9. Circle Measures

Questions Pg-157

1. Question

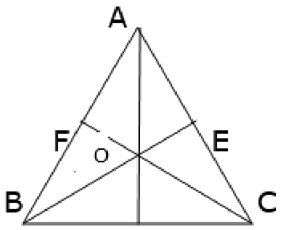
Prove that the circumcentre of an equilateral triangle is the same as its centroid.

i) Calculate the length of side of an equilateral triangle with vertices on a circle of diameter 1 centimetre.

ii) Calculate the perimeter of such a triangle.

Answer

The figure is given below:



Let ABC be a triangle

AD, BE and CF are medians intersecting at point O.

In triangles ABD and ACD,

BD = CD (AD is the median)

$\angle ABD = \angle ACD = 60°$

AB = AC (Equilateral Triangle)

∴ triangles ABD and ACD are congruent.

∴ $\angle BAD = \angle CAD = \frac{60}{2} = 30°$

In triangle ABD

Sum of all angles of a triangle = 180°

∴ $\angle ABD + \angle DAB + \angle ADB = 180°$

∴ $60° + 30° + \angle ADB = 180°$

$90° + \angle ADB = 180°$

∴ $\angle ADB = 180° - 90° = 90°$

∴ $\angle ADC = 180 - \angle ADB$ (Supplementary angles)

$= 180 - 90°$

$= 90°$

Similarly,

$\angle OBD = \angle OCD = 30°$ …(eq)1

In triangles OBD and OCD,

$\angle OBD = \angle OCD = 30°$ (from eq 1)

$\angle ADB = \angle ADC = 90°$

OD = OC is a common side

By RHS criterion,

Triangles OBD and OCD are congruent.

$\therefore$ OB = OC

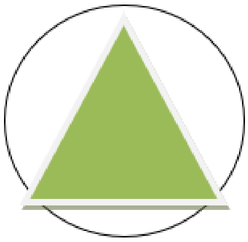
Similarly, OC = OA and OA = OB

$\therefore$ OA = OB = OC

Hence, pt. O is a circumcentre and also a intersection of medians.

Hence, the circumcentre of an equilateral triangle is the same

as its centroid.

i) 

Diameter of circle = 1 centimetre

$\therefore$ Radius of circle = $\frac{\text{diameter}}{2} = \frac{1}{2}$ cm = 0.5 cm

As proved above, the circumcentre of an equilateral triangle is the same as its centroid.

Also, centroid divides equilateral in the ratio 2:1

$\therefore$ radius $= \frac{2}{3} \times$ height of triangle

$\therefore 0.5 = \frac{2}{3} \times$ height of triangle

$\therefore$ height of triangle $= \frac{3}{2} \times 0.5 = \frac{3}{4}$

But, for an equilateral triangle,

Height of triangle $= \frac{\sqrt{3}}{2} \times$ side

$\frac{3}{4} = \frac{\sqrt{3}}{2} \times$ side

$\therefore$ side of triangle $= \frac{3}{4} \times \frac{2}{\sqrt{3}} = \frac{\sqrt{3}}{2}$

ii) Perimeter of triangle = Sum of all sides of triangle

As for an equilateral triangle, all sides are equal

Let a = side of triangle $= \frac{\sqrt{3}}{2}$

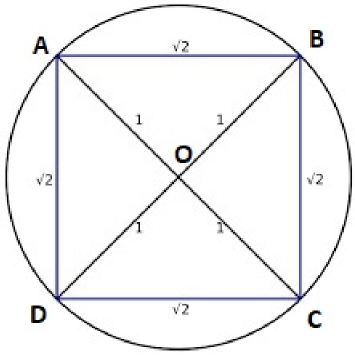
Perimeter of triangle = a + a + a = 3a $= 3 \times \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$ cm

2. Question

Calculate the perimeter of a square with vertices on a circle of diameter 1 centimetre.

Answer

The figure is shown below:



The square ABCD is inscribed within the circle.

Diameter of circle, DB = 1 cm

∴ Radius of circle = $\frac{\text{diameter}}{2} = \frac{1}{2}$ cm = 0.5 cm

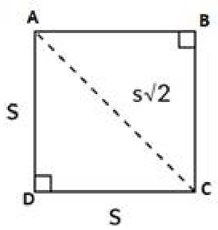
As seen in figure,

Diameter of circle = side of square

∴ the diagonal of the square = 1 cm

For a square,

If side = “s” cm, then,



Diagonal of square, $AC^2 = s^2 + s^2$

⇒ $AC^2 = 2s^2$

⇒ AC = √2 $\sqrt{2a} = 1$

∴ a = $\frac{1}{\sqrt{2}}$

Perimeter of a square = 4 × Side of square

$= 4 \times \frac{1}{\sqrt{2}}$

$= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

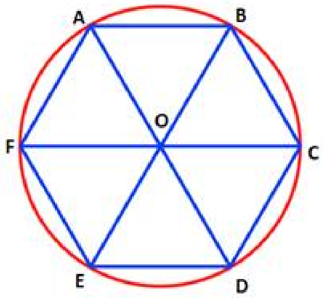
$= 4 \times \frac{\sqrt{2}}{2} = 2\sqrt{2}\text{ cm}$

3. Question

Calculate perimeter of a regular hexagon with vertices on a circle of diameter 1 centimetre.

Answer

The figure is shown below:



Diameter of circle, CF = 1 centimetre

∴ Radius of circle = $\frac{\text{diameter}}{2}$ = $\frac{1}{2}$ cm = 0.5 cm

As can be seen from figure,

Radius of circle = Side of Hexagon

∴ Side of Hexagon = 0.5 cm

Perimeter of Hexagon = 6 × Side of Hexagon

= 6 × 0.5

= 3 cm

Questions Pg-160

1. Question

The perimeter of a regular hexagon with vertices on a circle is 24 centimetres.

i) What is the perimeter of a square with vertices on this circle?

ii) What is the perimeter of a square with vertices on a circle of double the diameter?

iii) What is the perimeter of an equilateral triangle with vertices on a circle of half the diameter of the first circle?

Answer

Given:

Perimeter of Hexagon = 24 cm

Let a = side of Hexagon

But, Perimeter of Hexagon = 6 × side of Hexagon

24 = 6 × a

∴ a = 24/6=4

As can be seen from figure,

Radius of circle = Side of Hexagon

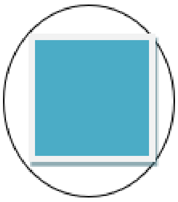
∴ Radius of circle = a = 4 cm

i) Diameter of circle = 2 × radius

= 2 × 4

= 8 cm

As seen in figure,



Diameter of circle = Diagonal of square

Diagonal of square = 8 cm

For a square,

If side = a cm

Diagonal of square = $\sqrt{2}a$ = 8

$\therefore a = \frac{8}{\sqrt{2}}$

Perimeter of square = 4 × side of square

$= 4 \times \frac{8}{\sqrt{2}}$

$= \frac{32}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{32\sqrt{2}}{2} = 16\sqrt{2}$ cm

ii) Diameter of circle =2 × Initial diameter

=2 × 8=16 cm

As seen in figure,

Diameter of circle = Diagonal of square

Diagonal of square = 16 cm

For a square,

If side = a cm

Diagonal of square = $\sqrt{2}a$ = 16

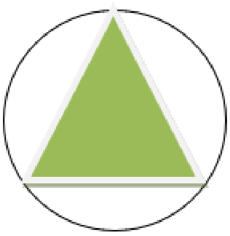
$\therefore a = \frac{16}{\sqrt{2}}$

Perimeter of square = 4 × side of square

$= 4 \times \frac{16}{\sqrt{2}}$

$= \frac{64}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{64\sqrt{2}}{2} = 32\sqrt{2}$ cm

iii) 

Diameter of circle =1/2 × Initial diameter

$=\frac{1}{2} \times 8 = 4$ cm

∴ Radius of circle $= \frac{\text{diameter}}{2} = \frac{4}{2}$ cm = 2 cm

As proved earlier, the circumcentre of an equilateral triangle is the same as its centroid.

Also, centroid divides equilateral in the ratio 2:1

∴ radius $= \frac{2}{3} \times$ height of triangle

∴ 2 $= \frac{2}{3} \times$ height of triangle

∴ height of triangle $= \frac{3}{2} \times 2 = 3$

But, for an equilateral triangle,

Height of triangle $= \frac{\sqrt{3}}{2} \times$ side

$3 = \frac{\sqrt{3}}{2} \times$ side

∴ side of triangle $= 3 \times \frac{2}{\sqrt{3}} = \frac{6}{\sqrt{3}}$

Perimeter of equilateral triangle = $3 \times$ side of triangle

$=3 \times \frac{6}{\sqrt{3}}$

$=\frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$=6\sqrt{3}$ cm

2. Question

A wire was bent into a circle of diameter 4 centimetres. What would be the diameter of a circle made by bending a wire of half the length?

Answer

Diameter of circle = 4 cm

Length of wire = Circumference of circle

If r = radius of circle

Circumference of circle = 2 × π × r=2 × π × 4

=8 × π

If length of wire is halved,

Circumference is also halved

∴ New Circumference = 1/2 × (8 × π)= 4 × π

Let r' be new radius

∴ 2 × π × r'= New Circumference = 4 × π

∴ $r' = 4 \times \frac{\pi}{2 \times \pi} = 2$

Hence, New diameter = 2 × r'= 2 × 2=4 cm

3. Question

The perimeter of a circle of diameter 2 metres was measured and found to be about 6.28 metres. How do we compute the perimeter of a circle of diameter 3 metres, without measuring?

Answer

If r = radius of circle

Perimeter = Circumference of circle = 2 × π × r

When, diameter=2 m

Radius = $\frac{\text{Diameter}}{2} = \frac{2}{2} = 1\text{ m}$

Perimeter = 2 × π × 1=6.28 ...(eq)1

When, diameter=3 m

Radius = $\frac{\text{Diameter}}{2} = \frac{3}{2} = 1.5\text{ m}$

Perimeter = 2 × π × 1.5=1.5 × (2 × π × 1)

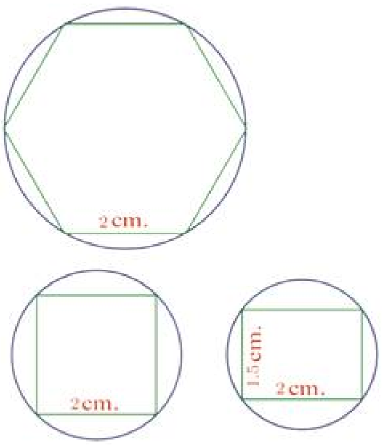
= 1.5 × 6.28 ...from (eq)1

= 9.42 m

Questions Pg-163

1. Question

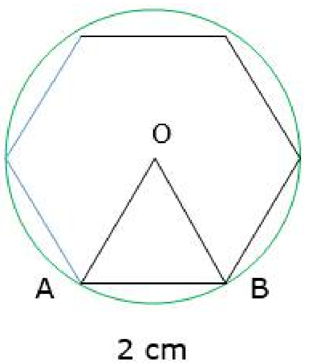
In the picture below, a regular hexagon, square and a rectangle are drawn with their vertices on a circle. Calculate the perimeter of each circle.



Answer

<u>Case (i) for regular hexagon:</u>

In the figure given below:



Side of regular hexagon = 2 cm

In this figure,

Radius of circle = Side of hexagon = 2cm

In triangle OAB,

∠OAB = ∠OBA = 60° (OA = OB = radius = 2 cm)

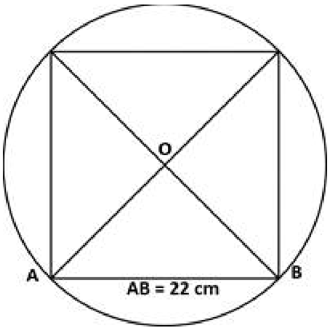
If r = radius of circle

Circumference = 2 × π × r

Hence, perimeter of each circle = 2 × π × 2≈4 × 3.14 = 12.56 cm

Case (ii) for square:

The figure is shown below:



In triangle OAB,

OA = OB = radius

In triangle OAB,

∠OAB = ∠OBA = 45°

Sum of all angles of a triangle = 180°

∠OAB + ∠OBA + ∠AOB = 180°

45° + 45° + ∠AOB = 180°

90° + ∠AOB = 180°

∴ ∠AOB = 180-90 = 90°

By Pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{One side})^2 + (\text{Other side})^2$

$(AB)^2 = (OA)^2 + (OB)^2$

$2^2 = 2 \times (OA)^2$ (OA = OB)

$\therefore (OA)^2 = \frac{4}{2} = 2$

OA = √2 = radius

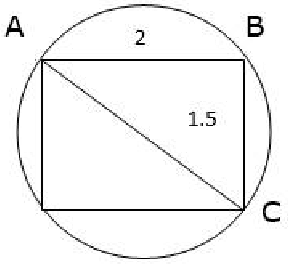
If r = radius of circle

Circumference = 2 × π × r

Hence, perimeter of each circle = 2 × π × √2

≈4 × 3.14 = 8.87 cm

Case (iii) for rectangle drawn within the circle. The figure is displayed below:



In triangle ABC,

∠ABC = 90° (angle subtended in the semicircle is right angle).

By pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{One side})^2 + (\text{Other side})^2$

$\therefore (AC)^2 = (AB)^2 + (AC)^2$

$\therefore (AC)^2 = 2^2 + 1.5^2$

$\therefore (AC)^2 = 4 + 2.25 = 6.25$

$\therefore AC = \sqrt{6.25} = 2.5 =$ Diameter of circle.

$\therefore \text{Radius} = \frac{\text{Diameter}}{2} = \frac{2.5}{2} = 1.25$

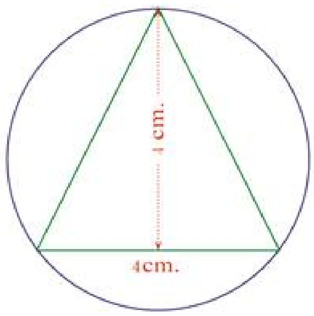
If r = radius of circle

Circumference $= 2 \times \pi \times r$

$\therefore \text{Perimeter} = 2 \times \pi \times r \approx 2 \times 3.14 \times 1.25 = 7.85$ cm

2. Question

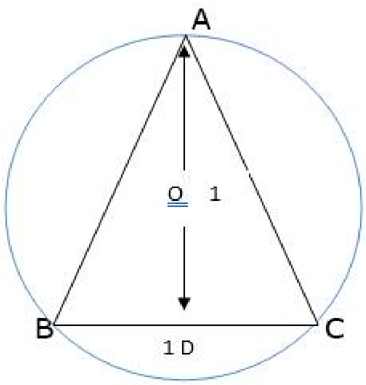
An isosceles triangle with its vertices on a circle is shown in this picture.



What is the perimeter of the circle?

Answer

The figure is shown below:



Let OA = radius of circle.

AD = 1 cm

OA = r

∴ OD = 1-r

OA = OB = r

In triangle OBD,

∠ODB = 90°

AB = AC and AD is perpendicular on BC from point A.

∴ BD = DC = $\frac{1}{2}$ cm

∴ By pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{One side})^2 + (\text{Other side})^2$

∴ $(OB)^2 = (OD)^2 + (BD)^2$

∴ $r^2 = (1-r)^2 + (\frac{1}{2})^2$

∴ $r^2 - (1-r)^2 = \frac{1}{4}$

∴ $(r + \{1-r\})(r-\{1-r\}) = \frac{1}{4}$ $\{a^2-b^2 = (a + b) \times (a-b)\}$

∴ $1 \times (2r-1) = \frac{1}{4}$

∴ $2r = 1 + \frac{1}{4} = \frac{5}{4}$

∴ $r = \frac{1}{2} \times \frac{5}{4} = \frac{5}{8}$

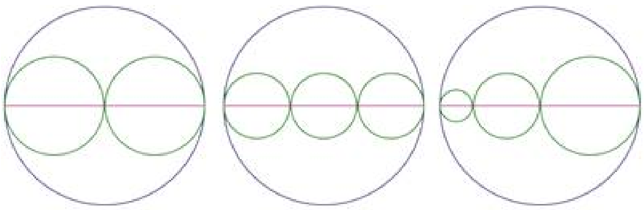
If r = radius of circle

Circumference = $2 \times \pi \times r$

∴ Perimeter = $2 \times \pi \times r \approx 2 \times 3.14 \times \frac{5}{8} = 3.925$ cm

3. Question

In all the pictures, below, centres of the circles are on the same line. In the first two pictures, the small circles are of the same diameter.



Prove that in all pictures, the perimeters of the large circle is the sum of the perimeters of the small circles.

Answer

Let R = radius of larger circle in all figures.

If r = radius of circle

Circumference = $2 \times \pi \times r$

∴ Perimeter of large circle = $2 \times \pi \times R$ ...(eq)1

In first figure,

Diameter of larger circle = 2 × Diameter of smaller circle(as seen from figure)

Hence,

Diameter of smaller circle = $\frac{1}{2} \times$ Diameter of larger circle

∴ Radius of smaller circle = $\frac{1}{2} \times$ Radius of larger circle (Radius = $\frac{1}{2} \times$ Diameter)

Let r = Radius of smaller circle

∴ $r = \frac{R}{2}$

Perimeter of a smaller circle = 2 × π × r

$= 2 \times \pi \times \frac{R}{2} = \pi \times R$

Perimeter of 2 smaller circles = 2 × (π × R) = …(eq)1

Hence, proved.

In second figure,

Diameter of larger circle = 3 × Diameter of smaller circle(as seen from figure)

Hence,

Diameter of smaller circle = $\frac{1}{3} \times$ Diameter of larger circle

∴ Radius of smaller circle = $\frac{1}{3} \times$ Radius of larger circle (Radius = $\frac{1}{2} \times$ Diameter)

Let r = Radius of smaller circle

∴ $r = \frac{R}{3}$

Perimeter of a smaller circle = 2 × π × r

$= 2 \times \pi \times \frac{R}{3} R$

Perimeter of 3 smaller circles = $3 \times (\frac{2}{3} \times \pi \times R)$ = …(eq)1

Hence, proved.

In 3rd figure,

There are 3 small circles.

Right half has radius = $\frac{R}{2}$

Perimeter of right half = $2 \times \pi \times \frac{R}{2} = \pi \times R$ …(eq)2

The left half's radius is divided in the ratio 1:2

Hence, radius of smallest circle = $\frac{1}{3} \times \frac{R}{2} = \frac{R}{6}$

Hence, radius of middle circle = $\frac{2}{3} \times \frac{R}{2} = \frac{R}{3}$

Perimeter of smallest circle = $2 \times \pi \times \frac{R}{6} = \pi \times \frac{R}{3}$…(eq)3

Perimeter of middle circle = $2 \times \pi \times \frac{R}{3}$…(eq)4

Sum of perimeter of all circles = (eq)2 + (eq)3 + (eq)4

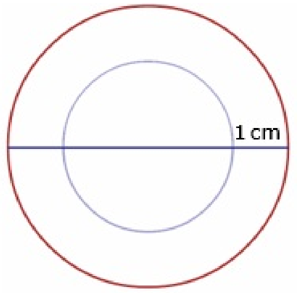
$= (\pi \times R) + (\pi \times \frac{R}{3}) + (2 \times \pi \times \frac{R}{3})$

$= \pi \times R$ ...(same as (eq)1)

Hence, proved.

4. Question

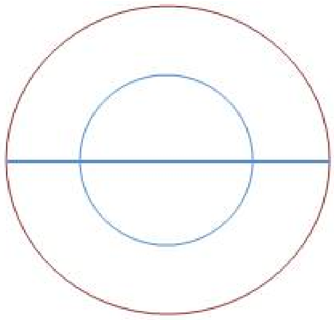
In this picture, the circles have the same centre and the line drawn is a diameter of the large circle. How much more is the perimeter of the large circle than the perimeter of the small circle?



Answer

The figure is shown below:

11 cm



Diameter of larger circle = 1 cm

Hence, diameter of smaller circle = $\frac{1}{2}$ cm

Let r = Radius of smaller circle = $\frac{1}{2} \times \frac{1}{2}\text{ cm} = \frac{1}{4}\text{ cm}$

If r = radius of circle

Circumference = $2 \times \pi \times r$

As seen from figure,

Radius of Larger circle = 2r

Perimeter of smaller circle = 2 × π × r ...(eq)2

Perimeter of larger circle = 2 × π × (2r) = 4 × π × r ...(eq)1

Perimeter of larger circle is = (eq)1 – (eq)2

More then smaller circle by

$= (4 \times \pi \times r) - (2 \times \pi \times r)$

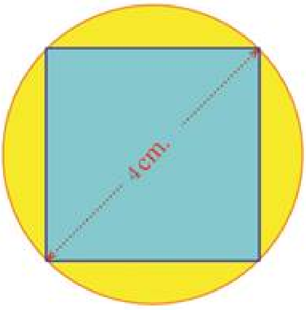
$= 2 \times \pi \times r$

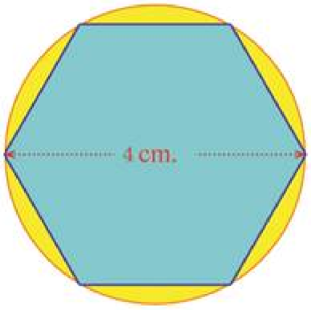
$\approx 2 \times 3.14 \times \frac{1}{4} = 1.57\text{ cm}$

Questions Pg-167

1. Question

In the pictures, below, find the difference between the areas of the circle and the polygon, up to two decimal places:





Answer

As seen in figure 1,

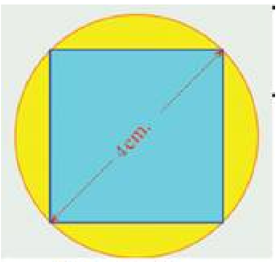


Figure 1

Diameter of circle = Diagonal of square

∴ Diameter of circle = 4 cm

∴ Radius of circle = $\frac{4}{2} = 2$ cm

If r = radius of circle

Area of circle = $\pi \times r^2$ cm^2

= $\pi \times 2^2$

= $4 \times \pi$ cm^2

As seen in figure 2,

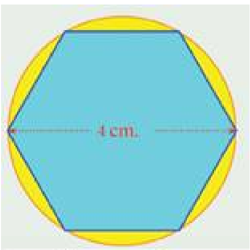


Figure 2

Diameter of circle = 4 cm

∴ Radius of circle = $\frac{4}{2} = 2$ cm

As can be seen from figure 2,

Radius of circle = Side of Hexagon

∴ Side of Hexagon = 2 cm

The regular hexagon is made of 6 equilateral triangles.

Area of polygon (here, regular hexagon) = 6 × Area of triangle

Side of Hexagon = Side of triangle = 2 cm

Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times 2^2$

$= \frac{\sqrt{3}}{4} \times 4 = \sqrt{3}\ \text{cm}^2$

Area of polygon (here, regular hexagon) = 6 × Area of triangle

$= 6 \times \sqrt{3}\ \text{cm}^2$

∴ Difference between = Area of circle – Area of polygon

Areas

= (4 × π)-(6 × √3)

$= 4 \times (3.14159) - 6 \times (1.73205)$

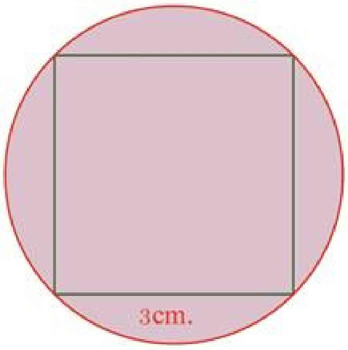
= 12.56636-10.3923

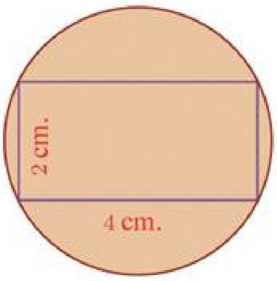
= 2.17406

≈2.17(upto 2 decimal places).

2. Question

The pictures below show circles through the vertices of a square and a rectangle:





Calculate the areas of the circles.

Answer

For figure 1,

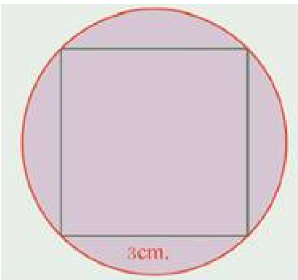


Figure 1

Side of square = 3 cm

If, Side of square = a cm

Diagonal of square = $\sqrt{2} \times a = \sqrt{2} \times 3$ cm

As seen in figure,

Diameter of circle = Diagonal of square

∴ Diameter of circle = $\sqrt{2} \times 3$ cm

∴ Radius of circle = $\frac{\sqrt{2} \times 3}{2} = \frac{3}{\sqrt{2}}$ cm

If r = radius of circle

Area of circle = $\pi \times r^2$ cm^2

$= \pi \times (\frac{3}{\sqrt{2}})^2$

$= \pi \times \frac{9}{2} cm^2$

$\approx 3.14 \times 4.5$

≈ 14.13 cm^2

For figure 2,

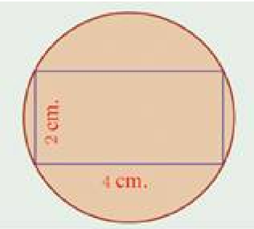


Figure 2

From Pythagoras theorem,

$(\text{Hypotenuse})^2 = (\text{one side})^2 + (\text{other side})^2$

Diagonal of rectangle = $(4^2 + 2^2)^{1/2}$

$= (16 + 4)^{1/2}$

$= \sqrt{20}$ cm

As seen from figure,

Diagonal of rectangle = Diameter of circle = $\sqrt{20}$ cm

∴ Radius of circle = $\frac{\sqrt{20}}{2}$ cm

If r = radius of circle

Area of circle = $\pi \times r^2$ cm^2

$= \pi \times (\frac{20}{\sqrt{2}})^2$

$= \pi \times \frac{400}{2} cm^2$

$\approx 3.1415 \times 200$

≈ 628.3 cm^2

Questions Pg-173

1. Question

In a circle, the length of an arc of central angle 40° is 3π centimetres. What is the perimeter of the circle? What is its radius?

Answer

X = 40°

Length of arc = 3cm

In a circle of radius r,

If central angle = x°

Length of arc = $\frac{x}{360} \times 2 \times \pi \times r$ cm

Putting in above equation,

$3 = \frac{40}{360} \times 2 \times \pi \times r$

$\therefore 3 = \frac{1}{9} \times 2 \times \pi \times r$

$\therefore 9 \times \frac{3}{2 \times \pi} = r$

$\therefore r \approx 3$ cm

If r = radius of circle,

Perimeter = $2 \times \pi \times r$

Hence, Perimeter = $2 \times \pi \times 3 \approx 18.84$ cm

2. Question

In the same circle, what is the length of an arc of central angle 25° is 4 centimetres.

i) In the same circle, what is the length of an arc of central angle 75°?

ii) In a circle of radius one and a half times the radius of this circle, what is the length of an arc of central angle 75°?

Answer

In a circle of radius r,

If central angle = x°

Length of arc = $\frac{x}{360} \times 2 \times \pi \times r$ cm

Here,

x = 25°

r = 4 cm

Length of arc = $\frac{25}{360} \times 2 \times \pi \times r$

$\approx \frac{5}{36} \times 3.14 \times 4$

≈ 1.74 cm

i) Here,

x = 75°

r = 4 cm

Length of arc = $\frac{75}{360} \times 2 \times \pi \times r$

$\approx \frac{15}{36} \times 3.14 \times 4 \approx 5.22$ cm

ii) Here,

x = 25°

r = 1.5 × 4 = 6 cm

Length of arc = $\frac{25}{360} \times 2 \times \pi \times r$

$\approx \frac{5}{36} \times 3.14 \times 6 \approx 2.62$ cm

3. Question

From a bangle of radius 3 centimetres, a piece is to be cut out to make a ring of radius $\frac{1}{2}$ centimetres.

i) What should be the central angle of the piece to be cut out?

ii) The remaining part of the bangle was bent to make a smaller bangle. What is its radius?

Answer

i) Radius of bangle = 3 cm

Radius of ring = $\frac{1}{2}$ cm

If r = radius of circle

Circumference of circle = 2 × π × r cm

∴ Length of ring = 2 × π × $\frac{1}{2}$ = π cm

Hence, length of arc = π cm

In a circle of radius r,

If central angle = x°

Length of arc = $\frac{x}{360} \times 2 \times \pi \times r$ cm

Here r = 3 cm

$\pi = \frac{x}{360} \times 2 \times \pi \times 3$

$\therefore \frac{\pi \times 360}{2 \times \pi \times 3} = x$

$\therefore x = \frac{360}{6} = 60°$

ii) Length of remaining part = Length of bangle-Length of smaller part

= (2 × π × 3)-(π)

= 5 × π ≈15.7 cm

Let a be the new radius

Since,

Circumference = 2 × π × a

15.7 = 2 × π × a

$\therefore a = \frac{15.7}{2 \times \pi} \approx 2.5$ cm

Questions Pg-176

1. Question

What is the area of a sector of central angle 120° in a circle of radius 3 centimetres? What is the area of a sector of the same central angle in a circle of radius 6 centimetres?

Answer

Radius = r = 3 cm

If x° is the angle at subtended at the sector,

Area of sector = $\frac{x}{360} \times \pi \times r^2$

Here, x = 120°

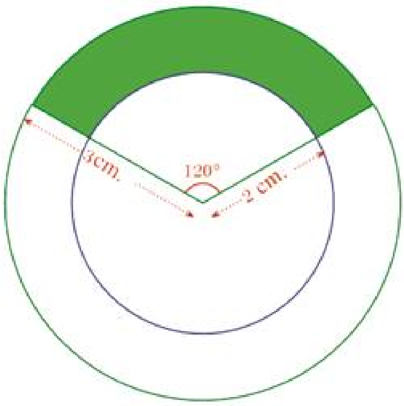
∴ Area = $\frac{120}{360} \times \pi \times 3^2 \approx 9 \times \frac{3.14}{3} \approx 9.42\ cm^2$

If r_2 = 6 cm

∴ Area = $\frac{120}{360} \times \pi \times 6^2 = \approx 36 \times \frac{3.14}{3} \approx 37.68\ cm^2$

2. Question

Calculate the area of the shaded part of this picture.



Answer

Inner radius = r = 2 cm

Outer radius = R = 4 cm

Area of shaded part = Area of larger sector –Area of smaller sector

If x° is the angle at subtended at the sector,

Area of sector = $\frac{x}{360} \times \pi \times r^2$

Here, x = 120°

Area of larger sector = $\frac{120}{360} \times 4^2 \times \pi = \frac{16}{3} \times \pi$

Area of smaller sector = $\frac{120}{360} \times 2^2 \times \pi = \frac{4}{3} \times \pi$

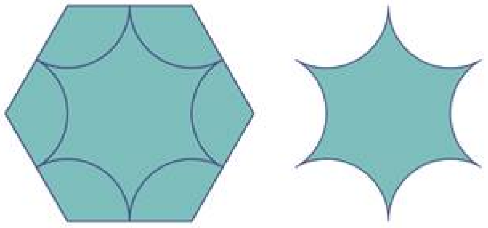
Area of shaded part = Area of larger sector –Area of smaller sector

$= \frac{16}{3} \times \pi - \frac{4}{3} \times \pi$

$\approx 12.56\ cm^2$

3. Question

Centred at each corner of a regular hexagon, a part of a circle is drawn and a figure is cut out as shown below:



What is the area of this figure?

Answer

As seen from the figure,

Radius of each arc = $\frac{2}{2} = 1$ cm

For a regular polygon with n sides,

Sum of angles = (n-2)× 180°

Here, n = 6

∴ Sum of angles = (6-2)× 180

= 4× 180

= 720°

Hence, each angle = $\frac{720}{6} = 120°$

Thus for each sector,

X = 120°

r = 1 cm

If x° is the angle at subtended at the sector,

Area of sector = $\frac{x}{360} \times \pi \times r^2$

∴ Area of sector = $\frac{120}{360} \times \pi \times 1^2 = \frac{\pi}{3}$ cm^2

Area of 6 sectors = $6 \times \frac{\pi}{3} = 2\pi$ cm^2

As the hexagon is made up of 6 equilateral triangles,

Area of polygon = 6× Area of equilateral triangle with side as 2 cm

If a = side

Then area of triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$

Here, side = 2 cm

∴ area of a triangle = $\frac{\sqrt{3}}{4} \times (2)^2 = \frac{\sqrt{3}}{4} \times 4 = \sqrt{3}$ cm^2

∴ area of polygon = 6× √3 = 6√3 cm^2

∴ area of shaded region = area of polygon-area of 6 sectors

= 6√3 – 2π = 4.112 cm^2