

Applications of Integrals



TOPIC 1

Curve & X-axis Between two Ordinates, Area of the Region Bounded by a Curve & Y-axis Between two Abscissa



- The area (in sq. units) of the region $A = \{(x, y) : (x-1)[x] \leq y \leq 2\sqrt{x}, 0 \leq x \leq 2\}$, where $[t]$ denotes the greatest integer function, is :
[Sep. 05, 2020 (II)]
(a) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (b) $\frac{4}{3}\sqrt{2} + 1$
(c) $\frac{8}{3}\sqrt{2} - 1$ (d) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$
- The area (in sq. units) of the region $\{(x, y) : 0 \leq y \leq x^2 + 1, 0 \leq y \leq x + 1, \frac{1}{2} \leq x \leq 2\}$ is :
[Sep. 03, 2020 (I)]
(a) $\frac{23}{16}$ (b) $\frac{79}{24}$ (c) $\frac{79}{16}$ (d) $\frac{23}{6}$
- Given: $f(x) = \begin{cases} x & , 0 \leq x < \frac{1}{2} \\ \frac{1}{2} & , x = \frac{1}{2} \\ 1-x & , \frac{1}{2} < x \leq 1 \end{cases}$
and $g(x) = \left(x - \frac{1}{2}\right)^2$, $x \in \mathbb{R}$. Then the area (in sq. units) of the region bounded by the curves, $y = f(x)$ and $y = g(x)$ between the lines, $2x = 1$ and $2x = \sqrt{3}$, is :
[Jan. 9, 2020 (II)]
(a) $\frac{1}{3} + \frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{4} - \frac{1}{3}$ (c) $\frac{1}{2} - \frac{\sqrt{3}}{4}$ (d) $\frac{1}{2} + \frac{\sqrt{3}}{4}$
- The area (in sq. units) of the region $A = \{(x, y) \in \mathbb{R} \times \mathbb{R} \mid 0 \leq x \leq 3, 0 \leq y \leq 4, y \leq x^2 + 3x\}$ is :
[April 8, 2019 (I)]
(a) $\frac{53}{6}$ (b) 8 (c) $\frac{59}{6}$ (d) $\frac{26}{3}$
- The area of the region $A = \{(x, y) : 0 \leq y \leq x|x| + 1 \text{ and } -1 \leq x \leq 1\}$ in sq. units is :
[Jan. 09, 2019 (II)]
(a) $\frac{2}{3}$ (b) 2 (c) $\frac{4}{3}$ (d) $\frac{1}{3}$
- Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (g \circ f)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is :
[2018]
(a) $\frac{1}{2}(\sqrt{3} + 1)$ (b) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$
(c) $\frac{1}{2}(\sqrt{2} - 1)$ (d) $\frac{1}{2}(\sqrt{3} - 1)$
- Let $f : [-2, 3] \rightarrow [0, \infty)$ be a continuous function such that $f(1-x) = f(x)$ for all $x \in [-2, 3]$.
If R_1 is the numerical value of the area of the region bounded by $y = f(x)$, $x = -2$, $x = 3$ and the axis of x and $R_2 = \int_{-2}^3 x f(x) dx$, then :
[Online April 25, 2013]
(a) $3R_1 = 2R_2$ (b) $2R_1 = 3R_2$
(c) $R_1 = R_2$ (d) $R_1 = 2R_2$
- Let $f(x)$ be a non-negative continuous function such that the area bounded by the curve $y = f(x)$, x -axis and the ordinates $x = \frac{\pi}{4}$ and $x = \beta > \frac{\pi}{4}$ is $\left(\beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2}\beta\right)$. Then $f\left(\frac{\pi}{2}\right)$ is :
[2005]

(a) $\left(\frac{\pi}{4} + \sqrt{2} - 1\right)$ (b) $\left(\frac{\pi}{4} - \sqrt{2} + 1\right)$

(c) $\left(1 - \frac{\pi}{4} - \sqrt{2}\right)$ (d) $\left(1 - \frac{\pi}{4} + \sqrt{2}\right)$

9. The area enclosed between the curve $y = \log_e(x+e)$ and the coordinate axes is [2005]

(a) 1 (b) 2 (c) 3 (d) 4

10. If $y = f(x)$ makes +ve intercept of 2 and 0 unit on x and y axes and encloses an area of $3/4$ square unit with the axes

then $\int_0^2 xf'(x)dx$ is

[2002]

(a) $3/2$ (b) 1 (c) $5/4$ (d) $-3/4$

TOPIC 2

Different Cases of Area Bounded Between the Curves



11. The area (in sq. units) of the region $A = \{(x, y) : |x| + |y| \leq 1, 2y^2 \geq |x|\}$ is : [Sep. 06, 2020 (I)]

(a) $\frac{1}{3}$ (b) $\frac{7}{6}$ (c) $\frac{1}{6}$ (d) $\frac{5}{6}$

12. The area (in sq. units) of the region enclosed by the curves $y = x^2 - 1$ and $y = 1 - x^2$ is equal to: [Sep. 06, 2020 (II)]

(a) $\frac{4}{3}$ (b) $\frac{8}{3}$ (c) $\frac{7}{2}$ (d) $\frac{16}{3}$

13. Consider a region $R = \{(x, y) \in \mathbf{R}^2 : x^2 \leq y \leq 2x\}$. If a line $y = \alpha$ divides the area of region R into two equal parts, then which of the following is true? [Sep. 02, 2020 (II)]

(a) $\alpha^3 - 6\alpha^2 + 16 = 0$ (b) $3\alpha^2 - 8\alpha^{3/2} + 8 = 0$
(c) $3\alpha^2 - 8\alpha + 8 = 0$ (d) $\alpha^3 - 6\alpha^{3/2} - 16 = 0$

14. The area of the region, enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and the straight line $y = x$, is:

[Jan. 7, 2020 (I)]

(a) $(24\pi - 1)$ (b) $(6\pi - 1)$
(c) $(12\pi - 1)$ (d) $(12\pi - 1)/6$

15. The area (in sq. units) of the region $\{(x, y) \in \mathbf{R}^2 : 4x^2 \leq y \leq 8x + 12\}$ is: [Jan. 7, 2020 (II)]

(a) $\frac{125}{3}$ (b) $\frac{128}{3}$ (c) $\frac{124}{3}$ (d) $\frac{127}{3}$

16. For $a > 0$, let the curves $C_1: y^2 = ax$ and $C_2: x^2 = ay$ intersect at origin O and a point P . Let the line $x = b$ ($0 < b < a$) intersect the chord OP and the x-axis at points Q and R , respectively. If the line $x = b$ bisects the area bounded by

the curves, C_1 and C_2 , and the area of $\Delta OQR = \frac{1}{2}$, then

'a' satisfies the equation:

[Jan. 8, 2020 (I)]

(a) $x^6 - 6x^3 + 4 = 0$ (b) $x^6 - 12x^3 + 4 = 0$
(c) $x^6 + 6x^3 - 4 = 0$ (d) $x^6 - 12x^3 - 4 = 0$

17. The area (in sq. units) of the region $\{(x, y) \in \mathbf{R}^2 : x^2 \leq y \leq |3 - 2x|\}$ is: [Jan. 8, 2020 (II)]

(a) $\frac{32}{3}$ (b) $\frac{34}{3}$ (c) $\frac{29}{3}$ (d) $\frac{31}{3}$

18. If the area (in sq. units) of the region $\{(x, y) : y^2 \leq 4x, x + y \leq 1, x \geq 0, y \geq 0\}$ is $a\sqrt{2} + b$, then $a - b$ is equal to : [April 12, 2019 (I)]

(a) $\frac{10}{3}$ (b) 6 (c) $\frac{8}{3}$ (d) $-\frac{2}{3}$

19. If the area (in sq. units) bounded by the parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$, $\lambda > 0$, is $\frac{1}{9}$, then λ is equal to :

[April 12, 2019 (II)]

(a) $2\sqrt{6}$ (b) 48 (c) 24 (d) $4\sqrt{3}$

20. The region represented by $|x - y| \leq 2$ and $|x + y| \leq 2$ is bounded by a : [April 10, 2019 (I)]

(a) square of side length $2\sqrt{2}$ units
(b) rhombus of side length 2 units
(c) square of area 16 sq. units
(d) rhombus of area $8\sqrt{2}$ sq. units

21. The area (in sq. units) of the region bounded by the curves $y = 2^x$ and $y = |x + 1|$, in the first quadrant is :

[April 10, 2019 (II)]

(a) $\log_e 2 + \frac{3}{2}$ (b) $\frac{3}{2}$
(c) $\frac{1}{2}$ (d) $\frac{3}{2} - \frac{1}{\log_e 2}$

22. The area (in sq. units) of the region $A = \{(x, y) : x^2 \leq y \leq x + 2\}$ is: [April 9, 2019 (I)]

(a) $\frac{10}{3}$ (b) $\frac{9}{2}$ (c) $\frac{31}{6}$ (d) $\frac{13}{6}$

23. The area (in sq. units) of the region

$A = \{(x, y) : \frac{y^2}{2} \leq x \leq y + 4\}$ is: [April 09, 2019 (II)]

(a) $\frac{53}{3}$ (b) 30 (c) 16 (d) 18

24. Let $S(\alpha) = \{(x, y) : y^2 \leq x, 0 \leq x \leq \alpha\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a λ , $0 < \lambda < 4$, $A(\lambda) : A(\alpha) = 2 : 5$, then λ equals : [April 08, 2019 (II)]

(a) $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ (b) $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$

(c) $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$ (d) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$

25. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is :
[Jan. 12, 2019 (I)]
- (a) $\frac{15}{4}$ (b) $\frac{21}{2}$ (c) $\frac{17}{4}$ (d) $\frac{15}{2}$
26. The area (in sq. units) of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is :
[Jan. 11, 2019 (I)]
- (a) $\frac{5}{4}$ (b) $\frac{9}{8}$ (c) $\frac{7}{8}$ (d) $\frac{3}{4}$
27. The area (in sq. units) in the first quadrant bounded by the parabola, $y = x^2 + 1$, the tangent to it at the point (2, 5) and the coordinate axes is :
[Jan. 11, 2019 (II)]
- (a) $\frac{8}{3}$ (b) $\frac{37}{24}$ (c) $\frac{187}{24}$ (d) $\frac{14}{3}$
28. If the area enclosed between the curves $y = kx^2$ and $x = ky^2$, ($k > 0$), is 1 square unit. Then k is:
[Jan. 10, 2019 (I)]
- (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\frac{2}{\sqrt{3}}$
29. The area (in sq. units) bounded by the parabola $y = x^2 - 1$, the tangent at the point (2, 3) to it and the y -axis is:
[Jan. 9, 2019 (I)]
- (a) $\frac{8}{3}$ (b) $\frac{32}{3}$ (c) $\frac{56}{3}$ (d) $\frac{14}{3}$
30. If the area of the region bounded by the curves, $y = x^2$, $y = \frac{1}{x}$ and the lines $y = 0$ and $x = t$ ($t > 1$) is 1 sq. unit, then t is equal to
[Online April 16, 2018]
- (a) $\frac{4}{3}$ (b) $e^{2/3}$ (c) $\frac{3}{2}$ (d) $e^{3/2}$
31. The area (in sq. units) of the region $\{x \in R : x \geq 0, y \geq 0, y \geq x - 2 \text{ and } y \leq \sqrt{x}\}$, is
[Online April 15, 2018]
- (a) $\frac{13}{3}$ (b) $\frac{10}{3}$ (c) $\frac{5}{3}$ (d) $\frac{8}{3}$
32. The area (in sq. units) of the region $\{(x, y) : x \geq 0, x + y \leq 3, x^2 \leq 4y \text{ and } y \leq 1 + \sqrt{x}\}$ is :
[2017]
- (a) $\frac{5}{2}$ (b) $\frac{59}{12}$ (c) $\frac{3}{2}$ (d) $\frac{7}{3}$
33. The area (in sq. units) of the smaller portion enclosed between the curves, $x^2 + y^2 = 4$ and $y^2 = 3x$, is :
[Online April 8, 2017]
- (a) $\frac{1}{2\sqrt{3}} + \frac{\pi}{3}$ (b) $\frac{1}{\sqrt{3}} + \frac{2\pi}{3}$
- (c) $\frac{1}{2\sqrt{3}} + \frac{2\pi}{3}$ (d) $\frac{1}{\sqrt{3}} + \frac{4\pi}{3}$
34. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is :
[2016]
- (a) $\pi - \frac{4\sqrt{2}}{3}$ (b) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$
- (c) $\pi - \frac{4}{3}$ (d) $\pi - \frac{8}{3}$
35. The area (in sq. units) of the region described by $A = \{(x, y) | y \geq x^2 - 5x + 4, x + y \geq 1, y \leq 0\}$ is:
[Online April 9, 2016]
- (a) $\frac{19}{6}$ (b) $\frac{17}{6}$ (c) $\frac{7}{2}$ (d) $\frac{13}{6}$
36. The area (in sq. units) of the region described by $\{(x, y) : y^2 \leq 2x \text{ and } y \geq 4x - 1\}$ is
[2015]
- (a) $\frac{15}{64}$ (b) $\frac{9}{32}$ (c) $\frac{7}{32}$ (d) $\frac{5}{64}$
37. The area (in square units) of the region bounded by the curves $y + 2x^2 = 0$ and $y + 3x^2 = 1$, is equal to :
[Online April 10, 2015]
- (a) $\frac{3}{5}$ (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{3}{4}$
38. The area of the region described by $A = \{(x, y) : x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$ is:
[2014]
- (a) $\frac{\pi}{2} - \frac{2}{3}$ (b) $\frac{\pi}{2} + \frac{2}{3}$ (c) $\frac{\pi}{2} + \frac{4}{3}$ (d) $\frac{\pi}{2} - \frac{4}{3}$
39. The area of the region above the x -axis bounded by the curve $y = \tan x$, $0 \leq x \leq \frac{\pi}{2}$ and the tangent to the curve at $x = \frac{\pi}{4}$ is:
[Online April 19, 2014]
- (a) $\frac{1}{2} \left(\log 2 - \frac{1}{2} \right)$ (b) $\frac{1}{2} \left(\log 2 + \frac{1}{2} \right)$
- (c) $\frac{1}{2} (1 - \log 2)$ (d) $\frac{1}{2} (1 + \log 2)$
40. Let $A = \{(x, y) : y^2 \leq 4x, y - 2x \geq -4\}$. The area (in square units) of the region A is:
[Online April 9, 2014]
- (a) 8 (b) 9 (c) 10 (d) 11
41. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x -axis, and lying in the first quadrant is :
[2013]
- (a) 9 (b) 36 (c) 18 (d) $\frac{27}{4}$

42. The area under the curve $y = |\cos x - \sin x|$, $0 \leq x \leq \frac{\pi}{2}$, and above x -axis is : **[Online April 23, 2013]**
 (a) $2\sqrt{2}$ (b) $2\sqrt{2} - 2$
 (c) $2\sqrt{2} + 2$ (d) 0
43. The area of the region (in sq. units), in the first quadrant bounded by the parabola $y = 9x^2$ and the lines $x = 0$, $y = 1$ and $y = 4$, is : **[Online April 22, 2013]**
 (a) $\frac{7}{9}$ (b) $\frac{14}{3}$ (c) $\frac{7}{3}$ (d) $\frac{14}{9}$
44. The area bounded by the curve $y = \ln(x)$ and the lines $y = 0$, $y = \ln(c)$ and $x = 0$ is equal to : **[Online April 9, 2013]**
 (a) 3 (b) $3 \ln(c) - 2$
 (c) $3 \ln(c) + 2$ (d) 2
45. The area between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line $y = 2$ is : **[2012]**
 (a) $20\sqrt{2}$ (b) $\frac{10\sqrt{2}}{3}$ (c) $\frac{20\sqrt{2}}{3}$ (d) $10\sqrt{2}$
46. The area bounded by the parabola $y^2 = 4x$ and the line $2x - 3y + 4 = 0$, in square unit, is **[Online May 26, 2012]**
 (a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) 1 (d) $\frac{1}{2}$
47. The area of the region bounded by the curve $y = x^3$, and the lines, $y = 8$, and $x = 0$, is **[Online May 19, 2012]**
 (a) 8 (b) 12 (c) 10 (d) 16
48. If a straight line $y - x = 2$ divides the region $x^2 + y^2 \leq 4$ into two parts, then the ratio of the area of the smaller part to the area of the greater part is **[Online May 12, 2012]**
 (a) $3\pi - 8 : \pi + 8$ (b) $\pi - 3 : 3\pi + 3$
 (c) $3\pi - 4 : \pi + 4$ (d) $\pi - 2 : 3\pi + 2$
49. The area enclosed by the curves $y = x^2$, $y = x^3$, $x = 0$ and $x = p$, where $p > 1$, is $\frac{1}{6}$. The p equals **[Online May 12, 2012]**
 (a) $\frac{8}{3}$ (b) $\frac{16}{3}$ (c) 2 (d) $\frac{4}{3}$
50. The parabola $y^2 = x$ divides the circle $x^2 + y^2 = 2$ into two parts whose areas are in the ratio **[Online May 7, 2012]**
 (a) $9\pi + 2 : 3\pi - 2$ (b) $9\pi - 2 : 3\pi + 2$
 (c) $7\pi - 2 : 2\pi - 3$ (d) $7\pi + 2 : 3\pi + 2$
51. The area bounded by the curves **[2011 RS]**
 $y^2 = 4x$ and $x^2 = 4y$ is:
 (a) $\frac{32}{3}$ sq units (b) $\frac{16}{3}$ sq units
 (c) $\frac{8}{3}$ sq. units (d) 0 sq. units
52. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive x -axis is **[2011]**
 (a) 1 square unit (b) $\frac{3}{2}$ square units
 (c) $\frac{5}{2}$ square units (d) $\frac{1}{2}$ square unit
53. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is **[2010]**
 (a) $4\sqrt{2} + 2$ (b) $4\sqrt{2} - 1$
 (c) $4\sqrt{2} + 1$ (d) $4\sqrt{2} - 2$
54. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent of the parabola at the point $(2, 3)$ and the x -axis is: **[2009]**
 (a) 6 (b) 9 (c) 12 (d) 3
55. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to **[2008]**
 (a) $\frac{5}{3}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$
56. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is **[2007]**
 (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 1
57. The parabolas $y^2 = 4x$ and $x^2 = 4y$ divide the square region bounded by the lines $x = 4$, $y = 4$ and the coordinate axes. If S_1 , S_2 , S_3 are respectively the areas of these parts numbered from top to bottom; then $S_1 : S_2 : S_3$ is **[2005]**
 (a) $1 : 2 : 1$ (b) $1 : 2 : 3$ (c) $2 : 1 : 2$ (d) $1 : 1 : 1$
58. The area of the region bounded by the curves $y = |x - 2|$, $x = 1$, $x = 3$ and the x -axis is **[2004]**
 (a) 4 (b) 2 (c) 3 (d) 1
59. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is **[2003]**
 (a) 6 sq. units (b) 2 sq. units
 (c) 3 sq. units (d) 4 sq. units.
60. The area bounded by the curves $y = \ln x$, $y = \ln |x|$, $y = |\ln x|$ and $y = |\ln |x||$ is **[2002]**
 (a) 4sq. units (b) 6 sq. units
 (c) 10 sq. units (d) none of these

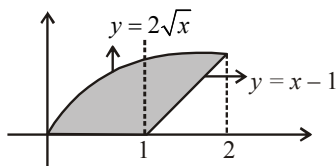


Hints & Solutions



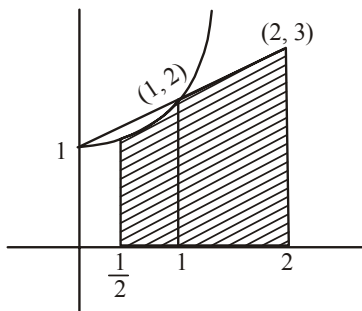
1. (a) $[x] = 0$ when $x \in [0, 1)$ and $[x] = 1$ when $x \in [1, 2)$

$$y = \begin{cases} 0 & 0 \leq x < 1 \\ x-1 & 1 \leq x < 2 \end{cases}$$



$$\begin{aligned} \therefore A &= \int_0^2 2\sqrt{x} \, dx - \frac{1}{2}(1)(1) \\ &= \frac{4x^{3/2}}{3} \Big|_0^2 - \frac{1}{2} = \frac{8\sqrt{2}}{3} - \frac{1}{2} \end{aligned}$$

2. (b)



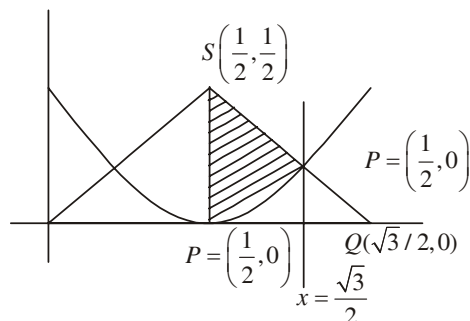
$$\text{Required area} = \int_{1/2}^1 (x^2 + 1) \, dx + \int_1^2 (x + 1) \, dx$$

$$= \left[\frac{x^3}{3} + x \right]_{1/2}^1 + \left[\frac{x^2}{2} + x \right]_1^2$$

$$= \left[\frac{4}{3} - \frac{13}{24} \right] + \frac{5}{2} = \frac{79}{24}.$$

3. (b) Coordinates of $P\left(\frac{1}{2}, 0\right)$, $Q\left(\frac{\sqrt{3}}{2}, 0\right)$, $R\left(\frac{\sqrt{3}}{2}, 1 - \frac{\sqrt{3}}{2}\right)$

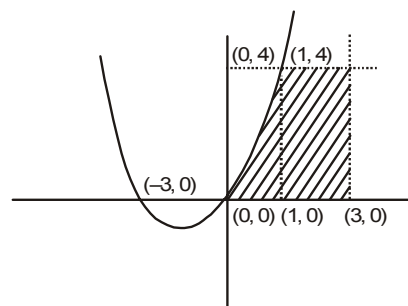
$$\text{and } S\left(\frac{1}{2}, \frac{1}{2}\right)$$



Required area = Area of trapezium $PQRS$

$$\begin{aligned} &= \int_{1/2}^{\sqrt{3}/2} \left(x - \frac{1}{2} \right)^2 \, dx \\ &= \frac{1}{2} \left(\frac{\sqrt{3}-1}{2} \right) \left(\frac{1}{2} + 1 - \frac{\sqrt{3}}{2} \right) - \frac{1}{3} \left(\left(x - \frac{1}{2} \right)^3 \right)_{1/2}^{\sqrt{3}/2} \\ &= \frac{\sqrt{3}}{4} - \frac{1}{3} \end{aligned}$$

4. (c) Since, the relation $y \leq x^2 + 3x$ represents the region below the parabola in the 1st quadrant



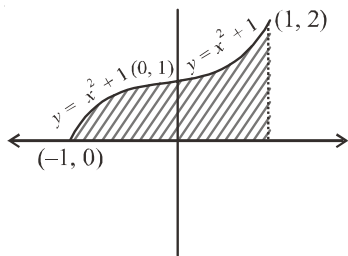
$$\therefore y = 4$$

$$\Rightarrow x^2 + 3x = 4 \Rightarrow x = 1, -4$$

$\therefore$ the required area = area of shaded region

$$\begin{aligned} &= \int_0^1 (x^2 + 3x) \, dx + \int_1^3 4 \, dx = \left[\frac{x^3}{3} + \frac{3x^2}{2} \right]_0^1 + [4x]_1^3 \\ &= \frac{1}{3} + \frac{3}{2} + 8 = \frac{59}{6} \end{aligned}$$

5. (b) Given $A = \{(x, y) : 0 \leq y \leq x|x| + 1 \text{ and } -1 \leq x \leq 1\}$



$\therefore$ Area of shaded region

$$= \int_{-1}^0 (-x^2 + 1) dx + \int_0^1 (x^2 + 1) dx$$

$$= \left(-\frac{x^3}{3} + x \right)_{-1}^0 + \left(\frac{x^3}{3} + x \right)_0^1$$

$$= 0 - \left(\frac{1}{3} - 1 \right) + \left(\frac{1}{3} + 1 \right) - (0 + 0)$$

$$= \frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2 \text{ square units}$$

6. (d) Here, $18x^2 - 9\pi x + \pi^2 = 0$
 $\Rightarrow (3x - \pi)(6x - \pi) = 0$

$$\Rightarrow \alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$$

Also, $\cos x = \cos x$

$$\therefore \text{Req. area} = \int_{\pi/6}^{\pi/3} \cos x dx = \frac{\sqrt{3} - 1}{2}$$

7. (d) We have

$$R_2 = \int_{-2}^3 x f(x) dx = \int_{-2}^3 (1-x) f(1-x) dx$$

$$\left[\text{Using } \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right]$$

$$\Rightarrow R_2 = \int_{-2}^3 (1-x) f(x) dx$$

$$(\because f(x) = f(1-x) \text{ on } [-2, 3])$$

$$\therefore R_2 + R_2 = \int_{-2}^3 x f(x) dx + \int_{-2}^3 (1-x) f(x) dx$$

$$= \int_{-2}^3 f(x) dx = R_1$$

$$\Rightarrow 2R_2 = R_1$$

8. (d) From given condition

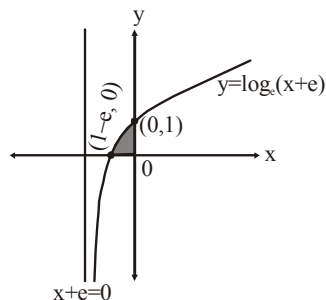
$$\int_{\pi/4}^{\beta} f(x) dx = \beta \sin \beta + \frac{\pi}{4} \cos \beta + \sqrt{2} \beta$$

Differentiating w. r. t β , we get

$$f(\beta) = \beta \cos \beta + \sin \beta - \frac{\pi}{4} \sin \beta + \sqrt{2}$$

$$f\left(\frac{\pi}{2}\right) = \beta \cdot 0 + \left(1 - \frac{\pi}{4}\right) \sin \frac{\pi}{2} + \sqrt{2} = 1 - \frac{\pi}{4} + \sqrt{2}$$

9. (a)



$$\text{Required area } A = \int_{1-e}^0 y dx = \int_{1-e}^0 \log_e(x+e) dx$$

put $x+e = t \Rightarrow dx = dt$ also when $x = 1-e$, $t = 1$ and when $x = 0$, $t = e$

$$\therefore A = \int_1^e \log_e t dt = [t \log_e t - t]_1^e$$

$$e - e - 0 + 1 = 1$$

Hence the required area is 1 square unit.

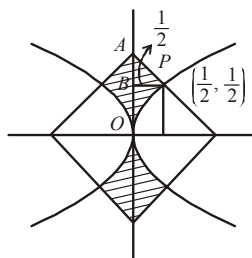
10. (d) Given that $\int_0^2 f(x) dx = \frac{3}{4}$; Now,

$$\int_0^2 x f'(x) dx = x \int_0^2 f'(x) dx - \int_0^2 f(x) dx$$

$$= [x f(x)]_0^2 - \frac{3}{4} = 2f(2) - \frac{3}{4}$$

$$= 0 - \frac{3}{4} (\because f(2) = 0) = -\frac{3}{4}$$

11. (d)

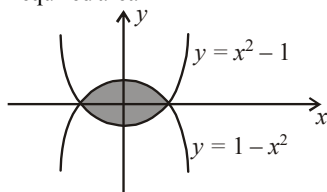


$$\text{Required area} = 4 \left[\int_0^{\frac{1}{2}} 2y^2 dy + \frac{1}{2} \text{area}(\Delta PAB) \right]$$

$$= 4 \left[\frac{2}{3} \left[y^3 \right]_0^{\frac{1}{2}} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \right] = 4 \left[\frac{2}{3} \times \frac{1}{8} + \frac{1}{8} \right]$$

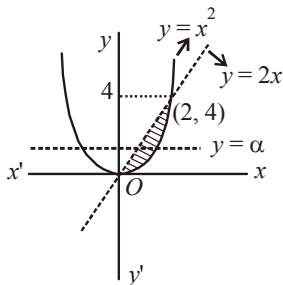
$$= 4 \times \frac{5}{24} = \frac{5}{6}$$

12. (b) Required area



$$\begin{aligned}\text{Area} &= 2 \int_0^1 \left((1-x^2) - (x^2-1) \right) dx \\ &= 4 \int_0^1 (1-x^2) dx \\ &= 4 \left(x - \frac{x^3}{3} \right) \Big|_0^1 = 4 \left(1 - \frac{1}{3} \right) = 4 \cdot \frac{2}{3} = \frac{8}{3} \text{ sq. units}\end{aligned}$$

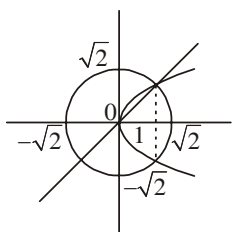
13. (b) Let
- $y = x^2$
- and
- $y = 2x$



According to question

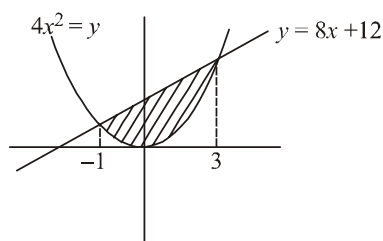
$$\begin{aligned}\therefore \int_0^\alpha \left(\sqrt{y} - \frac{y}{2} \right) dy &= \int_\alpha^4 \left(\sqrt{y} - \frac{y}{2} \right) dy \\ \Rightarrow \left[\frac{y^{3/2}}{3/2} \right]_0^\alpha - \left[\frac{y^2}{4} \right]_0^\alpha &= \left[\frac{y^{3/2}}{3/2} \right]_\alpha^4 - \left[\frac{y^2}{4} \right]_\alpha^4 \\ \Rightarrow \frac{2}{3} \alpha^{3/2} - \frac{\alpha^2}{4} &= \frac{2}{3} (8 - \alpha^{3/2}) - \frac{1}{4} (16 - \alpha^2) \\ \Rightarrow \frac{4}{3} \alpha^{3/2} - \frac{\alpha^2}{2} &= \frac{4}{3} \\ \Rightarrow 8\alpha^{3/2} - 3\alpha^2 &= 8 \\ \therefore 3\alpha^2 - 8\alpha^{3/2} + 8 &= 0\end{aligned}$$

14. (d) Total area – enclosed area between line and parabola



$$\begin{aligned}&= 2\pi - \int_0^1 \sqrt{x} - x dx \\ &= 2\pi - \left(\frac{2x^{3/2}}{3} - \frac{x^2}{2} \right) \Big|_0^1 \\ &= 2\pi - \left(\frac{2}{3} - \frac{1}{2} \right) = 2\pi - \left(\frac{1}{6} \right) = \frac{12\pi - 1}{6}\end{aligned}$$

15. (b)



Given curves are

$$\begin{aligned}4x^2 &= y & \dots(i) \\ y &= 8x + 12 & \dots(ii)\end{aligned}$$

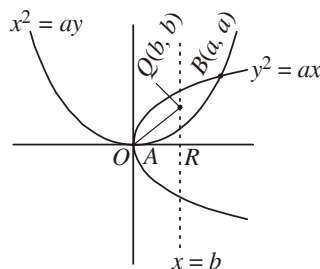
From eqns. (i) and (ii),

$$\begin{aligned}4x^2 &= 8x + 12 \\ \Rightarrow x^2 - x - 3 &= 0 \\ \Rightarrow x^2 - 2x - 3 &= 0 \\ \Rightarrow x^2 - 3x + x - 3 &= 0 \\ \Rightarrow (x+1)(x-3) &= 0 \\ \Rightarrow x &= -1, 3\end{aligned}$$

Required area bounded by curves is given by

$$\begin{aligned}A &= \int_{-1}^3 (8x + 12 - 4x^2) dx \\ A &= \left[\frac{8x^2}{2} + 12x - \frac{4x^3}{3} \right]_{-1}^3 \\ &= (4(9) + 36 - 36) - \left(4 - 12 + \frac{4}{3} \right) \\ &= 36 + 8 - \frac{4}{3} = 44 - \frac{4}{3} = \frac{132 - 4}{3} = \frac{128}{3}\end{aligned}$$

16. (b) Given eqns. are,
- $x^2 = ay$
- and
- $y^2 = ax$



After solving, we get $x = a, y = a$

Now, coordinates of B is (a, a) and A is $(0, 0)$

Now, coordinates of Q is (b, b)

$$\therefore \frac{1}{2}b^2 = \frac{1}{2} \Rightarrow b = 1$$

Area bounded by curves and $x = 1$ is

$$\int_0^1 \left(\sqrt{ax^{1/2}} - \frac{x^2}{a} \right) dx = \frac{1}{2} \int_0^a \left(\sqrt{ax^{1/2}} - \frac{x^2}{a} \right) dx$$

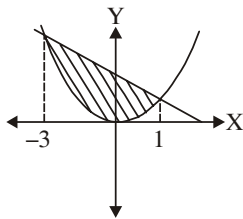
$$\Rightarrow \frac{2}{3}\sqrt{a} - \frac{1}{3a} = \frac{a^2}{6}$$

$$\Rightarrow 4a\sqrt{a} - 2 = a^3$$

$$\Rightarrow a^6 + 4a^3 + 4 = 16a^3$$

$$\Rightarrow a^6 - 12a^3 + 4 = 0$$

17. (a) Point of intersection of $y = x^2$ and $y = -2x + 3$ is obtained by $x^2 + 2x - 3 = 0$



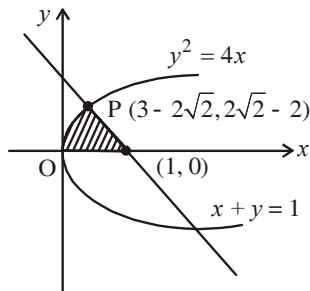
$$\Rightarrow x = -3, 1$$

$$\text{So, required area} = \int_{-3}^1 (\text{line} - \text{parabola}) dx$$

$$= \int_{-3}^1 (3 - 2x - x^2) dx = \left[3x - x^2 - \frac{x^3}{3} \right]_{-3}^1$$

$$= (3)4 - 2 \left(\frac{1^2 - 3^2}{2} \right) - \left(\frac{1^3 + 3^3}{3} \right) = 12 + 8 - \frac{28}{3} = \frac{32}{3}$$

18. (b) Consider $y^2 = 4x$ and $x + y = 1$



Substituting $x = 1 - y$ in the equation of parabola,

$$y^2 = 4(1 - y) \Rightarrow y^2 + 4y - 4 = 0$$

$$\Rightarrow (y + 2)^2 = 8 \Rightarrow y + 2 = \pm 2\sqrt{2}$$

Hence, required area

$$= \int_0^{3-2\sqrt{2}} 2\sqrt{x} dx + \frac{1}{2} \times (2\sqrt{2} - 2) \times (2\sqrt{2} - 2)$$

$$= \left[2 \times \frac{2}{3} x^{3/2} \right]_0^{3-2\sqrt{2}} + \frac{1}{2} (8 + 4 - 8\sqrt{2})$$

$$= \frac{4}{3} \times (3 - 2\sqrt{2}) \sqrt{3 - 2\sqrt{2}} + 6 - 4\sqrt{2}$$

$$= \frac{4}{3} (3 - 2\sqrt{2})(\sqrt{2} - 1) + 6 - 4\sqrt{2}$$

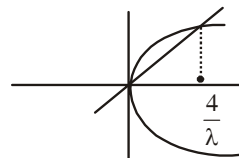
$$[\because (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2}]$$

$$= \frac{4}{3} (3\sqrt{2} - 3 - 4 + 2\sqrt{2}) + 6 - 4\sqrt{2}$$

$$= -\frac{10}{3} + \frac{8}{3}\sqrt{2} = a\sqrt{2} + b$$

$$\therefore a = 8/3 \text{ and } b = -10/3 \Rightarrow a - b = \frac{10}{3} + \frac{8}{3} = 6$$

19. (c) Given parabola $y^2 = 4\lambda x$ and the line $y = \lambda x$



Putting $y = \lambda x$ in $y^2 = 4\lambda x$, we get $x = 0, \frac{4}{\lambda}$

$$\therefore \text{required area} = \int_0^{\frac{4}{\lambda}} (2\sqrt{\lambda x} - \lambda x) dx$$

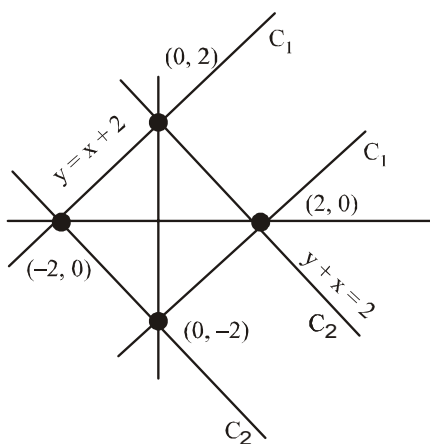
$$= \frac{2\sqrt{\lambda} x^{3/2}}{3/2} - \frac{\lambda x^2}{2} \Big|_0^{4/\lambda} = \frac{32}{3\lambda} - \frac{8}{\lambda}$$

$$= \frac{8}{3\lambda} = \frac{1}{9} \Rightarrow \lambda = 24$$

20. (a) Let, $C_1: |y - x| \leq 2$

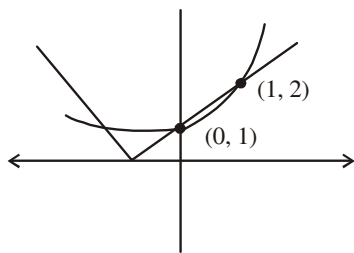
$$C_2: |y + x| \leq 2$$

By the diagram, region is square



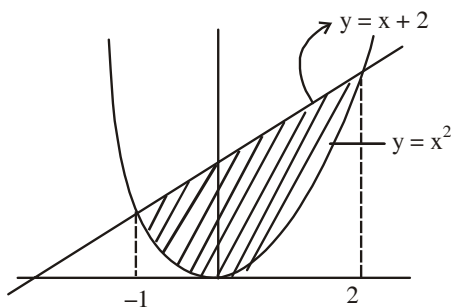
Now, length of side = $\sqrt{2^2 + 2^2} = 2\sqrt{2}$

21. (d)



$$\begin{aligned} \text{Area} &= \int_0^1 ((x+1) - x^2) dx \quad (\because \text{Area} = \int y dx) \\ &= \left[\frac{x^2}{2} + x - \frac{x^3}{3} \right]_0^1 = \left(\frac{1}{2} + 1 - \frac{1}{3} \right) - \left(\frac{0}{2} + 0 - \frac{0}{3} \right) = \frac{3}{2} - \frac{1}{3} = \frac{9}{6} - \frac{2}{6} = \frac{7}{6} \end{aligned}$$

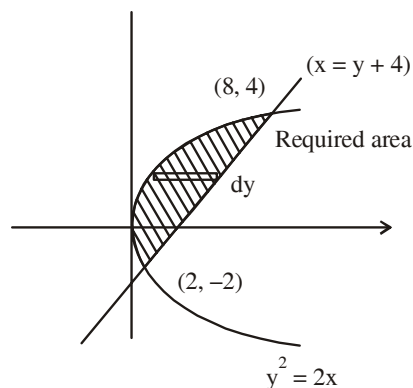
22. (b)



Required area is equal to the area under the curves $y \geq x^2$ and $y \leq x + 2$

$$\begin{aligned} \therefore \text{required area} &= \int_{-1}^2 ((x+2) - x^2) dx \\ &= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2 = \left(2 + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right) = \frac{9}{2} \end{aligned}$$

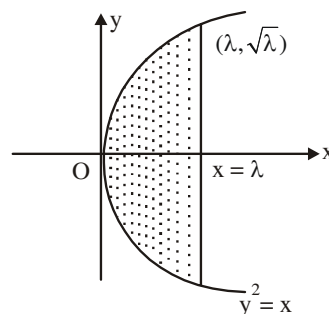
23. (d)



$$\text{Given region, } A = \left\{ (x, y) : \frac{y^2}{2} \leq x \leq y + 4 \right\}$$

$$\begin{aligned} \text{Hence, area} &= \int_{-2}^4 x dy = \int_{-2}^4 \left(y + 4 - \frac{y^2}{2} \right) dy \\ &= \left[\frac{y^2}{2} + 4y - \frac{y^3}{6} \right]_{-2}^4 = \left(8 + 16 - \frac{64}{6} \right) - \left(2 - 8 + \frac{8}{6} \right) \\ &= \left(24 - \frac{32}{3} \right) - \left(-6 + \frac{4}{3} \right) = \frac{40}{3} + \frac{14}{3} = \frac{54}{3} = 18 \end{aligned}$$

24. (d)



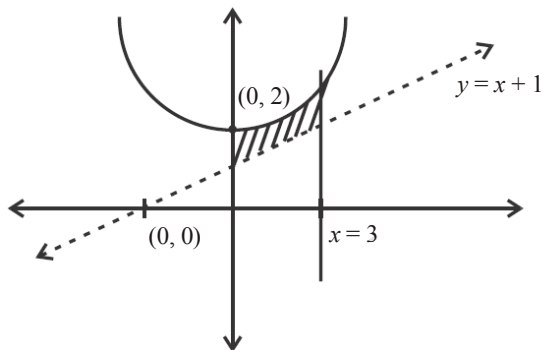
$$\begin{aligned} \text{Area of the region} &= 2 \times \int_0^{\lambda} y dx = 2 \int_0^{\lambda} \sqrt{x} dx \\ &= 2 \times \frac{2}{3} \pi^{\frac{3}{2}} \end{aligned}$$

$$A(\lambda) = 2 \times \frac{2}{3} (\lambda \times \sqrt{\lambda}) = \frac{4}{3} \lambda^{\frac{3}{2}}$$

$$\text{Given, } \frac{A(\lambda)}{A(4)} = \frac{2}{5} \Rightarrow \frac{\lambda^{\frac{3}{2}}}{8} = \frac{2}{5}$$

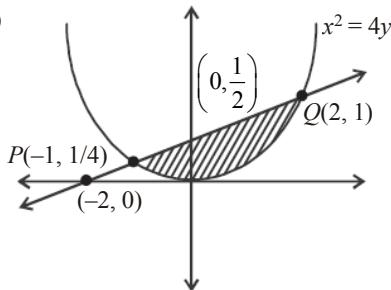
$$\lambda = \left(\frac{16}{5} \right)^{\frac{2}{3}} = 4 \left(\frac{4}{25} \right)^{\frac{1}{3}}$$

25. (d)



$$\begin{aligned} \text{Area of the bounded region} &= \int_0^3 [(x^2 + 2) - (x + 1)] dx \\ &= \left[\frac{x^3}{3} - \frac{x^2}{2} + x \right]_0^3 \\ &= 9 - \frac{9}{2} + 3 = \frac{15}{2} \end{aligned}$$

26. (b)



Let points of intersection of the curve and the line be P and Q

$$x^2 = 4\left(\frac{x+2}{4}\right)$$

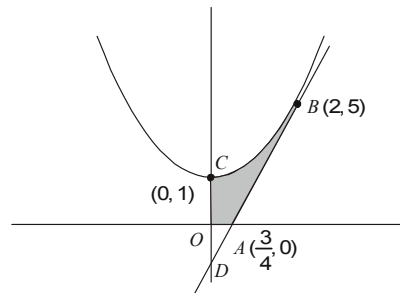
$$x^2 - x - 2 = 0$$

$$x = 2, -1$$

Point are $(2, 1)$ and $\left(-1, \frac{1}{4}\right)$

$$\begin{aligned} \text{Area} &= \int_{-1}^2 \left[\left(\frac{x+2}{4}\right) - \left(\frac{x^2}{4}\right) \right] dx = \left[\frac{x^2}{8} + \frac{1}{2}x - \frac{x^3}{12} \right]_{-1}^2 \\ &= \left(\frac{1}{2} + 1 - \frac{2}{3} \right) - \left(\frac{1}{8} - \frac{1}{2} + \frac{1}{12} \right) = \frac{9}{8} \end{aligned}$$

27. (b)



The equation of parabola $x^2 = y - 1$

The equation of tangent at $(2, 5)$ to parabola is

$$y - 5 = \left(\frac{dy}{dx} \right)_{(2,5)} (x - 2)$$

$$y - 5 = 4(x - 2)$$

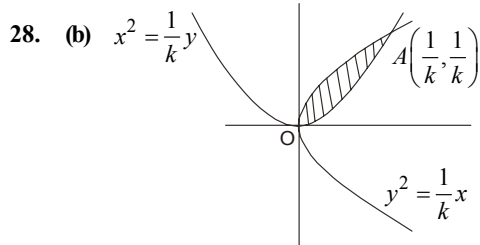
$$4x - y = 3$$

Then, the required area

$$= \int_0^2 \{(x^2 + 1) - (4x - 3)\} dx - \text{Area of } \triangle AOD$$

$$= \int_0^2 (x^2 - 4x + 4) dx - \frac{1}{2} \times \frac{3}{4} \times 3$$

$$= \left[\frac{(x-2)^3}{3} \right]_0^2 - \frac{9}{8} = \frac{37}{24}$$



Two curves will intersect in the 1st quadrant at $A\left(\frac{1}{R}, \frac{1}{R}\right)$

$\therefore$ area of shaded region = 1.

$$\therefore \int_0^{\frac{1}{k}} \left(\frac{\sqrt{x}}{\sqrt{k}} - kx^2 \right) dx = 1$$

$$\Rightarrow \left(\frac{1}{\sqrt{k}} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \right)_{\frac{1}{k}} - \left(k \cdot \frac{x^3}{3} \right)_{\frac{1}{k}} = 1$$

$$\Rightarrow \frac{2}{3\sqrt{k}} \cdot \frac{1}{k^{\frac{3}{2}}} - \frac{k}{3k^3} = 1$$

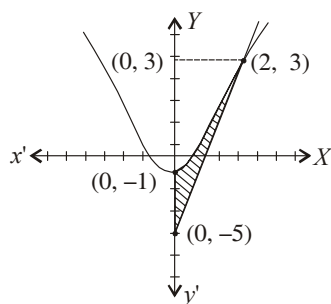
$$\Rightarrow \frac{2}{3k^2} - \frac{1}{3k^2} = 1$$

$$\Rightarrow 3k^2 = 1$$

$$\Rightarrow k = \pm \frac{1}{\sqrt{3}}$$

$$\therefore k = \frac{1}{\sqrt{3}} \quad (\because k > 0)$$

29. (a)



$\therefore$ Curve is given as :

$$y = x^2 - 1$$

$$\Rightarrow \frac{dy}{dx} = 2x$$

$$\Rightarrow \left(\frac{dy}{dx} \right)_{(2,3)} = 4$$

$\therefore$ equation of tangent at (2, 3)

$$(y - 3) = 4(x - 2)$$

$$\Rightarrow y = 4x - 5$$

$$\text{but } x = 0$$

$$\Rightarrow y = -5$$

Here the curve cuts Y-axis

$\therefore$ required area

$$= \frac{1}{4} \int_{-5}^3 (y+5) dy - \int_{-1}^3 \sqrt{y+1} dy$$

$$= \frac{1}{4} \left[\frac{y^2}{2} + 5y \right]_{-5}^3 - \frac{2}{3} \left[(y+1)^{3/2} \right]_{-1}^3$$

$$= \frac{1}{4} \left[\frac{9}{2} + 15 - \frac{25}{2} + 25 \right]$$

$$= -\frac{2}{3} [4^{3/2} - 0]$$

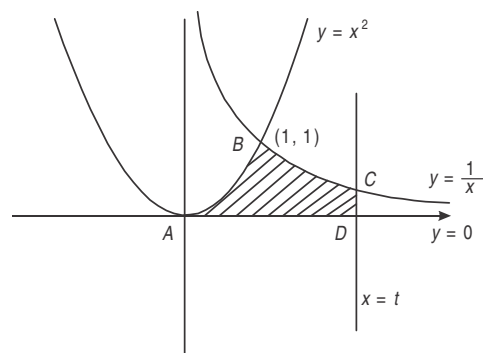
$$= \frac{32}{4} - \frac{16}{3} = \frac{8}{3} \text{ sq-units.}$$

30. (b) The intersection point of $y = x^2$ and $y = \frac{1}{x}$ is (1, 1)

Area bounded by the curves is the region ABCDA is given as:

$$\text{Area} = \int_0^1 x^2 dx + \int_1^t \frac{1}{x} dx$$

$$= \left[\frac{x^3}{3} \right]_0^1 + [\ln(x)]_1^t = \frac{1}{3} + \ln(t)$$

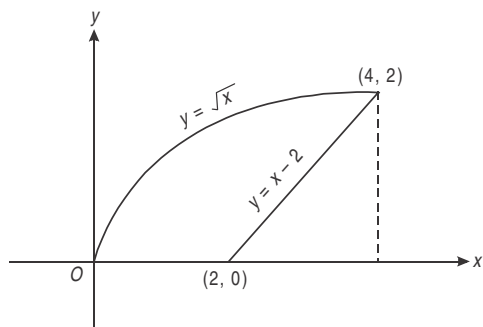


$$\therefore \text{area} = 1$$

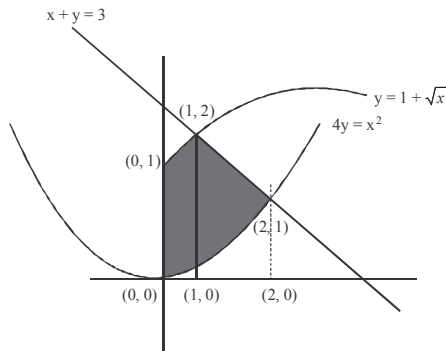
$$\Rightarrow \frac{1}{3} + \ln(t) = 1 \Rightarrow \ln(t) = \frac{2}{3} \Rightarrow t = e^{\frac{2}{3}}$$

31. (b) The intersection point of $y = x - 2$ and $y = \sqrt{x}$ is (4, 2).
The required area

$$= \int_0^4 \sqrt{x} dx - \frac{1}{2} \times 2 \times 2 = \frac{16}{3} - 2 = \frac{10}{3}$$



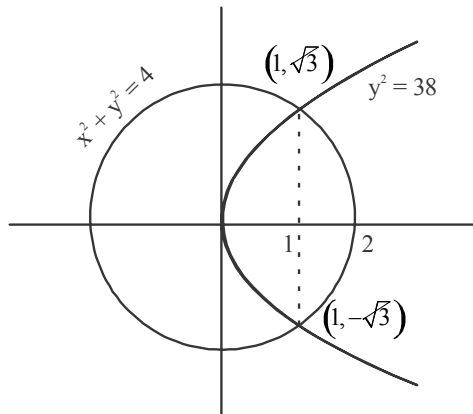
32. (a)



Area of shaded region

$$\begin{aligned}
 &= \int_0^1 (1 + \sqrt{x}) dx + \int_1^2 (3 - x) dx - \int_0^2 \frac{x^2}{4} dx \\
 &= [x]_0^1 + \left[\frac{3}{2} \sqrt{x} \right]_0^1 + [3x - \frac{x^2}{2}]_1^2 - \left[\frac{x^3}{12} \right]_0^2 = \frac{5}{2} \text{ sq. units}
 \end{aligned}$$

33. (d)



From the equations we get;

$$x^2 + 3x - 4 = 0$$

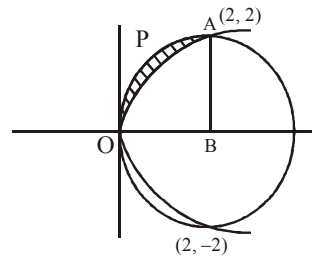
$$\Rightarrow (x+4)(x-1) = 0 \Rightarrow x = -4, x = 1$$

when $x = 1$, $y = \sqrt{3}$

$$\begin{aligned}
 \text{Area} &= \int_0^1 \left(\int_0^{\sqrt{3}} \sqrt{x} \, dy + \int_{\sqrt{3}}^2 \sqrt{4-x^2} \, dy \right) dx \times 2 \\
 &= \left(\sqrt{3} \left(\frac{x^{3/2}}{3/2} \right)_0^1 + \left(\frac{x}{2} \sqrt{4-x^2} + 2 \sin^{-1} \frac{x}{2} \right)_1^2 \right) \times 2 \\
 &= \left(\sqrt{3} \left(\frac{2}{3} \right) + \left\{ 2 \cdot \frac{\pi}{2} - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3} \right) \right\} \right) \times 2
 \end{aligned}$$

$$\begin{aligned}
 &= \left(\frac{2}{\sqrt{3}} - \frac{\sqrt{3}}{2} + \frac{2\pi}{3} \right) \times 2 \\
 &= \left(\frac{1}{2\sqrt{3}} + \frac{2\pi}{3} \right) \times 2 = \frac{1}{\sqrt{3}} + \frac{4\pi}{3}
 \end{aligned}$$

34. (d)

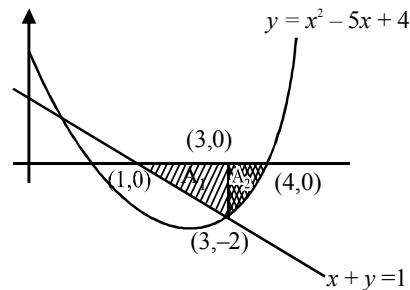


Points of intersection of the two curves are (0, 0), (2, 2) and (2, -2)

Area = Area (OPAB) – area under parabola (0 to 2)

$$= \frac{\pi \times (2)^2}{4} - \int_0^2 \sqrt{2} \sqrt{x} \, dx = \pi - \frac{8}{3}$$

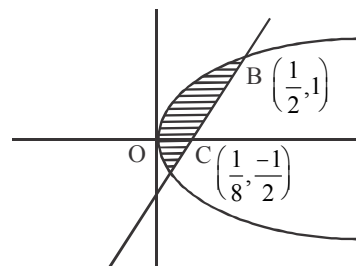
35. (a)

Required area = $A_1 + A_2$

$$= \frac{1}{2} \times 2 \times 2 + \left| \int_3^4 (x^2 - 5x + 4) dx \right|$$

$$= 2 + \frac{7}{6} = \frac{19}{6} \text{ sq. units}$$

36. (b) Required area



$$= \int_{-1/2}^1 \frac{y+1}{4} dy - \int_{-1/2}^1 \frac{y^2}{2} dy$$

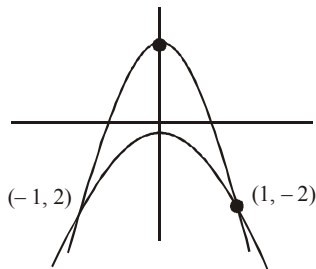
$$= \frac{1}{4} \left[\frac{y^2}{2} + y \right]_{-1/2}^1 - \frac{1}{2} \left[\frac{y^3}{3} \right]_{-1/2}^1$$

$$= \frac{1}{4} \left[\frac{3}{2} + \frac{3}{8} \right] - \frac{9}{48} = \frac{15}{32} - \frac{9}{48} = \frac{27}{96} = \frac{9}{32}$$

37. (c) Solving

$$y + 2x^2 = 0$$

$$y + 3x^2 = 1$$



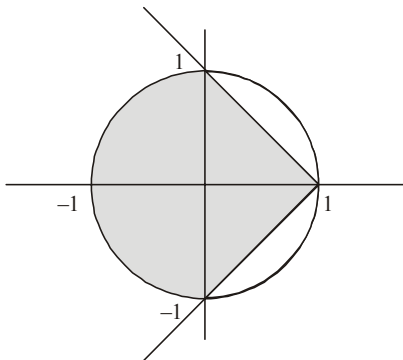
Point of intersection $(1, -2)$ and $(-1, -2)$

$$\text{Area} = 2 \int_0^1 \left((1 - 3x^2) - (-2x^2) \right) dx$$

$$2 \int_0^1 (1 - x^2) dx = 2 \left(x - \frac{x^3}{3} \right)_0^1 = \frac{4}{3}$$

$$= 15 - 6 = 9 \text{ sq units}$$

38. (c) Given curves are $x^2 + y^2 = 1$ and $y^2 = 1 - x$.
Intersecting points are $x = 0, 1$



Area of shaded portion is the required area.

So, Required Area = Area of semi-circle
+ Area bounded by parabola

$$= \frac{\pi r^2}{2} + 2 \int_0^1 \sqrt{1-x} dx = \frac{\pi}{2} + 2 \int_0^1 \sqrt{1-x} dx$$

($\because$ radius of circle = 1)

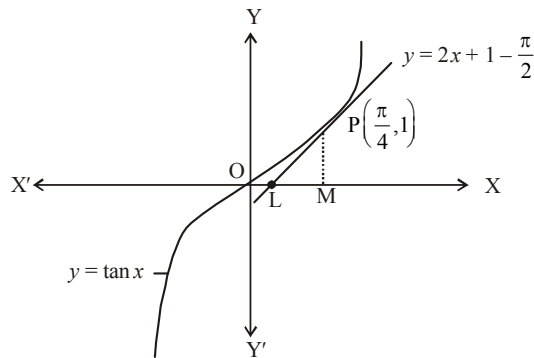
$$= \frac{\pi}{2} + 2 \left[\frac{(1-x)^{3/2}}{-3/2} \right]_0^1 = \frac{\pi}{2} - \frac{4}{3}(-1) = \frac{\pi}{2} + \frac{4}{3} \text{ sq. unit}$$

39. (a) The given curve is $y = \tan x$... (1)

$$\text{when } x = \frac{\pi}{4}, y = 1$$

Equation of tangent at P is

$$y - 1 = \left(\sec^2 \frac{\pi}{4} \right) \left(x - \frac{\pi}{4} \right)$$



$$\text{or } y = 2x + 1 - \frac{\pi}{2} \quad \dots (2)$$

Area of shaded region

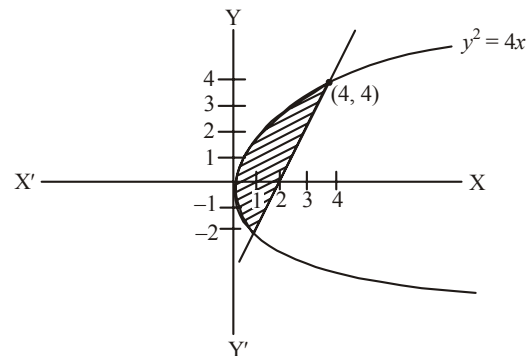
= area of OPMO - ar (Δ PLM)

$$= \int_0^{\pi/4} \tan x dx - \frac{1}{2} (OM - OL) PM$$

$$= [\log \sec x]_0^{\pi/4} - \frac{1}{2} \left\{ \frac{\pi}{4} - \frac{\pi-2}{4} \right\} \times 1$$

$$= \frac{1}{2} \left[\log 2 - \frac{1}{2} \right] \text{ sq unit}$$

40. (b) Area of shaded portion



$$= \left| \int_2^4 \left(\frac{y+4}{2} \right) dy \right| - \left| \int_{-2}^4 \frac{y^2}{4} dy \right|$$

$$= \left| \frac{1}{2} \left[\frac{y^2}{2} + 4y \right]_{-2}^4 \right| - \left| \frac{1}{4} \left[\frac{y^3}{3} \right]_{-2}^4 \right|$$

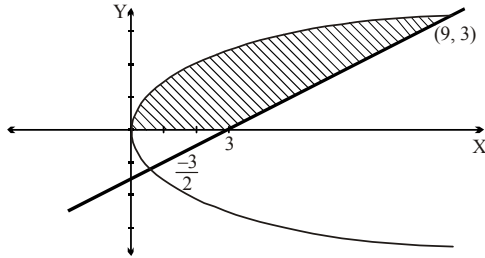
$$= \frac{1}{2} [\{8+16\} - \{2-8\}] - \left| \frac{1}{4} \left\{ \frac{64}{3} + \frac{8}{3} \right\} \right| = 9$$

41. (a) Given curves are

$$y = \sqrt{x} \quad \dots(1)$$

$$\text{and } 2y - x + 3 = 0 \quad \dots(2)$$

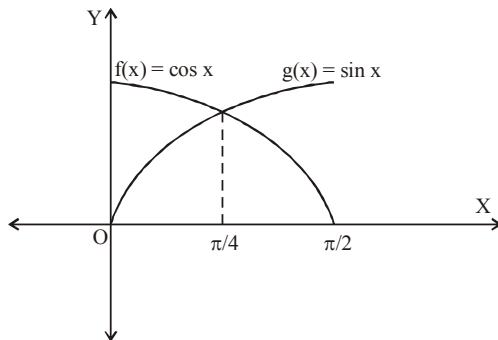
On solving both we get $y = -1, 3$



$$\text{Required area} = \int_0^3 \{ (2y+3) - y^2 \} dy$$

$$= \left[y^2 + 3y - \frac{y^3}{3} \right]_0^3 = 9.$$

42. (b)
- $y = |\cos x - \sin x|$



$$\text{Required area} = 2 \int_0^{\pi/4} (\cos x - \sin x) dx$$

$$= 2 [\sin x + \cos x]_0^{\pi/4}$$

$$= 2 \left[\frac{2}{\sqrt{2}} - 1 \right] = (2\sqrt{2} - 2) \text{ sq. units}$$

43. (d) Required area =
- $\int_{y=1}^4 \sqrt{\frac{y}{9}} dy$

$$= \frac{1}{3} \int_{y=1}^4 y^{1/2} dy = \frac{1}{3} \times \frac{2}{3} (y^{3/2}) \Big|_1^4$$

$$= \frac{2}{9} [(4^{1/2})^3 - (1^{1/2})^3] = \frac{2}{9} [8 - 1]$$

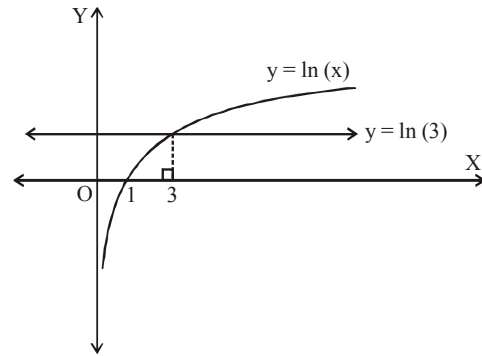
$$= \frac{2}{9} \times 7 = \frac{14}{9} \text{ sq. units.}$$

44. (d) To find the point of intersection of curves
- $y = \ln(x)$
- and
- $y = \ln(3)$
- , put
- $\ln(x) = \ln(3)$

$$\Rightarrow \ln(x) - \ln(3) = 0$$

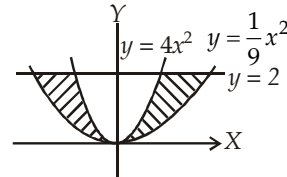
$$\Rightarrow \ln(x) - \ln(3) = \ln(1)$$

$$\Rightarrow \frac{x}{3} = 1, \Rightarrow x = 3$$



$$\begin{aligned} \text{Required area} &= \int_0^3 \ln(3) dx - \int_1^3 \ln(x) dx \\ &= [x \ln(3)]_0^3 - [x \ln(x) - x]_1^3 = 2 \end{aligned}$$

45. (c)



$$\text{Required area} = 2 \int_0^2 \left(\sqrt{9y} - \sqrt{\frac{y}{4}} \right) dy$$

$$= 2 \int_0^2 \left(3\sqrt{y} - \frac{\sqrt{y}}{2} \right) dy = 2 \left[\frac{2}{3} \times 3 \cdot y^{3/2} - \frac{1}{2} \times \frac{2}{3} \cdot y^{3/2} \right]_0^2$$

$$= 2 \left[2y^{3/2} - \frac{1}{3}y^{3/2} \right]_0^2 = 2 \times \left[\frac{5}{3}y^{3/2} \right]_0^2$$

$$= 2 \cdot \frac{5}{3} \cdot 2\sqrt{2} = \frac{20\sqrt{2}}{3}$$

46. (b) Intersecting points are
- $x = 1, 4$

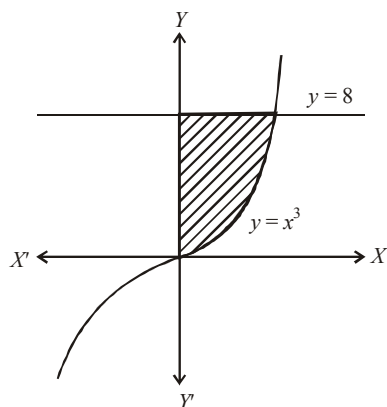
$$\therefore \text{Required area} = \int_1^4 \left[2\sqrt{x} - \left(\frac{2x+4}{3} \right) \right] dx$$

$$= \left[\frac{2x^{3/2}}{3/2} - \frac{2x^2}{3 \times 2} - \frac{4}{3}x \right]_1^4$$

$$= \frac{4}{3} \left(4^{3/2} - 1^{3/2} \right) - \frac{1}{3} (16 - 1) - \left[\frac{4}{3}(4) - \frac{4}{3} \right]$$

$$= \frac{4}{3}(7) - 5 - 4 = \frac{28}{3} - 9 = \frac{28-27}{3} = \frac{1}{3}$$

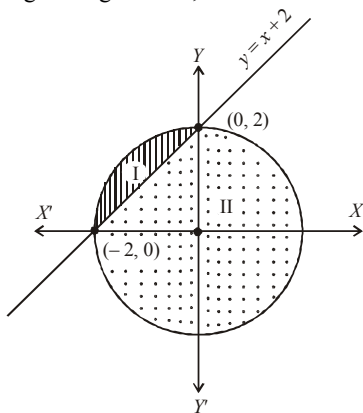
47. (b) Required Area = $\int_{y=0}^8 y^{1/3} dy$



$$= \frac{y^{1/3+1}}{\frac{1}{3}+1} \bigg|_0^8 = \frac{3}{4} \left(y^{4/3} \right) \bigg|_0^8$$

$$= \frac{3}{4} \left[(8)^{4/3} - 0 \right] = \frac{3}{4} [2^4] = \frac{3}{4} \times 16 = 12 \text{ sq. unit.}$$

48. (d) Let I be the smaller portion and II be the greater portion of the given figure then,



$$\text{Area of I} = \int_{-2}^0 \left[\sqrt{4-x^2} - (x+2) \right] dx$$

$$= \left[\frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \left(\frac{x}{2} \right) \right]_{-2}^0 - \left[\frac{x^2}{2} + 2x \right]_{-2}^0$$

$$= \left[2 \sin^{-1}(-1) \right] - \left[-\frac{4}{2} + 4 \right] = 2 \times \frac{\pi}{2} - 2 = \pi - 2$$

Now, area of II = Area of circle - area of I.

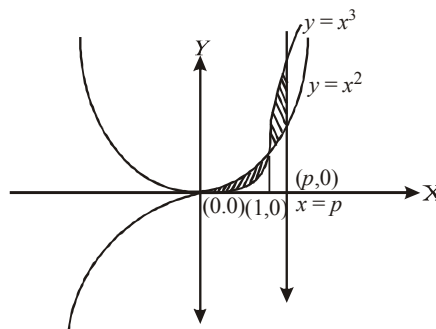
$$= 4\pi - (\pi - 2) = 3\pi + 2$$

Hence, required ratio = $\frac{\text{area of I}}{\text{area of II}} = \frac{\pi - 2}{3\pi + 2}$

49. (d) Given curves are $y = x^2$ and $y = x^3$

Also, $x = 0$ and $x = p, p > 1$

Now, intersecting point is (1, 1)



$$\text{Required Area} = \int_0^1 (x^2 - x^3) dx + \int_1^p (x^3 - x^2) dx$$

$$\frac{1}{6} = \frac{x^3}{3} - \frac{x^4}{4} \bigg|_0^1 + \frac{x^4}{4} - \frac{x^3}{3} \bigg|_1^p$$

$$\Rightarrow \frac{1}{6} = \left(\frac{1}{3} - \frac{1}{4} \right) + \left(\frac{p^4}{4} - \frac{p^3}{3} - \frac{1}{4} + \frac{1}{3} \right)$$

$$\Rightarrow \frac{1}{6} - \frac{1}{3} + \frac{1}{4} + \frac{1}{4} - \frac{1}{3} = \frac{3p^4 - 4p^3}{12}$$

$$\Rightarrow \frac{p^3(3p-4)}{12} = 0 \Rightarrow p^3(3p-4) = 0$$

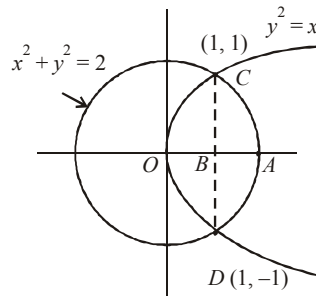
$$\Rightarrow p = 0 \text{ or } \frac{4}{3}$$

Since, it is given that $p > 1$

$\therefore p$ can not be zero.

$$\text{Hence, } p = \frac{4}{3}$$

50. (b)



$$\text{Area of circle} = \pi (\sqrt{2})^2 = 2\pi$$

$$\text{Area of } OCADO = 2 \{ \text{Area of } OCAO \}$$

$$= 2 \{ \text{area of } OCB + \text{area of } BCA \}$$

$$= 2 \int_0^1 y_p dx + 2 \int_1^{\sqrt{2}} y_c dx$$

$$\text{where } y_p = \sqrt{x} \text{ and } y_c = \sqrt{2-x^2}$$

$$\therefore \text{Required Area} = 2 \int_0^1 \sqrt{x} dx + 2 \int_1^{\sqrt{2}} \sqrt{2-x^2} dx$$

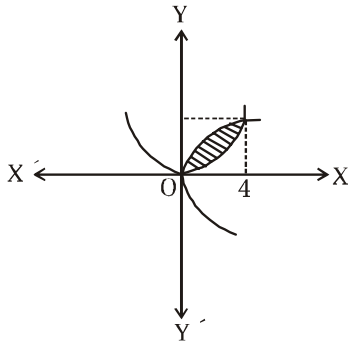
$$= 2 \left[\frac{2}{3} \cdot 1 - 0 \right] + 2 \left[\frac{x\sqrt{2-x^2}}{2} + \sin^{-1} \frac{x}{\sqrt{2}} \right]_1^{\sqrt{2}}$$

$$= \frac{4}{3} + 2 \left\{ \frac{\pi}{2} - \frac{\pi}{4} - \frac{1}{2} \right\} = \frac{4}{3} + 2 \left\{ \frac{\pi}{4} - \frac{1}{2} \right\} = \frac{3\pi+2}{6}$$

$$\text{Bigger area} = 2\pi - \frac{3\pi+2}{6} = \frac{9\pi-2}{6}$$

$$\therefore \text{Required Ratio} = \frac{9\pi-2}{3\pi+2} \text{ i.e., } 9\pi-2 : 3\pi+2$$

51. (b)



$$\text{Required area} = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4} \right) dx$$

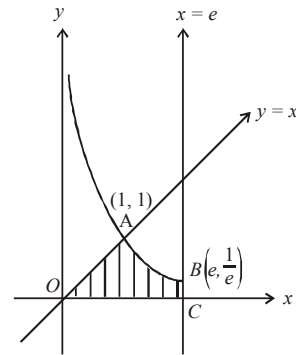
$$= \left[2 \left(\frac{x^{3/2}}{3/2} \right) - \frac{x^3}{12} \right]_0^4$$

$$= \frac{4}{3} \times 8 - \frac{64}{12} = \frac{32}{3} - 16 = \frac{16}{3} \text{ sq. units}$$

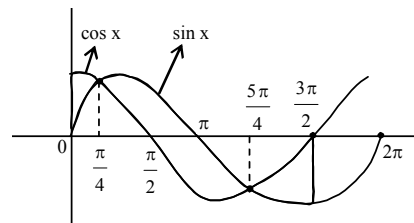
52. (b) Area of required region $AOCBO$

$$= \int_0^1 x dx + \int_1^e \frac{1}{x} dx = \left[\frac{x^2}{2} \right]_0^1 + [\log x]_1^e$$

$$= \frac{1}{2} + 1 = \frac{3}{2} \text{ sq. units}$$



53. (d)



Area above x-axis = Area below x-axis

 $\therefore$ Required area

$$= 2 \left[\int_0^{\pi/4} (\cos x - \sin x) dx + \int_{\pi/4}^{\pi} \sin x dx - \int_{\pi/4}^{\pi/2} \cos x dx \right]$$

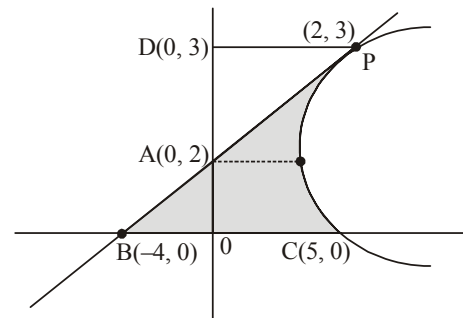
$$= 2 \left[(\sin x + \cos x)_0^{\pi/4} + (-\cos x)_{\pi/4}^{\pi} - (\sin x)_{\pi/4}^{\pi/2} \right]$$

$$= 2 \left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) - (0+1) + \left(1 + \frac{1}{\sqrt{2}} \right) - \left(1 - \frac{1}{\sqrt{2}} \right) \right]$$

$$= 2 \left[\sqrt{2} - 1 + 1 + \frac{1}{\sqrt{2}} - 1 + \frac{1}{\sqrt{2}} \right]$$

$$= 2[\sqrt{2} + \sqrt{2} - 1] = 4\sqrt{2} - 2$$

54. (b)



For slope of tangents at (2, 3)

$$(y-2)^2 = x-1$$

$$2(y-2)\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2(y-2)}$$

$$m = \left(\frac{dy}{dx}\right)_{(2,3)} = \frac{1}{2(3-2)} = \frac{1}{2}$$

Equation of tangent

$$y-3 = \frac{1}{2}(x-2)$$

$$\Rightarrow x-2y+4=0 \quad \dots(i)$$

The given parabola is $(y-2)^2 = x-1$... (ii)

vertex (1, 2) and it meets x-axis at (5, 0)

Then required area = Ar ΔBOA + Ar (OCPD) - Ar (ΔAPD)

$$= \frac{1}{2} \times 4 \times 2 + \int_0^3 x dy - \frac{1}{2} \times 2 \times 1$$

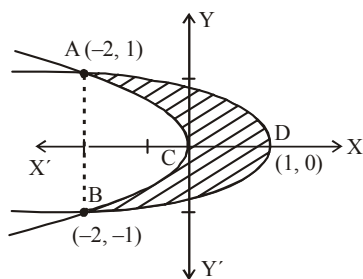
$$= 3 + \int_0^3 (y-2)^2 + 1 dy = 3 + \left[\frac{(y-2)^3}{3} + y \right]_0^3$$

$$= 3 + \left[\frac{1}{3} + 3 + \frac{8}{3} \right] = 3 + 6 = 9 \text{ sq. units}$$

55. (d) Given $x+2y^2=0 \Rightarrow y^2 = -\frac{x}{2}$

and $x+3y^2=1 \Rightarrow y^2 = -\frac{1}{3}(x-1)$

On solving these two equations we get the points of intersection as $(-2, 1)$, $(-2, -1)$



The required area is ACBDA, given by

$$A = 2 \left\{ \int_{-2}^0 \frac{1}{\sqrt{3}} \sqrt{1-x} dx - \frac{1}{\sqrt{2}} \int_{-2}^0 \sqrt{-x} dx \right\}$$

$$\Rightarrow 2 \left\{ \frac{1}{\sqrt{3}} \left[\frac{2}{3} (1-x)^{3/2} \right]_{-2}^0 - \frac{1}{\sqrt{2}} \left[\frac{2}{3} (-x)^{3/2} \right]_{-2}^0 \right\}$$

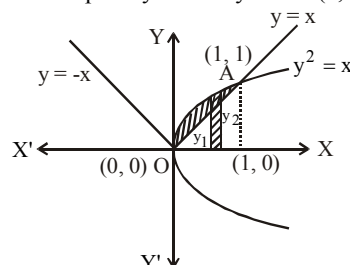
$$\Rightarrow 2 \left\{ \left[-\frac{1}{\sqrt{3}} \times \frac{2}{3} (0-3^{3/2}) \right] - \left[\frac{-1}{\sqrt{2}} \times \frac{2}{3} (0-2^{3/2}) \right] \right\}$$

$$\Rightarrow 2 \left\{ \frac{2}{3\sqrt{3}} \times 3\sqrt{3} - \frac{1}{\sqrt{2}} \times \frac{2}{3} \cdot 2\sqrt{2} \right\}$$

$$\Rightarrow 2 \left\{ 2 - \frac{4}{3} \right\} = 2 \left\{ \frac{6-4}{3} \right\} = \frac{4}{3} \text{ sq. units}$$

56. (a) It is clear from the figure, area lies between $y^2 = x$ and $y = x$

Intersection point $y = x$ and $y^2 = x$ is (1, 1)

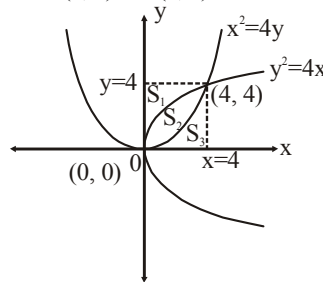


$$\therefore \text{Required area} = \int_0^1 (y_2 - y_1) dx$$

$$= \int_0^1 (\sqrt{x} - x) dx = \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2} \right]_0^1$$

$$= \frac{2}{3} \left[x^{3/2} \right]_0^1 - \frac{1}{2} \left[x^2 \right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$

57. (d) On solving, we get intersection points of $x^2 = 4y$ and $y^2 = 4x$ are (0, 0) and (4, 4).



By symmetry, we observe

$$S_1 = S_3 = \int_0^4 y dx$$

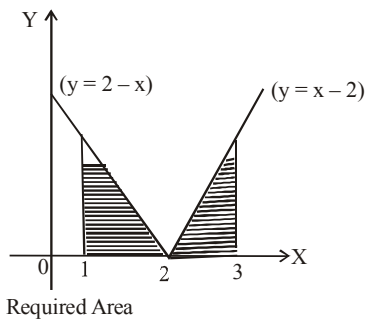
$$= \int_0^4 \frac{x^2}{4} dx = \left[\frac{x^3}{12} \right]_0^4 = \frac{16}{3} \text{ sq. units}$$

$$\text{Also } S_2 = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4} \right) dx = \left[\frac{2x^{3/2}}{3/2} - \frac{x^3}{12} \right]_0^4$$

$$= \frac{4}{3} \times 8 - \frac{16}{3} = \frac{16}{3} \text{ sq. units}$$

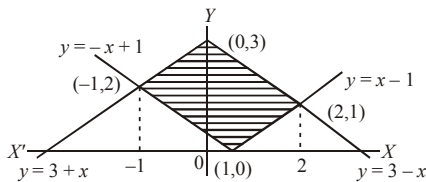
$$\therefore S_1 : S_2 : S_3 = 1 : 1 : 1$$

58. (d)



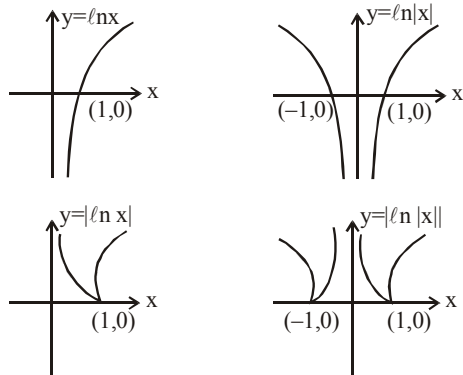
$$A = 2 \int_1^3 (x-2) dx = 2 \left[\frac{x^2}{2} - 2x \right]_1^3 = 1$$

59. (d) Intersection point of $y = x - 1$ and $y = 3 - x$ is $(2, 1)$ and eqns. $y = -x + 1$ and $y = 3 + x$ is $(-1, 2)$



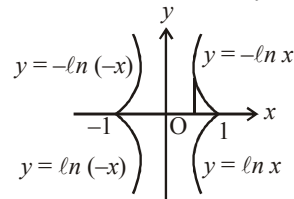
$$\begin{aligned} A &= \int_{-1}^0 \{(3+x) - (-x+1)\} dx + \\ &\int_0^1 \{(3-x) - (-x+1)\} dx + \int_1^2 \{(3-x) - (x-1)\} dx \\ &= \int_{-1}^0 (2+2x) dx + \int_0^1 2 dx + \int_1^2 (4-2x) dx \\ &= \left[2x + x^2 \right]_{-1}^0 + \left[2x \right]_0^1 + \left[4x - x^2 \right]_1^2 \\ &= 0 - (-2+1) + (2-0) + (8-4) - (4-1) \\ &= 1+2+4-3 = 4 \text{ sq. units} \end{aligned}$$

60. (a) Separate graph of each curve



[Note: Graph of $y = |f(x)|$ can be obtained from the graph of the curve $y = f(x)$ by drawing the mirror image of the portion of the graph below x -axis, with respect to x -axis.

Hence the bounded area is as shown by combined all figure.



$$\begin{aligned} \text{Required area} &= 4 \int_0^1 (-\ln x) dx \\ &= -4 [x \ln x - x]_0^1 = 4 \text{ sq. units} \end{aligned}$$