DPP - Daily Practice Problems

Date : | Start Time : | End Time :

MATHEMATICS CM02

SYLLABUS : Relations and Functions

Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQs. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. For the following relation
$R = \{(0, 0), (0, 1), (1, 1), (2, 1), (2, 2), (2, 0), (1, 0), (0, 2), (0, 1)\}$
(a) domain = $\{0, 1\}$ (b) range = $\{0, 1, 2\}$
(c) both correct (d) None of these

2. The domain of the function $\sqrt{x^2 - 5x + 6} + \sqrt{2x + 8 - x^2}$ is
(a) $[2, 3]$ (b) $[-2, 4]$
(c) $[-2, 2] \cup [3, 4]$ (d) $[-2, 1] \cup [2, 4]$

3. If $3f(x) - f\left(\frac{1}{x}\right) = \log x^4$, then $f(e^{-x})$ is
(a) $1 + x$ (b) $1/x$
(c) x (d) $-x$

4. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is
(a) $(0, \infty)$ (b) $(-\infty, 0)$
(c) $(-\infty, \infty) - \{0\}$ (d) $(-\infty, \infty)$

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

5. $f(x) = \sqrt{|x|^2 - 5|x| + 6} + \sqrt{8 + 2|x| - |x|^2}$ is real for all x in

(a) $[-4, -3]$ (b) $[-3, -2]$

(c) $[-2, 2]$ (d) $[3, 4]$

6. $f(x) = \frac{x(x-p)}{q-p} + \frac{x(x-q)}{p-q}$, $p \neq q$. What is the value of $f(p) + f(q)$?

(a) $f(p-q)$ (b) $f(p+q)$

(c) $f(p(p+q))$ (d) $f(q(p-q))$

7. A real valued function $f(x)$ satisfies the functional equation

$$f(x-y) = f(x)f(y) - f(a-x)f(a+y)$$

where a is a given constant and $f(0) = 1$, $f(2a-x)$ is equal to

(a) $-f(x)$ (b) $f(x)$

(c) $f(a) + f(a-x)$ (d) $f(-x)$

8. Domain of definition of the function

$f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$, is

(a) $(-1,0) \cup (1,2) \cup (2,\infty)$ (b) $(a, 2)$

(c) $(-1,0) \cup (a,2)$ (d) $(1,2) \cup (2,\infty)$.

9. Let $A = \{1, 2, 3, 4, 5\}$; $B = \{2, 3, 6, 7\}$. Then the number of elements in $(A \times B) \cap (B \times A)$ is

(a) 18 (b) 6

(c) 4 (d) 0

10. A relation R is defined in the set Z of integers as follows $(x, y) \in R$ iff $x^2 + y^2 = 9$. Which of the following is false?

(a) $R = \{(0, 3), (0, -3), (3, 0), (-3, 0)\}$

(b) Domain of $R = \{-3, 0, 3\}$

(c) Range of $R = \{-3, 0, 3\}$

(d) None of these

11. Let $f(x) = \sqrt{1+x^2}$, then

(a) $f(xy) = f(x).f(y)$ (b) $f(xy) \geq f(x).f(y)$

(c) $f(xy) \leq f(x).f(y)$ (d) None of these

12. The domain of the function $f(x) = \sqrt{x - \sqrt{1-x^2}}$ is

(a) $\left[-1, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$

(b) $[-1, 1]$

(c) $\left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{\sqrt{2}}, +\infty\right)$

(d) $\left[\frac{1}{\sqrt{2}}, 1\right]$

13. Period of the function $\left|\sin^3 \frac{x}{2}\right| + \left|\cos^5 \frac{x}{5}\right|$ is :

(a) 2π (b) 10π

(c) 8π (d) 5π

14. If $n(A) = 4$, $n(B) = 3$, $n(A \times B \times C) = 24$, then $n(C) =$

(a) 288 (b) 1

(c) 12 (d) 2

RESPONSE GRID					
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	10. ⓐⓑⓒⓓ	11. ⓐⓑⓒⓓ	12. ⓐⓑⓒⓓ	13. ⓐⓑⓒⓓ	14. ⓐⓑⓒⓓ

15. If S = {1, 2, 3, 4, 5,} and R = {(x, y) : x + y < 6} then n (R) =

(a) 8 (b) 10
(c) 6 (d) 5

16. The function $f(x) = \log\left(x + \sqrt{x^2 + 1}\right)$, is

(a) neither an even nor an odd function
(b) an even function
(c) an odd function
(d) a periodic function

17. Let $f(x) = \frac{x}{1-x}$ and 'a' be a real number. If $x_0 = a$, $x_1 = f(x_0), x_2 = f(x_1), x_3 = f(x_2)$....... If $x_{2009} = 1$, then the value of a is

(a) 0 (b) $\frac{2009}{2010}$
(c) $\frac{1}{2009}$ (d) $\frac{1}{2010}$

18. The domain of the function

$f(x) = \log_2\left(-\log_{1/2}\left(1 + \frac{1}{x^{1/4}}\right) - 1\right)$ is

(a) (0, 1) (b) (0, 1]
(c) [1, ∞) (d) (1, ∞)

19. The domain of the function $f(x) = \frac{1}{\sqrt{x^2 - 3x + 2}}$ is

(a) $(-\infty, 1)$ (b) $(-\infty, 1) \cup (2, \infty)$
(c) $(-\infty, 1] \cup [2, \infty)$ (d) $(2, \infty)$

20. If (1, 3), (2, 5) and (3, 3) are three elements of A × B and the total number of elements in A × B is 6, then the remaining elements of A × B are

(a) (1, 5); (2, 3); (3, 5) (b) (5, 1); (3, 2); (5, 3)
(c) (1, 5); (2, 3); (5, 3) (d) None of these

21. If $f(x) = \ln\left(\frac{x^2 + e}{x^2 + 1}\right)$, then range of $f(x)$ is

(a) (0, 1) (b) (0, 1]
(c) [0, 1) (d) {0, 1}

22. The function $f(x) = \log\left(\frac{1+x}{1-x}\right)$ satisfies the equation

(a) $f(x+2) - 2f(x+1) + f(x) = 0$
(b) $f(x+1) + f(x) = f(x(x+1))$
(c) $f(x_1) \cdot f(x_2) = f(x_1 + x_2)$
(d) $f(x_1) + f(x_2) = f\left(\frac{x_1 + x_2}{1 + x_1 x_2}\right)$

23. If $f : R \to R$ satisfies $f(x+y) = f(x) + f(y)$, for all x, $y \in R$ and $f(1) = 7$, then $\sum_{r=1}^{n} f(r)$ is

(a) $\frac{7n(n+1)}{2}$ (b) $\frac{7n}{2}$
(c) $\frac{7(n+1)}{2}$ (d) $7n + (n+1)$.

24. If { } denotes the fractional part of x, the range of the function $f(x) = \sqrt{\{x\}^2 - 2\{x\}}$ is

(a) φ (b) [0, 1/2]
(c) {0, 1/2} (d) {0}

RESPONSE GRID					
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	20. ⓐⓑⓒⓓ	21. ⓐⓑⓒⓓ	22. ⓐⓑⓒⓓ	23. ⓐⓑⓒⓓ	24. ⓐⓑⓒⓓ

25. If $f(x)=\frac{x-1}{x+1}$, then $f(2x)$ is equal to

(a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{3f(x)+1}{f(x)+3}$

(c) $\frac{f(x)+3}{f(x)+1}$ (d) $\frac{f(x)+3}{3f(x)+1}$

26. The range of the function $f(x)=\frac{x^2-x+1}{x^2+x+1}$ where $x \in R$, is

(a) $(-\infty, 3]$ (b) $(-\infty, \infty)$

(c) $[3, \infty)$ (d) $\left[\frac{1}{3}, 3\right]$

27. The domain of the function $f(x)=\exp(\sqrt{5x-3-2x^2})$ is

(a) $[3/2, \infty)$ (b) $[1, 3/2]$

(c) $(-\infty, 1]$ (d) $(1, 3/2)$

28. If $f(x+y)=f(x)+2y^2+kxy$ and $f(a)=2, f(b)=8$, then $f(x)$ is of the form

(a) $2x^2$ (b) $2x^2+1$

(c) $2x^2-1$ (d) x^2

29. The relation R defined on the set $A=\{1, 2, 3, 4, 5\}$ by $R=\{(x, y): |x^2-y^2|<16\}$ is given by

(a) $\{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$

(b) $\{(2, 2), (3, 2), (4, 2), (2, 4)\}$

(c) $\{(3, 3), (3, 4), (5, 4), (4, 3), (3, 1)\}$

(d) None of these

30. Which of the following relation is NOT a function

(a) $f=\{(x, x) \mid x \in R\}$ (b) $g=\{(x, 3) \mid x \in R\}$

(c) $h=\{(n, \frac{1}{n}) \mid n \in I\}$ (d) $t=\{(n, n^2) \mid n \in N\}$

RESPONSE GRID	25. ⓐⓑⓒⓓ 26. ⓐⓑⓒⓓ 27. ⓐⓑⓒⓓ 28. ⓐⓑⓒⓓ 29. ⓐⓑⓒⓓ 30. ⓐⓑⓒⓓ

DAILY PRACTICE PROBLEM DPP CHAPTERWISE 2 - MATHEMATICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	38	Qualifying Score	50
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

1. **(b)** We have domain of R = the set of first components of the ordered pairs in R = {0, 0, 1, 2, 2, 2, 1, 0, 0} = {0, 1, 2}
range of R = the set of second components of the ordered pairs in R
= {0, 1, 1, 1, 2, 0, 0, 2, 1} = {0, 1, 2}

2. **(c)** $f(x) = \sqrt{(x-2)(x-3)} + \sqrt{-(x-4)(x+2)}$
The first part is real outside (2, 3) and the second is real in [–2, 4] so that the domain is $[-2, 2] \cup [3, 4]$.

3. **(d)** $3f(x) - f\left(\frac{1}{x}\right) = \log x^4$; $x \equiv \frac{1}{x}$
$3f\left(\frac{1}{x}\right) - f(x) = \log\left(\frac{1}{x}\right)^4$
After solving we get $f(x) = \log x$
$f(e^{-x}) = \log_e e^{-x} = -x$

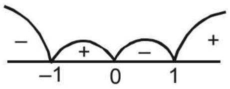
4. **(b)** $f(x) = \frac{1}{\sqrt{|x| - x}}$, define if $|x| - x > 0$
$\Rightarrow |x| > x, \Rightarrow x < 0$
Hence domain of $f(x)$ is $(-\infty, 0)$

5. **(d)** $\sqrt{|x^2| - 5|x| + 6} = \sqrt{(|x| - 2)(|x| - 3)}$
is real for $0 \le |x| \le 4$
$\therefore$ $f(x)$ is real for all $0 \le |x| \le 2$ or $3 \le |x| \le 4$.

6. **(b)** In the definition of function
$$f(x) = \frac{x(x-p)}{q-p} + \frac{x(p-q)}{(p-q)} = p$$
Putting p and q in place of x, we get
$$f(p) = \frac{p(p-p)}{q-p} + \frac{p(p-q)}{(p-q)} = p$$
$\Rightarrow f(p) = p$
and $f(q) = \frac{q(q-p)}{q-p} + \frac{q(p-q)}{(p-q)} = q$
$\Rightarrow f(q) = q$
Putting x = (p + q)
$$f(p+q) = \frac{(p+q)(p+q-p)}{(q-p)} + \frac{(p+q)(p+q-q)}{(p-q)}$$
$$= \frac{(p+q)q}{(q-p)} + \frac{(p+q)(p)}{(p-q)} = \frac{pq + q^2 - p^2 - pq}{(q-p)}$$
$$= \frac{q^2 - p^2}{q-p} = \frac{(q-p)(q+p)}{(q-p)} = p + q = f(q) + f(p)$$
So, f (p) + f (q) = f (p + q)

7. **(a)** $f(2a-x) = f(a-(x-a)) = f(a)f(x-a) - f(0)f(x)$
$= f(a)f(x-a) - f(x) = -f(x)$
$[\because x = 0, y = 0,\ f(0) = f^2(0) - f^2(a)$
$\Rightarrow f^2(a) = 0 \Rightarrow f(a) = 0]$
$\Rightarrow f(2a-x) = -f(x)$

8. **(a)** $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$
$4 - x^2 \ne 0;\ x^3 - x > 0;$
$x \ne \pm\sqrt{4}$ and $-1 < x < 0$ or $1 < x < \infty$
$\therefore D = (-1, 0) \cup (1, \infty) - \{\sqrt{4}\}$
$D = (-1, 0) \cup (1, 2) \cup (2, \infty)$.



9. **(c)** Here A and B sets having 2 elements in common, so A × B and B × A have 2^2 i.e., 4 elements in common.
Hence, $n[(A \times B) \cap (B \times A)] = 4$

10. **(d)** $x^2 + y^2 = 9 \Rightarrow y^2 = 9 - x^2 \Rightarrow y = \pm\sqrt{9 - x^2}$
$x = 0 \Rightarrow y = \pm\sqrt{9-0} = \pm 3 \in Z$
$x = \pm 1 \Rightarrow y = \pm\sqrt{9-1} = \pm\sqrt{8} \notin Z$
$x = \pm 2 \Rightarrow y = \pm\sqrt{9-4} = \pm\sqrt{5} \notin Z$
$x = \pm 3 \Rightarrow y = \pm\sqrt{9-9} = 0 \in Z$
$x = \pm 4 \Rightarrow y = \pm\sqrt{9-16} = \pm\sqrt{-7} \notin Z$ and so on.
$\therefore$ R = {(0, 3), (0, –3), (3, 0), (–3, 0)}
Domain of R = {x : (x, y) ∈ R} = {0, 3, –3}
Range of R = {y : (x, y) ∈ R} = {3, –3, 0}.

11. **(c)** $f(xy) = \sqrt{1 + x^2y^2}$
$f(x)f(y) = \sqrt{1+x^2}\sqrt{1+y^2} = \sqrt{1 + x^2y^2 + x^2 + y^2}$
$\ge \sqrt{1 + x^2y^2} = f(xy)$
$\therefore$ $f(xy) \le f(x)f(y)$

12. **(d)** For $f(x)$ to be defined, we must have
$x - \sqrt{1-x^2} \ge 0$ or $x \ge \sqrt{1-x^2} > 0$
$\therefore$ $x^2 \ge 1 - x^2$ or $x^2 \ge \frac{1}{2}$.
Also, $1 - x^2 \ge 0$ or $x^2 \le 1$.
Now, $x^2 \ge \frac{1}{2} \Rightarrow \left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right) \ge 0$
$\Rightarrow x \le -\frac{1}{\sqrt{2}}$ or $x \ge \frac{1}{\sqrt{2}}$
Also, $x^2 \le 1 \Rightarrow (x-1)(x+1) \le 0$
$\Rightarrow -1 \le x \le 1$
Thus, $x > 0,\ x^2 \ge \frac{1}{2}$ and $x^2 \le 1$
$\Rightarrow x \in \left[\frac{1}{\sqrt{2}}, 1\right]$

13. **(b)** Period of $\sin x = 2\pi \Rightarrow$ period of $\sin^3 x = 2\pi$

period of $|\sin^3 x| = \pi \Rightarrow$ period of $\left|\sin^3 \frac{x}{2}\right| = 2\pi$

period of $\cos^5 x = 2\pi \Rightarrow$ period of $|\cos^5 x| = \pi$

$\Rightarrow$ period of $\left|\cos^5 \frac{x}{5}\right| = 5\pi$

Thus required period $=$ LCM of 2π & $5\pi = 10\pi$

14. **(d)** $n(A)=4, n(B)=3$

$n(A) \times n(B) \times n(C) = n(A \times B \times C)$

$4 \times 3 \times n(C) = 24 \Rightarrow n(C) = 24/12 = 2$

15. **(b)** We have $(x, y) \in R$ iff $x + y < 6$

Given the value $x = 1$, we get possible values of $y = 1, 2, 3, 4$. Thus 1R1, 1R2, 1R3, 1R4. Similarly we may find other values. The set of such ordered pairs is

$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (4, 1)\}$

$\therefore n(R) = 10$

16. **(c)** $f(x) = \log(x + \sqrt{x^2+1})$

$f(-x) = \log\left\{-x + \sqrt{x^2+1}\right\} = \log\left\{\frac{-x^2+x^2+1}{x+\sqrt{x^2+1}}\right\}$

$= -\log(x+\sqrt{x^2+1}) = -f(x)$

$\Rightarrow f(x)$ is an odd function.

17. **(d)** $x_0 = a,\ x_1 = f(x) = \frac{x_0}{1-x_0} = \frac{a}{1-a}$;

$$x_2 = f(x_1) = \frac{x_1}{1-x_1} = \frac{\frac{a}{1-a}}{1-\frac{a}{1-a}} = \frac{a}{1-2a}$$

$\therefore\ x_{2009} = \frac{a}{1-2009a} = 1 \Rightarrow 1 - 2009a = a$

$\Rightarrow a = \frac{1}{2010}$

18. **(a)** $f(x)$ is defined if $-\log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) - 1 > 0$

$\Rightarrow \log_{1/2}\left(1+\frac{1}{x^{1/4}}\right) < -1$

$\Rightarrow 1 + \frac{1}{x^{1/4}} > \left(\frac{1}{2}\right)^{-1}$

$\Rightarrow \frac{1}{x^{1/4}} > 1$

$\Rightarrow 0 < x < 1$

19. **(b)** For $f(x)$ to be defined, we must have

$x^2 - 3x + 2 = (x-1)(x-2) > 0 \Rightarrow x < 1$ or > 2

Domain of $f = (-\infty, 1) \cup (2, \infty)$.

20. **(a)** It is obvious.

21. **(b)** $f(x) = \ln\left(\frac{x^2+e}{x^2+1}\right) = \ln\left(\frac{x^2+1-1+e}{x^2+1}\right) = \ln\left(1+\frac{e-1}{x^2+1}\right)$

Clearly range is (0, 1]

22. **(b)** Let $f(x) = \log(g(x))$

$\therefore\ f(x_1) + f(x_2) = \log(g(x_1)) + \log(g(x_2))$
$= \log(g(x_1) \cdot g(x_2))$

$\therefore$ Option (b) is correct

23. **(a)** $f(x+y) = f(x) + f(y)$.

Function should be $f(x) = mx$

$f(1) = 7;\ \therefore m = 7,\ f(x) = 7x$

$$\sum_{r=1}^{n} f(r) = 7\sum_{1}^{n} r = \frac{7n(n+1)}{2}$$

24. **(d)** $\{x^2\} - 2\{x\} \geq 0$

$\Rightarrow \{x\}(\{x\} - 2) \geq 0$

$\Rightarrow \{x\} \leq 0$ or $\{x\} \geq 2$

Second case is not possible.

Hence $\{x\} = 0$, as $\{x\} \leq [0, 1)$. Hence range of $f(x)$ contains only one element 0.

25. **(b)** Given $f(x) = \frac{x-1}{x+1}$

$\therefore f(2x) = \frac{2x-1}{2x+1}$

$= \frac{2(2x-1)}{2(2x+1)}$ (multiply and divide by 2)

$= \frac{4x-2}{4x+2} = \frac{3x+x-3+1}{3x+x+3-1} = \frac{3(x-1)+x+1}{3(x+1)+x-1}$

$$= \frac{3\left[\frac{x-1}{x+1}\right]+1}{\frac{x-1}{x+1}+3} = \frac{3f(x)+1}{f(x)+3}$$

26. **(d)** Let $y = \frac{x^2-x+1}{x^2+x+1}$

$\Rightarrow x^2(y-1) + x(y+1) + (y-1) = 0$

$\Rightarrow x = \frac{-(y+1) \pm \sqrt{(y+1)^2 - 4(y-1)^2}}{2(y-1)}$

$= \frac{-(y+1) \pm \sqrt{-3y^2+10y-3}}{2(y-1)}$ is real iff

$y - 1 \neq 0 \Rightarrow y \neq 1$

If $y = 1$ then original equation gives $x = 0$, so taking $y = 1$

Also $3y^2 - 10y + 3 \leq 0$

$\Rightarrow (3y-1)(y-3) \leq 0$

$\Rightarrow y \in \left[\frac{1}{3}, 3\right]\ \therefore$ Range is $\left[\frac{1}{3}, 3\right]$

27. **(b)** We have, $f(x) = \exp\left(\sqrt{5x-3-2x^2}\right)$

i.e., $f(x) = e^{\sqrt{5x-3-2x^2}}$

For Domain of $f(x)$, $\sqrt{5x-3-2x^2}$ should be +ve.

i.e., $\sqrt{5x-3-2x^2} \geq 0$

$\Rightarrow 2x^2 - 5x + 3 \leq 0$ (By taking –ve sign common)

$\Rightarrow\ 2x(x-1)-3(x-1)\le 0$

$\Rightarrow\ (2x-3)(x-1)\le 0$

$\Rightarrow\ 2x-3\le 0$ or $x-1\ge 0$

$\Rightarrow\ x\le \frac{3}{2}$ or $x\ge 1$

$\therefore\ 1\le x\le \frac{3}{2}$ i.e., $x\in\left[1,\frac{3}{2}\right]$

Hence, domain of the given function is $[1, \frac{3}{2}]$.

28. **(a)** $f(x+y)=f(x)+2y^2+kxy$

$f(1+y)=2+2y^2+ky$, putting $x=1$

putting $y=1$,

$f(2)=8=2+2+k \Rightarrow k=4$

$\therefore f(1+y)=2+2y^2+4y=2(y+1)^2$

$\therefore f(x)=2x^2$

29. **(d)** Here $R=\{(x,y):|x^2-y^2|<16\}$

and given $A=\{1,2,3,4,5\}$

$\therefore$ $R=\{(1,2)(1,3)(1,4);(2,1)(2,2)(2,3)(2,4);$
$(3,1);(3,2)(3,3)(3,4);(4,1)(4,2)(4,3);(4,4),(4,5),(5,4)$
$(5,5)\}$

30. **(c)** If $n=0$, then $h(n)$ is not defined, so, 'h' is not a function. All other are functions.