CHAPTER 8

BINOMIAL THEOREM

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. How many terms are present in the expansion of $\left(x^2+\frac{2}{x^2}\right)^{11}$?
(a) 11 (b) 12 (c) 10 (d) 11!

2. The total number of terms in the expansion of $(x+a)^{51}-(x-a)^{51}$ after simplification is
(a) 102 (b) 25 (c) 26 (d) None of these

3. The term independent of x in the expansion of $\left(2x+\frac{1}{3x^2}\right)^9$ is
(a) 2^{nd} (b) 3^{rd} (c) 4^{th} (d) 5^{th}

4. In the expansion of $\left(\sqrt[3]{\frac{x}{3}}-\sqrt{\frac{3}{x}}\right)^{10}$, $x>0$, the constant term is
(a) –70 (b) 70 (c) 210 (d) –210

5. The coefficient of x^{-12} in the expansion of $\left(x+\frac{y}{x^3}\right)^{20}$ is
(a) ${}^{20}C_8$ (b) ${}^{20}C_8\,y^8$ (c) ${}^{20}C_{12}$ (d) ${}^{20}C_{12}\,y^{12}$

6. In the binomial expansion of $(a-b)^n$, $n \geq 5$ the sum of the 5th and 6th terms is zero. Then a/b equals :
(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$ (c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$

7. If the coefficients of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal, then n is
(a) 56 (b) 55 (c) 45 (d) 15

8. The coefficient of the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is
(a) 5/4 (b) 7/4 (c) 9/4 (d) None of these

9. The coefficient of x^p and x^q (p and q are positive integers) in the expansion of $(1+x)^{p+q}$ are
(a) equal
(b) equal with opposite signs
(c) reciprocal of each other
(d) None of these

10. If t_r is the rth term in the expansion of $(1+x)^{101}$, then the ratio $\frac{t_{20}}{t_{19}}$ equal to
(a) $\frac{20x}{19}$ (b) $83x$ (c) $19x$ (d) $\frac{83x}{19}$

11. r and n are positive integers $r>1$, $n>2$ and coefficient of $(r+2)^{\text{th}}$ term and $3r^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals
(a) $3r$ (b) $3r+1$ (c) $2r$ (d) $2r+1$

12. In the expansion of $\left(x+\frac{2}{x^2}\right)^{15}$, the term independent of x is :
(a) ${}^{15}C_6.26$ (b) ${}^{15}C_5.2^5$
(c) ${}^{15}C_4.2^4$ (d) None of these

13. The formula
$(a+b)^m = a^m + ma^{m-1}b + \frac{m(m-1)}{1.2}a^{m-2}b^2 + \ldots$ holds when
(a) $b<a$ (b) $a<b$
(c) $|a|<|b|$ (d) $|b|<|a|$

14. $\frac{1}{\sqrt{5+4x}}$ can be expanded by binomial theorem, if
(a) $x<1$ (b) $|x|<1$
(c) $|x|<\frac{5}{4}$ (d) $|x|<\frac{4}{5}$

15. The expansion of $\frac{1}{(4-3x)^{1/2}}$ by binomial theorem will be valid, if
(a) $x<1$ (b) $|x|<1$
(c) $-\frac{2}{\sqrt{3}}<x<\frac{2}{\sqrt{3}}$ (d) None of these

16. If the coefficients of 2^{nd}, 3^{rd} and the 4^{th} terms in the expansion of $(1+x)^n$ are in A.P., then value of n is
(a) 3 (b) 7 (c) 11 (d) 14

17. If in the binomial expansion of $(1+x)^n$ where n is a natural number, the coefficients of the 5th, 6th and 7th terms are in A.P., then n is equal to:
(a) 7 or 13 (b) 7 or 14 (c) 7 or 15 (d) 7 or 17

18. The coefficient of the middle term in the expansion of $(2+3x)^4$ is :
(a) 6 (b) 5! (c) 8! (d) 216

19. If the r^{th} term in the expansion of $\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4, then r is equal to
(a) 2 (b) 3 (c) 4 (d) 5

20. What is the middle term in the expansion of $\left(\frac{x\sqrt{y}}{3}-\frac{3}{y\sqrt{x}}\right)^{12}$?
(a) $C(12, 7)\, x^3 y^{-3}$ (b) $C(12, 6)\, x^{-3} y^3$
(c) $C(12, 7)\, x^{-3} y^3$ (d) $C(12, 6)\, x^3 y^{-3}$

21. If x^4 occurs in the rth term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then what is the value of r ?
(a) 4 (b) 8 (c) 9 (d) 10

22. What is the coefficient of x^3y^4 in $(2x+3y^2)^5$?
(a) 240 (b) 360 (c) 720 (d) 1080

23. If the coefficient of x^7 in $\left[ax^2+\frac{1}{bx}\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation
(a) $a-b=1$ (b) $a+b=1$ (c) $\frac{a}{b}=1$ (d) $ab=1$

24. If A and B are coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ then :
(a) A = B (b) 2A = B (c) A = 2B (d) AB = 2

25. What is the coefficient of x^3 in $\frac{(3-2x)}{(1+3x)^3}$?
(a) −272 (b) −540 (c) −870 (d) −918

26. If 'n' is positive integer and three consecutive coefficient in the expansion of $(1+x)^n$ are in the ratio 6 : 33 : 110, then n is equal to :
(a) 9 (b) 6 (c) 12 (d) 16

27. $\sqrt{5}\,[(\sqrt{5}+1)^{50}-(\sqrt{5}-1)^{50}]$ is
(a) an irrational number (b) 0
(c) a natural number (d) None of these

28. The number of term in the expansion of $[(x+4y)^3\,(x-4y)^3]^2$ is
(a) 6 (b) 7 (c) 8 (d) 32

29. The term independent of x in the expansion of $\left(\sqrt[6]{x}-\frac{1}{\sqrt[3]{x}}\right)^9$ is
(a) $-{}^9C_3$ (b) $-{}^9C_4$ (c) $-{}^9C_5$ (d) $-{}^8C_3$

30. If the coefficients of r^{th} and $(r+1)^{th}$ terms in the expansion of $(3+7x)^{29}$ are equal, then the value of r is
(a) 31 (b) 11 (c) 18 (d) 21

31. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is
(a) 1594 (b) 792 (c) 924 (d) 2924

32. The coefficient of x^{-7} in the expansion of $\left[ax-\frac{1}{bx^2}\right]^{11}$ will be :
(a) $\frac{462}{b^5}a^6$ (b) $\frac{462a^5}{b^6}$ (c) $\frac{-462a^5}{b^6}$ (d) $\frac{-462a^6}{b^5}$

33. The coefficient of x^3 in the expansion of $\left(x-\frac{1}{x}\right)^7$ is :
(a) 14 (b) 21 (c) 28 (d) 35

34. Find the largest coefficient in the expansion of $(4+3x)^{25}$.
(a) $(3)^{25}\times{}^{25}C_{10}\left(\frac{4}{3}\right)^{11}$ (b) $20\times{}^{25}C_{11}\left(\frac{4}{3}\right)^{14}$
(c) $(2)^8\times{}^{25}C_{11}\left(\frac{5}{2}\right)^{11}$ (d) $(4)^{25}\times{}^{25}C_{11}\times\left(\frac{3}{4}\right)^{11}$

35. If $(1+x)^n=C_0+C_1x+C_2x^2+.....+C_nx^n$, then value of $\frac{(C_0+C_1)(C_1+C_2).....(C_{n-1}+C_n)}{C_0C_1C_2.....C_{n-1}}$ is
(a) $\frac{(n+3)^3}{(2n)!}$ (b) $\frac{(n+1)^n}{n!}$ (c) $\frac{(2n)!}{(n+1)!}$ (d) $\frac{(n-1)^n}{n!}$

36. Notation form of $(a+b)^n$ is
(a) $\sum_{k=0}^{n}{}^nC_k a^{n+k}b^k$ (b) $\sum_{k=0}^{n}{}^nC_k a^{n-k}b^k$
(c) $\sum_{k=0}^{n}{}^nC_k b^{n+k}a^k$ (d) None of these

37. In every term, the sum of indices of a and b in the expansion of $(a+b)^n$ is
(a) n (b) n + 1 (c) n + 2 (d) n − 1

38. The approximation of $(0.99)^5$ using the first three terms of its expansion is
(a) 0.851 (b) 0.751 (c) 0.951 (d) None of these

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

39. The largest term in the expansion of $(3+2x)^{50}$, where $x=\frac{1}{5}$, is
I. 5^{th} II. 3^{rd}
III. 7^{th} IV. 6^{th}
Choose the correct option
(a) Only I (b) Only II
(c) Both I and IV (d) Both III and IV

40. Consider the following statements.
I. Coefficient of x^r in the binomial expansion of $(1+x)^n$ is ${}^nC_r$.
II. Coefficient of $(r+1)^{th}$ term in the binomial expansion of $(1+x)^n$ is ${}^nC_r$.
Choose the correct option.
(a) Only I is correct (b) Only II is correct
(c) Both are correct. (d) Both are incorrect.

41. Consider the following statements.

I. General term of the expansion of $(x+y)^n$ is ${}^nC_r x^{n-r} y^r$.

II. The coefficients ${}^nC_r$ occuring in the binomial theorem are known as binomial coefficients.

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

42. Consider the following statements.

I. General term in the expansion of $(x^2-y)^6$ is $(-1)^r x^{12-2r} \cdot y^r$

II. 4^{th} term in the expansion of $(x-2y)^{12}$ is $-1760x^9y^3$.

Choose the correct option.

(a) Only I is false (b) Only II is false
(c) Both are false (d) Both are true

43. Consider the following statements.

Binomial expansion of $(x+a)^n$ contains $(n+1)$ terms.

I. If n is even, then $\left(\frac{n}{2}+1\right)$ th term is the middle term.

II. If n is odd, then $\left(\frac{n+1}{2}\right)$ th is the middle term.

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

44. Consider the following statements.

I. The number of terms in the expansion of $(x+a)^n$ is $n+1$.

II. The binomial expansion is briefly written as

$$\sum_{r=0}^{n} {}^nC_r x^{n-r}.a^r$$

Choose the correct option.

(a) Only I is true (b) Only II is true
(c) Both are true (d) Both are false

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

45.

Column I (Expression)	Column II (Expansion)
A. $(1-2x)^5$	1. $\frac{x^5}{243}+\frac{5}{81}.x^3+\frac{10}{27}.x+\frac{10}{9}.\frac{1}{x}+\frac{5}{3}.\frac{1}{x^3}+\frac{1}{x^5}$
B. $\left(\frac{2}{x}-\frac{x}{2}\right)^5$	2. $1-10x+40x^2-80x^3+80x^4-32x^5$
C. $(2x-3)^6$	3. $32x^{-5}-40x^{-3}+20x^{-1}-5x+\frac{5}{8}x^3-\frac{1}{32}x^5$
D. $\left(\frac{x}{3}+\frac{1}{x}\right)^5$	4. $64x^6-576x^5+2160x^4-4320x^3+4860x^2-2916x+729$

Codes

	A	B	C	D
(a)	2	4	3	1
(b)	2	3	4	1
(c)	1	3	4	2
(d)	1	4	3	2

46. Using Binomial Theorem, evaluate expression given in column-I and match with column-II.

Column I	Column II
A. $(96)^3$	1. 104060401
B. $(102)^5$	2. 9509900499
C. $(101)^4$	3. 11040808032
D. $(99)^5$	4. 884736

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	1	3	4
(d)	2	3	1	4

47.

Column-I	Column-II
A. Coefficient of x^5 in $(x+3)^8$ is	1. 18564
B. Coefficient of a^5b^7 in $(a-2b)^{12}$ is	2. $61236\,x^5y^5$
C. 13^{th} term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x \neq 0$, is	3. 1512
D. Middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$, is	4. -101376

Codes

	A	B	C	D
(a)	3	1	4	2
(b)	2	1	4	3
(c)	2	4	1	3
(d)	3	4	1	2

48.

Column-I	Column-II
A. Term independent of x in the expansion of $\left(x^2+\frac{1}{x}\right)^9$ is	1. 6^{th} term
B. Term independent of x in the expansion of $\left(x^2+\frac{1}{2x}\right)^{12}$ is	2. 10^{th} term
C. Term independent of x in the expansion of $\left(2x-\frac{1}{x}\right)^{10}$ is	3. 9^{th} term
D. Term independent of x in the expansion of $\left(x^3+\frac{3}{x^2}\right)^{15}$ is	4. 7^{th} term

Codes

	A	B	C	D
(a)	2	1	3	4
(b)	4	3	1	2
(c)	4	1	2	3
(d)	3	2	1	4

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

49. If the second, third and fourth terms in the expansion of $(a+b)^n$ are 135, 30 and 10/3 respectively, then the value of n is
(a) 6 (b) 5 (c) 4 (d) None of these

50. Coefficient of x^{13} in the expansion of $(1-x)^5(1+x+x^2+x^3)^4$ is
(a) 4 (b) 6 (c) 32 (d) 5

51. If x^4 occurs in the t^{th} term in the expansion of $\left(x^4+\frac{1}{x^3}\right)^{15}$, then the value of t is equal to :
(a) 7 (b) 8 (c) 9 (d) 10

52. In the expansion of $(1+x)^{18}$, if the coefficients of $(2r+4)^{th}$ and $(r-2)^{th}$ terms are equal, then the value of r is :
(a) 12 (b) 10 (c) 8 (d) 6

53. A positive value of m for which the coefficient of x^2 in the expansion $(1+x)^m$ is 6, is
(a) 3 (b) 4 (c) 0 (d) None of these

54. If the coefficients of 2^{nd}, 3^{rd} and the 4^{th} terms in the expansion of $(1+x)^n$ are in A.P., then value of n is
(a) 3 (b) 7 (c) 11 (d) 14

55. If the coefficient of x in $(x^2+k/x)^5$ is 270, then the value of k is
(a) 2 (b) 3 (c) 4 (d) 5

56. If the r^{th} term in the expansion of

$\left(\frac{x}{3}-\frac{2}{x^2}\right)^{10}$ contains x^4, then the value of r is
(a) 2 (b) 3 (c) 4 (d) 5

57. The number of zero terms in the expansion of

$(1+3\sqrt{2}x)^9+(1-3\sqrt{2}x)^9$ is
(a) 2 (b) 3 (c) 4 (d) 5

58. Number of terms in the expansion of

$\left(1+5\sqrt{2}x\right)^9+\left(1-5\sqrt{2}x\right)^9$ is
(a) 2 (b) 3 (c) 4 (d) 5

59. Value of 'a', if 17^{th} and 18^{th} terms in the expansion of $(2+a)^{50}$ are equal, is
(a) 1 (b) 2 (c) 3 (d) 4

60. One value of α for which the coefficients of the middle terms in the expansion of $(1+\alpha x)^4$ and $(1-\alpha x)^6$ are equal, is $\frac{-3}{10}$. Other value of 'α' is
(a) 0 (b) 1 (c) 2 (d) 3

61. Number of terms involving x^6 in the expansion of

$\left(2x^2-\frac{3}{x}\right)^{11}$, $r\neq 0$, is
(a) 1 (b) 2 (c) 6 (d) 0

62. If the fourth term in the expansion of $\left(ax+\frac{1}{x}\right)^n$ is $\frac{5}{2}$, then the value of $a\times n$ is
(a) 2 (b) 6 (c) 3 (d) 4

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.
(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

63. **Assertion :** The term independent of x in the expansion of

$\left(x+\frac{1}{x}+2\right)^m$ is $\frac{(4m)!}{(2m!)^2}$.

Reason : The coefficient of x^6 in the expansion of $(1+x)^n$ is ${}^nC_6$.

64. **Assertion :** If $(1+ax)^n=1+8x+24x^2+...$, then the values of a and n are 2 and 4 respectively.

Reason : $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+...$ for all $n\in Z^+$.

65. **Assertion :** If $a_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$, then $\sum_{r=0}^{n}\frac{r}{{}^nC_r}$ is equal to $\frac{n}{2}a_n$.

Reason : ${}^nC_r={}^nC_{n-r}$.

66. If $(1+x)^n=\sum_{r=0}^{n}C_rx^r$, then

Assertion : $\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)...\left(1+\frac{C_n}{C_{n-1}}\right)=\frac{(n+1)^n}{n!}$

Reason : ${}^nC_r=\frac{n(n-1)...(n-r+1)}{r(r-1)...1}$

67. **Assertion :** The r^{th} term from the end in the expansion of $(x+a)^n$ is ${}^nC_{n-r+1}x^{r-1}a^{n-r+1}$.
Reason : The r^{th} term from the end in the expansion of $(x+a)^n$ is $(n-r+2)^{th}$ term.

68. **Assertion :** In the expansion of $(x+2y)^8$, the middle term is 4^{th} term.
Reason : If n is even in the expansion of $(a+b)^n$, then $\left(\frac{n}{2}+1\right)^{th}$ term is the middle term.

69. **Assertion :** ${}^nC_0+{}^nC_2+{}^nC_4+.....=2^{n-1}$
Reason : ${}^nC_1+{}^nC_3+{}^nC_5+......=2^{n-1}$

70. **Assertion:** Number of terms in the expansion of $[(3x+y)^8-(3x-y)^8]$ is 4.

Reason: If n is even, then $\{(x+a)^n-(x-a)^n\}$ has $\frac{n}{2}$ terms.

71. **Assertion:** Number of terms in the expansion of

$\left(\sqrt{x}+\sqrt{y}\right)^{10}+\left(\sqrt{x}-\sqrt{y}\right)^{10}$ is 6.

Reason: If n is even, then the expansion of

$\{(x+a)^n+(x-a)^n\}$ has $\left(\frac{n}{2}+1\right)$ terms.

72. **Assertion:** General term of the expansion $(x+2y)^9$ is ${}^9C_r.2^r.x^{9-r}.y^r$.

Reason: General term of the expansion $(x+a)^n$ is given by $T_{r+1}={}^nC_r\, x^{n-r}.a^r$

73. **Assertion.** The coefficients of the expansions are arranged in an array. This array is called Pascal's triangle.

Reason: There are 11^{th} terms in the expansion of $(4x+7y)^{10}+(4x-7y)^{10}$.

74. **Assertion.** In the binomial expansion $(a+b)^n$, r^{th} term is ${}^nC_r.a^{n-r}.b^r$.

Reason. If n is odd, then there are two middle terms.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

75. After simplification, what is the number of terms in the expansion of $[(3x+y)^5]^4-[(3x-y)^4]^5$?

(a) 4 (b) 5 (c) 10 (d) 11

76. The term independent of x in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$, $x>0$, is 'a' times the corresponding binomial coefficient. Then 'a' is

(a) 3 (b) 1/3 (c) –1/3 (d) None of these

77. The term independent of x in the expansion of $\left(\frac{1-x}{1+x}\right)^2$ is

(a) 4 (b) 3 (c) 1 (d) None of these

78. The middle term in the expansion of

$\left(1+\frac{1}{x^2}\right)\left(1+x^2\right)^n$ is

(a) ${}^{2n}C_n\, x^{2n}$ (b) ${}^{2n}C_n\, x^{-2n}$
(c) ${}^{2n}C_n$ (d) ${}^{2n}C_{n-1}$

79. What are the values of k if the term independent of x in the expansion of $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$ is 405?

(a) ±3 (b) ±6 (c) ±5 (d) ±4

80. If 7^9+9^7 is divided by 64 then the remainder is

(a) 0 (b) 1 (c) 2 (d) 63

81. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is

(a) 6th term (b) 7th term
(c) 5th term (d) 8th term

82. The middle term in the expansion of $\left(1+\frac{1}{x^2}\right)^n\left(1+x^2\right)^n$ is

(a) ${}^{2n}C_n\, x^{2n}$ (b) ${}^{2n}C_n\, x^{-2n}$
(c) ${}^{2n}C_n$ (d) ${}^{2n}C_{n-1}$

83. The value of ${}^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3$ is

(a) ${}^{55}C_4$ (b) ${}^{55}C_3$ (c) ${}^{56}C_3$ (d) ${}^{56}C_4$

84. In the expansion of $(1+x)^{50}$, the sum of the coefficients of odd powers of x is :

(a) 0 (b) 2^{49} (c) 2^{50} (d) 2^{51}

85. Expand by using binomial and find the degree of polynomial $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$ is

(a) 7 (b) 6 (c) 5 (d) 4

86. Value of $\sum_{r=1}^{10} r.\frac{{}^nC_r}{{}^nC_{r-1}}$ is

(a) $10n-45$ (b) $10n+45$
(c) $10n-35$ (d) $10n^2-35$

87. If $(1+x)^{2n}=a_0+a_1x+a_2x^2+.....+a_{2n}x^{2n}$, then

(a) $a_0+a_2+a_4+....=\frac{1}{2}(a_0+a_1+a_2+a_3+....)$
(b) $a_{n+1}<a_n$
(c) $a_{n-3}=a_{n+3}$
(d) All of these

88. If $(1+ax)^n=1+8x+24x^2+\ldots$ then the values of a and n are

(a) n = 4, a = 2 (b) n = 5, a = 1
(c) n = 8, a = 3 (d) n = 8, a = 2

89. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is

(a) $(-1)^{n-1}n$ (b) $(-1)^n(1-n)$
(c) $(-1)^{n-1}(n-1)^2$ (d) $(n-1)$

90. The sum of the series

${}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....-.....+{}^{20}C_{10}$ is

(a) 0 (b) ${}^{20}C_{10}$ (c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$

91. The coefficient of x^{32} in the expansion of : $\left(x^4-\frac{1}{x^3}\right)^{15}$ is:

(a) ${}^{-15}C_3$ (b) ${}^{15}C_4$ (c) ${}^{-15}C_5$ (d) ${}^{15}C_2$

92. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as

(a) $1-\frac{3}{8}x^2$ (b) $3x+\frac{3}{8}x^2$
(c) $-\frac{3}{8}x^2$ (d) $\frac{x}{2}-\frac{3}{8}x^2$

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (b) 12 terms. [$\because$ No. of terms in $(x+a)^n = n+1$)]
2. (c) Since the total number of terms are 52 of which 26 terms get cancelled.
3. (c) Suppose $(r+1)^{th}$ term is independent of x. We have

$$T_{r+1} = {}^9C_r\,(2x)^{9-r}\left(\frac{1}{3x^2}\right)^r = {}^9C_r\,2^{9-r}\,\frac{1}{3^r}\,.\,x^{9-3r}$$

This term is independent of x if $9-3r=0$
i.e., $r=3$.
Thus, 4th term is independent of x.

4. (c) The constant term

$$= {}^{10}C_6\left(\sqrt[3]{\frac{x}{3}}\right)^6\left(-\sqrt{\frac{3}{x}}\right)^4 = {}^{10}C_4\,\frac{1}{3^2}\,.\,3^2 = 210$$

5. (b) Suppose x^{-12} occurs is $(r+1)^{th}$ term.
We have

$$T_{r+1} = {}^{20}C_r\,x^{20-r}\left(\frac{y}{x^3}\right)^r = {}^{20}C_r\,x^{20-4r}\,y^r$$

This term contains x^{-12} if $20-4r=-12$ or $r=8$.
$\therefore$ The coefficient of x^{-12} is ${}^{20}C_8\,y^8$.

6. (b) Given,
$T_5 + T_6 = 0$
$\Rightarrow {}^nC_4\,a^{n-4}b^4 - {}^nC_5\,a^{n-5}b^5 = 0$
$\Rightarrow {}^nC_4\,a^{n-4}b^4 = {}^nC_5\,a^{n-5}b^5$

$$\Rightarrow \frac{a}{b} = \frac{{}^nC_5}{{}^nC_4} = \frac{n-4}{5}$$

7. (b) Since $T_{r+1} = {}^nC_r\,a^{n-r}x^r$ in expansion of $(a+x)^n$,
Therefore,

$$T_8 = {}^nC_7(2)^{n-7}\left(\frac{x}{3}\right)^7 = {}^nC_7\,\frac{2^{n-7}}{3^7}x^7$$

and $$T_9 = {}^nC_8(2)^{n-8}\left(\frac{x}{3}\right)^8 = {}^nC_8\,\frac{2^{n-8}}{3^8}x^8$$

Therefore, $${}^nC_7\,\frac{2^{n-7}}{3^7} = {}^nC_8\,\frac{2^{n-8}}{3^8}$$

(since it is given that coefficient of x^7 = coefficient of x^8)

$$\Rightarrow \frac{n!}{7!\,(n-7)!}\times\frac{8!\,(n-8)!}{n!} = \frac{2^{n-8}}{3^8}\cdot\frac{3^7}{2^{n-7}}$$

$$\Rightarrow \frac{8}{n-7} = \frac{1}{6} \Rightarrow n = 55$$

8. (a) The $(r+1)$th term in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is given by

$$T_{r+1} = {}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r = {}^{10}C_r\,\frac{x^{5-(r/2)}}{3^{5-(r/2)}}\cdot\frac{3^r}{2^r x^{2r}}$$

$$= {}^{10}C_r\,\frac{3^{(3r/2)-5}}{2^r}\,x^{5-(5r/2)}.$$

For T_{r+1} to be independent of x, we must have $5-(5r/2)=0$ or $r=2$.
Thus, the 3rd term is independent of x and is equal to

$${}^{10}C_2\,\frac{3^{3-5}}{2^2} = \frac{10\times9}{2}\times\frac{3^{-2}}{4} = \frac{5}{4}$$

9. (a) Coefficient of x^p and x^q in the expansion of $(1+x)^{p+q}$ are ${}^{p+q}C_p$ and ${}^{p+q}C_q$.

and $${}^{p+q}C_p = {}^{p+q}C_q = \frac{(p+q)!}{p!\,q!}$$

10. (d) t_r is the rth term in the expansion of $(1+x)^{101}$.
$t_r = {}^{101}C_{r-1}.\,(x)^{(r-1)}$

$$\therefore\ \frac{t_{20}}{t_{19}} = \frac{{}^{101}C_{19}}{{}^{101}C_{18}}.\frac{x^{19}}{x^{18}} = \frac{{}^{101}C_{19}\,x}{{}^{101}C_{18}} = \frac{\frac{101!}{19!82!}}{\frac{101!}{18!83!}}x = \frac{83x}{19}$$

11. (c) $t_{r+2} = {}^{2n}C_{r+1}\,x^{r+1}$; $t_{3r} = {}^{2n}C_{3r-1}\,x^{3r-1}$
Given ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$;
$\Rightarrow {}^{2n}C_{2n-(r+1)} = {}^{2n}C_{3r-1}$
$\Rightarrow 2n-r-1 = 3r-1 \Rightarrow 2n = 4r \Rightarrow n = 2r$

12. (b) On comparing with the expansion of $(x+a)^n$, we get

$$x = x,\ a = \frac{2}{x^2},\ n = 15$$

Now, r^{th} term of $\left(x+\frac{2}{x^2}\right)^{15}$ is given as

$T_{r+1} = {}^nC_r\,x^{n-r}a^r$

$$= {}^{15}C_r\,(x)^{15-r}\left(\frac{2}{x^2}\right)^r$$

$= {}^{15}C_r\,x^{15-r}\,2^r\,.\,x^{-2r} = {}^{15}C_r\,x^{15-3r}\,2^r$

Now, in the expansion of $\left(x+\frac{2}{x^2}\right)^{15}$, the term is independent of x if $15-3r=0$
i.e., $r=5$
$\therefore$ Term independent of $x = {}^{15}C_5\,.\,2^5$

13. (d) The expression can be written as $a^m\left\{\left(1+\frac{b}{a}\right)^m\right\}$

14. (c) The given expression can be written as $5^{-1/2}\left(1+\frac{4}{5}x\right)^{-1/2}$

and it is valid only when $\left|\frac{4}{5}x\right| < 1 \Rightarrow |x| < \frac{5}{4}$

15. **(d)** The given expression can be written as $4^{-1/2}\left(1-\frac{3}{4}x\right)^{-1/2}$

and it is valid only when $\left|\frac{3}{4}x\right|<1 \Rightarrow -\frac{4}{3}<x<\frac{4}{3}$

16. **(b)** $2\,{}^nC_2 = {}^nC_1 + {}^nC_3$

$\Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow n = 2$ or 7

17. **(b)** In the binomial expansion of $(1 + x)^n$,

$T_r = {}^nC_{r-1}.(x)^{r-1}$

For $r = 5, T_5 = {}^nC_4x^4$

$r = 6, T_6 = {}^nC_5x^5$

and $r = 7, T_7 = {}^nC_6x^6$

Since, the coefficients of these terms are in A.P.

$\Rightarrow T_5 + T_7 = 2T_6$

$\Rightarrow {}^nC_4 + {}^nC_6 = 2 \times {}^nC_5$

$\Rightarrow \frac{n!}{(n-4)!4!} + \frac{n!}{(n-6)!6!} = \frac{2\times n!}{(n-5)!5!}$

$\Rightarrow \frac{n(n-1)(n-2)(n-3)}{4!} + \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!}$

$= \frac{2n(n-1)(n-2)(n-3)(n-4)}{5!}$

$\Rightarrow \frac{1}{4!} + \frac{(n-4)(n-5)}{6!} = \frac{2(n-4)}{5!}$

$\Rightarrow \frac{1}{1} + \frac{(n-4)(n-5)}{5\times 6} = \frac{2(n-4)}{5}$

$\Rightarrow \frac{30 + n^2 - 9n + 20}{5\times 6} = \frac{2n-8}{5}$

$\Rightarrow n^2 - 9n + 50 = 6(2n - 8)$

$\Rightarrow n^2 - 9n + 50 - 12n + 48 = 0$

$\Rightarrow n^2 - 21n + 98 = 0$

$\Rightarrow (n - 7)(n - 14) = 0$

$\Rightarrow n = 7$ or $n = 14$.

18. **(d)** When exponent is n then total number of terms are n + 1. So, total number of terms in $(2 + 3x)^4 = 5$

Middle term is 3rd.

$\Rightarrow T_3 = {}^4C_2(2)^2.(3x)^2$

$= \frac{4\times 3\times 2\times 1}{2\times 1\times 2}\times 4\times 9x^2 = 216x^2$

$\therefore$ Coefficient of middle term is 216

19. **(b)** $T_r = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-r+1}\left(\frac{-2}{x^2}\right)^{r-1}$

$= [{}^{10}C_{r-1}]\, x^{13-r-2r}.(-2)^{r-1}\left(\frac{1}{3}\right)^{10-r}$

r^{th} term contains x^4 when $13 - 3r = 4 \Rightarrow r = 3$

20. **(d)** In the expansion of $\left(\frac{x\sqrt{y}}{3} - \frac{3}{y\sqrt{x}}\right)^{12}$, n = 12 (even)

then middle term is $\frac{12}{2} + 1 = 7^{th}$ term.

$(r + 1)^{th}$ term,

$T_{r+1} = {}^{12}C_r\left[\frac{x\sqrt{y}}{3}\right]^{12-r}.\left(-\frac{3}{y\sqrt{x}}\right)^r$

$\therefore T_7 = T_{6+1} = {}^{12}C_6\left(\frac{x\sqrt{y}}{3}\right)^6\left(-\frac{3}{y\sqrt{x}}\right)^6$

$= {}^{12}C_6\,\frac{x^6y^3}{y^6x^3} = {}^{12}C_6x^3y^{-3} = C(12, 6)x^3y^{-3}$

21. **(c)** In the expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$, let T_r is the r^{th} term

$T_r = 15_{C_{r-1}}(x^4)^{15-r+1}\left(\frac{1}{x^3}\right)^{r-1}$

$= 15_{C_{r-1}}x^{64-4r-3r+3} = 15_{C_{r-1}}x^{67-7r}$

x^4 occurs in this term

$\Rightarrow 4 = 67 - 7r \Rightarrow 7r = 63 \Rightarrow r = 9$.

22. **(c)** $T_r = {}^nC_{r-1}(2x)^{r-1}(3y^2)^{n-r+1}$

$T_4 = T_{3+1} = {}^5C_3(2x)^3(3y^2)^2$

$= \frac{5!}{3!2!}2^3.x^3.9y^4 = \frac{5.4}{2.1}\times 8\times 9\times x^3y^4 = 720\,x^3y^4$

$\therefore$ Coefficient of $x^3y^4 = 720$

23. **(d)** T_{r+1} in the expansion

$\left[ax^2 + \frac{1}{bx}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$

$= {}^{11}C_r(a)^{11-r}(b)^{-r}(x)^{22-2r-r}$

For the coefficient of x^7, we have

$22 - 3r = 7 \Rightarrow r = 5$

$\therefore$ Coefficient of $x^7 = {}^{11}C_5(a)^6\,(b)^{-5}$ ...(i)

Again T_{r+1} in the expansion

$\left[ax - \frac{1}{bx^2}\right]^{11} = {}^{11}C_r(ax^2)^{11-r}\left(-\frac{1}{bx^2}\right)^r$

$= {}^{11}C_r(a)^{11-r}(-1)^r\times(b)^{-r}(x)^{-2r}(x)^{11-r}$

For the coefficient of x^{-7}, we have

$11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

$\therefore$ Coefficient of $x^{-7} = {}^{11}C_6\,a^5\times 1\times(b)^{-6}$

$\therefore$ Coefficient of x^7 = Coefficient of x^{-7}

$\Rightarrow {}^{11}C_5\,(a)^6\,(b)^{-5} = {}^{11}C_6\,a^5\times(b)^{-6}$

$\Rightarrow ab = 1$.

24. **(c)** We have

$(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1\,x + {}^{2n}C_2\,x^2$

$+ \ldots {}^{2n}C_nx^n + \ldots + {}^{2n}C_{2n}x^{2n}$ ...(i)

$(1+x)^{2n-1} = {}^{2n-1}C_0 + {}^{2n-1}C_1 x + {}^{2n-1}C_2 x^2 + \ldots + {}^{2n-1}C_n x^n + \ldots + {}^{2n-1}C_{2n-1} x^{2n-1}$ ...(ii)

According to the given data and equations (i) and (ii), we can claim that

$A = {}^{2n}C_n$ and $B = {}^{2n-1}C_n$

$\Rightarrow \dfrac{A}{B} = \dfrac{{}^{2n}C_n}{{}^{2n-1}C_n} = \dfrac{\dfrac{2n!}{n!n!}}{\dfrac{(2n-1)!}{n!(n-1)!}}$

$\Rightarrow \dfrac{A}{B} = \dfrac{2n(2n-1)!}{n(n-1)!} \times \dfrac{(n-1)!}{(2n-1)!} = 2$

$\Rightarrow A = 2B$

25. (d) $\dfrac{(3-2x)}{(1+3x)^3} = (3-2x)(1+3x)^{-3}$

$= (3-2x)\left[1 - 9x + \dfrac{(-3)(-4)}{2!}.9x^2 + \dfrac{(-3)(-4)(-5)}{3!}.27x^3 + \ldots\right]$

[Expanding $(1+3x)^{-3}$]

$= (3-2x)(1 - 9x + 54x^2 - 270x^3 + \ldots)$

$\therefore$ Coefficient of $x^3 = -270 \times 3 - 2 \times 54$
$= -810 - 108 = -918$

26. (c) Let the consecutive coefficient of $(1+x)^n$ are ${}^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

From the given condition,

${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6 : 33 : 110$

Now ${}^nC_{r-1} : {}^nC_r = 6 : 33$

$\Rightarrow \dfrac{n!}{(r-1)!\,(n-r+1)!} \times \dfrac{r!(n-r)!}{n!} = \dfrac{6}{33}$

$\Rightarrow \dfrac{r}{n-r+1} = \dfrac{2}{11} \Rightarrow 11r = 2n - 2r + 2$

$\Rightarrow 2n - 13r + 2 = 0$ ....(i)

and ${}^nC_r : {}^nC_{r+1} = 33 : 110$

$\Rightarrow \dfrac{n!}{r!(n-r)!} \times \dfrac{(r+1)!\,(n-r-1)!}{n!} = \dfrac{33}{110} = \dfrac{3}{10}$

$\Rightarrow \dfrac{(r+1)}{n-r} = \dfrac{3}{10} \Rightarrow 3n - 13r - 10 = 0$ ...(ii)

Solving (i) & (ii), we get $n = 12$

27. (c) $\sqrt{5}\left[\left(\sqrt{5}+1\right)^{50} - \left(\sqrt{5}-1\right)^{50}\right]$

$= 2\sqrt{5}\left[{}^{50}C_1\left(\sqrt{5}\right)^{49} + {}^{50}C_3\left(\sqrt{5}\right)^{47} + \ldots\right]$

$= 2\left[{}^{50}C_1(\sqrt{5})^{50} + {}^{50}C_3(\sqrt{5})^{48} + \ldots\right]$

= a natural number

28. (b) $[(x+4y)^3 (x-4y)^3]^2 = [\{x^2 - (4y)^2\}]^6$
$= (x^2 - 16y^2)^6$

$\therefore$ No. of terms in the expansion = 7

29. (a) $T_{r+1} = {}^9C_r\left(\sqrt[6]{x}\right)^{9-r}\left(-\dfrac{1}{\sqrt[3]{x}}\right)^r$

$= {}^9C_r(-1)^r . x^{\frac{9-r}{6} - \frac{r}{3}} = {}^9C_r . x^{\left(\frac{9-3r}{6}\right)}$

Now $\dfrac{9-3r}{6} = 0 \Rightarrow r = 3$;

Thus, term independent of $x = -{}^9C_3$

30. (d) $T_{r+1} = {}^{29}C_r.3^{29-r}.(7x)^r = ({}^{29}C_r . 3^{29-r} . 7^r)\,x^r$

$\therefore$ a_r = coefficient of $(r+1)^{th}$ term $= {}^{29}C_r . 3^{29-r} . 7^r$

Now, $a_r = a_{r-1}$

$\Rightarrow {}^{29}C_r . 3^{29-r} . 7^r = {}^{29}C_{r-1} . 3^{30-r} . 7^{r-1}$

$\Rightarrow \dfrac{{}^{29}C_r}{{}^{29}C_{r-1}} = \dfrac{3}{7} \Rightarrow \dfrac{30-r}{r} = \dfrac{3}{7} \Rightarrow r = 21$

31. (c) We have $2^n = 4096 = 2^{12} \Rightarrow n = 12$;

the greatest coeff = coeff of middle term.

So, middle term = t_7

Coeff of $t_7 = {}^{12}C_6 = \dfrac{12!}{6!6!} = 924$.

32. (b) Suppose x^{-7} occurs in $(r+1)^{th}$ term.

we have $T_{r+1} = {}^nC_r x^{n-r} a^r$ in $(x+a)^n$.

In the given question, $n = 1$, $x = ax$, $a = \dfrac{-1}{bx^2}$

$\therefore \quad T_{r+1} = {}^{11}C_r (ax)^{11-r}\left(\dfrac{-1}{bx^2}\right)^r$

$= {}^{11}C_r a^{11-r} b^{-r} x^{11-3r} (-1)^r$

This term contains x^{-7} if $11 - 3r = -7$

$\Rightarrow r = 6$

Therefore, coefficient of x^{-7} is

${}^{11}C_6 (a)^5 \left(\dfrac{-1}{b}\right)^6 = \dfrac{462}{b^6}a^5$

33. (b) Given, $\left(x - \dfrac{1}{x}\right)^7$ and the $(r+1)^{th}$ term in the expansion of $(x+a)^n$ is $T_{(r+1)} = {}^nC_r(x)^{n-r}a^r$

$\therefore$ $(r+1)^{th}$ term in expansion of

$\left(x - \dfrac{1}{x}\right)^7 = {}^7C_r(x)^{7-r}\left(-\dfrac{1}{x}\right)^r$

$= {}^7C_r(x)^{7-2r}(-1)^r$

Since x^3 occurs in T_{r+1}

$\therefore$ $7 - 2r = 3 \Rightarrow r = 2$

thus the coefficient of $x^3 = {}^7C_2(-1)^2 = \dfrac{7\times 6}{2\times 1} = 21$.

34. (d) $(4+3x)^{25} = 4^{25}\left(1 + \dfrac{3}{4}x\right)^{25}$

Let $(r+1)^{th}$ term will have largest coefficient

$\Rightarrow \dfrac{\text{Coefficient of } T_{r+1}}{\text{Coefficient of } T_r} \geq 1$

$$\Rightarrow \frac{{}^{25}C_r\left(\frac{3}{4}\right)^r}{{}^{25}C_{r-1}\left(\frac{3}{4}\right)^{r-1}} \ge 1$$

$$\Rightarrow \left(\frac{25-r+1}{r}\right)\frac{3}{4} \ge 1 \Rightarrow r \le \frac{78}{7}$$

Largest possible value of r is 11

$\therefore$ Coefficient of $T_{12} = 4^{25} \times {}^{25}C_{11} \times \left(\frac{3}{4}\right)^{11}$

35. **(b)** The given expression,

$$\left(1+\frac{C_1}{C_0}\right)\left(1+\frac{C_2}{C_1}\right)\left(1+\frac{C_3}{C_2}\right).....\left(1+\frac{C_n}{C_{n-1}}\right)$$

$$=\left(1+\frac{n}{1}\right)\left(1+\frac{n-1}{2}\right)\left(1+\frac{n-2}{3}\right)...\left(1+\frac{1}{n}\right)$$

$(\because C_0 = C_n = 1)$

$$=\frac{(n+1)^n}{n!}$$

36. **(b)** The notation $\sum_{k=0}^{n} {}^nC_k a^{n-k} b^k$ stands for

${}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + ... + {}^nC_r a^{n-r} b^r + ... + {}^nC_n a^{n-n} b^n$

where, $b^0 = 1 = a^{n-n}$.

Hence, the notation form of $(a + b)^n$ is

$$(a+b)^n = \sum_{k=0}^{n} {}^nC_k a^{n-k} b^k$$

37. **(a)** In the expansion of $(a + b)^n$, the sum of the indices of a and b is $n + 0 = n$ in the first term, $(n - 1) + 1 = n$ in the second term and so on.

Thus, it can be seen that the sum of the indices of a and b is n in every term of the expansion.

38. **(c)** Now, $(0.99)^5 = (1 - 0.01)^5$

$= {}^5C_0(1)^5 - {}^5C_1(1)^4(0.01) + {}^5C_2(1)^3(0.01)^2$ (ignore the other terms)

$$= 1 - 5 \times 1 \times 0.01 + \frac{5\times4}{2} \times 1 \times 0.01 \times 0.01$$

$= 1 - 0.05 + 10 \times 0.0001 = 1 - 0.05 + 0.001$

$= 1.001 - 0.05 = 0.951$

STATEMENT TYPE QUESTIONS

39. **(d)** $\because (3+2x)^{50} = 3^{50}\left(1+\frac{2x}{3}\right)^{50}$

Here, $T_{r+1} = 3^{50}\ {}^{50}C_r\left(\frac{2x}{3}\right)^{r-1}$

and $T_r = 3^{50}\ {}^{50}C_{r-1}\left(\frac{2x}{3}\right)^{r-1}$

But $x = \frac{1}{5}$ [given]

$$\therefore \frac{T_{r+1}}{T_r} \ge 1 \Rightarrow \frac{{}^{50}C_r}{{}^{50}C_{r-1}}\frac{2}{3}\cdot\frac{1}{5} \ge 1$$

$\Rightarrow 102 - 2r \ge 15r \Rightarrow r \le 6$

$\Rightarrow r = 6$

Therefore, there are two greatest terms T_r and T_{r+1} i.e., T_6 and T_7.

40. **(c)** Both are correct.

41. **(c)**

42. **(a)** I. General term $= T_{r+1} = {}^6C_r (x^2)^{6-r}(-y)^r$

$$= (-1)^r \frac{6!}{r!(6-r)!} . x^{12-2r} . y^r$$

II. 4^{th} term $= T_{3+1}$ in the expansion of $(x + (-2y))^{12}$

$= {}^{12}C_3 x^{12-3}[-2y]^3$

$$= \frac{12.11.10}{1.2.3} x^9 (-1)^3 . 2^3 . y^3$$

$= -220 \times 8 . x^9 . y^3 = -1760\, x^9 y^3.$

43. **(a)** Statement II is false.

If n is odd, then $\left(\frac{n+1}{2}\right)$th and $\left(\frac{n+3}{2}\right)$th terms are the two middle terms.

44. **(c)**

MATCHING TYPE QUESTIONS

45. **(b)** (A) $(1 - 2x)^5$

$= {}^5C_0 . 1^5 + {}^5C_1 . 1^4 . (-2x) + {}^5C_2 . 1^3 (-2x)^2 + {}^5C_3 . 1^2 (-2x)^3 + {}^5C_4 . 1^1 (-2x)^4 + {}^5C_5 1^0 (-2x)^5$

$$= 1.1 + 5.1.(-2x) + \frac{5.4}{1.2}.1.4x^2 + \frac{5.4}{1.2}.1(-8x^3) + \frac{5}{1}.1.16x^4 + (-32x^5)$$

$= 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5.$

(B) $\left[\frac{2}{x} + \left(-\frac{x}{2}\right)\right]^5$

$$= C(5,0)\left(\frac{2}{x}\right)^5 + C(5,1)\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right) + C(5,2)\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2 + C(5,3)\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3 + C(5,4)\left(\frac{2}{x}\right)\left(-\frac{x}{2}\right)^4 + C(5,5)\left(-\frac{x}{2}\right)^5$$

$$= 1\left(\frac{2}{x}\right)^5 + 5\left(\frac{2}{x}\right)^4\left(-\frac{x}{2}\right) + 10\left(\frac{2}{x}\right)^3\left(-\frac{x}{2}\right)^2 + 10\left(\frac{2}{x}\right)^2\left(-\frac{x}{2}\right)^3 + 5\left(\frac{2}{x}\right)\left(-\frac{x}{2}\right)^4 + \left(-\frac{x}{2}\right)^5$$

$$= 32x^{-5} - 40x^{-3} + 20x^{-1} - 5x + \frac{5}{8}x^3 - \frac{1}{32}x^5$$

(C) $(2x-3)^6$

$$= {}^6C_0(2x)^6 + {}^6C_1(2x)^5(-3) + {}^6C_2(2x)^4(-3)^2 + {}^6C_3(2x)^3(-3)^3 + {}^6C_4(2x)^2(-3)^4 + {}^6C_5(2x)(-3)^5 + {}^6C_6(2x)^0(-3)^6$$

$$= 64x^6 + \frac{6}{1}(32x^5)(-3) + \frac{6\ .\ 5}{1\ .\ 2}(16x^4)9 + \frac{6\ .\ 5\ .\ 4}{1\ .\ 2\ .\ 3}(8x^3)(-27) + \frac{6\ .\ 5}{1\ .\ 2}(4x^2)81 + \frac{6}{1}(2x)(-243) + 729$$

$$= 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$$

(D) $\left(\frac{x}{3}+\frac{1}{x}\right)^5 = {}^5C_0\left(\frac{x}{3}\right)^5\left(\frac{1}{x}\right)^0 + {}^5C_1\left(\frac{x}{3}\right)^4\left(\frac{1}{x}\right) + {}^5C_2\left(\frac{x}{3}\right)^3\left(\frac{1}{x}\right)^2 + {}^5C_3\left(\frac{x}{3}\right)^2\left(\frac{1}{x}\right)^3 + {}^5C_4\left(\frac{x}{3}\right)\left(\frac{1}{x}\right)^4 + {}^5C_5\left(\frac{x}{3}\right)^0\left(\frac{1}{x}\right)^5$

$$= \frac{x^5}{243} + \frac{5}{1}.\frac{x^4}{81}.\frac{1}{x} + \frac{5.4}{1.2}.\frac{x^3}{27}.\frac{1}{x^2} + \frac{5.4}{1.2}.\frac{x^2}{9}.\frac{1}{x^3} + \frac{5}{1}.\frac{x}{3}.\frac{1}{x^4} + \frac{1}{x^5}$$

$$= \frac{x^5}{243} + \frac{5}{81}.x^3 + \frac{10}{27}.x + \frac{10}{9}.\frac{1}{x} + \frac{5}{3}.\frac{1}{x^3} + \frac{1}{x^5}$$

46. (a) (A) $(96)^3 = (100-4)^3$

$= {}^3C_0\,(100)^3 - {}^3C_1(100)^2(4) + {}^3C_2(100)\,(4)^2 - {}^3C_3(4)^3$

(B) $(102)^5 = (100+2)^5$

(C) $(101)^4 = (100+1)^4$

(D) $(99)^5 = (100-1)^5$

47. (d) (A) General term in $(x+3)^8 = {}^8C_r x^{8-r}.\ 3^r$

We have to find the coefficient of x^5

$8-r=5,\ r=8-5=3$

$\therefore$ Coefficient of x^5 (putting $r=3$)

$$= {}^8C_3\ .\ 3^3 = \frac{8.7.6}{1.2.3}\ .27 = 56.27 = 1512$$

(B) $(a-2b)^{12} = [\,a+(-2b)]^{12}$

General term $T_{r+1} = C(12, r)\, a^{12-r}(-2b)^r$.

Putting $12-r=5$ or $12-5=r \Rightarrow r=7$

$T_{7+1} = C\,(12, 7)\, a^{12-7}\,(-2b)^7$

$= C(12, 7)\, a^5\,(-2b)^7 = C(12, 7)\,(-2)^7\, a^5b^7$

Hence required coefficient is $C(12, 7)\,(-2)^7$

$$= -\frac{12!}{7!\ 5!}.2^7 = \frac{-12\times11\times10\times9\times8\times7!}{7!\times5\times4\times3\times2\times1}\times2^7$$

$= 8\times-11\times9\times2^7$

$= -99\times8\times128 = -101376$

(C) 13^{th} term, $T_{13} = T_{12+1}$

$$= {}^{18}C_{12}\,(9x)^{18-12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$$

$$= {}^{18}C_6 9^6.x^6(-1)^{12}.\frac{1}{3^{12}}\times\frac{1}{x^6}$$

$$= 18564\times(3^2)^6.\frac{1}{3^{12}}\times\frac{x^6}{x^6}$$

$$= 18564\times\frac{3^{12}}{3^{12}} = 18564$$

(D) Number of terms in the expansion is $10+1=11$ (odd)

Middle term of the expansion is $\left(\frac{n}{2}+1\right)^{th}$ term

$= (5+1)^{th}$ term $= 6^{th}$ term

$$T_6 = T_{5+1} = C(10, 5)\left(\frac{x}{3}\right)^{10-5}(9y)^5$$

$$= C(10, 5)\frac{x^5}{3^5}9^5y^5 = C(10, 5)3^5x^5y^5$$

$$= \frac{10!}{5!(10-5)!}3^5x^5y^5 = \frac{10!}{5!\,5!}3^5x^5y^5$$

$$= \frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!}3^5x^5y^5 = 61236x^5y^5$$

48. (b) (A) General term of

$\left(x^2+\frac{1}{x}\right)^9$ is $T_{r+1} = {}^9C_r\,(x^2)^{9-r}\left(\frac{1}{x}\right)^r$

$= (x^{18-2r}.\ x^{-r}).{}^9C_r = x^{18-3r}.{}^9C_r$

Term independent of x $\Rightarrow 18-3r=0 \Rightarrow r=6$ i.e. 7^{th} term.

(B) General term $= {}^{12}C_r(x^2)^{12-r}(2x)^{-r}$

$= {}^{12}C_r\, x^{24-2r-r}\cdot 2^{-r}$

Term independent of x $\Rightarrow 24-3r=0 \Rightarrow r=8$ i.e. 9^{th} term.

(C) General term $= {}^{10}C_r\,(2x)^{10-r}\left(-\frac{1}{x}\right)^r$

$= {}^{10}C_r\, 2^{10-r}.x^{10-r}.(-1)^r.x^{-r}$

Term independent of x $\Rightarrow 10-2r=0 \Rightarrow r=5$ i.e. 6^{th} term.

(D) General term $= {}^{15}C_r\,(x^3)^{15-r}\left(\frac{3}{x^2}\right)^r$

$= {}^{15}C_r.x^{45-3r}.3^r.x^{-2r}$

$= {}^{15}C_r.x^{45-5r}.3^r$

Term independent of x $\Rightarrow 45-5r=0 \Rightarrow r=9$ i.e., 10^{th} term

INTEGER TYPE QUESTIONS

49. (b) $T_2 = {}^nC_1\, ab^{n-1} = 135$ ...(i)

$T_3 = {}^nC_2\, a^2b^{n-2} = 30$ ...(ii)

$T_4 = {}^nC_3\, a^3b^{n-3} = \frac{10}{3}$ ...(iii)

Dividing (i) by (ii)

$$\frac{{}^nC_1ab^{n-1}}{{}^nC_2a^2b^{n-2}} = \frac{135}{30}$$

$$\frac{n}{\frac{n}{2}(n-1)} \cdot \frac{b}{a} = \frac{9}{2} \quad \text{...(iv)}$$

$$\frac{b}{a} = \frac{9}{4}(n-1) \quad \text{...(v)}$$

Dividing (ii) by (iii)

$$\frac{\frac{n(n-1)}{2}}{\frac{n(n-1)(n-2)}{3.2}} \cdot \frac{b}{a} = 9 \quad \text{...(vi)}$$

Eliminating a and b from (v) and (vi), we get

$n = 5$

50. (a) Expression $= (1-x)^5 .(1+x)^4 (1+x^2)^4$

$= (1-x)(1-x^2)^4(1+x^2)^4$

$= (1-x)(1-x^4)^4$

$\therefore$ Coefficient of $x^{13} = -{}^4C_3(-1)^3 = 4$

51. (c) The binomial expansion of $(x+a)^n$ gives $(t+1)^{th}$ term $= T_{t+1} = {}^nC_t x^{n-t} a^t$

We have expansion of $\left(x^4 + \frac{1}{x^3}\right)^{15}$.

On comparing with $(x+a)^n$, we get

$x = x^4,\ a = \frac{1}{x^3},\ n = 15$

$\therefore$ t^{th} term

$= T_t = {}^{15}C_{t-1}(x^4)^{15-(t-1)} . \left(\frac{1}{x^3}\right)^{t-1}$

$= {}^{15}C_{t-1}(x)^{60-4t+4} . (x)^{-3t+3}$

$= {}^{15}C_{t-1}(x)^{67-7t}$

Since, x^4 occurs in the t^{th} term

$\therefore$ $67 - 7t = 4 \Rightarrow 7t = 63 \Rightarrow t = 9$

52. (d) Since the coefficient of $(r+1)^{th}$ term in the expansion of $(1+x)^n = {}^nC_r$

$\therefore$ In the expansion of $(1+x)^{18}$

coefficient of $(2r+4)^{th}$ term $= {}^{18}C_{2r+3,}$

Similarly, coefficient of $(r-2)^{th}$ term in the expansion of $(1+x)^{18} = {}^{18}C_{r-3}$

If ${}^nC_r = {}^nC_s$ then $r + s = n$

So, ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$ gives

$2r + 3 + r - 3 = 18$

$\Rightarrow 3r = 18 \Rightarrow r = 6.$

53. (a) Given expansion is $(1+x)^m$. Now,

General term $= T_{r+1} = {}^mC_r\, x^r$

Put r = 2, we have

$T_3 = {}^mC_2 . x^2$

According to the question $C(m, 2) = 6$

or $\frac{m(m-1)}{2!} = 6$

$\Rightarrow m^2 - m = 12$

or $m^2 - m - 12 = 0$

$\Rightarrow m^2 - 4m + 3m - 12 = 0$

or $(m-4)(m+3) = 0$

$\therefore$ $m = 4$, since $m \neq -3$

54. (b) $2\,{}^nC_2 = {}^nC_1 + {}^nC_3 \Rightarrow n^2 - 9n + 14 = 0$

$\Rightarrow n = 2$ or 7

55. (b) **Hint :** $T_{r+1} = {}^5C_r(x^2)^{5-r}(k/x)^r = {}^5C_r\, k^r x^{10-3r}$

For coefficient of x, $10 - 3r = 1 \Rightarrow r = 3$

coefficient of $x = {}^5C_3\, k^3 = 270$

$\Rightarrow k^3 = \frac{270}{10} = 27 \ \therefore k = 3$

56. (b) **Hint :** $T_r = {}^{10}C_{r-1}\left(\frac{x}{3}\right)^{10-(r-1)}\left(-\frac{2}{x^2}\right)^{r-1}$

$= {}^{10}C_{r-1}\left(\frac{1}{3}\right)^{11-r} .(-2)^{r-1} x^{13-3r}$

for coefficient of x^4, $13 - 3r = 4 \Rightarrow r = 3$

57. (d) **Hint :** Given expression

$= 2[1 + {}^9C_2(3\sqrt{2}x)^2 + {}^9C_4(3\sqrt{2}x)^4 + {}^9C_6(3\sqrt{2}x)^6 + {}^9C_8(3\sqrt{2}x)^8]$

$\therefore$ the number of non-zero terms is 5

58. (d) If n is odd, then the expansion of $(x+a)^n + (x-a)^n$ contains $\left(\frac{n+1}{2}\right)$ terms. So, the expansion of $\left(1+5\sqrt{2}x\right)^9 + \left(1-5\sqrt{2}x\right)^9$ has $\left(\frac{9+1}{2}\right) = 5$ terms.

59. (a) $T_{17} = {}^{50}C_{16} \times 2^{34} \times a^{16}$

$T_{18} = {}^{50}C_{17} \times 2^{33} \times a^{17}$

Given $T_{17} = T_{18}$

$\Rightarrow \frac{{}^{50}C_{16}}{{}^{50}C_{17}} \times 2 = \frac{a^{17}}{a^{16}}$

$\Rightarrow a = \frac{50!}{34!16!} \times \frac{33!17! \times 2}{50!} = \frac{17}{34} \times 2 = 1$

60. (a) In the expansion of $(1+\alpha x)^4$

Middle term $= {}^4C_2(\alpha x)^2 = 6\alpha^2x^2$

In the expansion of $(1-\alpha x)^6$,

Middle term $= {}^6C_3(-\alpha x)^3 = -20\alpha^3x^3$

It is given that

Coefficient of the middle term in $(1+\alpha x)^4$ = Coefficient of the middle term in $(1-\alpha x)^6$

$\Rightarrow 6\alpha^2 = -20\alpha^3$

$\Rightarrow \alpha = 0,\ \alpha = -\frac{3}{10}$

61. **(d)** Suppose x^6 occurs in $(r+1)^{th}$ term in the expansion of $\left(2x^2-\dfrac{3}{x}\right)^{11}$

Now, $T_{r+1} = {}^{11}C_r\,(2x^2)^{11-r}\left(-\dfrac{3}{x}\right)^r$

$= {}^{11}C_r(-1)^r\,2^{11-r}\,3^r x^{22-3r}$

For this term to contain x^6, we must have

$22-3r=6 \Rightarrow r=\dfrac{16}{3}$, which is a fraction.

But, r is a natural number. Hence, there is no term containing x^6.

62. **(c)** $T_{3+1}=\dfrac{5}{2}$

$\Rightarrow {}^nC_3\,(ax)^{n-3}\left(\dfrac{1}{x}\right)^3=\dfrac{5}{2}$

$\Rightarrow {}^nC_3\,a^{n-3}.x^{n-6}=\dfrac{5}{2}$ ...(i)

$\Rightarrow n-6=0 \Rightarrow n=6$

($\because$ RHS of above equality is independent of x)

Put n = 6 in (i), we get

${}^6C_3\,a^3=\dfrac{5}{2}\Rightarrow a^3=\dfrac{1}{8}$

$\Rightarrow a=\dfrac{1}{2}$ and n = 6

Hence, $a\times\dfrac{1}{n}=\dfrac{1}{2}\times 6=3$

ASSERTION - REASON TYPE QUESTIONS

63. **(d)** $\left(x+\dfrac{1}{x}+2\right)^m=\left(\dfrac{x^2+2x+1}{x}\right)^m=\dfrac{(1+x)^{2m}}{x^m}$

Term independent of x is coefficient of x^m in the expansion of $(1+x)^{2m}={}^{2m}C_m=\dfrac{(2m)!}{(m!)^2}$

Coefficient of x^6 in the expansion of $(1+x)^n$ is ${}^nC_6$

64. **(a)** Given that, $(1+ax)^n=1+8x+24x^2+...$

$\Rightarrow\ 1+\dfrac{n}{1}ax+\dfrac{n(n-1)}{1.2}a^2x^2+...=1+8x+24x^2+...$

On comparing the coefficient of x, x^2, we get

$na=8, \dfrac{n(n-1)}{2}a^2=24$

$\Rightarrow\ na\,(na-a)=48\Rightarrow 8(8-a)=48$

$\Rightarrow\ 8-a=6\Rightarrow a=2 \therefore n\times 2=8\Rightarrow n=4$

65. **(a)** Let $b=\sum\limits_{r=0}^{n}\dfrac{r}{{}^nC_r}=\sum\limits_{r=0}^{n}\dfrac{n-(n-r)}{{}^nC_r}$

$=n\sum\limits_{r=0}^{n}\dfrac{1}{{}^nC_r}-\sum\limits_{r=0}^{n}\dfrac{n-r}{{}^nC_r}$

$=na_n-\sum\limits_{r=0}^{n}\dfrac{n-r}{{}^nC_{n-r}}$ $\left(\because {}^nC_r={}^nC_{n-r}\right)$

$=n\,a_n-b$

$\therefore 2b=na_n\Rightarrow b=\dfrac{n}{2}a_n$

66. **(b)** We have, $\left(1+\dfrac{C_1}{C_0}\right)\left(1+\dfrac{C_2}{C_1}\right)...\left(1+\dfrac{C_n}{C_{n-1}}\right)$

$=\left(1+\dfrac{n}{1}\right)\left[1+\dfrac{\frac{n(n-1)}{2!}}{n}\right]...\left(1+\dfrac{1}{n}\right)$

$=\dfrac{(1+n)}{1}.\dfrac{(1+n)}{2}.\dfrac{(1+n)}{3}...\dfrac{(1+n)}{n}=\dfrac{(1+n)^n}{n!}$

67. **(a)** There are (n + 1) terms in the expansion of $(x+a)^n$. Observing the terms, we can say that the first term from the end is the last term, i.e., $(n+1)^{th}$ term of the expansion and n + 1 = (n + 1) – (1 – 1). The second term from the end is the n^{th} term of the expansion and n = (n + 1) – (2 – 1).

The third term from the end is the $(n-1)^{th}$ term of the expansion and n – 1 = (n + 1) – (3 – 1), and so on. Thus, r^{th} term from the end will be term number (n + 1) – (r – 1) = (n – r + 2) of the expansion and the $(n-r+2)^{th}$ term is ${}^nC_{n-r+1}x^{r-1}a^{n-r+1}$.

68. **(d)** In the expansion of $(x+2y)^8$, the middle term is $\left(\dfrac{8}{2}+1\right)^{th}$ i.e., 5th term.

69. **(a)** In the binomial expression, we have

$(a+b)^n={}^nC_0a^n+{}^nC_1a^{n-1}b+{}^nC_2a^{n-2}b^2+...+{}^nC_nb^n$ ...(i)

The coefficients ${}^nC_0, {}^nC_1, {}^nC_2, ..., {}^nC_n$ are known as binomial or combinatorial coefficients.

Putting a = b = 1 in (i), we get

${}^nC_0+{}^nC_1+{}^nC_2+...+{}^nC_n=2^n$

Thus, the sum of all binomial coefficients is equal to 2^n.

Again, putting a = 1 and b = – 1 in Eq. (i), we get

${}^nC_0+{}^nC_2+{}^nC_4+...={}^nC_1+{}^nC_3+{}^nC_5+...$

Thus, the sum of all the odd binomial coefficients is equal to the sum of all the even binomial coefficients and each is equal to $\dfrac{2^n}{2}=2^{n-1}$.

$\Rightarrow {}^nC_0+{}^nC_2+{}^nC_4+...={}^nC_1+{}^nC_3+{}^nC_5+...=2^{n-1}$

70. **(a)** Both Assertion and Reason are correct.

Also, Reason is the correct explanation for the Assertion.

71. **(a)** Both are correct and Reason is the correct explanation.

72. **(a)** **Assertion:** $(x+2y)^9$

n = 9, a = 2y

$\therefore\ T_{r+1}={}^9C_r.x^{9-r}.(2y)^r$

$={}^9C_r.2^r.x^{9-r}.y^r$

73. (c) Assertion is correct. Reason is false.

Total number of terms $=\left(\frac{n}{2}+1\right)=5+1=6$

74. (d) Assertion is false and Reason is true.

CRITICALTHINKING TYPE QUESTIONS

75. (c) Given expression is :
$[(3x+y)^5]^4-[(3x-y)^4]^5=[(3x+y)]^{20}-[(3x-y)]^{20}$
First and second expansion will have 21 terms each but odd terms in second expansion be Ist, 3rd, 5th,.....21st will be equal and opposite to those of first expansion.
Thus, the number of terms in the expansion of above expression is 10.

76. (d) $T_{r+1}={}^{18}C_r(9x)^{18-r}\left(-\frac{1}{3\sqrt{x}}\right)^r$

$=(-1)^r\,{}^{18}C_r\,9^{18-\frac{3r}{2}}\,x^{18-\frac{3r}{2}}$
is independent of x provided $r=12$ and then $a=1$.

77. (c) $(1-x)^2(1+x)^{-2}=(1-2x+x^2)(1-2x+3x^2+..........)$
The term independent of x is 1.

78. (c)

79. (a) Given expansion is $\left(\sqrt{x}+\frac{k}{x^2}\right)^{10}$

$(r+1)_{th}$ term, $T_{r+1}={}^{10}C_r(\sqrt{x})^{10-r}\left(\frac{k}{x^2}\right)^r$

$\Rightarrow T_{r+1}={}^{10}C_r x^{5-r/2}.(k)^r.x^{-2r}$

$\therefore T_{r+1}={}^{10}C_r x^{(10-5r)/2}(k)^r$

Since, T_{r+1} is independent of x

$\therefore\ \frac{10-5r}{2}=0\ \Rightarrow\ r=2$

$\therefore\ 405={}^{10}C_2(k)^2$
$405=45\times k^2$
$\Rightarrow k^2=9\Rightarrow k=\pm 3$

80. (a) We have
$7^9+9^7=(8-1)^9+(8+1)^7=(1+8)^7-(1-8)^9$

$=[1+{}^7C_1 8+{}^7C_2 8^2+.....+{}^7C_7 8^7]$

$-[1-{}^9C_1 8+{}^9C_2 8^2-.....-{}^9C_9 8^9]$

$={}^7C_1 8+{}^9C_1 8+[{}^7C_2+{}^7C_3.8+....-{}^9C_2+{}^9C_3.8-...]8^2$
$=8(7+9)+64k=8..16+64k=64q$,
where $q=k+2$
Thus, 7^9+9^7 is divisible by 64.

81. (d) $T_{r+1}=\frac{n(n-1)(n-2).........(n-r+1)}{r!}(x)^r$
For first negative term,
$n-r+1<0\Rightarrow r>n+1$

$\Rightarrow r>\frac{32}{5}\ \therefore r=7.\left(\because n=\frac{27}{5}\right)$

Therefore, first negative term is T_8.

82. (c) $\left(1+\frac{1}{x^2}\right)^n(1+x^2)^n=\frac{\left(1+x^2\right)^{2n}}{x^{2n}}$,
numerator has $(2n+1)$ terms.

$\therefore$ The middle terms is $\frac{1}{x^{2n}}\ [{}^{(2n)}C_n(x^2)^n]={}^{(2n)}C_n$.

83. (d) ${}^{50}C_4+\sum_{r=1}^{6}{}^{56-r}C_3$

$={}^{50}C_4+\left[\begin{array}{r}{}^{55}C_3+{}^{54}C_3+{}^{53}C_3+{}^{52}C_3\\+{}^{51}C_3+{}^{50}C_3\end{array}\right]$

We know $\left[{}^nC_r+{}^nC_{r-1}={}^{n+1}C_r\right]$

$=({}^{50}C_4+{}^{50}C_3)+{}^{51}C_3+{}^{52}C_3+{}^{53}C_3+{}^{54}C_3+{}^{55}C_3$

$=({}^{51}C_4+{}^{51}C_3)+{}^{52}C_3+{}^{53}C_3+{}^{54}C_3+{}^{55}C_3$

Proceeding in the same way, we get

${}^{55}C_4+{}^{55}C_3={}^{56}C_4$.

84. (b) Binomial expansion of
$(1+x)^{50}=C_0+C_1x+C_2x^2+C_3x^3+\ldots+C_{50}x^{50}$
and in given expression
Putting $x=1$, we get
$2^{50}=C_0+C_1+C_2+C_3\ldots+C_{50}$ … (i)
and putting $x=-1$
$0=C_0-C_1+C_2-C_3.....+C_{50}$ … (ii)
Subtracting (ii) from (i), we get
$2^{50}=2(C_1+C_3+C_5+\ldots C_{49})$

$\Rightarrow\ C_1+C_3+C_5+\ldots C_{49}=\frac{2^{50}}{2}=2^{49}$

Sum of the coefficient of odd powers of $x=2^{49}$

85. (a) $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$=2[x^5+{}^5C_2x^3(x^3-1)+{}^5C_4x(x^3-1)^2]$
$=2[x^5+10x^3(x^3-1)+5x(x^6-2x^3+1]$
$=10x^7+20x^6+2x^5-20x^4-20x^3+10x$
$\therefore$ polynomial has degree 7.

86. (a) $\frac{r.{}^nC_r}{{}^nC_{r-1}}=\frac{n.{}^{n-1}C_{r-1}}{{}^nC_{r-1}}$

$=n.\frac{(n-1)!}{(r-1)!(n-r)!}\times\frac{(r-1)!(n-r+1)!}{n!}$

$=n-r+1$

Sum $=n+(n-1)+....+(n-9)=10n-45$

87. (d) $a_0+a_1+a_2+.....=2^{2n}$ and $a_0+a_2+a_4+....=2^{2n-1}$
$a_n={}^{2n}C_n=$ the greatest coefficient, being the middle coefficient
$a_{n-3}={}^{2n}C_{n-3}={}^{2n}C_{2n-(n-3)}={}^{2n}C_{n+3}=a_{n+3}$

88. (a) $na = 8 \Rightarrow n^2a^2 = 64, \dfrac{n(n-1)}{2}a^2 = 24$

since $\dfrac{2n}{n-1} = \dfrac{8}{3} \Rightarrow 6n = 8n - 8$

$\Rightarrow n = 4, a = 2$

89. (b) Coeff. of x^n in $(1+x)(1-x)^n$ = coeff. of x^n in

$(1+x)(1 - {}^nC_1x + {}^nC_2x^2 - .... + (-1)^n\, {}^nC_nx^n)$

$= (-1)^n\, {}^nC_n + (-1)^{n-1}\, {}^nC_{n-1} = (-1)^n + (-1)^{n-1}.n$

$= (-1)^n(1-n)$

90. (d) We know that, $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + ...\, {}^{20}C_{10}x^{10} + ...\, {}^{20}C_{20}x^{20}$

Put $x = -1$, $(0) = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} - {}^{20}C_{11} .... + {}^{20}C_{20}$

$\Rightarrow 0 = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9] + {}^{20}C_{10}$

$\Rightarrow {}^{20}C_{10} = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - {}^{20}C_9 + {}^{20}C_{10}]$

$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... + {}^{20}C_{10} = \dfrac{1}{2}\, {}^{20}C_{10}$

91. (b) We know by Binomial expansion, that $(x+a)^n$

$= {}^nC_0\, x^n a^0 + {}^nC_1x^{n-1}.a + {}^nC_2\, x^{n-2}a^2 + {}^nC_3\, x^{n-3}a^3. + {}^nC_4x^{n-4}.\, a^4 + ... + {}^nC_nx^0a^n$

Given expansion is $\left(x^4 - \dfrac{1}{x^3}\right)^{15}$

On comparing we get $n = 15, x = x^4, a = \left(-\dfrac{1}{x^3}\right)$

$\therefore$ We have

$$\left(x^4 - \frac{1}{x^3}\right)^{15} = {}^{15}C_0(x^4)^{15}\left(-\frac{1}{x^3}\right)^0 + {}^{15}C_1(x^4)^{14}\left(-\frac{1}{x^3}\right) + {}^{15}C_2(x^4)^{13}\left(-\frac{1}{x^3}\right)^2 + {}^{15}C_3(x^4)^{12}\left(-\frac{1}{x^3}\right)^3 + {}^{15}C_4(x^4)^{11}\left(-\frac{1}{x^3}\right)^4 + ...$$

$T_{r+1} = {}^{15}C_r(x^4)^{15-r}.\left(-\dfrac{1}{x^3}\right)^r = -{}^{15}C_r\, x^{60-7r}$

$\Rightarrow x^{60-7r} = x^{32} \quad \Rightarrow 60 - 7r = 32$

$\Rightarrow 7r = 28 \Rightarrow r = 4$

So, 5th term, contains x^{32}

$= {}^{15}C_4(x^4)^{11}\left(-\dfrac{1}{x^3}\right)^4 = {}^{15}C_4\, x^{44}x^{-12} = {}^{15}C_4x^{32}$.

Thus, coefficient of $x^{32} = {}^{15}C_4$.

92. (c) $\because$ x^3 and higher powers of x may be neglected

$$\therefore \frac{(1+x)^{\frac{3}{2}} - \left(1 + \frac{x}{2}\right)^3}{(1-x)^{\frac{1}{2}}}$$

$$= (1-x)^{\frac{-1}{2}}\left[\left(1 + \frac{3}{2}x + \frac{\frac{3}{2}.\frac{1}{2}}{2!}x^2\right) - \left(1 + \frac{3x}{2} + \frac{3.2}{2!}\frac{x^2}{4}\right)\right]$$

$$= \left[1 + \frac{x}{2} + \frac{\frac{1}{2}.\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right] = \frac{-3}{8}x^2$$

(as x^3 and higher powers of x can be neglected)