $\vec{r} = \overrightarrow{r_0} + \vec{\bar{v}}t$

$$\vec{r} = \overrightarrow{r_0} + \left(\frac{\overrightarrow{v_0} + \vec{v}}{2}\right)t \qquad \text{[From equation (ii)]}$$

$$\Rightarrow \quad \vec{r} = \overrightarrow{r_0} + \left(\frac{\overrightarrow{v_0} + \overrightarrow{v_0} + \vec{a}t}{2}\right)t \qquad \text{[From equation (i)]}$$

$$\vec{r} = \overrightarrow{r_0} + \overrightarrow{v_0}t + \frac{1}{2}\vec{a}t^2$$

Writing in component form

$$\Rightarrow \quad x\hat{i} + y\hat{j} = x_0\hat{i} + y_0\hat{j} + (v_{0_x}\hat{i} + v_{0_y}\hat{j})t + \frac{1}{2}(a_x\hat{i} + a_y\hat{j})t^2$$

Rearranging

$$\Rightarrow \quad x\hat{i} + y\hat{j} = \left(x_0 + v_{0x}t + \frac{1}{2}a_xt^2\right)\hat{i} + \left(y_0 + v_{0y}t + \frac{1}{2}a_yt^2\right)\hat{j}$$

Comparing two sides

$$\left.\begin{aligned} x &= x_0 + v_{0x}t + \frac{1}{2}a_xt^2 \quad \text{along x-axis} \\ y &= y_0 + v_{0y}t + \frac{1}{2}a_yt^2 \quad \text{along y-axis} \end{aligned}\right\} \qquad \text{...(iii)}$$

Thus we see that the motions in x and y-directions can be treated independently from each other. This result simplifies our study of motion in a plane.

A similar result can be obtained for the three-dimensional motion of an object. So for this case, we get a set of three equations as follows.

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

$$z = z_0 + v_{0z}t + \frac{1}{2}a_z t^2$$

40. Define the following, (i) Relative velocity (ii) Average velocity (iii) Average acceleration (iv) Centripetal acceleration

Sol. (i) **Relative velocity** : Let two objects A and B are moving in a plane with velocities $\vec{v}_A$ and $\vec{v}_B$ measured with respect to ground. The relative velocity of A with respect to B is given by

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ ...(i)

Similarly relative velocity of B with respect to A is given by

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$ ...(ii)

Comparing (i) and (ii)

$\vec{v}_{AB} = -\vec{v}_{BA}$

And $|\vec{v}_{AB}| = |\vec{v}_{BA}|$

(ii) **Average velocity** : Average velocity $\bar{\vec{v}}$ of an object is the ratio of its displacement and the corresponding time interval.

$$\therefore \quad \bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

Since $\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t}$, the direction of average velocity is the same as that of $\Delta\vec{r}$.

The instantaneous velocity or simply the velocity is the limiting value of the average velocity when the time interval approaches zero.

(iii) **Average Acceleration :** The average acceleration $\bar{\vec{a}}$ is the ratio of the change in velocity and the corresponding time interval. If the velocity of an object changes from $\vec{v}$ to $\vec{v}_1$ in time interval Δt, then the acceleration of the object is given by the relation

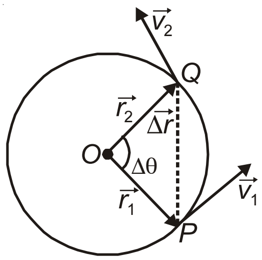
$$\bar{\vec{a}} = \frac{\vec{v}' - \vec{v}}{\Delta t}$$

$$\Rightarrow \quad \bar{\vec{a}} = \frac{\Delta\vec{v}}{\Delta t} \text{ [where } \Delta\vec{v} = \vec{v}' - \vec{v}\text{]}$$

From the above relation, we can see that $\bar{\vec{a}}$ is along the direction of $\Delta\vec{v}$. The direction of $\Delta\vec{v}$ is different from that of $\vec{v}'$ and $\vec{v}$ as long as the object moves along a curve and not along a straight line. Or we say that for the motion along a curve, the direction of average acceleration is different from that of the velocity of the object. They may have any angle between 0° and 180° between them.

(iv) **Centripetal Acceleration :** Consider a particle moving on a circular path of radius r and centre O, with a uniform speed v, as shown in the figure below. Let the particle be at point P at time t, and at Q at time $t + \Delta t$. Let $\vec{v}_1$ and $\vec{v}_2$ be the velocity vectors at P and Q directed along the tangents at P and Q respectively.

To find the change in velocity,



$\vec{v}_2 - \vec{v}_1 = \Delta\vec{v}$

This is the change in velocity during this time interval Δt.

By definition, the average acceleration is given by

$\vec{\bar{a}} = \dfrac{\Delta\vec{v}}{\Delta t}$ *i.e.*, $\vec{\bar{a}}$ is along Δv.

As $\Delta t \to 0$, the average acceleration becomes the instantaneous acceleration and $\Delta\theta$ also approaches zero.

Thus $\Delta\vec{v}$ and hence, $\vec{a}$ is perpendicular to velocity vector $\vec{v}_1$. But since $\vec{v}_1$ is directed along tangent at point P, so acceleration a_c acts along the radius towards the centre of the circle. That is why this acceleration is called **centripetal acceleration** which means 'centre seeking'. Numerically its value is $\dfrac{v^2}{r}$.

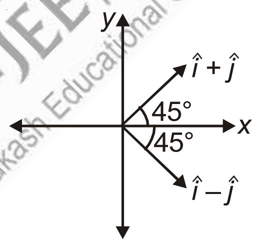
41. (i) Using graphical method, find the angle between $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$.

(ii) Express the unit vector along above mentioned vectors.

Sol. (i) Angle that $\hat{i} + \hat{j}$ makes with x-axis

$\tan\theta_1 = \dfrac{1}{1}$

$\theta_1 = 45°$



Similarly, the angle that $\hat{i} - \hat{j}$ make with x-axis.

$\tan\theta_2 = -\dfrac{1}{1}$

$\theta_2 = -45°$

$\therefore$ Required angle $= 90°$

(ii) $|\hat{i} + \hat{j}| = \sqrt{1^2 + 1^2}$

$= \sqrt{2}$

Therefore unit vector along $\hat{i} + \hat{j}$, is $= \dfrac{1}{\sqrt{2}}(\hat{i} + \hat{j}) = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{\sqrt{2}}\hat{j}$

Similarly $|\hat{i} - \hat{j}| = \sqrt{2}$

$\therefore$ Unit vector along $\hat{i} - \hat{j}$, is $= \dfrac{1}{\sqrt{2}}(\hat{i} - \hat{j}) = \dfrac{1}{\sqrt{2}}\hat{i} - \dfrac{1}{\sqrt{2}}\hat{j}$

42. The ceiling of a long hall is 30 m high. What is the maximum horizontal distance that a ball thrown with a speed 45 ms^{-1} can go without hitting the ceiling of the wall? [take g = 10 m/s^2]

Sol. v_0 = 4 5 ms^{-1}

Let θ_0 = angle for projection

Taking maximum height reached $H_{max} = \dfrac{v_0^2 \sin^2\theta_0}{2g}$

Taking maximum height = 30 m

We have $\dfrac{v_0^2\sin^2\theta_0}{2g} = 30$

$$\Rightarrow \quad \sin^2\theta_0 = \frac{30\times2\times10}{v_0^2}$$

$$\Rightarrow \quad \sin^2\theta_0 = \frac{30\times2\times10}{45\times45}$$

$$\Rightarrow \quad \sin^2\theta_0 = \frac{600}{45\times45} = \frac{8}{27}$$

$$\Rightarrow \quad \sin\theta_0 = \frac{2\sqrt{2}}{3\sqrt{3}} \qquad ...(i)$$

$$\cos\theta_0 = \sqrt{1-\sin^2\theta_0}$$

$$= \sqrt{1-\frac{8}{27}} = \sqrt{\frac{15}{27}} = \frac{\sqrt{15}}{3\sqrt{3}} = \frac{\sqrt{5}}{3}$$

$$\Rightarrow \quad \cos\theta_0 = \frac{\sqrt{5}}{3} \qquad ..(ii)$$

Horizontal range R for $\theta_0 = \sin^{-1}\left(\dfrac{2\sqrt{2}}{3\sqrt{3}}\right)$

$$R = \frac{v_0^2\sin2\theta_0}{g}$$

$$= \frac{v_0^2}{g}2\sin\theta_0\cos\theta_0$$

$$= \frac{v_0^2}{g}\times2\times\frac{2\sqrt{2}}{3\sqrt{3}}\times\frac{\sqrt{5}}{3} \qquad \text{[Using equation (i) and (ii)]}$$

$$= \frac{45\times45}{10}\times\frac{2\times2\sqrt{2}}{9\sqrt{3}}\times\sqrt{5}$$

= 164.3 m

43. A body of mass 10 kg revolves in a circle of diameter 0.8 m completing 420 revolutions in a minute. Calculate its (i) Angular speed (ii) Linear speed (iii) Time period and (iv) Centripetal acceleration

Sol. Given,

Diameter = 0.8 m

$\therefore$ Radius R = 0.4 m

$$\text{Frequency } (\nu) = \frac{\text{Number of revolutions}}{\text{Time taken}} = \frac{420}{60} \text{ rev/s} = 7 \text{ s}^{-1}$$

(i) Angular speed $\omega = 2\pi\nu = 2\times\frac{22}{7}\times 7$ = 44 rad/sec

(ii) Linear speed $v = \frac{2\pi R}{T}$

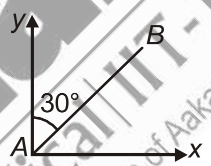
T = time period $= \frac{1}{\nu} = \frac{1}{7}$ s

$\therefore \quad v = 2\times\frac{22\times 0.4}{7\times 7} = \frac{44}{49}\times 0.4 = 0.3591 \text{ m s}^{-1}$

(iii) Time period $T = \frac{1}{7}$ = 0.142 s

(iv) Centripetal acceleration $a_C = \omega^2 R = (44)^2 \times 0.4 = 774.4 \text{ m/s}^2$

44. A bird is flying with velocity $5\hat{i}+6\hat{j}$ w.r.t. wind. Wind blows along y-axis with velocity v. If bird is initially at A, and after sometime reaches B as shown. Find v, and also find the velocity of bird with respect to ground.



Sol. Given that velocity of bird with respect to wind is $\vec{v}_{bw} = 5\hat{i}+6\hat{j}$

Velocity of wind w.r.t. ground $\vec{v}_{wG} = v\hat{j}$

Now $\vec{v}_{bw} = \vec{v}_{bG} - \vec{v}_{wG}$ ...(i)

$5\hat{i}+6\hat{j} = \vec{v}_{bG} - \vec{v}_{wG}$

$\Rightarrow \quad \vec{v}_{bG} = 5\hat{i}+6\hat{j}+v\hat{j}$

$\vec{v}_{bG} = 5\hat{i}+(6+v)\hat{j}$

$\vec{v}_{bG}$ is shown by the vector $\overline{AB}$ in figure. Since velocity $\vec{v}_{bG}$ makes angle 30° with y-axis, it is inclined at 60° with x-axis. Hence

$$\tan 60° = \frac{6+v}{5}$$

$$\Rightarrow \quad \sqrt{3} = \frac{6+v}{5}$$

$\Rightarrow$ v = 2.66 m s^{-1}

$\therefore$ Velocity of bird with respect to ground

$$v_{bG} = 5\hat{i} + 6\hat{j} + 2.66\,\hat{j}$$

$$= 5\hat{i} + 8.66\,\hat{j}$$

❑ ❑ ❑

Chapter 3

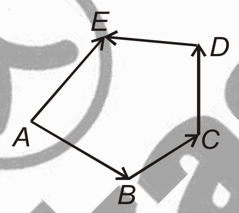
Motion in a Plane

Solutions (Set-2)

Paragraph for Q. Nos. 1 to 3

The resultant of any number of vectors can be determined by arranging them in head to tail fashion. The closing side of the figure taken in opposite order gives the resultant

In the figure,



$\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE} = \overrightarrow{AE}$

or $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}+\overrightarrow{EA} = 0$

If the vectors form a closed figure, the resultant is zero.

1. The minimum number of non coplanar (lying in different planes) vectors required whose sum is zero, is

(1) 2 (2) 3 (3) 4 (4) 5

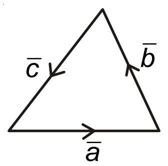
Sol. Answer (3)

Minimum number of non coplanar vectors whose sum is zero is four.

2. The minimum number of vectors with different magnitude required whose sum is zero is

(1) 2 (2) 3 (3) 4 (4) 5

Sol. Answer (2)



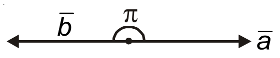
$\bar{a}+\bar{b}+\bar{c}=0$

Hence three vectors.

3. The minimum number of vectors required whose sum is zero is

(1) 2 (2) 3 (3) 4 (4) 5

Sol. Answer (1)



if $|\bar{a}| = |\bar{b}|$

$\Rightarrow \quad \bar{a} + \bar{b} = 0$

4. Given that $0.4\hat{i} + 0.8\hat{j} + b\hat{k}$ is unit vector. What is the value of b?

(1) ± 0.2 (2) $\pm\sqrt{0.2}$ (3) ± 0.8 (4) $\pm\sqrt{0.8}$

Sol. Answer (2)

We have,

$$\sqrt{(0.4)^2 + (0.8)^2 + (b)^2} = 1$$

$$\Rightarrow \quad b = \pm\sqrt{0.2}$$

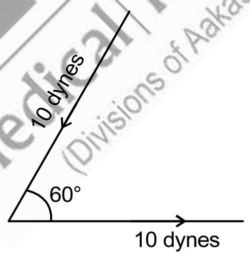
5. The position vector of particle changes from 10 m east to 10 m north in one second. What is the average velocity of the particle?

(1) 20 ms^{-1} (2) 5 ms^{-1} (3) $10\sqrt{2}$ ms^{-1} (4) $\left(\frac{10}{\sqrt{2}}\right)$ ms^{-1}

Sol. Answer (3)

$$\vec{V}_{avg} = \frac{\overrightarrow{\Delta s}}{\Delta t} = \frac{10\hat{j} - 10\hat{i}}{1} = 10\sqrt{2} \text{ m/s}$$

6. Two forces each numerically equal to 10 dynes are acting as shown in the following figure, then the resultant is



(1) 10 dynes (2) 20 dynes (3) $10\sqrt{3}$ dynes (4) 5 dynes

Sol. Answer (1)

$|\bar{F}_1| = |\bar{F}_2| = 10$

$\theta = 180 - 60 = 120$

$$|\bar{F}_R| = \sqrt{(10)^2 + (10)^2 - 2\times(10)(10)\frac{1}{2}}$$

$= 10$ dynes

7. A person moves 30 m North, then 20 m east, then $30\sqrt{2}$ m south west, his displacement is

(1) 14 m south-west (2) 28 m south (3) 10 m west (4) 15 m east

Sol. Answer (3)

$$\vec{D} = \vec{D}_1 + \vec{D}_2 + \vec{D}_3$$

$$= 30\hat{j} + 20\hat{i} + 30\sqrt{2}\cos 45(-\hat{i}) + 30\sqrt{2}\,\sin 45\,(-\hat{j})$$

$$= -10\,\hat{i}$$

8. The position co-ordinates of an object are given by $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$. The average velocity of the particle during the interval 0 – 1 s is

(1) 10 unit (2) 12 unit (3) $\sqrt{22}$ unit (4) 8 unit

Sol. Answer (3)

$$\vec{V} = \frac{\overrightarrow{\Delta X}}{t} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{\vec{X}_1 - \vec{X}_0}{\Delta t}$$

$$= \sqrt{22}\ \text{unit}$$

9. If two vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the angle between them is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) π

Sol. Answer (3)

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$\Rightarrow \quad \cos\theta = 0$$

$$\theta = \frac{\pi}{2}$$

10. At what angle the two vectors of magnitude $(P + Q)$ and $(P - Q)$ should act so that their resultant is $\sqrt{P^2 + Q^2}$?

(1) $\cos^{-1}\frac{P^2 - Q^2}{P^2 + Q^2}$ (2) $\cos^{-1}\frac{P^2 + Q^2}{P^2 - Q^2}$ (3) $\cos^{-1}\frac{P^2 - Q^2}{2(P^2 + Q^2)}$ (4) $\cos^{-1}\frac{P^2 + Q^2}{2(Q^2 - P^2)}$

Sol. Answer (4)

Let $|\vec{A}| = (P + Q)$

$|\vec{B}| = (P - Q)$

Given $|\vec{A} + \vec{B}| = \sqrt{P^2 + Q^2}$

$\Rightarrow \quad \sqrt{P^2+Q^2} = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$

$\Rightarrow \quad \cos\theta = \dfrac{P^2+Q^2}{2(Q^2-P^2)}$

11. Which of the following pair of displacement cannot be added to produce a resultant displacement of 2 m?

(1) 1 m and 1 m (2) 1 m and 2 m (3) 1 m and 3 m (4) 1 m and 4 m

Sol. Answer (4)

If $\vec{A}$ and $\vec{B}$ are two vectors, then

$\left||\vec{A}|-|\vec{B}|\right| \le |\vec{A}+\vec{B}| \le |\vec{A}|+|\vec{B}|$

12. Maximum and minimum magnitudes of the resultant of two vectors of magnitude p and q are in the ratio 3 : 1. Which of the following relations is true?

(1) $p = 2q$ (2) $p = q$ (3) $pq = 1$ (4) None of these

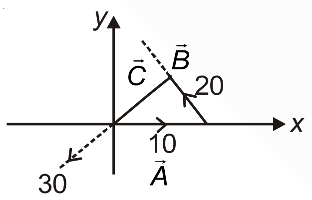
Sol. Answer (1)

Given, $\dfrac{p+q}{p-q} = 3 \quad \Rightarrow \quad p = 2q$

13. The directions of three forces 10 N, 20 N and 30 N acting at a point are parallel to the sides of an equilateral triangle taken in order. The magnitude of their resultant is

(1) $5\sqrt{3}$ N (2) 15 N (3) 0 N (4) $10\sqrt{3}$ N

Sol. Answer (4)



We can write $\vec{A}, \vec{B}$ and $\vec{C}$ in component form then we can find $|\vec{A}+\vec{B}+\vec{C}|$

14. A particle is thrown with the speed u at an angle α with the horizontal. When the particle makes an angle β with the horizontal, its speed will be

(1) $u\cos\alpha$ (2) $u\cos\alpha\sec\beta$ (3) $u\cos\alpha\cos\beta$ (4) $u\sec\alpha\cos\beta$

Sol. Answer (2)

$u\cos\alpha = v\cos\beta$

$\Rightarrow \quad v = (u\cos\alpha)\sec\beta$

15. A projectile has same range for two angles of projection. If times of flight in two cases are t_1 and t_2 then the range of the projectile is

(1) $\dfrac{1}{2}gt_1t_2$ (2) $\dfrac{1}{4}gt_1t_2$ (3) gt_1t_2 (4) $\dfrac{1}{8}gt_1t_2$

Sol. Answer (1)

If $R_1 = R_2$

$\Rightarrow$ Angles of projection will be α and $(90 - \alpha)$

$$t_1 = \frac{2u \sin\alpha}{g}$$

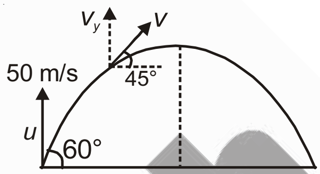
$$t_2 = \frac{2u \sin(90-\alpha)}{g}$$

16. A particle is projected with velocity 50 m/s at an angle 60° with the horizontal from the ground. The time after which its velocity will make an angle 45° with the horizontal is

(1) 2.5 s (2) 1.83 s (3) 2.37 s (4) 3.72 s

Sol. Answer (2)

$u \cos 60° = v \cos 45°$



$$\Rightarrow \quad 50 \times \frac{1}{2} = (v)\frac{1}{\sqrt{2}} \quad \Rightarrow \quad v = 25\sqrt{2}$$

use $v_y = u_y + a_y t$ to find t.

17. A projectile can have the same range for two angle of projection. If times of flight in two cases are t_1 and t_2 then velocity of projection of the projectile is

(1) $\frac{1}{2}g(t_1 + t_2)$ (2) $\frac{1}{4}g(t_1 + t_2)$ (3) $\frac{1}{2}g(t_1^2 + t_2^2)^{1/2}$ (4) $\frac{1}{2}g(t_1^2 + t_2^2)$

Sol. Answer (3)

If one angle is α then another angle will be $(90 - \alpha)$ for same range.

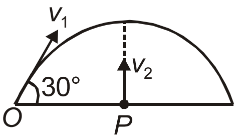
$$t_1 = \frac{2u \sin\alpha}{g}, \quad t_2 = \frac{2u \sin(90-\alpha)}{g}$$

$$\sin\alpha = \frac{gt_1}{2u}, \cos\alpha = \frac{gt_2}{2u}$$

$$\sin^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow \quad u = \frac{1}{2}g\,(t_1^2 + t_2^2)^{1/2}$$

18. A particle is projected from point O with velocity v_1 at an angle 30° with the horizontal and another particle is projected from point P which is vertically below the highest point with velocity v_2 in vertically upward direction as shown. For the two particles to collide the ratio $\frac{v_1}{v_2}$ will be



(1) 1 (2) 2 (3) 0.5 (4) 1.5

Sol. Answer (2)

$v_{y_1} = v_{y_2}$

$\Rightarrow \quad v_1 \sin 30 = v_2$

$(v_1)\frac{1}{2} = v_2$

$\frac{v_1}{v_2} = 2$

19. A particle projected at some angle with velocity 50 m/s crosses a 20 m high wall after 4 s from the time of projection. The angle of projection of the particle is

(1) 30° (2) 45° (3) 60° (4) 53°

Sol. Answer (1)

At t = 4 s

y = 20 m

$y = u_y t + \frac{1}{2} a_y t^2$

$20 = (u_y)(4) + \frac{1}{2}(-10)(4)^2$

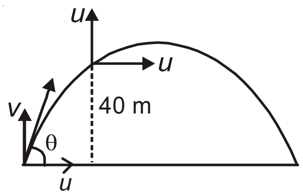
Also, $\sqrt{(u_x)^2 + (u_y)^2} = 50$

Angle of projection $\theta = \tan^{-1}\left(\frac{u_y}{u_x}\right)$

20. A projectile projected from the ground has its direction of motion making an angle $\frac{\pi}{4}$ with the horizontal at a height 40 m. Its initial velocity of projection is 50 m/s, the angle of projection is

(1) $\frac{1}{2}\cos^{-1}\left(-\frac{8}{25}\right)$ (2) $\frac{1}{2}\cos^{-1}\left(+\frac{8}{25}\right)$ (3) $\frac{1}{2}\cos^{-1}\left(-\frac{4}{5}\right)$ (4) $\frac{1}{2}\cos^{-1}\left(-\frac{1}{4}\right)$

Sol. Answer (1)



at h = 40 m

$u^2 = v^2 - 2g\,(40)$ ...(i)

Also, $u^2 + v^2 = (50)^2$ ...(ii)

From (i) & (ii) find angle of projection.

21. Two bullets are fired simultaneously from the same level and in the horizontal direction over a lake. The speed of one bullet is 196 ms^{-1}. Assume that air friction is negligible and the lake is still. The bullet which is faster will, compared to the slower one, fall in the water

(1) In half the time of the other (2) At the same time

(3) In twice the time of the other (4) None of these

Sol. Answer (2)

Since vertical velocity is zero hence both will fall at the same time.

22. An aeroplane is moving horizontally with a velocity of *u*. It drops a packet from a height *h*. The time taken the packed in reaching the ground will be

(1) $\sqrt{\left(\frac{2g}{h}\right)}$ (2) $\sqrt{\left(\frac{2u}{g}\right)}$ (3) $\sqrt{\left(\frac{h}{2g}\right)}$ (4) $\sqrt{\left(\frac{2h}{g}\right)}$

Sol. Answer (4)

$$t = \sqrt{\frac{2h}{g}}$$

23. A body of mass *m* thrown horizontally with a velocity *v* from the top of the tower of height *h* touches the ground at a distance 250 m from the foot of tower. Another body of mass 2 m thrown horizontally with half the velocity from a tower of height 4*h* will touch the ground at a distance

(1) 250 m (2) 1000 m (3) 125 m (4) 500 m

Sol. Answer (1)

$$t = \sqrt{\frac{2h}{g}}$$

$$t_1 = \sqrt{\frac{2h}{g}},\ t_2 = 2\sqrt{\frac{2h}{g}}$$

$$R = v_x t$$

$$R_1 = (v)\sqrt{\frac{2h}{g}}$$

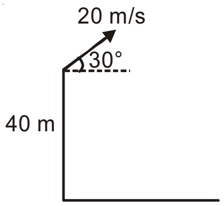
$$R_2 = \left(\frac{v}{2}\right)2\sqrt{\frac{2h}{g}} = R_1$$

24. From the top of a tower of height 40 m a ball is projected upward with a speed of 20 ms^{-1} at an elevation of 30°. The ratio of the total time taken by the ball to hit the ground to its time to come back at the same level is (g = 10 ms^{-2})

(1) 2 : 1 (2) 3 : 1 (3) 3 : 2 (4) 4 : 1

Sol. Answer (1)

$$T = \frac{(2)(20)\left(\frac{1}{2}\right)}{10}$$



T = 2 sec

Total time (T_1)

$$s_y = u_y t + \frac{1}{2} a_y t^2$$

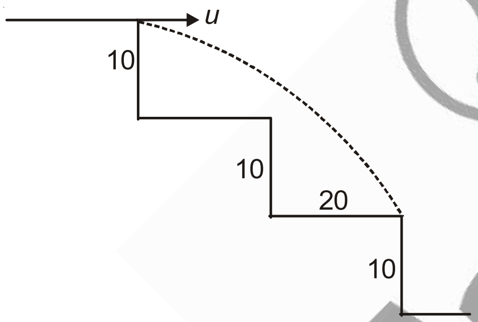
$$-40 = (20)\left(\frac{1}{2}\right)T_1 + \frac{1}{2}(-10)(T_1)^2$$

Find value of T_1

25. A stair case contains three steps having height 10 cm each. If each are 20 cm wide then the minimum horizontal velocity of a ball moving off the upper mark plane so as to hit the lowest plane directly is

(1) 4 m/s (2) 2 m/s (3) 1 m/s (4) 0.5 m/s

Sol. Answer (2)



$$t = \sqrt{\frac{(2)\,20}{(100)g}} = \left(\frac{2}{10}\right) \text{sec}$$

Now, $u \times \frac{2}{10} \geq \frac{40}{100}$

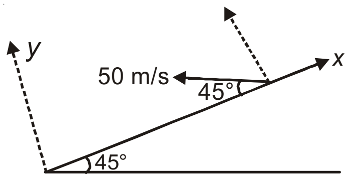
$\Rightarrow \quad u \geq 2 \text{ m/s}$

26. A projectile is fired horizontally from an inclined plane (of inclination 45° with horizontal) with speed 50 m/s. If g = 10 m/s^2, the range measured along the incline is

(1) 500 m (2) $500\sqrt{2}$ m (3) $200\sqrt{2}$ m (4) $500/\sqrt{2}$ m

Sol. Answer (2)

$$T = \frac{(2)(50)\left(\frac{1}{\sqrt{2}}\right)}{g\cos 45}$$



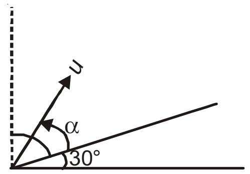
For range

$$x = u_x T + \frac{1}{2} a_x T^2$$

27. At what angle with an incline plane of inclination 30° a particle should be projected to have maximum range on the inclined plane?

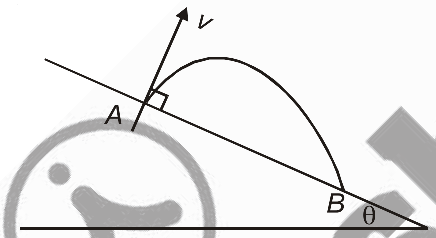
(1) 30° (2) 45° (3) 15° (4) 75°

Sol. Answer (1)



$$\alpha = \frac{(90-30)}{2} = 30$$

28. If time taken by the projectile to reach *B* is *T*, then *AB* is equal to



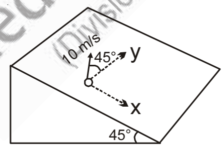
(1) $Tv \sin\theta$ (2) $Tv \cos\theta$ (3) $Tv \sec\theta$ (4) $Tv \tan\theta$

Sol. Answer (4)

$$T = \left(\frac{2v}{g\cos\theta}\right)$$

$$AB = \left(\frac{1}{2}(g\sin\theta)\right)(T^2)$$

29. A small marble is projected with a velocity of 10 m/s in a direction 45° from the horizontal *y*-direction, along the smooth wadge. Calculate the magnitude *v* of its velocity after 2 seconds. Take $g = 10$ m/s^2. (Note that the particle is moving in inclined plane of the wedge)



(1) 20 m/s (2) 15 m/s (3) $10\sqrt{3}$ m/s (4) 10 m/s

Sol. Answer (4)

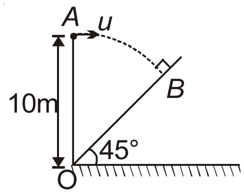
$$\vec{v} = \vec{u} + \vec{a}\,t$$

$$= \left(\frac{10}{\sqrt{2}}(-\hat{i}) + \frac{10}{\sqrt{2}}\hat{j}\right) + (g\sin 45)(-\hat{i})2$$

$$|\vec{v}| = 10 \text{ m/s}$$

Paragraph for Q. Nos. 30 to 32

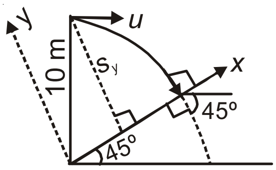
A particle is projected horizontally with speed *u* from point *A*, which is 10 m above the ground. If the particle hits the inclined plane perpendicularly at point *B*. [g = 10 m/s^2]



30. Find horizontal speed with which the particle was projected.

(1) $\frac{20}{3}$ (2) $20\sqrt{\frac{20}{3}}$ (3) $u = 10\sqrt{\frac{2}{3}}$ (4) $10\sqrt{\frac{3}{2}}$

Sol. Answer (3)



Particle collides perpendicular to inclined plane hence particle makes an angle θ = 45° from horizontal.

$v_y = u_y + a_y\, t$

$t = \left(\frac{u}{g}\right)$

$\frac{-(10)}{\sqrt{2}} = \left(\frac{-u}{\sqrt{2}}\right)\left(\frac{u}{g}\right) - \frac{1}{2}\left(\frac{g}{\sqrt{2}}\right)\left(\frac{u}{g}\right)^2$

$u = 10\sqrt{\frac{2}{3}}$

31. Find the length *OB* along the inclined plane.

(1) $\frac{10}{\sqrt{3}}$ (2) $20\sqrt{3}$ (3) $\frac{20}{\sqrt{3}}$ (4) $\frac{20\sqrt{2}}{3}$

Sol. Answer (4)

OB is the displacement along inclined plane in time $t = \frac{u}{g}$

32. Speed at point *B* will be

(1) $\frac{20}{\sqrt{3}}$ (2) $20\sqrt{3}$ (3) $\frac{10}{\sqrt{3}}$ (4) $10\sqrt{3}$

Sol. Answer (1)

$$V_B = \sqrt{u^2 + \left(g\frac{u}{g}\right)^2}$$

$$= u\sqrt{2}$$

$$= \left(\frac{20}{\sqrt{3}}\right)$$

Paragraph for Q. Nos. 33 to 35

When a boat travels in a river(strictly in a straight line), it can go either in the direction of flow of river (*i.e.,* downstream) or in the direction opposite the flow of river (*i.e.*, upstream). Thus the boat's actual speed is more than by which it can move in stationary water while travelling downstream (as river's flow speed is added to it) and less while travelling upstream (as the boat moves against the flow of river). Based on the given information answer the following questions

A boat going downstream in a flowing river overcame a raft at a point *P*. 1 h later it turned back and after some time passed the raft at a distance 6 km from point *P*.

33. After reversing its direction, how much time was taken by the boat to meet the raft again (*i.e.*, 2nd time)?

    (1) 2 h (2) 1 h (3) 30 min (4) 3 h

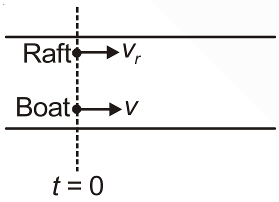
34. Find the speed of river.

    (1) 4 km/h (2) 2 km/h (3) 3 km/h (4) 1 km/h

35. Now, if instead of 6 km they have met at 8 km from point *P*, find the speed of river

    (1) 1 km/h (2) 2 km/h (3) 3 km/h (4) 4 km/h

Solution of Q.Nos. 33 to 35



Since speed of boat is constant with respect to water

Hence,

33. Answer (2)

    T (time when it will meet again the raft

    = 2 hr

34. Answer (3)

$$v_r = \frac{6}{2} = 3\text{ km/hr}$$

35. Answer (4)

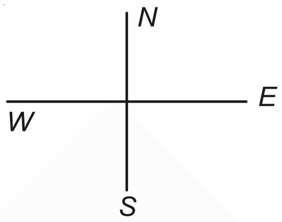
$$v_r^1 = \frac{8}{2} = 4\text{ km/hr}$$

36. Three ships A, B & C are in motion. The motion of A as seen by B is with speed v towards north - east. The motion of B as seen by C is with speed v towards the north - west. Then as seen by A, C will be moving towards

(1) North (2) South (3) East (4) West

Sol. Answer (2)

$$\vec{V}_{A,B} = v\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$



$$\vec{V}_{B,C} = v\left(\frac{-\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}\right)$$

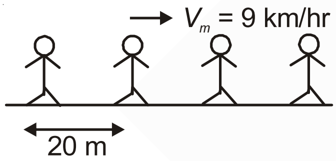
$$\vec{V}_{A,C} = \vec{V}_A - \vec{V}_C$$

$$\vec{V}_{A,C} = \vec{V}_{A,B} - \vec{V}_{B,C}$$

37. Men are running in a line along a road with velocity 9 km/hr behind one another at equal distances of 20 m. Cyclists are also riding along the same line in the same direction at 18 km/hr at equal intervals of 30 m. The speed with which an observer must travel along the road in opposite direction of so that whenever he meets a runner he also meets a cyclist is

(1) 9 km/h (2) 12 km/h (3) 18 km/h (4) 6 km/h

Sol. Answer (1)



Let observer must travel with speed v_0

We must have

$$\frac{20}{9+v_0} = \frac{30}{18+v_0}$$

$$\Rightarrow \quad v_0 = 9 \text{ km/hr}$$

38. A man holding a flag is running in North-East direction with speed 10 m/s. Wind is blowing in east direction with speed $5\sqrt{2}$ m/s. Find the direction in which flag will flutter.

(1) South (2) East (3) West (4) North

Sol. Answer (1)

We know

$$\vec{V}_{A,B} = \vec{V}_A - \vec{V}_B$$

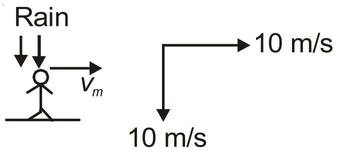
Paragraph for Q. Nos. 39 to 41

Raindrops are falling with velocity $10\sqrt{2}$ m/s making an angle 45º with the vertical. The drops appear to be falling vertically to a man running with constant velocity. The velocity of rain drops change such that the rain drops now appear to be falling vertically with $\sqrt{3}$ times the velocity it appeared earlier to the same person running with same velocity.

39. The magnitude of velocity of man w.r.t. ground is

(1) $10\sqrt{2}$ m/s (2) $10\sqrt{3}$ m/s (3) 10 m/s (4) 20 m/s

Sol. Answer (3)



$V_m = 10$ m/s

40. After the velocity of rain drops change the magnitude of velocity of raindrops with respect to ground is

(1) $20\sqrt{3}$ m/s (2) 10 m/s (3) $10\sqrt{3}$ m/s (4) 20 m/s

Sol. Answer (4)

Given, $v_y^1 = 10\sqrt{3}$

$$v_{\text{Rain}} = \sqrt{(10\sqrt{3})^2 + (10)^2}$$

$$= 20 \text{ m/s}$$

41. The angle (in degrees) between the initial and final velocity of rain drops with respect to ground

(1) 15º (2) 8º (3) 22.5º (4) 37º

Sol. Answer (1)

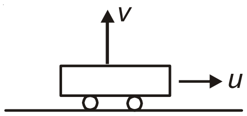
$$\tan\theta = \left(\frac{v_y}{v_x}\right)$$

42. A boy standing in a trolley car projects a stone perpendicular to the trolley car with a speed v. If the trolley car moves with a constant velocity u, the time of flight of the stone is

(1) $\frac{2v}{g}$ (2) $\frac{2u}{g}$ (3) $\frac{u+v}{g}$ (4) None of these

Sol. Answer (1)

$T = \frac{2v}{g}$ vertical motion



43. Rain is falling vertically with a speed of 4 m/s. After some time, wind starts blowing with a speed of 3 m/s in the north to south direction. In order to protect himself from rain, a man standing on the ground should hold his umbrella at an angle θ given by

(1) $\theta = \tan^{-1}3/4$ with the vertical towards south

(2) $\theta = \tan^{-1}3/4$ with the vertical towards north

(3) $\theta = \cot^{-1}3/4$ with the vertical towards south

(4) $\theta = \cot^{-1}3/4$ with the vertical towards north

Sol. Answer (2)

Using relative motion we get

$v = \tan^{-1}\left(\frac{3}{4}\right)$ with vertical towards north.

44. A gun fitted on the top of a moving car is aimed in the backward direction at an angle 60° with the horizontal. If the muzzle velocity of the bullet fired from the gun is 8 m/s, then the speed of the car for which the bullet comes out vertically is

(1) 2 m/s (2) 4 m/s (3) $2\sqrt{2}$ m/s (4) $4\sqrt{2}$ m/s

Sol. Answer (2)

Speed of car will be 4 m/s.

45. A particle is moving in a circular path of radius R_0 with a constant speed v_0. If the particle's speed is decreased by a factor of β *i.e.*, $v' = \frac{v_0}{\beta}$ and the radius of circular path is doubled, then, the centripetal acceleration

(1) Increases by a factor of β^2

(2) Decreases by a factor of $2\beta^2$

(3) Decreases by a factor of β

(4) Also doubles

Sol. Answer (2)

$$a_c = \left(\frac{v^2}{R}\right) = \left(\frac{1}{\beta^2}\right)\frac{v_0^2}{2R_0} = \left(\frac{1}{2\beta^2}\right)$$

46. A train 1 moves from east to west (clockwise) and another train 2 moves from west to east (anticlockwise) on the equator with equal speeds relative to ground. The ratio of their centripetal acceleration $\frac{a_1}{a_2}$ relative to centre of earth is

(1) > 1 (2) = 1 (3) < 1 (4) ≥ 1

Sol. Answer (3)

$$a = \frac{v^2}{R}$$

When R is the radius of circular path.

Hence $\frac{a_1}{a_2} < 1$

47. Two particles A and B are revolving on two coplanar circles with time periods 4 second and 6 second respectively. Time period of particle A with respect to B will be

(1) 6 second (2) 12 second (3) 18 second (4) 24 second

Sol. Answer (2)

$$T_1 = \frac{2\pi}{\omega}$$

$$\Rightarrow \quad \omega_1 = \frac{2\pi}{4} = \left(\frac{\pi}{2}\right)$$

$$\omega_2 = \left(\frac{2\pi}{6}\right) = \left(\frac{\pi}{3}\right)$$

Hence T = 12 sec.

48. Speed of a particle moving on a circular path of radius 2 m is varying with time as $v = (2t^2)$ m/s. Net acceleration of the particle at t = 2 second is

(1) $8\sqrt{17}$ m/s^2 (2) 4 m/s^2 (3) $2\sqrt{3}$ m/s^2 (4) $4\sqrt{3}$ m/s^2

Sol. Answer (1)

$$a_t = \frac{dv}{dt} = 4t$$

$$a_R = \frac{v^2}{R} = \left(\frac{4t^2}{R}\right)$$

$$a_{\text{total}} = \sqrt{(a_R)^2 + (a_t)^2}$$

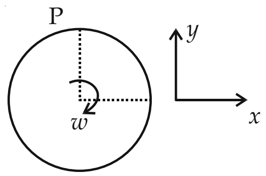
49. What is the maximum magnitude of change in velocity of a motorcycle moving with a uniform speed v_0 in a circular path of length $l = \frac{\pi}{3}R$ and radius R? Treat motorcycle as a particle

(1) $|\Delta\vec{v}| = \frac{v_0}{2}$ (2) $|\Delta\vec{v}| = \sqrt{3}\, v_0$ (3) $|\Delta\vec{v}| = 2v_0$ (4) $|\Delta\vec{v}| = v_0$

Sol. Answer (4)

Maximum magnitude of change in velocity can be v_0

50. A disc of radius 1 m is rotating about central axis with angular velocity 2 rad/s. If a point P lying on its circumference as shown is moving with velocity $4\hat{j}$ m/s, then velocity of centre of disc is



(1) $(2\hat{i} + 4\hat{j})$ m/s (2) $(4\hat{i} + 2\hat{j})$ m/s (3) $(-2\hat{i} + 4\hat{j})$ m/s (4) $(2\hat{i} - 4\hat{j})$ m/s

Sol. Answer (3)

$$\vec{V_P} = \vec{V_{P,C}} + \vec{V_C}$$

$$4\hat{j} = 2\hat{i} + \vec{V_C}$$

$$\vec{V_c} = (4\hat{j} - 2\hat{i})\text{ m/s}$$

51. A particle is performing motion whose position vector is given by $\vec{r} = 15\,(\cos pt\,\hat{i} + \sin pt\,\hat{j})$ m, when p is in rad/s and t is in seconds. The magnitude of centripetal acceleration at t = 3 s, is

(1) $30p^2$ m/s^2 (2) $15p^2$ m/s^2 (3) $45p^2$ m/s^2 (4) $90p^2$ m/s^2

Sol. Answer (2)

$$\vec{V} = \frac{\vec{dr}}{dt}$$

$$a_c = \left(\frac{v^2}{R}\right)$$

52. A particle is performing circular motion in a circle of radius R in such a way that at every moment magnitude of its tangential and radial acceleration is equal. If initial velocity of particle is v_0 then time period of 1st revolution is

(1) $\frac{R}{v_0}$ (2) $\frac{R}{v_0}\left(1-e^{-2\pi}\right)$ (3) $\frac{R}{v_0}\left(e^{-2\pi}\right)$ (4) $\frac{2\pi R}{v_0}$

Sol. Answer (2)

Given, $\frac{dV}{dt} = \frac{V^2}{r}$

$$\int_{V_0}^{V}\frac{dV}{V^2} = \int_0^t\frac{dt}{R}$$

$$\frac{1}{V_0} - \frac{1}{V} = \frac{t}{R} \Rightarrow \frac{1}{V} = \left(\frac{1}{V_0} - \frac{t}{R}\right)$$

$$V = \left(\frac{V_0R}{R - V_0t}\right) \Rightarrow \frac{dS}{dt} = \frac{V_0R}{R - V_0t}$$

$$\int_0^{2\pi R} dS = \int_0^{t_0}(V_0R)\frac{dt}{(R - V_0t)}$$

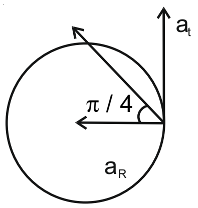
$$2\pi R = -\frac{(V_0R)}{V_0}\ln\left(\frac{R - V_0t_0}{R}\right)$$

53. A particle is performing circular motion in a circle of radius R with variable speed, $v = Kt$ ($K > 0$). Time when angle between acceleration vector and its velocity vector becomes $\frac{\pi}{4}$, is

(1) $2\sqrt{\frac{R}{K}}$ (2) $\sqrt{\frac{2R}{K}}$ (3) $4\sqrt{\frac{R}{K}}$ (4) $\sqrt{\frac{R}{K}}$

Sol. Answer (4)

$\Rightarrow a_R = a_t$



$$\frac{V^2}{R} = K$$

$$\Rightarrow \frac{K^2t^2}{R} = K$$

$$\Rightarrow t = \sqrt{\frac{R}{K}}$$

54. A particle moves along a curve $y = x^2$ with constant speed of 10 m/s. When the particle is at origin, then the radius of curvature of path is

(1) 0.5 m (2) 2.0 m (3) 1.5 m (4) 1.0 m

Sol. Answer (1)

$Y = x^2$

$V_y = 2x\, V_x$

$a_y = 2V_x^2 + 2xa_x$

$a_y = 200$ m/s^2

$$R = \frac{V^2}{a_y} = 0.5 \text{ m}$$

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