Races and Games of Skill

INTRODUCTION

Race A contest of speed in running, driving, riding sailing or rowing over a specified distance is called *race*.

Race Course The ground or path on which contests are arranged is called a *race course*.

Starting Point The point from where a race begins is called the *starting point*.

Winning Post (or Goal) The point where the race finishes is called the *winning post* or *finishing point* or *goal*.

Dead-heat Race If all the persons contesting a race reach the finishing point exactly at the same time, then the race is called a *dead-heat race*.

Winner The person who first reaches the finishing point is the *winner*.

Suppose A and B are two contestants in a race. We give below certain statements and their corresponding mathematical meanings, which are frequently used:

Statements	Mathematical Interpretation
1. A beats B by t s	A finishes the race t s before B finishes.
2. A gives B a start of t s	A starts t s after B starts from the same point.
3. A gives B a start of x m	While A starts at the starting point, B starts x m ahead from the starting point at the same time.
4. Game of 100	A game in which the participant scoring 100 points first is the winner.
5. In a game of 100, "A can give B 20 points"	While A scores 100 points, B scores only (100 – 20) or 80 points.

$$\text{Some Useful Short-cut Methods}$$

1. If A is n times as fast as B and A gives B a start of x m, then the length of the race course, so that both A and B reach the winning post at the same time, must be

$$x\left(\frac{n}{n-1}\right) \text{m.}$$

Illustration 1 A is $1\frac{2}{3}$ times as fast as B. If A gives B a start of 60 m, how long should the race course be so that both of them reach at the same time?

Solution: Here $n = \frac{5}{3}$ and $x = 60$

$\therefore$ Length of race course $= x\left(\frac{n}{n-1}\right)$

$$= 60\left(\frac{5/3}{5/3-1}\right) = 60\left(\frac{5}{5-3}\right) = 150 \text{ m}$$

2. If A can run xm race in t_1 s and B in t_2 s, where $t_1 < t_2$, then A beats B by a distance

$$\frac{x}{t_2} \times (t_2 - t_1)m.$$

Illustration 2 A can run 100 m in 27 seconds and B in 30 seconds. By what distance A beats B?

Solution: A beats B by a distance

$$= \frac{x}{t_2} \times (t_2 - t_1) = \frac{100}{30}(30-27) = \frac{300}{30} = 10 \text{ m}$$

Practice Exercises

1. Alok and Atul are competing in 800 m race. Alok gives Atul a lead of 100 m. Initially, Alok runs at x times of Atul's speed (x is an integer > 1), but after crossing 600 m, he slows down to a speed which is $\frac{1}{x}$ time of Atul's speed. Atul continues to run at his original speed. If both of them meet for the second time at a distance of 50 m from the finish line, then who wins the race and by what distance?
   (a) Atul, 33.33 m   (b) Alok, 37.5 m
   (c) Atul, 37.5 m   (d) Alok, 25 m

2. A racecourse is 400 m long. A and B run a race and A wins by 5 m. B and C run over the same course and B wins by 4 m. C and D run over it and D wins by 16 m. If A and D run over it, then who would win and by how much?
   (a) A by 8.4 m   (b) D by 8.4 m
   (c) D by 7.2 m   (d) A by 7.2 m

   [Based on MAT, 2002]

3. A and B start simultaneously from the same end of a 50 m long swimming pool. The ratio of their speeds was 2:1, respectively. If the race was one of 1,000 m, how many times after the start would they have met at the end of the race ?
   (a) 19   (b) 18
   (c) 16   (d) 15

4. In a kilometre race, A beats B by 100 m and B beats C by 150 m. In the same race, by how many metres does A beat C?
   (a) 250 m   (b) 240 m
   (c) 225 m   (d) 235 m

   [Based on IIT Joint Man. Ent. Test, 2004]

5. In a 1,000 m long race, Karun gives Varun a head start of 40 m, and still beats Varun by 10 m. Find the distance by which Karun will beat Varun, if Varun gives a start of 40 m to Karun.
   (a) 88 m   (b) 52 m
   (c) 40 m   (d) 50 m

6. Abha beats Mansi in a kilometre race by 50 seconds and Mansi beats Lata by 40 seconds in the same race. If Abha beats Lata by 450 m in a race of a 1 kilometre. How much time Lata takes to run a kilometre?
   (a) 200 seconds   (b) 150 seconds
   (c) 110 seconds   (d) 120 seconds

7. In a 100 m race, Sujit beats Rishi by 5 m and Rishi beats Praveen by 5 m. By what distance does Sujit beats Praveen?
   (a) 10 m   (b) 11 m
   (c) 9 m   (d) 9.75 m

8. A and B run a 1,760 m race ending in a dead heat. At first A runs 20% faster than B. B then quickens his pace, and for the remaining distance runs 20% faster than A. When B quickens his pace. A has already run:
   (a) 800 m   (b) 1,000 m
   (c) 790 m   (d) 960 m

9. In a running race, A gives a head start of 350 m to B. If the ratio of speeds of A and B is 20:13, how far must the winning post be so that A and B finish the race at the same time?
   (a) 1 Km   (b) 2 Km
   (c) 3 Km   (d) None of these

10. A beats B by 100 m in a race of 1200 m and B beats C by 200 m in a race of 1600 m. Approximately by how many metres can A beat C in a race of 9,600 m?
    (a) 1,600 m   (b) 1,800 m
    (c) 1,900 m   (d) 2,400 m

11. In an 800 m race around a stadium having the circumference of 200 m, the top runner meets the last runner on the 5th minute of the race. If the top runner runs at twice the speed of the last runner, what is the time taken by the top runner to finish the race?
    (a) 20 minutes   (b) 15 minutes
    (c) 10 minutes   (d) 5 minutes

12. In a race, Guninder was not the first. Joginder came in after Harinder. Inderjeet was not ahead of Maninder. Guninder was not in front of Joginder. Inderjeet was not fourth or fifth. Maninder was not the first. Who finished first and second in the race?
    (a) Harinder followed by Maninder
    (b) Harinder followed by Joginder
    (c) Harinder followed by Guninder
    (d) Cannot be determined

Directions (Q. 13–15): *Answer the questions based on the following information.*

A certain race is made up of three stretches: A, B and C, each 2 Km long, and to be covered by a certain mode of transport. The table given further gives these modes of transport for the stretches, and the minimum and the maximum possible speeds (in Km/h) over these stretches. The speed over a particular stretch is assumed to be constant. The previous record for the race is 10 minutes.

A	Car	40	60
B	Motorcycle	30	50
C	Bicycle	10	20

13. Anshuman travels at minimum speed by car over A and completes stretch B at the fastest speed. At what speed should he cover stretch C in order to break the previous record?
    (a) Maximum speed for C (b) Minimum speed for C
    (c) Cannot be determined   (d) None of these

14. Mr Hare completes the first stretch at the minimum speed and takes the same time for stretch B. He takes 50% more time than the previous record to complete the race. What is Mr Hare's speed for the stretch C?
    (a) 10.9 Km/h   (b) 13.3 Km/h
    (c) 17.1 Km/h   (d) None of these

15. Mr Tortoise completes the race at an average speed of 20 Km/h. His average speed for the first two stretches is 4 times that for the last stretch. Find the speed over stretch C.
    (a) 15 Km/h   (b) 12 Km/h
    (c) 10 Km/h   (d) Cannot be determined

16. In a race of 600 m, A can beat B by 60 m and in a race of 500 m, B can beat C by 25 m. By how many metres will A beat C in a 400 m race?
    (a) 56 m   (c) 60 m
    (c) 58 m   (d) 54 m
    [Based on MAT (Dec), 2009, 2008, 2007]

17. A can give B a start of 20 m and C a start of 39 m in a walking race of 400 m. How much start can B given C?
    (a) 22 m   (c) 20 m
    (c) 15 m   (d) 26 m
    [Based on MAT (Dec, Sept, May), 2009 (Dec), 2008]

18. A and B run a 5 Km race on a round course of 400 m. If their speeds be in the ratio 5:4, how often does the winner pass the other?
    (a) $4\dfrac{1}{2}$ times   (c) $2\dfrac{3}{4}$ times
    (c) $3\dfrac{1}{2}$ times   (d) $2\dfrac{1}{2}$ times
    [Based on MAT (Dec), 2008]

19. A long distance runner runs 9 laps of a 400 m track every day. His timings (in minute) for four consecutive days are 88, 96, 89 and 87 respectively. On an average, how many m/min does the runner cover?
    (a) 17.78   (b) 90
    (c) 40   (d) None of these
    [Based on MAT (Sept) 2008]

20. Muan and Sanjay ran a race which lasted a minute and a half. Muan gave Sanjay a start of 9 m and beat him by 1 m. Muan ran 40 m while Sanjay ran 39 m. The length of the course is:
    (a) 395 m   (b) 355 m
    (c) 400 m   (d) 410 m
    [Based on MAT (Sept), 2008]

21. There are two concentric circular tracks of radii 100 m and 102 m, respectively. A runs on the inner track and goes once round the track in 1 minute 30 seconds, while B runs on the other track in 1 minute 32 seconds. Who runs faster?
    (a) Both A and B are equal   (b) A
    (c) B (d) None of the above
    [Based on MAT (Sept), 2008]

22. A can run 22.5 m while B runs 25 m. In kilometre race B beats A by:
    (a) 100 m   (b) $111\dfrac{1}{9}$ m
    (c) 25 m   (d) 50 m
    [Based on FMS (MS), 2006]

23. In a 10 Km race, First beats Second by 2 Km and First beats Third by 4 Km. If the runners maintain constant speeds throughout the race, by how many kilometers does Second beat Third?
    (a) $2\dfrac{1}{4}$ (b) $2\dfrac{1}{2}$
    (c) $2\dfrac{3}{4}$ (d) 3
    [Based on FMS, 2010]

24. Hari and Ravi started a race from opposite ends of the pool. After a minute and a half, they passed each other in the centre of the pool. If they lost no time in turning and maintained their respective speeds, how many minutes after starting did they pass each other the second time?
    (a) 3   (b) $4\dfrac{1}{2}$
    (c) 6   (d) $7\dfrac{1}{2}$
    [Based on FMS, 2011]

25. The respective ratio between the speeds of a car, a train and a bus is 5:9:4. The average speed of the car, the bus and the train is 72 Km/h together. What is the average speed of the car and the train together?
    (a) 82 Km/h   (b) 78 Km/h
    (c) 84 Km/h   (d) Cannot be determined

26. In a thousand metre race A beats B by 100 m and B beats C by 100 m. How many metres start can A give to C?
    (a) 195 m   (b) 200 m
    (c) 205 m   (d) 190 m
    [Based on ATMA, 2005]

27. Four runners started running simultaneously from a point on a circular track. They took 200 seconds, 300 seconds, 360 seconds and 450 seconds to complete one round. After how much time do they meet at the starting point for the first time?
    (a) 1800 seconds   (b) 3600 seconds
    (c) 2400 seconds   (d) 4800 seconds
    [Based on SSC (GL), 2011]

28. In a 100 m race, Kamal defeats Bimal by 5 seconds. If the speed of Kamal is 18 Km/h, then the speed of Bimal is:
(*a*) 15.4 Km/h
(*b*) 14.5 Km/h
(*c*) 14.4 Km/h
(*d*) 14 Km/h

[Based on SSC (GL), 2010]

29. In a race of 200 m run, *A* beats *S* by 20 m and *N* by 40 m. If *S* and *N* are running a race of 100 m with exactly same speed as before, then by how many metres will *S* beat *N*?
(*a*) 11.11 m
(*b*) 10 m
(*c*) 12 m
(*d*) 25 m

30. In a mile race, Akshay can be given a start of 128 m by Bhairav. If Bhairav can give Chinmay a start of 4 m in a 100 m dash, then who out of Akshay and Chinmay will win a race of one and half miles, and what will be the final lead given by the winner to the loser?
(One mile is 1,600 m)

(*a*) Akshay, $\dfrac{1}{2}$ mile
(*b*) Chinmay, $\dfrac{1}{32}$ mile

(*c*) Akshay, $\dfrac{1}{24}$ mile
(*d*) Chinmay, $\dfrac{1}{16}$ mile

31. Three runners *A*, *B*, and *C* run a race, with runner A finish 12 m ahead of runner *B* and 18 m ahead of runners *C*, while runner *B* finishes 8 m ahead of runner *C*. Each runner travels the entire distance at a constant speed. What was the length of the race?
(*a*) 36 m
(*b*) 48 m
(*c*) 60 m
(*d*) 72 m

32. Karan and Arjun run a 100 metre race, where Karan beats Arjun by 10 m. To do a favour to Arjun, Karan starts 10 m behind the starting line in a second 100 metre race. They both run at their earlier speeds. Which of the following is true in connection with the second race?
(*a*) Karan and Arjun reach the finishing line simultaneously
(*b*) Arjun beats Karan by 1 m
(*c*) Arjun beats Karan by 11 m
(*d*) Karan beats Arjun by 1 m

33. A sprinter starts running on a circular path of radius *r* m. Her average speed (in m/min) is πr during the first 30 s, $\pi r/2$ during next 1 minute, $\pi r/4$ during next 2 minutes, $\pi r/8$ during next 4 minutes, and so on. What is the ratio of the time taken for the *n*th round to that for the previous round?
(*a*) 4
(*b*) 8
(*c*) 16
(*d*) 32

34. A runs 1 times as fast as *B*. If *A* gives *B* a start of 120 m and they reach the goal at the same time, the goal is at a distance of:
(*a*) 360 m
(*b*) 440 m
(*c*) 460 m
(*d*) 380 m

35. A can run 500 m in 30 seconds and *B* in 35 seconds. How many metres start can *A* give to *B* in a Km race so that the race may end in a dead-heat?

(*a*) $139\dfrac{5}{7}$
(*b*) $138\dfrac{5}{7}$

(*c*) $142\dfrac{6}{7}$
(*d*) $140\dfrac{5}{7}$

36. Ram and Shyam run a race of 2000 m. First, Ram gives Shyam a start of 200 m and beats him by 30 s. Next, Ram gives Shyam a start of 3 minutes and is beaten by 1000 m. Find the time in minutes in which Ram and Shyam can run the race separately?
(*a*) 8 minutes, 10 minutes
(*b*) 4 minutes, 5 minutes
(*c*) 5 minutes, 9 minutes
(*d*) 6 minutes, 9 minutes

[Based on MAT, 2014]

37. A race course is 400 metres long A and B run a race and A wins by 5 metres. B and C run over the same course and B wins by 4 metres. C and D run over it and D wins by 16 metres. If A and D run over it, then who would win and by how much?
(*a*) A by 8.4 metres
(*b*) D by 8.4 metres
(*c*) D by 7.3 metres
(*d*) A by 7.3 metres

[Based on MAT, 2012]

DIFFICULTY LEVEL-2
(BASED ON MEMORY)

1. In a race of 200 m A beats S by 20 m and N by 40 m. If S and N are running a race of 100 m with exactly the same speed as before, then by how many metres will S beat N?
(*a*) 11.11 m
(*b*) 10 m
(*c*) 12 m
(*d*) 25 m

[Based on CAT, 2010]

Answer Keys

DIFFICULTY LEVEL-1

1. (*a*)   **2.** (*c*)   **3.** (*d*)   **4.** (*d*)   **5.** (*a*)   **6.** (*a*)   **7.** (*d*)   **8.** (*a*)   **9.** (*a*)   **10.** (*c*)   **11.** (*c*)   **12.** (*c*)   **13.** (*c*)
14. (*b*)   **15.** (*c*)   **16.** (*c*)   **17.** (*b*)   **18.** (*d*)   **19.** (*c*)   **20.** (*c*)   **21.** (*a*)   **22.** (*a*)   **23.** (*b*)   **24.** (*b*)   **25.** (*c*)   **26.** (*b*)
27. (*a*)   **28.** (*c*)   **29.** (*a*)   **30.** (*d*)   **31.** (*b*)   **32.** (*d*)   **33.** (*c*)   **34.** (*b*)   **35.** (*c*)   **36.** (*b*)   **37.** (*c*)

DIFFICULTY LEVEL-2

1. (*a*)

Explanatory Answers

DIFFICULTY LEVEL-1

1. (*a*) It is clear that Atul wins the race.

So (*a*) and (*c*) are left. Since they meet at a distance of 50 m from the finish line, total distance covered by Atul = 650 m and Alok = 750 m

$\Rightarrow$ $\qquad 100 + \dfrac{600}{x} + 150x = 750$

(in the same time, distance is proportional to speed)

$\Rightarrow$ $\qquad (3x - 4)(x - 3) = 0$

$\Rightarrow$ $\qquad x = 3$ or, $\dfrac{4}{3}$

Since *x* is an integer, it has to be 3.

So, when Atul covers the remaining 50 m, Alok covers only $\dfrac{50}{3}$, i.e., 16.67 m

Therefore, Atul wins by 33.33 m.

2. (*c*) When *A* covers 400 m, *B* covers 395 m
When *B* covers 400 m, *C* covers 396 m
When *B* covers 395 m, *C* covers

$$\dfrac{396}{400} \times 395 = 391.05 \text{ m}$$

$\therefore$ When *A* covers 400 m, *B* covers 391.05 m
When *D* covers 400 m, *C* covers 384 m
When *C* covers 391.05 m, *D* covers 407.3 m

$\therefore$ When *A* covers 400 m, *D* covers 407.3 m
When *D* covers 400 m, *A* covers 392.8 m

$\therefore$ *D* wins by 7.2 m, when *D* and *A* run the race.

3. (*d*) After the start at the end of every 200 m of *A* they would have met thrice and both would be at the starting point again. Hence at the end of the 1,000 m race they would have met $3\left(\dfrac{1000}{200}\right) = 15$ times.

4. (*d*) By the time *A* covers 1000 m, *B* has covered 900 m.

$\therefore$ By the time *B* covers 900 m,

By the time *B* covers 1000 m, *C* has covered 850 m, *C* will cover

$$\dfrac{850}{1000} \times 900 = 765 \text{ m}$$

$\therefore$ *A* beats *C* by 235 m.

5. (*a*) As the race is of 1,000 m.

In first case, Karun runs 1,000 m while Varun runs $(960 - 10) = 950$ m

In second case, Karun runs 960 m then Varun will run $\dfrac{950 \times 960}{1000} = 912$ m.

$\therefore$ Karun will beat Varun by $1000 - 912 = 88$ m.

6. (*a*) In a race of 1,000 m if Abha takes *t* second, then Mansi takes $(t + 50)$ sec and Lata takes $(t + 90)$ s

Now, If Abha travels 1,000 m, Lata travels 550 m

It is thus clear that Lata travels 1,000 m in $(t + 90)$ second and 550 m in *t* second.

$\therefore$ $\qquad 1000{:}550 = (t + 90){:}t$

$\Rightarrow$ $\qquad 100t = 55t + 4950 \Rightarrow t = 110$ seconds

Therefore, Lata takes $(110 + 90) = 200$ seconds in travelling 1,000 m.

7. (*d*) When Sujit runs 100 m, Rishi runs 95 m.

When Rishi runs 100 m, Praveen runs 95 m.

$\therefore$ When Rishi runs 95 m, Praveen runs 90.25 m.

When Sujit runs 100 m, Praveen runs 90.25 m and is beaten by 9.75 m.

Alternative method:

The ratio of speeds of Sujit and Rishi
$$= 100:95$$
$$= 20:19.$$

Similarly, the ratio of speeds of Rishi and Parveen
$$= 20:19.$$

∴ The ratio of speeds of Sujit and Parveen
$$= 20^2 : 10^2$$

⇒ When Sujit goes 100 m, Parveen goes
$$\frac{361}{400} \times 100 = 90.25 \text{ m}$$

∴ The lead that can be given is 100 − 90.25
$$= 9.75 \text{ m}.$$

8. (*a*) Let t_1 be the time at which B switches the speed and $t_1 + t_2$ be the total time between start and finish. Let x be the speed of B initially. So A's speed = $1.2x$ and B's final speed = $1.44x$

Now lag of B in time t_1
$$= (1.2x - x)t_1 = 0.2 \times t_1 \qquad (1)$$

Also, gain of B in time t_2
$$= (1.44x - 1.2x)t_2 = 0.24 \times t_2 \qquad (2)$$

Since both reach at the same time,

∴   lag = lead

⇒ $\dfrac{t_1}{t_2} = \dfrac{0.24}{0.20} = \dfrac{6}{5}$

∴ A covers $\dfrac{1760 \times t_1}{t_1 + t_2}$ of the distance

$$= \frac{1760 \times 5}{11} = 800 \text{ m}.$$

9. (*a*) Let their speeds be $20x$ and $13x$. then relative speed of both of them moving in the same direction is $(20x - 13x) = 7x$

Now B is 350 m ahead of A and this distance has to be covered with a relative speed of $7x$. Therefore, time to cover this distance $= \dfrac{350}{7x}$

Distance covered by A in this time $= \dfrac{350}{7x}$

Speed × time $= 20x \times \dfrac{350}{7x} = 1{,}000 \text{ m} = 1 \text{ Km}.$

10. (*c*) Ratio of speeds of $A{:}B = 12{:}11$ and ratio of speeds of $B{:}C = 8{:}7$

Therefore ratio of speeds of $A{:}B{:}C = 96{:}88{:}77$

So in 9,600 m race A will beat C by 1,900 m.

11. (*c*)

12. (*c*)

13. (*c*) Total time taken to cover stretch A at a minimum

speed $= \left(\dfrac{2}{40}\right) = \dfrac{1}{20}$ hrs = 3 minutes.

Likewise total time taken to vover stretch B at a

maximum speed $= \left(\dfrac{2}{50}\right) = 2.4$ minutes. Total time taken in covering these two stretches = (3 + 2.4) = 5.4 minutes.

To break the previous record the third stretch will have to be covered in (10 − 5.4) = 4.6 minutes.

Required speed $= \dfrac{2}{4.6} = 0.434$ Km/min = 26.08 Km/h. Since the maximum speed is 20 Km/h hence it is not possible for C to break the previous record.

14. (*b*) Time taken to cover the stretch A at minimum

$$\text{speed} = \frac{2}{40} = 3 \text{ minutes}.$$

Time taken to cover stretch B = 3 minutes. Time taken by him in covering the entire race = (1.5 × 10) = 15 minutes.

Hence remaining time to cover stretch = (15 − 6) = 9 minutes.

Therefore required speed $= \left(\dfrac{2}{9}\right) = 0.22$ Km/min.

⇒ 13.3 Km/h.

15. (*c*) Let the average speed for the last stretch be x Km/h, hence his average speed for the first two stretches = $4x$. So, total time taken to cover the three stretches

$$= \frac{4}{4x} + \frac{2}{x}$$

∴ $\dfrac{4}{4x} + \dfrac{2}{x} = \dfrac{6}{20} \Rightarrow x = 10$ Km/h.

16. (*c*) In 600 m race, Ratio of distances

A	:	B
600	:	540
10	:	9

In 500 m race, Ratio of distances

B	:	C
500	:	475
20	:	19

$$A{:}B{:}C = (10 \times 20){:}(9 \times 20){:}(19 \times 9)$$
$$= 200{:}180{:}171$$

So, when A runs 200 m → C runs 171 m

When A runs 1 m → C runs $\dfrac{171}{200}$ m

When A runs 400 m → C runs = 342 m

A can beat C by = 400 − 342 = 58 m.

17. (*b*) In a 400 m race — Ratio of distances

$$A \ : \ B \ : \ C$$
$$400 : 380 : 361$$

So, when B runs 380 m → C runs 361 m

When B runs 400 m → C runs $\dfrac{361 \times 400}{380} = 380$ m

The start B can give $C = 400 - 380 = 20$ m.

18. (*d*) It is clear from the question that when A covers 500 m, B covers 400 m, i.e., A takes a lead of 100 m in every 500 m of distance. Therefore, a lead of 400 m will be taken in travelling a distance of 2000 m or in other words A passes B every after 2000 m.

Hence, total number of such pass

$$= \dfrac{5000}{2000} = \dfrac{5}{2} = 2\dfrac{1}{2} \text{ times.}$$
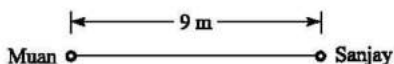
19. (*c*) $\dfrac{88 + 96 + 89 + 87}{4} = 90$ mm

It means on an average he runs $(400 \times 9 = 3600)$

m distance in 90 minutes or in other words he runs

$$\dfrac{3600}{90} = 40 \text{ m/min.}$$

20. (*c*) Speed of Muan $= \dfrac{40}{90} = \dfrac{4}{9}$ m/s

Speed of Sanjay $= \dfrac{39}{90} = \dfrac{13}{30}$ m/s

Muan ○⟷————⟷○ Sanjay (9 m)

Relative speed of Muan

$$= \left(\dfrac{4}{9} - \dfrac{13}{30} \right) = \dfrac{40 - 39}{90} = \dfrac{1}{90} \text{ m/s}$$

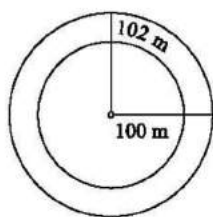Now, in order to cover a distance of 10 m, Muan has to run at a speed of $\dfrac{1}{90}$ m/s

$$t = 900 \text{ seconds}$$

It cover the distance of course, he covers it with a speed of $\dfrac{4}{9}$ m/s in 900 seconds.

$$\therefore \qquad D = \dfrac{4}{9} \times 900 = 400 \text{ m.}$$

21. (*a*)

Circumference of inner track $= 2 \times \pi \times 100 = 200\,\pi$m

Circumference of outer track $= 2 \times \pi \times 102 = 204\,\pi$m

Speed of $A = \dfrac{200}{1.5} = 133.33$ m/min

Speed of $B = \dfrac{204}{1.53} = 133.33$ m/min.

22. (*a*) In 25 m race B beats A by 2.5 m

In 1000 m race B beats A by $\dfrac{2.5}{25} \times 1000 = 100$ m.

23. (*b*) If the First runs 10 Km then the Second runs 8 Km and the Third runs 6 Km.

∴ When the Second runs 8 Km the Third runs 6 Km. When the second runs 10 Km then the third runs

$$\dfrac{6 \times 10}{8} = 7.5 \text{ Km}$$

∴ The Second beats the Third by 2.5 Km in a 10 Km race.

24. (*b*) To meet for the first time, both together cover the length of the pool whereas to meet for the second time, both together need to cover 3 times the length of the pool.

If to cover the length of the pool, they take $1\dfrac{1}{2}$ minutes, then to cover 3 times the length of the pool, they will take $4\dfrac{1}{2}$ minutes.

So, after $4\dfrac{1}{2}$ minutes, they will meet each other for the second time.

25. (*c*) Total speed of car, bus and train

$$= 72 \times 3 = 216 \text{ Km}$$

Speed of car and train $= \dfrac{5 + 9}{5 + 9 + 4} \times 216 = 168$ km

$$\text{Average} = \dfrac{168}{2} = 84 \text{ Km.}$$

26. (*b*)

C 100 m B 100 m A

$$\because \qquad \left. \begin{array}{l} A = 1000m \\ B = 900m \\ C = 800m \end{array} \right\} \quad \therefore \ A - C = 200.$$

27. (*a*) Required = L.C.M. of 200, 300, 350 and 450 seconds

= 1800 seconds.

28. (*c*) Time taken by Kamal to run 100 m

$$= \dfrac{100}{18 \times \dfrac{5}{18}} = 20 \text{ seconds}$$

Therefore, time taken by Bimal to run 100 m
$$= 20 + 5 = 25 \text{ seconds}$$
Hence, Bimal's speed
$$= \frac{100}{25} = 4 \text{ m/sec}$$
$$= \frac{4 \times 18}{5} \text{ Km/h} = 14.4 \text{ Km/h.}$$

29. (*a*) In the time when A runs 200 m, S runs 180 m and N runs 160 m. In other words, in the time when S runs 180 m, N runs 160 m.
Therefore, when S runs 100 m, N will run
$$= \left(100 \times \frac{160}{180}\right) = 88.89 \text{ m.}$$
Hence in a 100m race, S will beat N by $(100 - 88.89) = 11.11$ m.

30. (*d*) When Bhairav covers 1,600 m, Akshay covers $(1600 - 128)$ m. So when Bhairav covers $(1600/6) = 100$ m, Akshay covers $(128/16)$ m = 8 m less.
When Bhairav covers 100 m, Chinmay covers $(100 - 4) = 96$ m.
Thus, the ratio in which Akshay and Chinmay cover distance is 92:96. In 96 m, Chinmay gains $(96 - 92) = 4$ m over Akshay. So, in 1.5 miles, Chinmay gains 100 m = $\left(\frac{1}{16}\right)$ miles over Akshay.

31. (*b*) Let the distance of race be x m which is covered by A in t second. Then, in the same time B covers $(x - 12)$ m and C covers $(x - 18)$ m.
$$\therefore \quad \text{Speed of } A = \frac{x}{t}\text{m/s,}$$
$$\text{Speed of } B = \frac{(x - 12)}{t} \text{ m/s .}$$
$$\text{and,} \quad \text{Speed of } C = \frac{(x - 18)}{t}\text{m/s}$$
Time taken by B to finish the race
$$= \frac{x}{\frac{(x - 12)}{t}} = \frac{x}{(x - 12)} t \, s.$$
Now, distance travelled by C in this time
$$= \frac{x}{(x - 12)} \times t \times \frac{(x - 18)}{t} = x - 8$$
$$\Rightarrow \quad \frac{x(x - 18)}{(x - 12)} = x - 8$$
$$\Rightarrow \quad x = 48 \text{ metre.}$$

32. (*d*) In the first race when Karan runs 100 m, Arjun runs only 90 m. Hence, the ratio of speeds of Arjun and Karan is 90:100 = 9:10. In the second race, Karan has to run 110 m. When he finishes the race, Arjun would have run $\frac{9}{10} \times 110 = 99$ m
(i.e., 1m less than 100 m)
Hence Karan beats Arjun by 1 metre.

33. (*c*) The radius of the track is r m
$\therefore$ the circumference is $2\pi r$ m
The average speed for successive $\pi r, \dfrac{\pi r}{2}, \dfrac{\pi r}{4}, \dfrac{\pi r}{8}$, time intervals of 1/2, 1, 2, 4 etc. minutes is $\dfrac{\pi r}{2}, \dfrac{\pi r}{2}, \dfrac{\pi r}{2}, \dfrac{\pi r}{2}$ etc., m/min. Therefore in each interval (of increasing duration) the distances travelled are etc., (i.e., exactly the same). For such intervals are needed to cover one round. The next four intervals are needed for the next round. As each interval in the second group is 16 times the corresponding interval in the previous group, the total time for each round is 16 times taken for the previous round.

34. (*b*) The speed of A and B are in the ratio 11:8.
Let speeds be $11s$ and $8s$ (in m/sec)
Let race be of x m
Then time taken by A to run x m is same as that of B to run $(x - 120)$ m.
$$\therefore \quad \frac{x}{11s} = \frac{x - 120}{8s}$$
$$\therefore \quad 3x = 11 \times 120$$
$$\therefore \quad x = 440.$$

35. (*c*) Time taken by A to run 1 Km
$$= 30 \times 2 = 60 \text{ seconds}$$
Time taken by B to run 1 Km = $35 \times 2 = 70$ seconds
$\therefore$ A can give B a start of $(70 - 60) = 10$ seconds
In 35 seconds B runs 500 m
$$\therefore \text{ In 10 seconds } B \text{ runs} = \frac{500}{35} \times 10 = \frac{1000}{7}$$
$$= 142\frac{6}{7} \text{ m}$$
So, A can give B a start of $142\frac{6}{7}$ m in a Km race.

36. (*b*) Let the time taken by Ram to finish the race = t minute
In the first case:
Ram gives Shyam a start of 200 m and beats him by 30 seconds, i.e., $\frac{1}{2}$ minute.
$$\therefore \quad \text{Speed of Ram} = \frac{2000}{t}$$
$$\text{and} \quad \text{Speed of Shyam} = \frac{1800}{t + \frac{1}{2}}$$
In the second case:
Ram gives Shyam a start of 3 minutes and losses to Shyam by 1000 m.
$\therefore$ Time taken by Shyam − Time taken by Ram = 3 m

$$\Rightarrow \quad \frac{\frac{2000}{1800}}{t+\frac{1}{2}} - \frac{\frac{1000}{2000}}{t} = 3 \qquad \Rightarrow \quad \frac{\frac{2000}{1800 \times 2}}{2t+1} - \frac{t}{2} = 3$$

$$\Rightarrow \quad \frac{2000 \times (2t+1)}{1800 \times 2} - \frac{t}{2} = 3$$

$$\Rightarrow \quad \frac{5}{9}(2t+1) - \frac{t}{2} = 3$$

$$\Rightarrow \quad \frac{10(2t+1) - 9t}{18} = 3$$

$$20t + 10 - 9t = 18 \times 3$$

$$\Rightarrow \quad 11t = 54 - 10$$

$$\therefore \quad t = \frac{44}{11} = 4 \text{ minutes}$$

So, time taken by Ram to finish the race is 4 minutes.

Again, speed of Shyam $= \dfrac{1800}{t + \dfrac{1}{2}}$

$$= \frac{1800}{4 + \frac{1}{2}} = \frac{1800}{(8+1)} \times 2 = 400 \text{ m/min}$$

So, to cover 2000 m Shyam will take 5 minutes.

37. (c) The statements in the question can be reformulated as follows:

If A covers 400 m, B covers 395 m

If B covers 400 m, C covers 396 m

If D covers 400 m, C covers 384 m

Therefore, if B covers 395 m, then C will cover,

$$\frac{369}{400} \times 395 = 391.05 \text{ m}$$

Again, if C covers 391.05 m, then D will cover

$$\frac{400}{384} \times 391.05 = 407.34 \text{ m}$$

Thus, if A and D run over 400 m, then D wins by 7.3 m.

DIFFICULTY LEVEL-2

1. (a) In the time that A takes to run 200 m, S runs 180 m and N runs 160 m. So in the time

S takes to run 200 m, N runs $200\left(\dfrac{160}{180}\right) = 177.77$ m or is beaten by 22.22 m. So, in 100 m, N is beaten by 11.11 m.