CHAPTER – 4
QUADRATIC EQUATIONS

FACTORISATION METHODS TO FIND THE SOLUTION OF QUADRATIC EQUATIONS

Steps to find the solution of given quadratic equation by factorisation

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Find two numbers α and β such that sum of α and β is equal to b and product of α and β is equal to ac.
- Write the middle term bx as $\alpha x + \beta x$ and factorise it by splitting the middle term and let factors are (x + p) and (x + q) i.e. $ax^2 + bx + c = 0 \Rightarrow (x + p)(x + q) = 0$
- Now equate reach factor to zero and find the values of x.
- These values of x are the required roots/solutions of the given quadratic equation.

IMPORTANT QUESTIONS

Solve the quadratic equation by using factorization method: $x^2 + 2x – 8 = 0$

Solution: $x^2 + 2x – 8 = 0$

$\Rightarrow x^2 + 4x – 2x – 8 = 0 \Rightarrow x(x + 4) – 2(x + 4) = 0$

$\Rightarrow (x + 4)(x – 2) = 0 \Rightarrow x + 4 = 0, x – 2 = 0 \Rightarrow x = – 4, 2$

Questions for practice

1. Solve the quadratic equation using factorization method: $x^2 + 7x – 18 = 0$

2. Solve the quadratic equation using factorization method: $x^2 + 5x – 6 = 0$

3. Solve the quadratic equation using factorization method: $y^2 – 4y + 3 = 0$

4. Solve the quadratic equation using factorization method: $x^2 – 21x + 108 = 0$

5. Solve the quadratic equation using factorization method: $x^2 – 11x – 80 = 0$

6. Solve the quadratic equation using factorization method: $x^2 – x – 156 = 0$

7. Solve the following for x : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$.

8. Solve the following for x : $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

NATURE OF ROOTS

The roots of the quadratic equation $ax^2 + bx + c = 0$ by quadratic formula are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

where $D = b^2 - 4ac$ is called discriminant. The nature of roots depends upon the value of discriminant D. There are three cases –

Case – I

When $D > 0$ i.e. $b^2 - 4ac > 0$, then the quadratic equation has two distinct roots.

i.e. $x = \frac{-b + \sqrt{D}}{2a}$ *and* $\frac{-b - \sqrt{D}}{2a}$

Case – II

When $D = 0$, then the quadratic equation has two equal real roots.

i.e. $x = \frac{-b}{2a}$ *and* $\frac{-b}{2a}$

Case – III

When $D < 0$ then there is no real roots exist.

IMPORTANT QUESTIONS

Find the discriminant of the quadratic equation 2x2 – 4x + 3 = 0, and hence find the nature of its roots.

Solution : The given equation is of the form $ax^2 + bx + c = 0$, where a = 2, b = – 4 and c = 3.
Therefore, the discriminant, $D = b^2 – 4ac = (– 4)^2 – (4 \times 2 \times 3) = 16 – 24 = – 8 < 0$
So, the given equation has no real roots.

Questions for Practice

1. Find the discriminant and the nature of the roots of quadratic equation: $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$.
2. Find discriminant and the nature of the roots of quadratic equation: $4x^2 – 2x2 + 3 = 0$.
3. Find discriminant and the nature of the roots of quadratic equation: $4x^2 – 12x + 9 = 0$.
4. Find discriminant and the nature of the roots of quadratic equation: $5x^2 + 5x + 6 = 0$.
5. Write the nature of roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$.
6. Write the nature of roots of the quadratic equation $9x^2 – 6x – 2 = 0$.
7. Write the nature of roots of quadratic equation : $4x^2 + 6x + 3 = 0$
8. The roots of $ax^2 + bx + c = 0$, $a \neq 0$ are real and unequal. What is value of D?
9. If $ax^2 + bx + c = 0$ has equal roots, what is the value of c?

QUADRATIC FORMULA METHOD

Steps to find the solution of given quadratic equation by quadratic formula method:

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Write the values of a, b and c by comparing the given equation with standard form.
- Find discriminant $D = b^2 – 4ac$. If value of D is negative, then is no real solution i.e. solution does not exist. If value of $D \geq 0$, then solution exists follow the next step.
- Put the value of a, b and D in quadratic formula $x = \frac{-b \pm \sqrt{D}}{2a}$ and get the required roots/solutions.

IMPORTANT QUESTIONS

Solve the quadratic equation by using quadratic formula: $x^2 + x – 6 = 0$

Solution: Here, a = 1, b = 1, c = –6

$\Rightarrow D = b^2 – 4ac = 1 – 4(1)(–6) = 1 + 24 = 25 > 0$

Now, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{25}}{2(1)} = \frac{-1 \pm 5}{2} \Rightarrow x = \frac{-1-5}{2} or \frac{-1+5}{2} \Rightarrow x = \frac{-6}{2} or \frac{4}{2} \Rightarrow x = -3 \ or \ 2$

Questions for practice

1. Solve the quadratic equation by using quadratic formula: $x^2 – 7x + 18 = 0$
2. Solve the quadratic equation by using quadratic formula: $x^2 – 5x + 6 = 0$
3. Solve the quadratic equation by using quadratic formula: $y^2 + 4y + 3 = 0$
4. Solve the quadratic equation by using quadratic formula: $x^2 + 11x – 80 = 0$
5. Solve the quadratic equation by using quadratic formula: $x^2 + x – 156 = 0$
6. Solve for x by using quadratic formula : $9x^2 – 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

MCQ QUESTIONS (1 mark)

1. Which of the following is a quadratic equation?

(a) $x^2 + 2x + 1 = (4 - x)^2 + 3$ (b) $-2x^2 = (5 - x)(2x - \frac{2}{5})$

(c) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k \neq 1$ (d) $x^3 - x^2 = (x - 1)^3$.

2. Which of the following is not a quadratic equation?

(a) $2(x - 1)^2 = 4x^2 - 2x + 1$ (b) $2x - x^2 = x^2 + 5$

(c) $(\sqrt{2}x - \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (d) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$.

3. If is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(a) 2 (b) – 2 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

4. Which of the following equations has the sum of its roots as 3?

(a) $2x^2 - 3x + 6 = 0$ (b) $-x^2 + 3x - 3 = 0$

(c) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (d) $3x^2 - 3x + 3 = 0$

5. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(a) 0 only (b) 4 (c) 8 only (d) 0, 8

6. Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

(a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{4}$ (d) $\frac{9}{64}$

7. The quadratic equation $2x^2 - 5x + 1 = 0$ has

(a) two distinct real roots (b) two equal real roots

(c) no real roots (d) more than 2 real roots

8. Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ (b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$ (d) $5x^2 - 3x + 1 = 0$

9. Which of the following equations has no real roots?

(a) $x^2 - 4x + 3\sqrt{2} = 0$ (b) $x^2 + 4x - 3\sqrt{2} = 0$

(c) $x^2 - 4x - 3\sqrt{2} = 0$ (d) $3x^2 + 4\sqrt{3}x + 4 = 0$

10. $(x^2 + 1)^2 - x^2 = 0$ has

(a) four real roots (b) two real roots

(c) no real roots (d) one real root.

11. If 2 is the root of the equation $x^2 + bx + 12 = 0$ and the equation $x^2 + bx + q = 0$ has equal roots then q =

(a) 8 (b) 16 (c) –8 (d) –16

12. If the equation $(a^2 + b^2)x^2 – 2(ac + bd)x + c^2 + d^2 = 0$ has equal roots then

(a) ab = cd (b) ad = bc (c) ad = $\sqrt{bc}$ (d) ab = $\sqrt{cd}$

13. If a and b can take values 1, 2, 3, 4. Then the number of the equations of the form $ax^2 + bx + c = 0$ having real roots is

(a) 6 (b) 7 (c) 10 (d) 12

14. The number of quadratic equations having real roots and which do not change by squaring their roots is

(a) 4 (b) 3 (c) 2 (d) 1

15. If one of the roots of the quadratic equation $(k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other then k =

(a) 2 (b) 1 (c) –1 (d) – 2

16. If α, β are the roots of the quadratic equation $4x^2 + 3x + 7 = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta}$

(a) $\frac{7}{3}$ (b) $\frac{-7}{3}$ (c) $\frac{3}{7}$ (d) $\frac{-3}{7}$

17. If α, β are the roots of the quadratic equation $x^2 – p(x + 1) – c = 0$, then $(\alpha + 1)(\beta + 1)$ =

(a) c – 1 (b) 1 – c (c) c (d) 1 + c

18. Find the values of k for which the quadratic equation $2x^2 + kx + 3 = 0$ has real equal roots.

(a) $\pm 2\sqrt{6}$ (b) $2\sqrt{6}$ (c) 0 (d) ± 2

19. Find the values of k for which the quadratic equation $kx(x – 3) + 9 = 0$ has real equal roots.

(a) k = 0 or k = 4 (b) k = 1 or k = 4 (c) k = –3 or k = 3 (d) k = –4 or k = 4

20. Find the values of k for which the quadratic equation $4x^2 – 3kx + 1 = 0$ has real and equal roots.

(a) $\pm\frac{4}{3}$ (b) $\pm\frac{2}{3}$ (c) ± 2 (d) none of these

21. The value of k for which equation $9x^2 + 8xk + 8 = 0$ has equal roots is:

(a) only 3 (b) only –3 (c) ±3 (d) 9

22. Which of the following is not a quadratic equation?

(a) $x - \frac{3}{x} = 4$ (b) $3x - \frac{5}{x} = x^2$ (c) $x + \frac{1}{x} = 3$ (d) $x^2 - 3 = 4x^2 - 4x$

23. Which of the following is a solution of the quadratic equation $2x^2 + x – 6 = 0$?

(a) x = 2 (b) x = –12 (c) x = $\frac{3}{2}$ (d) x = –3

24. The value of k for which x = –2 is a root of the quadratic equation $kx^2 + x – 6 = 0$

(a) –1 (b) –2 (c) 2 (d) $-\frac{3}{2}$

25. The value of p so that the quadratics equation $x^2 + 5px + 16 = 0$ has no real root, is

(a) $p>8$ (b) $p<5$ (c) $\frac{-8}{5} < x < \frac{8}{5}$ (d) $\frac{-8}{5} \le x < 0$

26. If $px^2 + 3w + q = 0$ has two roots $x = -1$ and $x = -2$, the value of $q - p$ is

(a) –1 (b) –2 (c) 1 (d) 2

27. The common root of the quadratic equation $x^2 - 3x + 2 = 0$ and $2x^2 - 5x + 2 = 0$ is:

(a) $x = 2$ (b) $x = -2$ (c) $x = \frac{1}{2}$ (d) $x = 1$

28. If $x^2 - 5x + 1 = 0$, the value of $\left(x + \frac{1}{x}\right)$ is:

(a) –5 (b) –2 (c) 5 (d) 3

29. If $a - 3 = \frac{10}{a}$, the value of a are

(a) –5, 2 (b) 5, –2 (c) 5, 2 (d) 5, 0

30. If the roots of the quadratic equation $kx^2 + (a + b)x + ab = 0$ are $(-1, -b)$, the value of k is:

(a) –1 (b) –2 (c) 1 (d) 2