Vedantu
Learn LIVE Online

Table of Contents

Limits

LIMITS

LIMITS

1. INTRODUCTION

Calculus is the mathematics of motion and change, while algebra, geometry, and trigonometry are more static in nature. The development of calculus in the 17^{th} century by Newton, Leibniz and others grew out of attempts by these and earlier mathematicians to answer certain fundamental questions about dynamic real-world situations. These investigations led to two fundamental procedures- differentiation and integration; which can be formulated in terms of a concept called- limit.

In a very real sense, the concept of limit is the threshold to modern mathematics. You are about to cross that threshold, and beyond lies the fascinating world of calculus.

2. LIMIT OF A FUNCTION (INFORMAL DEFINITION)

The notation :

$$\lim_{x\to c} f(x) = L$$

is read "the limit of $f(x)$ as x approaches c is L" and means that the functional values $f(x)$ can be made arbitrarily close to a unique number L by choosing x sufficiently close to c (but not equal to c).

2.1 One-Sided Limits

2.1.1 Right-hand Limit : We write $\lim_{x\to c^+} f(x) = L$ if we can make the number $f(x)$ as close to L as we please by choosing x sufficiently close to c on a small interval (c, b) immediately to the right of c.

2.1.2 Left-hand limit : We write $\lim_{x\to c^-} f(x) = L$ if we can make the number $f(x)$ as close to L as we please by choosing x sufficiently close to c on a small interval (a, c) immediately to the left of c.

2.1.3 Limit of a function $f(x)$ is said to exist as $x \to a$ (x approaches a) when ;

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \ell \text{ (finite quantity)}$$

$$\Rightarrow \lim_{x\to a} f(x) = \ell$$

3. EVALUATION OF ALGEBRAIC LIMITS

Let $f(x)$ be an algebraic function and 'a' be a real number. Then $\lim_{x\to a} f(x)$ is known as an algebraic limit.

E.g. $\lim_{x\to 1}\frac{x^3-1}{x-1}$, $\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$, $\lim_{x\to 2}\frac{x^5-32}{x-2}$ etc.

are algebraic limits.

3.1 Direct substitution method

If by direct substitution of the point in the given expression we get a finite number, then the number obtained is the limit of the given expression.

If upon substituting the point in the given expression, we get the following forms. :

$$\frac{0}{0}, \frac{\infty}{\infty}, \infty-\infty, 0\times\infty, \infty^0, 0^0, 1^\infty$$

(Indeterminate Forms)

Then we can't find the value of limit by direct substitution. Following methods are followed to find the limit of the function.

3.2 Factorisation method

Consider $\lim_{x\to a}\frac{f(x)}{g(x)}$. If by putting $x = a$ the rational function $\frac{f(x)}{g(x)}$ takes the form $\frac{0}{0}, \frac{\infty}{\infty}$ etc, then (x–a) is a factor of both $f(x)$ & $g(x)$. In such a case we factorise the numerator and denominator, and then cancel out the common factor (x–a). After cancelling out the common factor (x–a), we again put x = a in the given expression and see whether we get a meaningful number or not. This process is repeated till we get a meaningful number.

3.3 Rationalisation method

This is particularly used when either numerator or denominator, or both involve expressions consisting of square roots (radical signs)

3.4 Method of evaluating algebraic limits when $x \to \infty$

To evaluate this type of limits we follow the following procedure.

Step–1 : Write down the given expression in the form of a rational function, i.e., $\frac{f(x)}{g(x)}$, if it is not so.

Step–2 : If k is the highest power of x in numerator and denominator both, then divide each term in numerator and denominator by x^k.

Step–3 : Use the result $\lim_{x\to\infty} \frac{1}{x^n} = 0$, where $n > 0$.

3.4.1 Important Results : If m, n are positive integers and a_0, $b_0 \neq 0$ are non-zero numbers, then

$$\lim_{x\to\infty} \frac{a_0x^m + a_1x^{m-1} + ... + a_{m-1}x + a_m}{b_0x^n + b_1x^{n-1} + ... + b_{n-1}x + b_n}$$

$$= \begin{cases} \frac{a_0}{b_0}, & \text{if} \quad m = n \\ 0, & \text{if} \quad m < n \\ \infty, & \text{if} \quad m > n, \ (a_0 \times b_0) > 0 \\ -\infty, & \text{if} \quad m > n, \ (a_0 \times b_0) < 0 \end{cases}$$

4. TRIGONOMETRIC LIMITS

To evaluate trigonometric limits the following results are very useful.

1. $\lim_{x\to 0} \frac{\sin x}{x} = 1$

2. $\lim_{x\to 0} \frac{\tan x}{x} = 1$

(in 1 & 2, x is measured in radians)

3. $\lim_{x\to 0} \frac{\sin^{-1} x}{x} = 1$

4. $\lim_{x\to 0} \frac{\tan^{-1} x}{x} = 1$

5. $\lim_{x\to 0} \frac{\sin x^0}{x} = \frac{\pi}{180}$

6. $\lim_{x\to 0} \cos x = 1$

7. $\lim_{x\to a} \frac{\sin(x-a)}{(x-a)} = 1$

8. $\lim_{x\to a} \frac{\tan(x-a)}{(x-a)} = 1$

5. EXPONENTIAL & LOGARITHMIC LIMITS

To evaluate the exponential and logarithmic limits we use the following results.

1. $\lim_{x\to 0} \frac{a^x - 1}{x} = \log_e a$

2. $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

3. $\lim_{x\to 0} \frac{\log(1+x)}{x} = 1$

4. $\lim_{x\to a} \left(\frac{x^n - a^n}{x - a}\right) = n.a^{n-1}$

6. EXPONENTIAL LIMITS OF THE FORM $^\infty$

To evaluate the exponential limits of the form 1^∞ we use the following results.

1. If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$, then

$$\lim_{x\to a} [1 + f(x)]^{1/g(x)} = e^{\lim_{x\to a} \frac{f(x)}{g(x)}},$$

2. If $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = \infty$, then

$$\lim_{x\to a} [f(x)]^{g(x)} = \lim_{x\to a} [1 + f(x) - 1]^{g(x)} = e^{\lim_{x\to a} (f(x)-1)g(x)}$$

1.1 Particular Cases

1. $\lim_{x\to 0} (1+x)^{1/x} = e$

2. $\lim_{x\to\infty} \left(1 + \frac{1}{x}\right)^x = e$

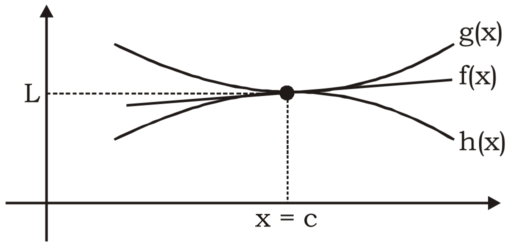
3. $\lim_{x\to 0} (1+\lambda x)^{1/x} = e^\lambda$

4. $\lim_{x\to\infty} \left(1 + \frac{\lambda}{x}\right)^x = e^\lambda$

7. SQUEEZE RULE/SANDWICH RULE

If $g(x) \le f(x) \le h(x)$ on an open interval containing 'c', and if:

$$\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L, \text{ then } \lim_{x\to c} f(x) = L$$



In words : If a function can be squeezed/sandwiched between two functions whose limits at a particular point c have the same value L, then that function must also have limit L at x = c.

8. THE ALGEBRA OF LIMITS

Let f and g be two real functions with domain D. We define four new functions $f \pm g$, fg, f/g on domain D by setting $(f \pm g)(x) = f(x) \pm g(x)$, $(fg)(x) = f(x) \times g(x)$
$(f/g)(x) = f(x)/g(x)$, if $g(x) \ne 0$ for any $x \in D$.
Following are some results concerning the limits of these functions.

Let $\lim_{x\to a} f(x) = \ell$ and $\lim_{x\to a} g(x) = m$, if ℓ and m exist.

1. $\lim_{x\to a} (f \pm g)(x) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x) = l \pm m$

2. $\lim_{x\to a} (f g)(x) = \lim_{x\to a} f(x) \times \lim_{x\to a} g(x) = \ell m$

3. $\lim_{x\to a} \left(\frac{f}{g}\right)(x) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{\ell}{m}$, provided $m \ne 0$.

4. $\lim_{x\to a} k f(x) = k . \lim_{x\to a} f(x)$, where k is constant.

5. $\lim_{x\to a} (f(x))^{g(x)} = \ell^m$; (provided $\lim_{x\to a} f(x) > 0$)

6. $\lim_{x\to a} fog(x) = f\left(\lim_{x\to a} g(x)\right) = f(m)$; provided $f(x)$ is continuous at $g(x) = m$.

In particular

(a) $\lim_{x\to a} \log f(x) = \log\left(\lim_{x\to a} f(x)\right) = \log \ell$; (provided $l > 0$)

(b) $\lim_{x\to a} e^{f(x)} = e^{\lim_{x\to a} f(x)} = e^{\ell}$.

7. If $\lim_{x\to a} f(x) = +\infty$ or $-\infty$, then $\lim_{x\to a} \frac{1}{f(x)} = 0$.

9. LIMITS BY DE'L' HOSPITAL'S RULE

If $f(x)$ and $g(x)$ be two functions of x such that

1. $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$
2. both are continuous at x = a,
3. both are differentiable at x = a,
4. $f'(x)$ and $g'(x)$ are continuous at the point x = a, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}, \text{ provided that } g(a) \ne 0.$$

The above rule is also applicale,

if $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$.

Generalisation : If $\lim_{x\to a} \frac{f'(x)}{g'(x)}$ assumes the indeterminate form (0/0) or (∞/∞) and $f'(x)$, $g'(x)$ satisfy all the conditions embodied in De'L'Hospitals rule, we can repeat the application of this rule on $\frac{f'(x)}{g'(x)}$ to get

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)} = \lim_{x\to a} \frac{f''(x)}{g''(x)} = \dots .$$

Sometimes it may be necessary to repeat this process a numer of times till our goal of evaluating limit is achieved.

10. EXPANSIONS TO EVALUATE LIMITS

1. $(1+x)^n = 1+nx+\frac{n(n-1)}{2!}+\frac{n(n-1)(n-2)}{3!}x^3+....$

(for rational or integral *n*, Rule is not applicable for irrational *n*.)

2. $e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...$

3. $a^x = 1+x(log_e a)+\frac{x^2}{2!}(log_e a)^2+....$

4. $log(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+...$

5. $\sin x = x-\frac{x^3}{3!}+\frac{x^5}{5!}-....$

6. $\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-....$

7. $\tan x = x+\frac{x^3}{3}+\frac{2}{15}x^5+....$

8. $\sin^{-1}x = x+\frac{1}{2}.\frac{x^3}{3}+\frac{1}{2}.\frac{3}{4}.\frac{x^5}{5}+\frac{1}{2}.\frac{3}{4}.\frac{5}{6}\frac{x^7}{7}+...$

9. $\tan^{-1}x = x-\frac{1}{3}x^3+\frac{1}{5}x^5-...$

SOLVED EXAMPLES

Example – 1

Evaluate $\lim\limits_{x\to 2}\dfrac{x^3-6x^2+11x-6}{x^2-6x+8}$.

Sol. When x = 2, the expression

$\dfrac{x^3-6x^2+11x-6}{x^2-6x+8}$ of the form $\dfrac{0}{0}$.

Now, $\lim\limits_{x\to 2}\dfrac{x^3-6x^2+11x-6}{x^2-6x+8}\left(\dfrac{0}{0}\text{form}\right)$

$=\lim\limits_{x\to 2}\dfrac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}\left(\dfrac{0}{0}\text{ form}\right)$

$=\lim\limits_{x\to 2}\dfrac{(x-1)(x-3)}{(x-4)}=\dfrac{(2-1)(2-3)}{(2-4)}=\dfrac{1}{2}$.

Example – 2

Evaluate $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$.

Sol. When x = a, $\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ assumes the indeterminate form $\dfrac{0}{0}$.

Now, $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

$=\lim\limits_{x\to a}\dfrac{\left(\sqrt{a+2x}-\sqrt{3x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\left(\sqrt{3a+x}-2\sqrt{x}\right)\left(\sqrt{3a+x}+2\sqrt{x}\right)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$

$=\lim\limits_{x\to a}\dfrac{(a-x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{(a-x)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$

$=\lim\limits_{x\to a}\dfrac{\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\left(\sqrt{a+2x}+\sqrt{3x}\right)}=\dfrac{4\sqrt{a}}{2\left(3\sqrt{3a}\right)}=\dfrac{2}{3\sqrt{3}}$.

Example – 3

Evaluate the following limit :

$\lim\limits_{n\to\infty}\dfrac{1^2+2^2+3^2+.....+n^2}{n^3}$.

($x\to\infty$ type problem)

Sol. $\lim\limits_{n\to\infty}\dfrac{1^2+2^2+.....+n^2}{n^3}=\lim\limits_{n\to\infty}\dfrac{n(n+1)(2n+1)}{6n^3}$

$=\dfrac{1}{6}\lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)\left(2+\dfrac{1}{n}\right)=\dfrac{1}{6}(1+0)(2+0)=\dfrac{1}{3}$.

Example – 4

Evaluate the following limits :

$\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin(\cos x)\cos x}{\sin x-\operatorname{cosec} x}$.

$\left(\text{using }\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1\right)$

Sol. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin(\cos x)\cos x}{\sin x-\operatorname{cosec} x}$

$=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin(\cos x)\sin x\cos x}{\sin^2 x-1}$

$=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\sin(\cos x)\sin x}{\cos x}$

$=-1$.

Example – 5

Solve : $\lim\limits_{x\to\pi/3}\dfrac{\tan^3 x - 3\tan x}{\cos\left(x+\dfrac{\pi}{6}\right)}$.

Sol. Let $L = \lim\limits_{x\to\pi/3}\dfrac{\tan^3 x - 3\tan x}{\cos\left(x+\dfrac{\pi}{6}\right)}$ and $x-\dfrac{\pi}{3} = t$

$$\Rightarrow \quad L = \lim_{t\to 0}\frac{\tan^3\left(t+\dfrac{\pi}{3}\right) - 3\tan\left(t+\dfrac{\pi}{3}\right)}{\cos\left(t+\dfrac{\pi}{2}\right)}$$

$$\Rightarrow \quad L = \lim_{t\to 0}\frac{\tan(3t+\pi)\left[3\tan^2\left(t+\dfrac{\pi}{3}\right)-1\right]}{-\sin t}$$

$$\Rightarrow \quad L = \lim_{t\to 0}\frac{+\tan(3t)}{-\sin t}.\lim_{t\to 0}\left[3\tan^2\left(t+\frac{\pi}{3}\right)-1\right]$$

$$\Rightarrow \quad L = -3\lim_{t\to 0}\frac{\tan(3t)}{3t}\times\lim_{t\to 0}\frac{t}{\sin t}\times\lim_{t\to 0}\left[3\tan^2\left(t+\frac{\pi}{3}\right)-1\right]$$

$\Rightarrow \quad L = -3 \times 1 \times 1 \times 8 = -24.$

Example – 6

Evaluate the following limits :

$$\lim_{x\to a}\frac{e^{\sqrt{x}} - e^{\sqrt{a}}}{x - a}.$$

$$\left(\text{using } \lim_{x\to 0}\frac{a^x - 1}{x} = \log a\right)$$

Sol. $\lim\limits_{x\to a}\dfrac{e^{\sqrt{x}} - e^{\sqrt{a}}}{x-a} = \lim\limits_{x\to a}\dfrac{e^{\sqrt{a}}\left(e^{\sqrt{x}-\sqrt{a}} - 1\right)}{x-a}$

$$= e^{\sqrt{a}}\lim_{x\to a}\frac{e^{\sqrt{x}-\sqrt{a}} - 1}{\sqrt{x}-\sqrt{a}}\lim_{x\to a}\frac{\sqrt{x}-\sqrt{a}}{x-a}$$

$$= e^{\sqrt{a}}(1)\lim_{x\to a}\frac{(x-a)}{(x-a)\left(\sqrt{x}+\sqrt{a}\right)} = \frac{e^{\sqrt{a}}}{2\sqrt{a}}$$

Example – 7

Evaluate the following limits :

$$\lim_{x\to 2}\frac{3^x + 3^{3-x} - 12}{3^{3-x} - 3^{x/2}}.$$

Sol. $L = \lim\limits_{x\to 2}\dfrac{3^x + 3^{3-x} - 12}{3^{3-x} - 3^{x/2}}$

$$\Rightarrow \quad L = \lim_{x\to 2}\frac{3^x + \dfrac{27}{3^x} - 12}{\dfrac{27}{3^x} - 3^{x/2}}$$

$$\Rightarrow \quad L = \lim_{x\to 2}\frac{3^{2x} - 12.3^x + 27}{-\left(3^{x/2}\cdot 3^x - 3^3\right)}$$

$$\Rightarrow \quad L = -\lim_{x\to 2}\frac{(3^x-9)(3^x-3)}{\left(3^{x/2}-3\right)\left(3^x+9+3.3^{x/2}\right)}$$

$$\Rightarrow \quad L = \lim_{x\to 2} -\frac{(3^{x/2}+3)(3^x-3)}{\left(3^x+3.3^{x/2}+9\right)}$$

$$\Rightarrow \quad L = \frac{-6.6}{9+3.3+9} = \frac{-36}{27} = \frac{-4}{3}.$$

Example – 8

Evaluate : $\lim\limits_{x\to a}\left(2-\dfrac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$

[1^∞ type of indeterminate form]

Sol. Since, $\lim\limits_{x\to a}\left(2-\dfrac{a}{x}\right) = 1$ and $\lim\limits_{x\to a}\left(\dfrac{\pi x}{2a}\right) = \infty.$

$\Rightarrow \quad \lim\limits_{x\to a}\left(2-\dfrac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$ of the form (1^∞)

Hence, $\lim\limits_{x\to a}\left(2-\dfrac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$

$$= e^{\lim\limits_{x \to a}\left(2-\frac{a}{x}-1\right)\times \tan\frac{\pi x}{2a}}.$$

$$= e^{\lim\limits_{x \to a}\left(\frac{x-a}{x}\right)\times \tan\left(\frac{\pi x}{2a}\right)}.$$

$$= e^{\lim\limits_{h \to 0}\frac{h}{a+h}\times \tan\left(\frac{\pi}{2}+\frac{\pi h}{2a}\right)} \qquad \text{(putting, } x-a=h)$$

$$= e^{-\lim\limits_{h \to 0}\frac{h}{a+h}\times \cot\left(\frac{\pi h}{2a}\right)} \qquad \left(\tan\left(\frac{\pi}{2}+\theta\right) = -\cot\theta\right)$$

$$= e^{-\lim\limits_{h \to 0}\frac{(\pi h/2a)}{\tan(\pi h/2a)}\times \lim\limits_{h \to 0}\frac{(2a/h)}{(a+h)}}$$

$\therefore \quad e^{-2/\pi}$.

Example – 9

Evaluate : $\lim\limits_{x \to 0}\dfrac{\sin 3x^2}{\ln\cos(2x^2-x)}$ using LH rule.

($\frac{0}{0}$ type of indeterminate form)

Sol. Let $\lim\limits_{x \to 0}\dfrac{\sin 3x^2}{\ln\cos(2x^2-x)}$ [$\frac{0}{0}$ form]

Apply LH rule to get :

$$L = \lim_{x \to 0}\frac{-6x\cos 3x^2\cos(2x^2-x)}{(4x-1)\sin(2x^2-x)}$$

$$= -6\lim_{x \to 0}\frac{\cos 3x^2\cos(2x^2-x)}{4x-1}$$

$$\lim_{x \to 0}\frac{x}{\sin(2x^2-x)}$$

The limit of the factor is computed directly, the limit of the second one, which represents an indeterminate form of the 0/0 is found with the aid of the L'Hospital's rule. Again consider.

$$L = -6\lim_{x \to 0}\frac{\cos 3x^2\cos(2x^2-x)}{4x-1}$$

$$\lim_{x \to 0}\frac{x}{\sin(2x^2-x)}$$

$$\Rightarrow \quad L = -6.\frac{1.1}{-1}\lim_{x \to 0}\frac{1}{(4x-1)\cos(2x^2-x)}$$

$$\Rightarrow \quad L = -6.\frac{1}{-1.1} = -6$$

Example – 10

Evaluate : $\lim\limits_{x \to 1}\left(\dfrac{1}{\ln x}-\dfrac{1}{x-1}\right)$ using LH rule.

[$\infty - \infty$ type of indeterminate form]

Sol. Let $L = \lim\limits_{x \to 1}\left(\dfrac{1}{\ln x}-\dfrac{1}{x-1}\right)$ [$\infty - \infty$ form]

Let us reduce it to an indeterminate form of the type 0/0.

$$L = \lim_{x \to 1}\frac{x-1-\ln x}{(x-1)\ln x} \qquad \text{[0/0 form]}$$

Apply LH rule to get,

$$L = \lim_{x \to 1}.\frac{1-1/x}{\ln x+1-1/x}$$

$$\Rightarrow \quad L = \lim_{x \to 1}.\frac{x-1}{x\ln x+x-1}$$

Apply LH rule again,

$$\Rightarrow \quad L = \lim_{x \to 1}.\frac{x-1}{\ln x+2} = \frac{1}{2}$$

Example – 11

Evaluate the left hand and right hand limits of the function defined by

$$f(x)=\begin{cases}1+x^2, \text{ if } 0\le x\le 1\\ 2-x, \text{ if } \quad x>1\end{cases} \text{ at } x=1.$$

Also, show that $\lim\limits_{x\to 1} f(x)$ does not exist.

Sol. We have,

(LHL of f(x) at x = 1)

$$=\lim_{x\to 1^-} f(x)=\lim_{h\to 0} f(1-h)$$

$$=\lim_{h\to 0} 1+(1-h)^2=\lim_{h\to 0} 2-2h+h^2=2.$$

and,

(RHL of f(x) at x = 1)

$$\lim_{x\to 1^+} f(x)=\lim_{h\to 0} f(1+h)$$

$$=\lim_{h\to 0} 2-(1+h)=\lim_{h\to 0} 1-h=1$$

Clearly, $\lim\limits_{x\to 1^-} f(x)\ne \lim\limits_{x\to 1^+} f(x).$

So, $\lim\limits_{x\to 1} f(x)$ does not exist.

Example – 12

If $f(x)=\begin{cases}\dfrac{x-|x|}{x}, & x\ne 0\\ 2 \quad , & x=0\end{cases}$ show that $\lim\limits_{x\to 0} f(x)$ does not exist.

Sol. We have,

(LHL of f(x) at x = 0)

$$=\lim_{x\to 0^-} f(x)$$

$$=\lim_{h\to 0} f(0-h)=\lim_{h\to 0}\frac{-h-|-h|}{(-h)}$$

$$=\lim_{h\to 0}\frac{-h-h}{-h}=\lim_{h\to 0}\frac{-2h}{-h}=\lim_{h\to 0} 2=2$$

(RHL of f(x) at x = 0)

$$=\lim_{x\to 0^+} f(x)$$

$$=\lim_{h\to 0} f(0+h)=\lim_{h\to 0}\frac{h-|h|}{h}$$

$$=\lim_{h\to 0}\frac{h-h}{h}=\lim_{h\to 0}\frac{0}{h}=\lim_{h\to 0} 0=0$$

Clearly, $\lim\limits_{x\to 0^-} f(x)\ne \lim\limits_{x\to 0^+} f(x)$

So, $\lim\limits_{x\to 0} f(x)$ does not exist.

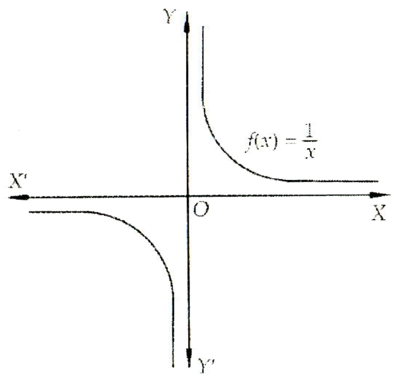
Example – 13

Discuss the existence of each of the following limits :

(i) $\lim\limits_{x\to 0}\dfrac{1}{x}$ (ii) $\lim\limits_{x\to 0}\dfrac{1}{|x|}$

Sol.(i) The graph of $f(x)=\dfrac{1}{x}$ is as shown in Fig. We observe that as x approaches to 0 from the LHS i.e. x is negative and very close to zero, then the values of $\dfrac{1}{x}$ are negative and very large in magnitude.

$$\therefore \quad \lim_{x\to 0^-}\frac{1}{x}\to -\infty$$



Similarly, when x approaches to 0 from the right i.e. x is positive and very close to 0, then the values of $\dfrac{1}{x}$ are very large and positive.

$\therefore \quad \lim_{x \to 0^+} \frac{1}{x} \to \infty \qquad$ Thus we have, $\lim_{x \to 0^-} \frac{1}{x} \neq \lim_{x \to 0^+} \frac{1}{x}$

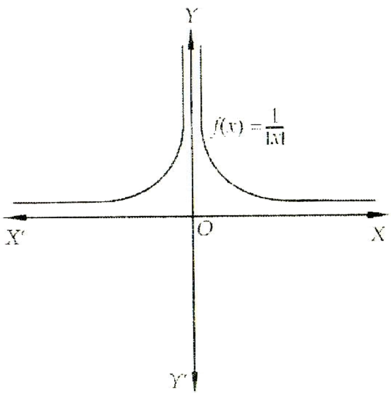
Hence, $\lim_{x \to 0} \frac{1}{x}$ does not exist.

(ii) The graph of $f(x) = \frac{1}{|x|}$ is shown in Fig.. We observe that as x approaches to 0 from LHS i.e. x is negative and close to 0, then | x | is close to zero and is positive. Consequently, $\frac{1}{|x|}$ is large and positive.

$\therefore \quad \lim_{x \to 0^-} \frac{1}{|x|} \to \infty$

Also, if x approaches to 0 from RHS i.e. x is positive and close to 0, then | x | is close to zero and is positive.

Consequently, $\frac{1}{|x|}$ is large and positive.



$\therefore \quad \lim_{x \to 0^+} \frac{1}{|x|} \to \infty$

Thus, we have

$$\lim_{x \to 0^-} \frac{1}{|x|} = \lim_{x \to 0^+} \frac{1}{|x|}$$

Hence, $\lim_{x \to 0} \frac{1}{|x|}$ exists and it tends to infinity.

Example – 14

If $f(x) = \begin{cases} mx^2 + n, x < 0 \\ nx + m, 0 \le x \le 1 \\ nx^3 + m, x > 1 \end{cases}$

For what values of integers m, n does the limits $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$ exist.

Sol. It is given that

$\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$ both exist

$\Leftrightarrow \quad \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$ and $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$\Leftrightarrow \quad \lim_{h \to 0} f(0-h) = \lim_{h \to 0} f(0+h)$ and $\lim_{h \to 0} f(1-h) = \lim_{h \to 0} f(1+h)$

$\Leftrightarrow \quad \lim_{h \to 0} m(-h)^2 + n = \lim_{h \to 0} n\,(h) + m$ and

$\lim_{h \to 0} n(1-h) + m = \lim_{h \to 0} n(1+h)^3 + m$

$\Leftrightarrow \quad n = m$ and $n + m = n + m$

$\Leftrightarrow \quad m = n$

Hence, $\lim_{x \to 0} f(x)$ and $\lim_{x \to 1} f(x)$ both sides for $n = m$.

Example – 15

If $f(x) = \begin{cases} |x|+1 & , \quad x < 0 \\ 0 & , \quad x = 0 \\ |x|-1 & , \quad x > 0 \end{cases}$. For what value (s) of a does $\lim_{x \to a} f(x)$ exist ?

Sol. We have,

$$f(x) = \begin{cases} |x|+1 & , \quad x < 0 \\ 0 & , \quad x = 0 \\ |x|-1 & , \quad x > 0 \end{cases}$$

$$\Rightarrow \quad f(x) = \begin{cases} -x+1 & , \quad x < 0 \\ 0 & , \quad x = 0 \\ x-1 & , \quad x > 0 \end{cases}$$

$$\left[\because |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}\right]$$

Clearly, $\lim\limits_{x \to a} f(x)$ exists for all $a \neq 0$.

So, let us see whether $\lim\limits_{x \to 0} f(x)$ exist or not.

We have,

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} f(0-h) = \lim_{h \to 0} -(-h)+1 = 1$$

$$\lim_{x \to 0+} f(x) = \lim_{x \to 0} f(0+h) = \lim_{h \to 0} h - 1 = -1$$

$$\therefore \quad \lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$$

So, $\lim\limits_{x \to 0}$ f(x) does not exist. Hence, $\lim\limits_{x \to a}$ f(x) exists for all $a \neq 0$.

Example – 16

Suppose $f(x) = \begin{cases} a+bx & , & x<1 \\ 4 & , & x=1 \\ b-ax & , & x>1 \end{cases}$

and if $\lim\limits_{x \to 1} f(x) = f(1)$. What are possible values of a and b ?

Sol. We have,

$$\lim_{x \to 1} f(x) = f(1)$$

$$\Leftrightarrow \quad \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$\Leftrightarrow \quad \lim_{x \to 1^-} f(x) = f(1) \text{ and } \lim_{x \to 1^+} f(x) = f(1)$$

$$\Leftrightarrow \quad \lim_{h \to 0} f(1-h) = 4 \text{ and } \lim_{h \to 0} f(1+h) = 4$$

$$\Leftrightarrow \quad \lim_{h \to 0} \{a + b(1-h)\} = 4 \text{ and } \lim_{h \to 0} \{b - a(1+h)\} = 4$$

$$\Leftrightarrow \quad a + b = 4 \text{ and } b - a = 4$$

$$\Leftrightarrow \quad a = 0, b = 4$$

Example – 17

Evaluate the left hand and right hand *l*imits of the function

$$f(x) = \begin{cases} \dfrac{\sqrt{(x^2-6x+9)}}{(x-3)} & , & x \neq 3 \\ 0 & , & x = 3 \end{cases}$$

at x = 3 and hence comment on the existence of *l*imit at x = 3.

Sol. The given function can be written as

$$f(x) = \begin{cases} \dfrac{|x-3|}{(x-3)} & , & x \neq 3 \\ 0 & , & x = 3 \end{cases}$$

$$\therefore \text{ L.H.L.} = \lim_{x \to 3^-} f(x)$$

$$= \lim_{h \to 0} f(3-h)$$

$$= \lim_{h \to 0} \frac{|3-h-3|}{(3-h-3)}$$

$$= \lim_{h \to 0} \frac{|-h|}{(-3)} = \lim_{h \to 0} \frac{h}{-h} = -1$$

$$\& \text{ R.H.L.} = \lim_{x \to 3^+} f(x)$$

$$= \lim_{h \to 0} f(3+h)$$

$$= \lim_{h \to 0} \frac{|3+h-3|}{(3+h-3)}$$

$$= \lim_{h \to 0} \frac{|h|}{h} = \lim_{h \to 0} \frac{h}{h} = -1$$

Hence left hand limit and right hand limit of $f(x)$ at $x = 3$ are -1 and 1 respectively.

As left Hand Limit ≠ Right Hand Limit, Limiting value at x = 3.

i.e. $\lim\limits_{x \to 3} f(x)$ does not exist.

Example – 18

If $f(x)=\begin{cases}\dfrac{x-|x|}{x}, & x\neq 0\\ 2, & x=0\end{cases}$

show that $\lim\limits_{x\to 0} f(x)$ does not exist.

Sol. We have,

(L.H.L. of $f(x)$ at $x=0$)

$= \lim\limits_{h\to 0} f(0-h)$

$= \lim\limits_{h\to 0}\dfrac{-h-|-h|}{(-h)} = \lim\limits_{h\to 0}\dfrac{-h-h}{-h}$

$= \lim\limits_{h\to 0}\dfrac{-2h}{-h} = \lim\limits_{h\to 0} 2 = 2$

(R.H.L. of $f(x)$ at $x=0$)

$= \lim\limits_{h\to 0^+} f(0+h) = \lim\limits_{h\to 0} f(0+h)$

$= \lim\limits_{h\to 0}\dfrac{h-|h|}{h} = \lim\limits_{h\to 0}\dfrac{h-h}{h}$

$= \lim\limits_{h\to 0}\dfrac{0}{h} = \lim\limits_{h\to 0} 0 = 0.$

Since, $\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x).$

So, $\lim\limits_{x\to 0} f(x)$ does not exist.

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

Factorization

1. $\lim_{x\to 2}\frac{x^3-8}{x^2-4}$ is

(a) 3 (b) $\frac{3}{2}$

(c) 1 (d) 0

2. $\lim_{x\to 1^+}\frac{\sqrt{x^2-1}+\sqrt{x-1}}{\sqrt{x^2-1}}$

(a) $\frac{1}{2}$ (b) $\sqrt{2}+1$

(c) 1 (d) $1+\frac{1}{\sqrt{2}}$

3. $\lim_{x\to\sqrt{3}}\frac{x^2-3}{x^2+3\sqrt{3}x-12}$

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{\sqrt{3}}{5}$ (d) $\frac{3}{5}$

4. $\lim_{x\to 1}\frac{1-x^{-2/3}}{1-x^{-1/3}}$

(a) 2 (b) 1

(c) $\frac{12}{3}$ (d) none of these

5. $\lim_{x\to 1}\frac{(\sqrt{2}-1)(2x-3)}{2x^2+x-3}$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{-1}{10}$

(c) 1 (d) None of these

Rationalization

6. The value of $\lim_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}$ is

(a) 2/9 (b) -2/49

(c) 1/56 (d) -1/56

7. $\lim_{x\to 0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3}$

(a) 3 (b) 4

(c) 1 (d) 2

8. $\lim_{x\to b}\frac{\sqrt{x-a}-\sqrt{b-a}}{x^2-b^2}$

(a) $\frac{1}{4b\sqrt{a-b}}$ (b) $\frac{1}{4b\sqrt{b-a}}$

(c) $\frac{1}{4a\sqrt{a-b}}$ (d) $\frac{1}{b\sqrt{b-a}}$

9. $\lim_{x\to 2}\frac{\sqrt{1+\sqrt{2+x}}-\sqrt{3}}{x-2}$

(a) $\frac{1}{8\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$

(c) $8\sqrt{3}$ (d) $\sqrt{3}$

Exponential & Logarithmic

10. $\text{Limit}_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x}=$

(a) 1/2 (b) 0

(c) 1 (d) none

11. $\lim_{x\to 0}\frac{3^x-1}{\sqrt{x+1}-1}$ is equal to

(a) $\log_e 9$ (b) $\log_e 3$

(c) 0 (d) 1

12. $\lim_{n\to\infty}\frac{4^{1/n}-1}{3^{1/n}-1}$ is equal to

(a) $\log_4 3$ (b) 1

(c) $\log_3 4$ (d) none of these

13. $\lim_{x\to 0}\frac{3^x-2^x}{4^x-3^x}$ is equal to

(a) 1 (b) –1

(c) 0 (d) none of these

14. The value of $\lim_{x\to 3}\left(\log_a \frac{x-3}{\sqrt{x+6}-3}\right)$ is

(a) $\log_a 6$ (b) $\log_a 3$

(c) $\log_a 2$ (d) None of these

15. If α, β are the roots of $x^2 - ax + b = 0$, then $\lim_{x\to\alpha}\frac{e^{x^2-ax+b}-1}{x-\alpha}$ is

(a) $\beta-\alpha$ (b) $\alpha-\beta$

(c) 2α (d) 2β

16. $\lim_{x\to 0}\frac{2^x-1}{(1+x)^{\frac{1}{2}}-1}=$

(a) log2 (b) log4

(c) $\log\sqrt{2}$ (d) None of these

Limit tending to ∞

17. The value of $\lim_{x\to\infty}\frac{(x+1)(3x+4)}{x^2(x-8)}$ is equal to

(a) 2 (b) 3

(c) 1 (d) 0

18. $\lim_{x\to\infty}\frac{(2+x)^{40}(4+x)^5}{(2-x)^{45}}$

(a) –1 (b) 1

(c) 16 (d) 32

19. $\underset{n\to\infty}{\text{Limit}}\frac{(n+2)!+(n+1)!}{(n+3)!}$, $n\in N=$

(a) 0 (b) 1

(c) 2 (d) –1

20. $\lim_{n\to\infty}\left(\frac{1}{1-n^2}+\frac{2}{1-n^2}+...+\frac{n}{1-n^2}\right)$ is equal to

(a) 0 (b) – 1/2

(c) 1/2 (d) None of these

21. The value of $\lim_{n\to\infty}\frac{1+2+3+...+n}{n^2+100}$ is equal

(a) ∞ (b) 1/2

(c) 2 (d) 0

22. $\lim_{n\to\infty}\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+...+\frac{1}{2^n}$ equals

(a) 2 (b) -1

(c) 1 (d) 3

23. $\underset{n\to\infty}{\text{Limit}}\frac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}}=$

(a) 5 (b) 3

(c) 1 (d) zero

24. $\lim_{n\to\infty}(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+...\text{to n terms}\right)}$ is equal to

(a) 2 (b) 4

(c) 8 (d) 0

25. $\lim_{x\to-\infty}3x+\sqrt{9x^2-x}$

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$

(c) $-\frac{1}{6}$ (d) $-\frac{1}{3}$

Trigonometric Limit

26. $\lim_{x\to 0}\frac{2\sin^2 3x}{x^2}=$

(a) 6 (b) 9

(c) 18 (d) 3

27. $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$

(a) $\frac{\pi}{2}$ (b) $\pi+2$

(c) $\frac{2}{\pi}$ (d) none of these

28. $\lim_{x\to\infty}\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$

(a) 0 (b) ∞

(c) 1 (d) none of these

29. $\lim_{x\to 0}\frac{x^2\sin\left(\frac{1}{x}\right)}{\sin x}$

(a) 1 (b) 0

(c) $\frac{1}{2}$ (d) none of these

30. $\lim_{\theta \to \frac{\pi}{2}} (\sec\theta - \tan\theta) =$

(a) 0 (b) 1
(c) –1 (d) 2

31. $\lim_{x \to \infty} x.\cos\left(\frac{\pi}{4x}\right)\sin\left(\frac{\pi}{4x}\right)$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) π (d) 0

32. $\lim_{x \to 1} (1+\cos\pi x)\cot^2 \pi x$

(a) –1 (b) $\frac{1}{2}$
(c) 1 (d) none of these

33. $\lim_{h \to 0} \frac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)\right]}{\sqrt{3}h\left(\sqrt{3}\cosh - \sinh\right)}$

(a) -2/3 (b) -3/4
(c) -2 $\sqrt{3}$ (d) 4/3

34. $\lim_{\theta \to 0} \frac{4\theta(\tan\theta - \sin\theta)}{(1-\cos 2\theta)^2}$ is

(a) $\frac{1}{\sqrt{2}}$ (b) 1/2
(c) 1 (d) 2

35. $\lim_{x \to 0} \frac{\operatorname{cosec} x - \cot x}{x}$ is equal to

(a) $\frac{-1}{2}$ (b) 1
(c) $\frac{1}{2}$ (d) 1

36. The value of $\text{Limit}_{x \to 0} \frac{\cos(\sin x) - \cos x}{x^4}$ is equal to

(a) 1/5 (b) 1/6
(c) 1/4 (d) 1/2

37. $\lim_{x \to 0} \frac{1-\cos x}{x\,(2^x - 1)}$ is equal to

(a) $\frac{1}{2}\log_2 e$ (b) $\frac{1}{2}\log_e 2$
(c) 1 (d) none of these

38. $\text{Limit}_{x \to 2} \frac{\sin\left(e^{x-2}-1\right)}{\ln(x-1)} =$

(a) 0 (b) – 1
(c) 2 (d) 1

39. The value of $\lim_{n \to \infty} \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)...\cos\left(\frac{x}{2^n}\right)$ is

(a) 1 (b) $\frac{\sin x}{x}$
(c) $\frac{x}{\sin x}$ (d) None of these

40. $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n} =$ finite non-zero, then n is

(a) 1 (b) 2
(c) 3 (d) 4

1^∞ Form

41. $\lim_{x \to \pi/4} (2-\tan x)^{\log \tan x}$ is equal to

(a) 0 (b) 1
(c) e (d) e^{-1}

42. $\lim_{x \to 0} \left\{\tan\left(\frac{\pi}{4}-x\right)\right\}^{1/x}$ is equal to

(a) 1 (b) e
(c) e^2 (d) e^{-2}

43. $\lim_{x \to 0} \left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}$ is equal to

(a) e (b) $e^{1/2}$
(c) e^{-2} (d) none of these

44. $\lim_{x \to \infty} \left(\frac{x-1}{x+1}\right)^{x+2}$ is equal to

(a) e (b) e^{-1}
(c) e^{-2} (d) none of these

45. $\underset{x\to 0^+}{\text{Limit}}\left(1+\tan^2\sqrt{x}\right)^{5/x} =$

(a) e^5 (b) e^2

(c) e (d) none

46. The limiting value of $(\cos x)^{1/\sin x}$ as $x \to 0$ is

(a) 1 (b) e

(c) 0 (d) none

47. The $\lim_{x\to 0}(\cos x)^{\cot x}$ is

(a) - 1 (b) 0

(c) 1 (d) None of these

48. $\underset{x\to 0}{\text{Limit}}\,|x|^{\sin x} =$

(a) 0 (b) 1

(c) -1 (d) none

Existence of Limit

49. $\underset{x\to 5}{\text{Limit}}\dfrac{x^2-9x+20}{x-[x]}$ where [x] is the greatest integer not greater than x

(a) is equal to 1 (b) 0

(c) 4 (d) none

50. $\lim_{x\to 3}([x-3]+[3-x]-x)$, where [.] denotes the greatest integer function, is equal to

(a) 4 (b) -4

(c) 0 (d) Does not exist

51. $\underset{x\to 1}{\text{Limit}}(1-x+[x-1]+[1-x]) =$ where [x] denotes greatest integer function

(a) 0 (b) 1

(c) -1 (d) does not exist

52. $\lim_{x\to -1}\{[x]+|x|\}$, where [.] denotes the greatest integer function,

(a) is 0 (b) is 1

(c) does not exist (d) none of these

53. $\lim_{x\to 0}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} =$

(a) 0 (b) 1

(c) -1 (d) Does not exist

54. If $f(x)=\begin{cases}\dfrac{\sin[x]}{[x]} & [x]\neq 0\\ 0 & [x]\neq 0\end{cases}$, where [.] denotes the greatest integer function, then $\lim_{x\to 0} f(x)$ is equal to

(a) 1 (b) 0

(c) -1 (d) Does not exist

L. H. Rule

55. The value of $\lim_{x\to 0}\dfrac{e^x+\log(1+x)-(1-x)^{-2}}{x^2}$ is equal to

(a) 0 (b) -3

(c) -1 (d) infinity

56. $\lim_{x\to 0}$ x log sinx is equal to

(a) ∞ (b) Zero

(c) 1 (d) Cannot be determined

57. $\lim_{x\to\frac{\pi}{4}}$ tanx logsinx =

(a) 0 (b) 1

(c) -1 (d) None of these

58. $\lim_{x\to 0}\left\{\dfrac{\sin x-x+\dfrac{x^3}{6}}{x^5}\right\} =$

(a) 1/120 (b) -1/120

(c) 1/20 (d) None of these

59. The value of $\lim_{x\to 0}\dfrac{x\cos x-\log(1+x)}{x^2}$ is

(a) 1/2 (b) 0

(c) 1 (d) None of these

60. If $f(9)=9$ and $f'(9)=1$, then $\lim_{x\to 9}\dfrac{3-\sqrt{f(x)}}{3-\sqrt{x}}$ is equal to

(a) 0 (b) 1

(c) -1 (d) None of these

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. $\lim_{x\to 0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}\,x}$ is **(2002)**

(a) λ (b) -1

(c) zero (d) does not exist

2. $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+2}\right)^x$ is equal to **(2002)**

(a) e^4 (b) e^2

(c) e^3 (d) e

3. For $x \in R$, $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to **(2002)**

(a) e (b) e^{-1}

(c) e^{-5} (d) e^5

4. Let $f(2)=4$ and $f'(2)=4$. Then, $\lim_{x\to 2}\frac{xf(2)-2f(x)}{x-2}$ is given by **(2002)**

(a) 2 (b) –2

(c) –4 (d) 3

5. $\lim_{x\to\frac{\pi}{2}}\frac{\left[1-\tan\left(\frac{x}{2}\right)\right](1-\sin x)}{\left[1+\tan\left(\frac{x}{2}\right)\right](\pi-2x)^3}$ is **(2003)**

(a) $\frac{1}{8}$ (b) 0

(c) $\frac{1}{32}$ (d) ∞

6. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$, the value of k is **(2003)**

(a) 0 (b) –1/3

(c) 2/3 (d) –2/3

7. If $\lim_{x\to\infty}\left(1+\frac{a}{x}+\frac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b are **(2004)**

(a) $a\in R, b\in R$ (b) $a=1, b\in R$

(c) $a\in R, b=2$ (d) $a=1, b=2$

8. Let α and β be the distinct roots of $ax^2+bx+c=0$, then $\lim_{x\to\alpha}\frac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to **(2005)**

(a) $\frac{1}{2}(\alpha-\beta)^2$ (b) $-\frac{a^2}{2}(\alpha-\beta)^2$

(c) 0 (d) $\frac{a^2}{2}(\alpha-\beta)^2$

9. Let $f : R \to R$ be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}=1$. Then, $\lim_{x\to\infty}\frac{f(2x)}{f(x)}$ is equal to **(2010)**

(a) 1 (b) $\frac{2}{3}$

(c) $\frac{3}{2}$ (d) 3

10. $\lim_{x\to 2}\left(\frac{\sqrt{1-\{\cos 2(x-2)\}}}{x-2}\right)$ **(2011)**

(a) equals $\sqrt{2}$ (b) equals $-\sqrt{2}$

(c) equals $\frac{1}{\sqrt{2}}$ (d) does not exist

11. Let $f : R \to [0, \infty)$ be such that $\lim_{x\to 5} f(x)$ exists and $\lim_{x\to 5}\frac{[f(x)]^2-9}{\sqrt{|x-5|}}=0$. Then, $\lim_{x\to 5} f(x)$ equals to **(2011)**

(a) 3 (b) 0

(c) 1 (d) 2

12. If function f (x) is differentiable at x = a, then $\lim_{x\to a}\frac{x^2f(a)-a^2f(x)}{x-a}$ **(2011)**

(a) $2a\,f(a)+a^2f'(a)$ (b) $-a^2f'(a)$

(c) $af(a)-a^2f'(a)$ (d) $2af(a)-a^2f'(a)$

13. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to **(2013)**

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$

(c) 1 (d) 2

14. $\lim_{x\to 0}\frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to : **(2014)**

(a) π (b) $\frac{\pi}{2}$

(c) 1 (d) $-\pi$

15. If $\lim_{x\to 2}\frac{\tan(x-2)\{x2+(k-2)x-2k\}}{x^2-4x+4}=5,$ then k is equal to: **(2014/Online Set–2)**

(a) 0 (b) 1

(c) 2 (d) 3

16. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to: **(2015)**

(a) 2 (b) $\frac{1}{2}$

(c) 4 (d) 3

17. $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{\sin^2 x}$ is equal to : **(2015/Online Set–1)**

(a) 2 (b) 3

(c) $\frac{5}{4}$ (d) $\frac{3}{2}$

18. $\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ is **(2015/Online Set–2)**

(a) 2 (b) -2

(c) 1/2 (d) -1/2

19. Let $p=\lim_{x\to 0+}(1+\tan^2\sqrt{x})^{\frac{1}{2x}}$ then log p is equal to : **(2016)**

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) 2

20. If $\lim_{x\to\infty}\left(1+\frac{a}{x}-\frac{4}{x^2}\right)^{2x}=e^3$, then ‘a’ is equal to: **(2016/Online Set–1)**

(a) 2 (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{4}$

21. $\lim_{x\to 0}\frac{(1-\cos 2x)^2}{2x\tan x-x\tan 2x}$ is : **(2016/Online Set–2)**

(a) -2 (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) 2

22. $\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$ equals : **(2017)**

(a) $\frac{1}{24}$ (b) $\frac{1}{16}$

(c) $\frac{1}{8}$ (d) $\frac{1}{4}$

23. $\lim_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}}$ is equal to : **(2017/Online Set–1)**

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2\sqrt{2}}$

24. For each $t \in R$, let [t] be the greatest integer less than or equal to t. Then

$$\lim_{x\to 0+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+.....+\left[\frac{15}{x}\right]\right)$$ **(2018)**

(a) does not exist (in R) (b) is equal to 0.

(c) is equal to 15. (d) is equal to 120.

25. $\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ equals: **(2018/Online Set–2)**

(a) $\frac{1}{4}$ (b) 1

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

26. $\lim_{x\to 0}\frac{(27+x)^{\frac{1}{3}}-3}{9-(27+x)^{\frac{2}{3}}}$ equals : **(2018/Online Set–3)**

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) $-\frac{1}{6}$ (d) $\frac{1}{6}$

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

SINGLE ANSWER QUESTIONS

1. $\lim_{x\to 0}\dfrac{e^x-e^{-x}-2x}{x-\sin x}$ is equal to

(a) 1 (b) –1
(c) 2 (d) 0

2. $\text{Limit}_{x\to 0}\dfrac{\left(4^x-1\right)^3}{\sin\left(\dfrac{x}{p}\right)ln\left(1+\dfrac{x^2}{3}\right)}=$

(a) $9\,p\,(log\,4)$ (b) $3\,p\,(log\,4)^3$
(c) $12\,p\,(log\,4)^3$ (d) $27\,p\,(log\,4)^2$

3. The value of $\text{Limit}_{x\to 0}\dfrac{\cos(\sin x)-\cos x}{x^4}$ is equal to

(a) 1/5 (b) 1/6
(c) 1/4 (d) 1/2

4. If $f(9)=9$ and $f'(9)=1$, then $\lim_{x\to 9}\dfrac{3-\sqrt{f(x)}}{3-\sqrt{x}}$ is equal to

(a) 0 (b) 1
(c) –1 (d) None of these

5. If $\lim_{x\to\infty}\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b, are

(a) $a\in R, b=2$ (b) $a=1$ and $b\in R$
(c) $a\in R, b\in R$ (d) $a=1$ and $b=2$

6. $\lim_{h\to 0}\dfrac{\sin(a+3h)-3\sin(a+2h)+3\sin(a+h)-\sin a}{h^3}$ is

(a) sin a (b) –sin a
(c) cos a (d) –cos a

7. $\text{Limit}_{x\to 3}\dfrac{\left(x^3+27\right)ln(x-2)}{\left(x^2-9\right)}=$

(a) – 8 (b) 8
(c) 9 (d) – 9

8. If α, β are the roots of the quadratic equation $ax^2+bx+c=0$ then,

$$\text{Limit}_{x\to\alpha}\frac{1-\cos\left(ax^2+bx+c\right)}{(x-\alpha)^2}=$$

(a) 0 (b) $\dfrac{1}{2}(\alpha-\beta)^2$
(c) $\dfrac{a^2}{2}(\alpha-\beta)^2$ (d) $-\dfrac{a^2}{2}(\alpha-\beta)^2$

9. $\text{Limit}_{x\to 5}\dfrac{x^2-9x+20}{x-[x]}$ where [x] is the greatest integer not greater than x

(a) is equal to 1 (b) 0
(c) 4 (d) none

10. $\text{Limit}_{x\to 2}\dfrac{\sin\left(e^{x-2}-1\right)}{ln(x-1)}=$

(a) 0 (b) – 1
(c) 2 (d) 1

11. The value of $\lim_{x\to 3}\left(log_a\dfrac{x-3}{\sqrt{x+6}-3}\right)$ is

(a) $log_a 6$ (b) $log_a 3$
(c) $log_a 2$ (d) None of these

12. $\text{Limit}_{y\to 0}\left(\text{Limit}_{x\to\infty}\dfrac{\exp\left(x\,ln\left(1+\dfrac{ay}{x}\right)\right)-\exp\left(x\,ln\left(1+\dfrac{by}{x}\right)\right)}{y}\right)=$

(a) a + b (b) a – b
(c) b – a (d) – (a + b)

13. The value of $\lim\limits_{x\to a}\sqrt{a^2-x^2}\cot\frac{\pi}{2}\sqrt{\frac{a-x}{a+x}}$ is

(a) $\frac{2a}{\pi}$ (b) $-\frac{2a}{\pi}$

(c) $\frac{4a}{\pi}$ (d) $-\frac{4a}{\pi}$

14. If [x] denotes the greatest integer ≤ x, then $\text{Limit}_{n\to\infty}\frac{1}{n^4}\left([1^3x]+[2^3x]+...+[n^3x]\right)$ equals

(a) x/2 (b) x/3

(c) x/6 (d) x/4

15. If $f(x)=\begin{cases}\frac{\sin[x]}{[x]} & \text{if } [x]\neq 0\\ 0 & \text{if } [x]=0\end{cases}$ where [x] denotes the greatest integer less than or equal to x, then $\text{Limit}_{x\to 0} f(x)$ equals

(a) 1 (b) 0

(c) –1 (d) none

16. If $f(x)=\begin{Bmatrix}\frac{\tan[x]}{[x]}, & [x]\neq 0\\ 0, & [x]=0\end{Bmatrix}$ where [x] denotes the greatest integer less than or equal to x, then $\lim\limits_{x\to 0} f(x)$ equals

(a) 1 (b) –1

(c) 0 (d) Does not exist

17. $\lim\limits_{h\to 0}\frac{2\left[\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)\right]}{\sqrt{3}h\left(\sqrt{3}\cos h-\sin h\right)}$ is equal to

(a) 4/3 (b) – 4/3

(c) 2/3 (d) 3/4

18. $\text{Limit}_{x\to -1}\frac{\cos 2-\cos 2x}{x^2-|x|}=$

(a) 2 cos 2 (b) – 2 cos 2

(c) 2 sin 2 (d) – 2 sin 2

19. $\text{Limit}_{n\to\infty} n\cos\left(\frac{\pi}{4n}\right)\sin\left(\frac{\pi}{4n}\right)$ has the value equal to

(a) π/3 (b) π/4

(c) π/6 (d) none

20. The value of $\lim\limits_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ is

(a) 2a sin a + a^2cos a (b) 2a sin a – a^2 cos a

(c) 2a cos a + a^2sin a (d) None of these

21. If $f(a)=2, f'(a)=1, g(a)=-1, g'(a)=2$, then $\lim\limits_{x\to a}\frac{g(x)f(a)-g(a)f(x)}{x-a}$ is equal to

(a) 3 (b) 5

(c) –3 (d) 0

22. If $\lim\limits_{x\to 0}\frac{\sin 2x+a\sin x}{x^3}$ be finite, then the value of a and the limit are given by

(a) –2, 1 (b) –2, –1

(c) 2, 1 (d) 2, –1

23. The value of

$\lim\limits_{x\to\frac{\pi}{2}}\left[1^{1/\cos^2 x}+2^{1/\cos^2 x}+...+n^{1/\cos^2 x}\right]^{\cos^2 x}$ is

(a) 0 (b) n

(c) ∞ (d) $\frac{n(n+1)}{2}$

24. $\lim\limits_{n\to\infty}\frac{1}{n}\left(1+e^{1/n}+e^{2/n}+...+e^{\frac{n-1}{n}}\right)$ is equal to

(a) e (b) –e

(c) e – 1 (d) 1 – e

25. The values of a, b and c such that

$\lim\limits_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$ are

(a) a = 1, b = –2, c = 1 (b) a = 1, b = 2, c = –1

(c) a = 1, b = 2, c = 1 (d) a = –1, b = 2, c = 1

26. $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin x-(\sin x)^{\sin x}}{1-\sin x+\ln\sin x}$ equals

(a) 1 (b) 2

(c) 3 (d) 4

27. $\text{Limit}_{x\to 0} \dfrac{e^{-x^2/2}-\cos x}{x^3\sin x}=$

(a) 1/4 (b) 1/6

(c) 1/12 (d) 1/8

28. $\text{Limit}_{x\to 0} \sin^{-1}(\sec x)$

(a) is equal to $\pi/2$ (b) is equal to 1

(c) is equal to zero (d) none of these

29. $\text{Lim}_{x\to 0}|x(x-1)|^{[\cos 2x]}$, where [.] denotes greatest integer function, is equal to

(a) 1 (b) 0

(c) e (d) Does not exists

30. If $\text{Lim}_{x\to 0}\dfrac{x^3}{\sqrt{a+x}(bx-\sin x)}=1, a>0$, then $a+2b$ is equal to

(a) 36 (b) 37

(c) 38 (d) 40

31. $\lim\limits_{x\to 0}\dfrac{\sin x^4-x^4\cos x^4+x^{20}}{x^4\left(e^{2x^4}-1-2x^4\right)}$ is equal to

(a) 0 (b) –1/6

(c) 1/6 (d) does not exist

32. If $f(n+1)=\dfrac{1}{2}\left\{f(n)+\dfrac{9}{f(n)}\right\}$, $n\in N$ and $f(n)>0$ for all $n\in N$ then $\lim\limits_{n\to\infty} f(n)$ is equal to

(a) 3 (b) – 3

(c) 1/2 (d) None of these

33. The value of $\lim\limits_{x\to 0}\left(\left[\dfrac{100x}{\sin x}\right]+\left[\dfrac{99\sin x}{x}\right]\right)$, where [.] represents greatest integral function is

(a) 199 (b) 198

(c) 0 (d) None of these

34. The limiting value of $(\cos x)^{1/\sin x}$ as $x\to 0$ is

(a) 1 (b) e

(c) 0 (d) none

MULTIPLE CHOICE QUESTIONS

35. If $\lim\limits_{x\to 0}(\cos x+a\sin bx)^{1/x}=e^2$, then the values of a and b are

(a) $a=1, b=2$ (b) $a=2, b=1/2$

(c) $a=2\sqrt{2},\ b=\dfrac{1}{\sqrt{2}}$ (d) $a=4, b=2$

36. If $\lim\limits_{x\to 0}\left(\dfrac{a^x+b^x+c^x}{3}\right)^{\lambda/x}$, $(a, b, c, \lambda>0)$ is equal to

(a) 1, if $\lambda=1$ (b) abc, if $\lambda=1$

(c) abc, if $\lambda=3$ (d) $(abc)^{2/3}$, if $\lambda=2$

37. The value of a for which

$$\lim_{x\to 0}\frac{(e^x-1)^4}{\sin\left(\frac{x^2}{a^2}\right)\log_e\left\{1+\frac{x^2}{2}\right\}}=8,\text{ is}$$

(a) –2 (b) –1

(c) 1 (d) 2

38. If $\lim\limits_{x\to 0}\dfrac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then

(a) a = 1 (b) b = 2

(c) c = –2 (d) c = 0

39. If $\lim\limits_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}=1$, then

(a) $b=\dfrac{-3}{2}$ (b) $a=\dfrac{5}{2}$

(c) $b=\dfrac{-1}{2}$ (d) $a=\dfrac{-5}{2}$

40. If x is a real number in [0, 1] then the value of $\lim_{m\to\infty}\lim_{n\to\infty}[1+\cos^{2m}(n!\pi x)]$ is given by, where [x] represents greatest integer $\le$ x.

(a) 2 if x is rational

(b) 1 for all x

(c) 1 if x is irrational (d) 2 for all x

41. The limit of sequence $\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}$ .... is

(a) a rational number (b) 2

(c) is an irrational number (d) $2\sqrt{2}$

Using the following passage, solve Q.42 to Q.44

Passage –1

AP is a diameter of a unit circle with centre at O. Let AC be an arc of this circle, which subtends angle θ radian at centre O. A tangent line is drawn to the circle at the point A and a segement AB on this tangent is laid off whose length is equal to that of the arc AC. A straight line BC is drawn to intersect the extension of the diameter AP at Q. CD is the perpendicular let fall from the point C upon the diameter AP.

42. The area of the trapezoid ABCD is

(a) $\frac{1-\cos\theta}{\theta-\sin\theta}$ (b) $(\theta+\sin\theta)\sin^2\frac{\theta}{2}$

(c) $2\cos^2\frac{\theta}{2}(\theta-\sin\theta)$ (d) $\theta(\theta+\sin\theta)$

43. The length AQ equal to

(a) $\frac{\theta(1-\cos\theta)}{\theta-\sin\theta}$ (b) $\frac{\theta(1-\cos\theta)}{\theta+\sin\theta}$

(c) $\frac{\theta(1+\cos\theta)}{\theta-\sin\theta}$ (d) $\frac{\theta(1+\cos\theta)}{\theta+\sin\theta}$

44. The value of the $\lim_{\theta\to 0^+} AQ$ is

(a) 0 (b) 1

(c) 2 (d) 3

Using the following passage, solve Q.45 to Q.47

Passage –2

Consider two functions $f(x)=\lim_{n\to\infty}\left(\cos\frac{x}{\sqrt{n}}\right)^n$ and $g(x)=-x^{4b}$ where $b=\lim_{x\to\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2+1}\right)$ Then.

45. $f(x)$ is

(a) e^{-x^2} (b) $e^{\frac{-x^2}{2}}$

(c) e^{x^2} (d) $e^{\frac{x^2}{2}}$

46. $g(x)$ is

(a) $-x^2$ (b) x^2

(c) x^4 (d) $-x^4$

47. Number of solutions of $f(x)+g(x)=0$ is

(a) 2 (b) 4

(c) 0 (d) 1

ASSERTION REASON

(A) If ASSERTION is true, REASON is true, REASON is a correct explanation for ASSERTION.

(B) If ASSERTION is true, REASON is true, REASON is not a correct explanation for ASSERTION.

(C) If ASSERTION is true, REASON is false.

(D) If ASSERTION is false, REASON is true.

48. **Assertion** : If a and b are positive and [x] denotes greatest integer $\le$ x, then

$$\lim_{x\to 0^+}\frac{x}{a}\left[\frac{b}{x}\right]=\frac{b}{a}$$

Reason : $\lim_{x\to\infty}\frac{\{x\}}{x}=0$ where {x} denotes fractional part of x.

(a) A (b) B

(c) C (d) D

(e) E

49. **Assertion** : For the existence of $\lim_{x\to a}\frac{f(x)}{g(x)}$. It is necessary that $\lim_{x\to a}\frac{f'(x)}{g'(x)}$ exists.

Reason : If $f(a)=g(a)=0$ then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}.$$

(a) A (b) B

(c) C (d) D

(e) E

Match the Following

50. **Column–I** **Column–II**

(A) $\underset{x\to\infty}{\text{Lt}}\ x\cos\frac{\pi}{8x}.\sin\frac{\pi}{8x}=$ (P) $\frac{\pi}{8}$

(B) $\underset{x\to 0}{\text{Lt}}\ \frac{\tan\left[-\pi^2\right]x^2-\left[-\pi^2\right]x^2}{\sin^2(x)}=$ (Q) $\sqrt{2}$

(C) $\underset{x\to\infty}{\text{Lt}}\sqrt{\frac{2x-\sin x+\cos x}{x+\cos^2 x+\sin^2 x}}=$ (R) $\frac{8}{\pi}$

(D) $\underset{x\to 1}{\text{Lt}}\left(\frac{x^n-1}{n(x-1)}\right)^{\frac{1}{x-1}}=$ (S) $e^{\frac{n-1}{2}}$

(T) 0

Subjective

51. The value of $\underset{x\to 0}{\text{Lim}}\frac{\sin x-\tan x}{\tan^{-1}x-\sin^{-1}x}$ is

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

Single Answers Questions

1. If $f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2$, then the value of

$\lim_{x \to a} \frac{g(x) f(a) - g(a) f(x)}{x - a}$ is **(1983)**

(a) -5 (b) $\frac{1}{5}$

(c) 5 (d) None of these

2. $\lim_{n \to \infty} \left\{ \frac{1}{1-n^2} + \frac{2}{1-n^2} + ... \frac{n}{1-n^2} \right\}$ is equal to : **(1984)**

(a) 0 (b) $-\frac{1}{2}$

(c) $\frac{1}{2}$ (d) none of these

3. If $f(x) = \begin{cases} \frac{\sin [x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$ where $[x]$ denotes the greatest integer less than or equal to x, then $\lim_{x \to 0} f(x)$ equals **(1985)**

(a) 1 (b) 0

(c) -1 (d) None of these

4. The value of $\lim_{x \to 0} \frac{\sqrt{\frac{1}{2}(1 - \cos^2 x)}}{x}$ **(1991)**

(a) 1 (b) -1

(c) 0 (d) None of these

5. $\lim_{x \to 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x - 1}$ **(1998)**

(a) exists and it equals $\sqrt{2}$

(b) exists and it equals $-\sqrt{2}$

(c) does not exist because $x - 1 \to 0$

(d) does not exist because left hand limit is not equal to right hand limit

6. $\lim_{x \to 0} \frac{x \tan 2x - 2x \tan x}{(1 - \cos 2x)^2}$ **(1999)**

(a) 2 (b) -2

(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

7. For $x \in R$, $\lim_{x \to \infty} \left(\frac{x-3}{x+2} \right)^x$ is equal to **(2000)**

(a) e (b) e^{-1}

(c) e^{-5} (d) e^5

8. $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ equals **(2001)**

(a) $-\pi$ (b) π

(c) $\pi/2$ (d) 1

9. The integer n for which $\lim_{x \to 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is a finite non-zero number, is **(2002)**

(a) 1 (b) 2

(c) 3 (d) 4

10. Let $f : R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$, then

$\lim_{x \to 0} \left(\frac{f(1+x)}{f(1)} \right)^{1/x}$ equals : **(2002)**

(a) 1 (b) $e^{\frac{1}{2}}$

(c) e^2 (d) e^3

11. If $\lim_{x \to 0} \frac{\{(a-n) nx - \tan x\} \sin nx}{x^2} = 0$, where n is non zero real number, then a is equal to **(2003)**

(a) 0 (b) $\frac{n+1}{n}$

(c) n (d) $n + \frac{1}{n}$

12. The value of $\lim\limits_{x\to 0}((\sin x)^{1/x}+(1/x)^{\sin x})$, where $x>0$ is **(2006)**

(a) 0 (b) –1
(c) 1 (d) 2

13. If $\lim\limits_{x\to 0}[1+x\log(1+b^2)]^{\frac{1}{x}}=2b\sin^2\theta$, $b>0$ and $\theta\in(-\pi,\pi]$, then the value of θ is **(2011)**

(a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$
(c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

14. If $\lim\limits_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then **(2012)**

(a) $a=1, b=4$ (b) $a=1, b=-4$
(c) $a=2, b=-3$ (d) $a=2, b=3$

15. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation

$$\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0,$$

where $a>-1$. Then, $\lim\limits_{a\to 0^+}\alpha(a)$ and $\lim\limits_{a\to 0^+}\beta(a)$ are **(2012)**

(a) $-\frac{5}{2}$ and 1 (b) $-\frac{1}{2}$ and -1
(c) $-\frac{7}{2}$ and 2 (d) $-\frac{9}{2}$ and 3

16. $\lim\limits_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to **(2013)**

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

Multiple Answers Questions

17. Let $L=\lim\limits_{x\to 0}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}$, $a>0$. If L is finite, them **(2009)**

(a) $a=2$ (b) $a=1$
(c) $L=\frac{1}{64}$ (d) $L=\frac{1}{32}$

18. Let $f(x)=\frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x\neq 1$. then **(2017)**

(a) $\lim_{x\to 1^+}f(x)=0$
(b) $\lim_{x\to 1^-}f(x)=0$
(c) $\lim_{x\to 1^+}f(x)$ does not exist
(d) $\lim_{x\to 1^-}f(x)$ does not exist

Fill in the Blanks

19. $\lim\limits_{x\to 1}(1-x)\tan\frac{\pi x}{2}=\ldots$ **(1978)**

20. $\lim\limits_{x\to -\infty}\left[\frac{\left(x^4\sin\left(\frac{1}{x}\right)+x^2\right)}{(1+|x|^3)}\right]=\ldots.$ **(1987)**

(1988)

21. ABC is an isosceles triangle inscribed in a circle of radius r. If AB = AC and h is the altitude from A to BC, then the triangle ABC has perimeter $P=2(\sqrt{2hr-h^2}+\sqrt{2hr})$ and area A = ... Also, $\lim\limits_{h\to 0}\frac{A}{P^3}=\ldots.$ **(1989)**

22. $\lim\limits_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}=\ldots..$ **(1991)**

23. $\lim\limits_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2}=\ldots.$ **(1996)**

24. $\lim_{h\to 0}\frac{\log(1+2h)-2\log(1+h)}{h^2}=\ldots$ **(1997)**

25. For each positive integer *n*, let

$y_n=\frac{1}{n}\left((n+1)(n+2)\ldots(n+n)\right)^{\frac{1}{n}}$.

For $x\in\mathbb{R}$, let [x] be the greatest integer less than or equal to *x*. If $\lim_{n\to\infty}y_n=L$, then the value of [L] is ______.

(2018)

True/False

26. If $\lim_{x\to a}[f(x)g(x)]$ exists, then both $\lim_{x\to a}f(x)$ and $\lim_{x\to a}g(x)$ exist. **(1981)**

Subjective Problems

27. Evaluate the following limit $\lim_{x\to 1}\left(\frac{x-1}{2x^2-7x+5}\right)$ **(1978)**

28. Evaluate $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$ **(1978)**

29. Evaluate $\lim_{x\to 0}\sqrt{\frac{x-\sin x}{x+\cos^2 x}}$ **(1979)**

30. Evaluate $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$ **(1980)**

31. Use the formula $\lim_{x\to 0}\frac{a^x-1}{x}=\log_e a$, to find

$\lim_{x\to 0}\frac{2^x-1}{(1+x)^{1/2}-1}$ **(1982)**

32. Find $\lim_{x\to 0}\{\tan(\frac{\pi}{4}+x)\}^{1/x}$. **(1993)**

33. The largest value of the non-negative integer a for which

$\lim_{x\to 1}\left\{\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}\right\}^{\frac{1-x}{1-\sqrt{x}}}=\frac{1}{4}$ is **(2014)**

34. Let m and n be two positive integers greater than 1. If

$\lim_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$ then the value of $\frac{m}{n}$ is **(2015)**

35. Let $\alpha,\beta\in R$ be such that $\lim_{x\to 0}\frac{x^2\sin(\beta x)}{\alpha x-\sin x}=1$. Then $6(\alpha+\beta)$ equals **(2016)**

ANSWER KEY

EXERCISE - 1 : BASIC OBJECTIVE QUESTIONS

1. (a)	2. (d)	3. (b)	4. (a)	5. (b)	6. (d)	7. (a)	8. (b)	9. (a)	10. (c)
11. (a)	12. (c)	13. (d)	14. (a)	15. (b)	16. (b)	17. (d)	18. (a)	19. (a)	20. (b)
21. (b)	22. (c)	23. (d)	24. (b)	25. (b)	26. (c)	27. (c)	28. (c)	29. (b)	30. (a)
31. (a)	32. (b)	33. (d)	34. (b)	35. (c)	36. (b)	37. (a)	38. (d)	39. (b)	40. (c)
41. (b)	42. (d)	43. (d)	44. (c)	45. (a)	46. (a)	47. (c)	48. (b)	49. (d)	50. (b)
51. (c)	52. (c)	53. (d)	54. (d)	55. (b)	56. (b)	57. (a)	58. (a)	59. (a)	60. (b)

EXERCISE - 2 : PREVIOUS YEAR JEE MAINS QUESTIONS

1. (d)	2. (a)	3. (c)	4. (c)	5. (c)	6. (c)	7. (b)	8. (d)	9. (a)	10. (d)
11. (a)	12. (d)	13. (d)	14. (a)	15. (d)	16. (a)	17. (d)	18. (c)	19. (b)	20. (b)
21. (a)	22. (b)	23. (b)	24. (d)	25. (c)	26. (c)				

EXERCISE - 3 : ADVANCED OBJECTIVE QUESTIONS

1. (c)	2. (b)	3. (b)	4. (b)	5. (b)	6. (d)	7. (c)	8. (c)	9. (d)	10. (d)
11. (a)	12. (b)	13. (c)	14. (d)	15. (d)	16. (d)	17. (a)	18. (c)	19. (b)	20. (a)
21. (b)	22. (b)	23. (b)	24. (c)	25. (c)	26. (b)	27. (c)	28. (d)	29. (a)	30. (c)
31. (c)	32. (a)	33. (b)	34. (a)	35. (a,c)	36. (c,d)	37. (a,d)	38. (a,b)	39. (a,d)	40. (a,c)
41. (a,b)	42. (b)	43. (a)	44. (d)	45. (b)	46. (a)	47. (a)	48. (a)	49. (e)	

50. $(A \to P,\ B \to T,\ C \to Q,\ D \to S)$ 51. 0001

EXERCISE - 4 : PREVIOUS YEAR JEE ADVANCED QUESTIONS

1. (c)	2. (b)	3. (d)	4. (d)	5. (d)	6. (c)	7. (c)	8. (b)	9. (c)	10. (c)
11. (d)	12. (c)	13. (d)	14. (b)	15. (b)	16. (d)	17. (a,c)	18. (b,c)	19. $\frac{2}{\pi}$	20. –1

21. $h\left(\sqrt{2hr-h^2}\right), \frac{1}{128r}$ 22. e^5 23. e^2 24. –1 25. (1) 26. False 27. $-\frac{1}{3}$ 28. $\frac{2}{\pi}$ 29. 0

30. $a^2 \cos a + 2a \sin a$ 31. $2\ell n 2$ 32. e^2 33. (2) 34. (0002) 35. (7)

Dream on !!