Circle

Chapter 26

CIRCLE

A circle is the locus of point which moves in a plane such that its distance from a fixed point is constant. The fixed point is called the *centre* and the constant distance is called the *radius* of the circle.

STANDARD EQUATION OF A CIRCLE

1. The equation of a circle with the centre at (α, β) and radius a, is $(x-\alpha)^2+(y-\beta)^2=a^2$
2. If the centre of the circle is at the origin and the radius is a, then the equation of circle is $x^2+y^2=a^2$.

GENERAL EQUATION OF A CIRCLE

The general equation of a circle is of the form $x^2+y^2+2gx+2fy+c=0$, … (1)

Where g, f and c are constants.

The coordinates of the centre are $(-g, -f)$ and radius $=\sqrt{g^2+f^2-c}$.

CONDITIONS FOR GENERAL EQUATION OF SECOND DEGREE TO REPRESENT A CIRCLE

A general equation of second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$ in x, y represents a circle if

1. Coefficient of x^2 = coefficient of y^2 i.e. $a=b$,
2. Coefficient of xy is zero i.e. $h=0$.

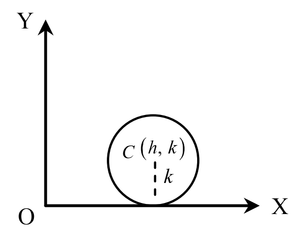
DIFFERENT FORMS OF THE EQUATION OF A CIRCLE

1. **Circle with centre at the point *(h, k)* and which touches the axis of *x***

   Since the circle touches the x-axis, the radius of the circle $=k$.

   $\therefore$ Equation of the circle is $(x-h)^2+(y-k)^2=k^2$

   or $x^2+y^2-2hx-2ky+h^2=0$

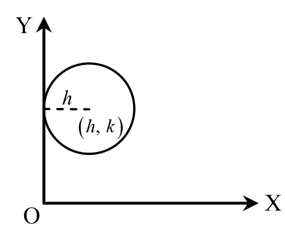


2. **Circle with centre at the point *(h, k)* and which touches the axis of *y***

   Since the circle touches the y-axis, the radius of the circle $=h$

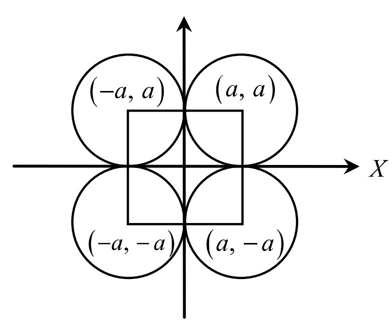
   $\therefore$ Equation of the circle is $(x-h)^2+(y-k)^2=h^2$

   or $x^2+y^2-2hx-2ky+k^2=0$.



3. **Circle with radius *a* and which touches both the coordinate axes**

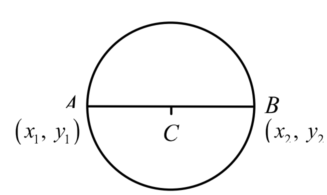
   Since the centre of the circle may be in any of the four quadrants, therefore it will be any one of the four points $(\pm a, \pm a)$. Thus, there are four circles of radius a touching both the coordinate axes, and their equations are $(x\pm a)^2+(y\pm a)^2=a^2$ or $x^2+y^2\pm 2ax\pm 2ay+a^2=0$.



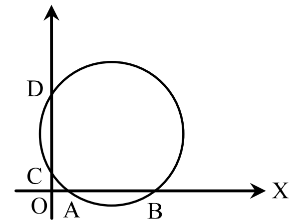
CIRCLE ON A GIVEN DIAMETER

The equation of the circle drawn on the line segment joining two given points (x_1, y_1) and (x_2, y_2) as diameter is $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.

Its centre is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ and radius is $\frac{1}{2}\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$



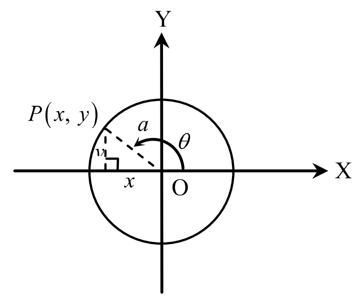
INTERCEPTS MADE BY A CIRCLE ON THE AXES



1. The length of the intercept made by the circle $x^2+y^2+2gx+2fy+c=0$ on

   (a) x-axis $=AB=2\sqrt{g^2-c}$ (b) y-axis $=CD=2\sqrt{f^2-c}$
2. Intercepts are always positive.
3. If the circle touches x-axis then $AB=0$ $\therefore$ $c=g^2$.
4. If the circle touches y-axis, then $CD=0$ $\therefore$ $c=f^2$.
5. If the circle touches both the axes, then $AB=0=CD$ $\therefore$ $c=g^2=f^2$.

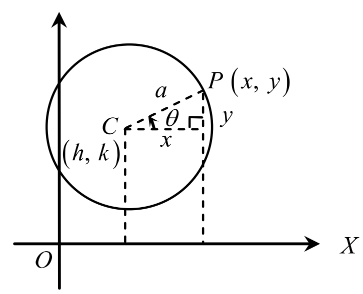
PARAMETRIC EQUATIONS OF A CIRCLE



1. The parametric equations of a circle $x^2+y^2=a^2$ are $x=a\cos\theta$, $y=a\sin\theta,\ 0\le\theta<2\pi$.

   θ is called parameter and the point $P(a\cos\theta,\ a\sin\theta)$ is called the point 'θ' on the circle $x^2+y^2=a^2$.

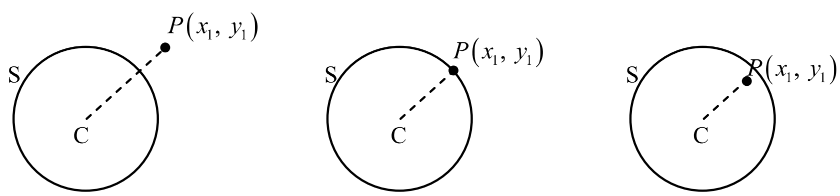
   Thus, the coordinates of any point on the circle $x^2+y^2=a^2$ may be taken as $(a\cos\theta,\ a\sin\theta)$.



2. The parametric equations of a circle $(x-h)^2+(y-k)^2=a^2$ are $x=h+a\cos\theta,\ y=k+a\sin\theta,\ \ 0\le\theta<2\pi$ is called the point 'θ' on this circle. Thus the coordinates of any point on this circle may be taken as $(h+a\cos\theta,\ k+a\sin\theta)$.

POSITION OF A POINT WITH RESPECT TO A CIRCLE

Let $S\equiv x^2+y^2+2gx+2fy+c=0,$ be a circle and $P(x_1,\ y_1)$ be a point in the plane of S, then $S_1\equiv x_1^2+y_1^2+2gx_1+2fy_1+c.$ The point $P(x_1,\ y_1)$ lies outside, on or inside the circle S according as $S_1>,=$ or <0.



Note : Let S be a circle and $P(x_1,\ y_1)$ and $Q(x_2,\ y_2)$ be two points in the plane of S, then they lie

(a) on the same side of S iff S_1 and S_2 have same sign,

(b) on the opposite sides of S iff S_1 and S_2 have opposite signs.

CIRCLE THROUGH THREE POINTS

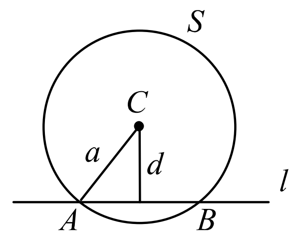
The equation of the circle through three non-collinear points $(x_1,\ y_1),(x_2,\ y_2)$ and $(x_3,\ y_3)$ is

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix}=0$$

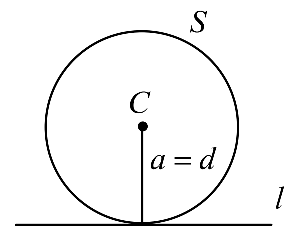
INTERSECTION OF A LINE AND A CIRCLE

Let S be a circle with centre C and radius a. Let l be any line in the plane of the circle and d be the perpendicular distance from C to the line l, then

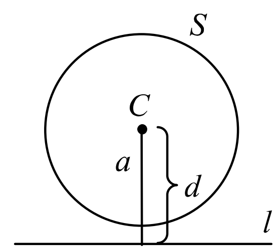
(a) l intersects S in two distinct points iff $d < a$.



(b) l intersects S in one and only one point iff $d = a$ i.e. the line touches the circle iff perpendicular distance from the centre to the line is equal to the radius of the circle.

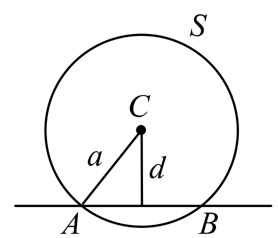


(c) l does not intersect S iff $d > a$.



LENGTH OF THE INTERCEPT MADE BY A CIRCLE ON A LINE

If the line l meets the circle S with centre C and radius 'a' in two distinct points A and B and if d is the perpendicular distance of C from the line l, the length of the intercept made by the circle on the line $= |AB| = 2\sqrt{a^2 - d^2}$.



To find the point of intersection of a line $y = mx + c$ with a circle $x^2 + y^2 = a^2$ we need to solve both the curves i.e. roots of equation $x^2 + (mx+c)^2 = a^2$ gives x coordinates of the point of intersection. Now following cases arise :

(i) Discriminant $> 0 \Rightarrow$ two distinct and real points of intersection.

(ii) Discriminant $= 0 \Rightarrow$ coincident roots i.e. line is tangent to the circle.

(iii) Discriminant $< 0 \Rightarrow$ no real point of intersection.

TANGENT TO A CIRCLE AT A GIVEN POINT

1. Equation of the tangent to the circle $x^2 + y^2 = a^2$ at the point (x_1, y_1) on it is

$$xx_1 + yy_1 = a^2.$$

2. Equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at the point (x_1, y_1) on it is

$$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0.$$

3. Equation of the tangent to the circle $x^2 + y^2 = a^2$ at the point $(a\cos\theta, a\sin\theta)$ on it is

$$x\cos\theta + y\sin\theta = a$$ [Parametric form of equation of tangent]

Note : The equation of the tangent at the point (x_1, y_1) on the circle $S = 0$ is $T = 0$.

EQUATION OF THE TANGENT IN SLOPE FORM

The equation of a tangent of slope m to the circle $x^2 + y^2 = a^2$ is $y = mx \pm a\sqrt{1 + m^2}$.

The coordinates of the point of contact are $\left(\pm\frac{am}{\sqrt{1+m^2}}, \mp\frac{a}{\sqrt{1+m^2}}\right)$.

CONDITION OF TANGENCY

The straight line $y = mx + c$ will be a tangent to the circle $x^2 + y^2 = a^2$ if $c = \pm a\sqrt{1+m^2}$.

Note : A line will touch a circle if and only if the length of the $\perp$ from the centre of the circle to the line is equal to the radius of the circle.

NORMAL TO A CIRCLE AT A GIVEN POINT

The normal to a circle, at any point on the circle, is a straight line which is $\perp$ to the tangent to the circle at that point and always passes through the centre of the circle.

1. Equation of the normal to the circle $x^2 + y^2 = a^2$ at the point (x_1, y_1) on it is

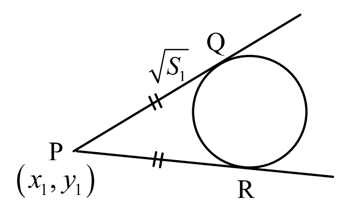
$$\frac{x}{x_1} = \frac{y}{y_1}.$$

2. Equation of the normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at the point (x_1, y_1) on it is

$$\frac{x - x_1}{x_1 + g} = \frac{y - y_1}{y_1 + f}.$$

LENGTH OF TANGENTS

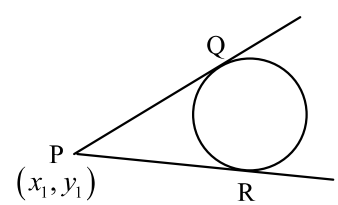
Let PQ and PR be two tangents drawn from $P(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$. Then PQ = PR and the length of tangent drawn from point P is given by



$$PQ = PR = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c} = \sqrt{S_1}.$$

PAIR OF TANGENTS

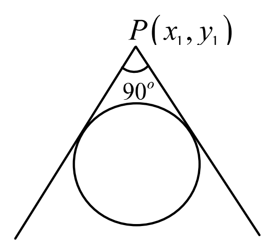
Form a given point $P(x_1, y_1)$ two tangents PQ and PR can be drawn to the circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$ Their combined equation is $SS_1 = T^2$ where S= 0 is the equation of circle, T=0 is the equation of tangents at (x_1, y_1) and S_1 is obtained by replacing x by x_1 and y by y_1 in S.



DIRECTOR CIRCLE

The locus of the point of intersection of two perpendicular tangents to a circle is called the Director circle.

Let the circle be $x^2 + y^2 = a^2$. then equation of director circle is $x^2 + y^2 = 2a^2$.Obviously director circle is a concentric circle whose radius is $\sqrt{2}$ times the radius of the given circle.



Director circle of circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is

$$x^2 + y^2 + 2gx + 2fy + 2c - g^2 - f^2 = 0$$

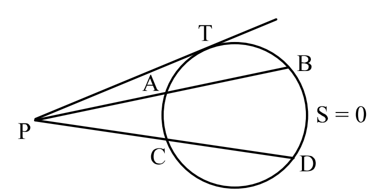
POWER OF A POINT WITH RESPECT TO A CIRCLE

Let $P(x_1, y_1)$ be point and secant PAB, drawn.

The power of $P(x_1, y_1)$ w.r.t.

$$S = x^2 + y^2 + 2gx + 2fy + c = 0$$

is equal to $PA.PB$, which is $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

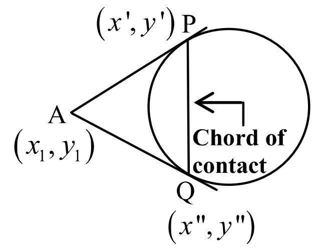


$\therefore$ Power remains constant for the circle i.e. independent of A and B

$\therefore$ $PA.PB = PC.PD = PT^2 =$ square of the length of a tangent.

[Power of a point P is positive, negative or zero according to position of P outside, inside or on the circle respectively]

CHORD OF CONTACT OF TANGENTS



1. **Chord of contact:** The chord joining the points of contact of the two tangents to a conic drawn from a given point, outside it, is called the chord of contact of tangents

2. **Equation of chord of contact:**
The equation of the chord of contact of tangents drawn from a point (x_1, y_1) to the circle $x^2+y^2=a^2$ is $xx_1+yy_1=a^2$

Equation of chord of contact at (x_1, y_1) to the circle $x^2+y^2+2gx+2fy+c=0$ is $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$. It is clear from above that the equation to the chord of contact coincides with the equation of the tangent, if point (x_1, y_1) lies on the circle.

The length of chord of contact = $2\sqrt{r^2-p^2}$; (p being length of perpendicular from centre of the chord)

Area of ΔAPQ *is given by* $\dfrac{a\left(x_1^2+y_1^2-a^2\right)^{3/2}}{x_1^2+y_1^2}$

3. **Equation of the chord bisected at a given point:**
The equation of the chord of the circle $S \equiv x^2+y^2+2gx+2fy+c=0$ bisected at the point (x_1, y_1) is given by $T = S_1$ *. i.e.,* $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=x_1^2+y_1^2+2gx_1+2fy_1+c.$

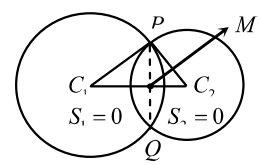
COMMON CHORD OF TWO CIRCLES

1. **Definition :** The chord joining the points of intersection of two given circles is called their common chord.
2. **Equation of common chord :** The equation of the common chord of two circles

$S_1 \equiv x^2+y^2+2g_1x+2f_1y+c_1=0$ … (i)

and $S_2 \equiv x^2+y^2+2g_2x+2f_2y+c_2=0$ … (ii)

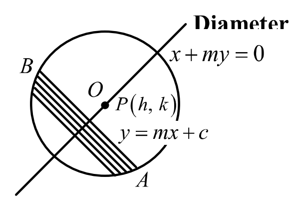
is $2x(g_1-g_2)+2y(f_1-f_2)+c_1-c_2=0$ i.e., $S_1-S_2=0.$



3. **Length of the common chord :** $PQ=2(PM)=2\sqrt{C_1P^2-C_1M^2}$
Where $C_1P=$ radius of the circle $S_1=0$ and $C_1M=$ length of the perpendicular from the centre C_1 to the common chord PQ.

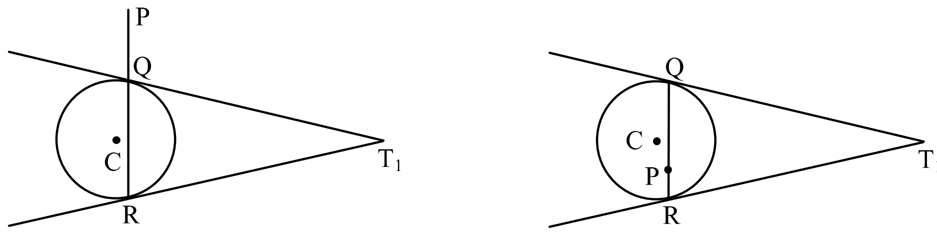
DIAMETER OF A CIRCLE

The locus of the middle points of a system of parallel chords of a circle is called a diameter of the circle.
The equation of the diameter bisecting parallel chords $y=mx+c$ (c is a parameter) of the circle $x^2+y^2=a^2$ is $x+my=0$.



POLE AND POLAR

If from a point P any straight line is drawn to meet the circle in Q and R and if tangents to the circle at Q and R meet in T_1, then the locus of T_1 is called the polar of P with respect to the circle.



The point P is called the pole of its polar.

The polar of the point $P(x_1, y_1)$ w.r.t. the circle $S=0$ is given by $T=0$.

i.e. $xx_1 + yy_1 + = a^2$ for the circle $x^2 + y^2 = a^2$ and

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$ for the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

Note :

(a) If the point P lies outside the circle, then the polar and the chord of contact of this point P are same straight line.

(b) If the point P lies on the circle, then the polar and the tangent to the circle at P are same straight line.

(c) The coordinates of the pole of the line $lx + my + n = 0$ with respect to the circle $x^2 + y^2 = a^2$ are $\left(-\frac{a^2 l}{n}, -\frac{a^2 m}{n}\right)$.

CONJUGATE POINTS

Two points are said to be conjugate points with respect to a circle if the polar of either passes through the other

CONJUGATE LINES

Two straight lines are said to be conjugate lines if the pole of either lies on the other.

Common tangents to two circles

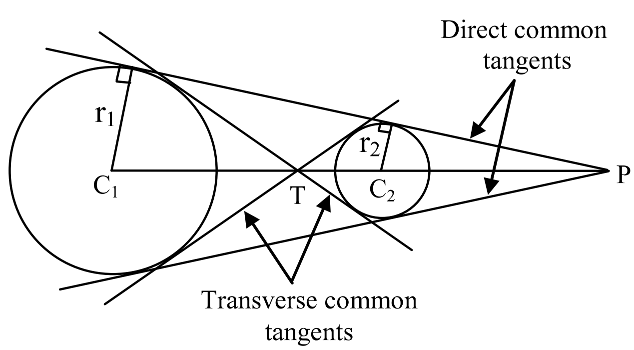
Different cases of intersection of two circle :

Let the two circles be $(x - x_1)^2 + (y - y_1)^2 = r_1^2$ … (i)

and $(x - x_2)^2 + (y - y_2)^2 = r_2^2$ … (ii)

With centres $C_1(x_1, y_1)$ *and* $C_2(x_2, y_2)$ and radii r_1 and r_2 respectively. Then following cases may arise :

Case I : When $C_1C_2 > r_1 + r_2$ *i.e.*, the distance between the centres is greater than the sum of radii.



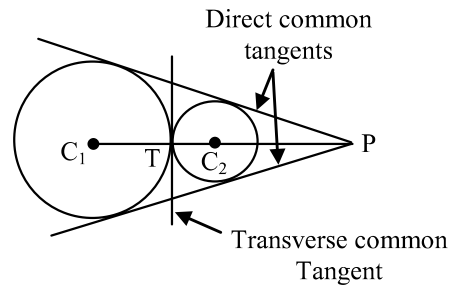
In this case four common tangents can be drawn to the two circles, in which two are direct common tangents and the other two are transverse common tangents.

The points P, T of intersection of direct common tangents and transverse common tangents respectively, always lie on the line joining the centres of the two circles and divide it externally and internally respectively in the ratio of their radii.

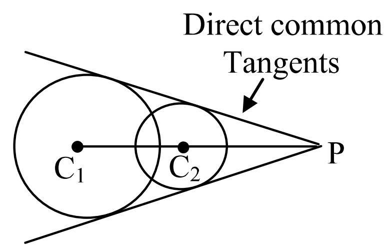
$\frac{C_1P}{C_2P} = \frac{r_1}{r_2}$ (externally) and $\frac{C_1T}{C_2T} = \frac{r_1}{r_2}$ (internally) Hence, the ordinates of P and T are

$$P \equiv \left(\frac{r_1x_2 - r_2x_1}{r_1 - r_2}, \frac{r_1y_2 - r_2y_1}{r_1 - r_2} \right) \text{ and } T \equiv \left(\frac{r_1x_2 + r_2x_1}{r_1 + r_2}, \frac{r_1y_2 + r_2y_1}{r_1 + r_2} \right).$$

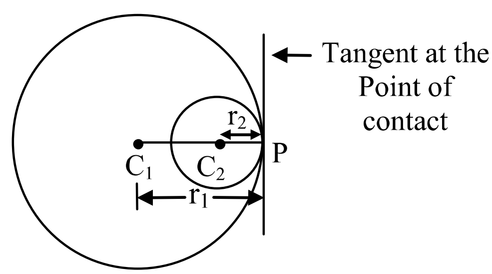
Case II : When $C_1C_2 = r_1 + r_2$ *i.e.*, the distance between the centres is equal to the sum of radii.
In this case two direct common tangents are real and distinct while transverse tangents are coincident.



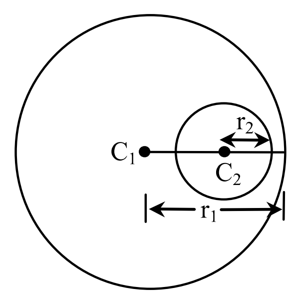
Case III : When $C_1C_2 < r_1 + r_2$ *i.e.*, the distance between the centres is less than sum of radii.
In this case two direct common tangents are real and distinct while the transverse tangents are imaginary.



Case IV : When $C_1C_2 = |r_1 - r_2|$, *i.e.*, the distance between the centres is equal to the difference of the radii.
In this case two tangents are real and coincident while the other two tangents are imaginary.



Case V : When $C_1C_2 < |r_1 - r_2|$, *i.e.*, the distance between the centres is less than the difference of the radii.
In this case, all the four common tangents are imaginary.



WORKING RULE TO FIND DIRECT COMMON TANGENTS

Step 1 : Find the coordinates of centres C_1, C_2 and radii r_1, r_2 of the two given circles.

Step 2 : Find the coordinates of the point, say P dividing C_1C_2 externally in the ratio $r_1 : r_2$. Let $P \equiv (h, k)$.

Step 3 : Write the equation of any line through $P(h, k)$ i.e. $y - k = m(x - h)$ … (1)

Step 4 : Find the two values of m, using the fact that the length of the perpendicular on (1) from the centre C_1 of one circle is equal to its radius r_1.

Step 5 : Substituting these values of 'm' in (1), the equation of the two direct common tangents can be obtained.

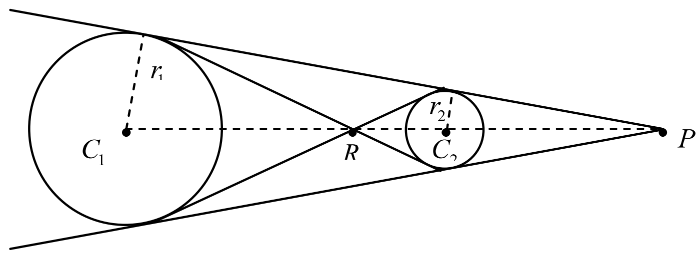
Note :

(a) The direct common tangents to two circles meet on the line joining centres C_1 and C_2, and divide it externally in the ratio of the radii.

(b) The transverse common tangents also meet on the line of centres C_1 and C_2, and divide it internally in the ratio of the radii.

WORKING RULE TO FIND TRANSVERSE COMMON TANGENTS

All the steps except the 2nd step are the same as above. Here in the second step the point $R(h, k)$ will divide C_1C_2 internally in the ratio $r_1 : r_2$.



Note :

(a) When two circles are real and non-intersecting, 4 common tangents can be drawn.

(b) When two circles touch each other externally, 3 common tangents can be drawn to the circles.

(c) When two circles intersect each other at two real and distinct points, two common tangents can be drawn to the circles.

(d) When two circle touch each other internally one common tangent can be drawn to the circles.

IMAGE OF THE CIRCLE BY THE LINE MIRROR

Let the circle be $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ and the line be $L = lx + my + n = 0$.

The radius of the image circle will be the same as that of the given circle.

Let the centre of the image circle be (x_1, y_1).

Slope of C_1C_2 $\times$ slope of line $L = -1$ … (1)

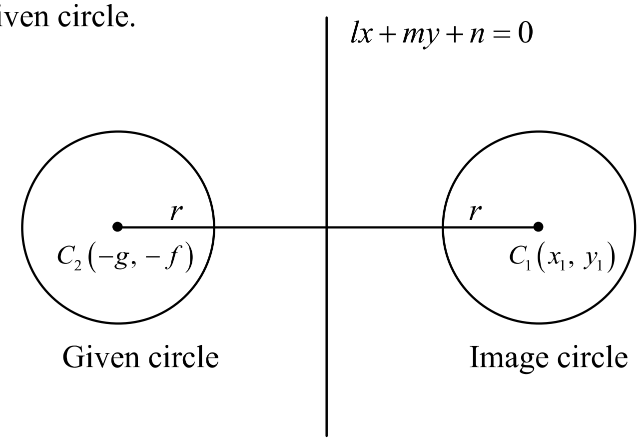
and midpoint of C_1C_2 lies on $lx + my + n = 0$

$\Rightarrow$ $l\left(\dfrac{x_1 - g}{2}\right) + m\left(\dfrac{y_1 - f}{2}\right) + n = 0$ … (2)

Solving (1) and (2), we get (x_1, y_1).

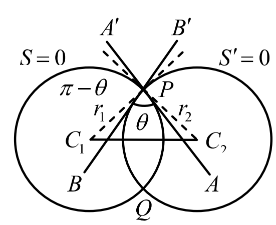
$\Rightarrow$ Required image circle will be

$$(x - x_1)^2 + (y - y_1)^2 = \left(\sqrt{g^2 + f^2 - c}\right)^2.$$



ANGLE OF INTERSECTION OF TWO CIRCLES

The angle of intersection between two circles $S = 0$ and $S' = 0$ is defined as the angle between their tangents at their point of intersection.



If $S \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

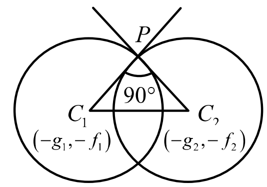
$S' \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

are two circles with radii r_1, r_2 and d be the distance between their centres then the angle of intersection θ between them is given by

$$\cos\theta = \left|\frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}\right|$$

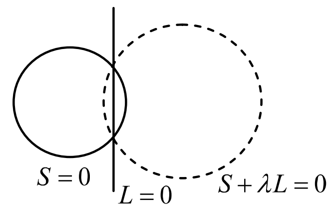
or $$\cos\theta = \left|\frac{2(g_1g_2 + f_1f_2) - (c_1 + c_2)}{2\sqrt{g_1^2 + f_1^2 - c_1}\sqrt{g_2^2 + f_2^2 - c_2}}\right|.$$

Condition of Orthogonality : If the angle of intersection of the two circles is a right angle $(\theta = 90°)$, then such circles are called orthogonal circles and condition for orthogonality is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

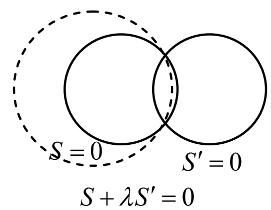


FAMILY OF CIRCLES

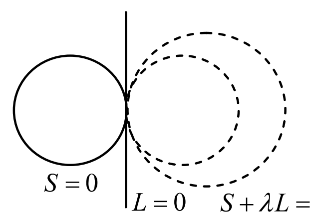
1. The equation of the family of circles passing through the point of intersection of circle $S = 0$ and a line $L = 0$ is given as $S + \lambda L = 0$, (where λ is a parameter)



2. The equation of the family of circles passing through the point of intersection of two given circles $S = 0$ and $S' = 0$ is given as $S + \lambda S' = 0$, (where λ is a parameter, $\lambda \neq -1$). But it is better to find first the equation of common $S - S' = 0$ and then use $S + \lambda(S - S') = 0$

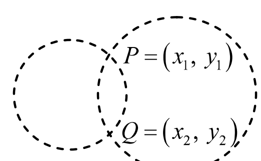


3. The equation of the family of circles touching the circle $S = 0$ and the line $L = 0$ at their point of contact P is
$S + \lambda L = 0$, (where λ is a parameter)

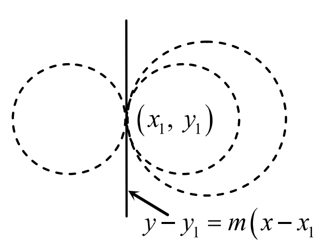


4. The equation of a family of circles passing through two given points $P(x_1, y_1)$ and $Q(x_2, y_2)$ can be written in the form

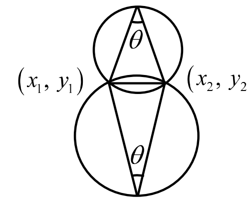
$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0, \quad \text{(where } \lambda \text{ is a parameter)}$$



5. The equation of family of circles, which touch $y - y_1 = m(x - x_1)$ at (x_1, y_1) for any finite m is $(x - x_1)^2 + (y - y_1)^2 + \lambda\{(y - y_1) - m(x - x_1)\} = 0$
And if m is infinite, the family of circles is
$(x - x_1)^2 + (y - y_1)^2 + \lambda(x - x_1) = 0$, (where λ is a parameter)

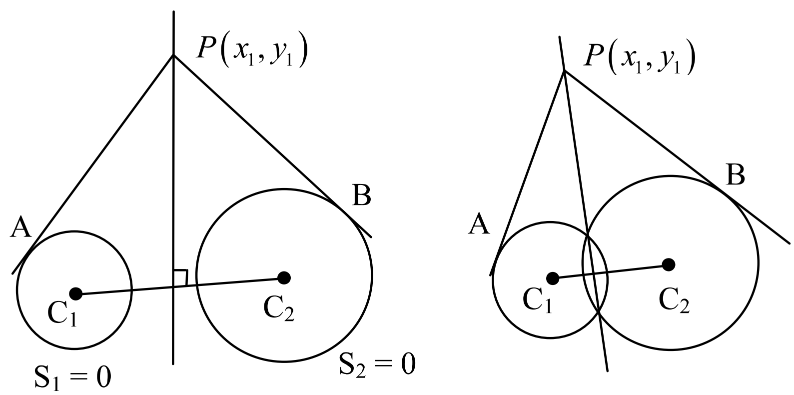


6. Equation of the circles given in diagram is
$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)\pm\cot\theta\{(x-x_1)(y-y_2)-(x-x_2)(y-y_1)\}=0$$



RADICAL AXIS

The radical axis of two circles is the locus of a point which moves such that the lengths of the tangents drawn from it to the two circles are equal.



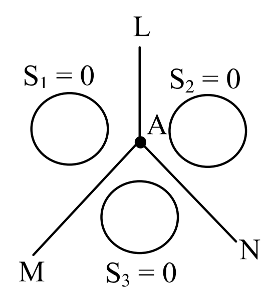
The equation of the radical axis of the two circle is $S_1 - S_2 = 0$ i.e.,
$$2x(g_1-g_2)+2y(f_1-f_2)+c_1-c_2=0, \qquad \text{which is a straight line.}$$

PROPERTIES OF RADICAL AXIS

1. The radical axis and common chord are identical for two intersecting circles.
2. The radical axis of two circles is perpendicular to the line joining their centres.
3. Radical centre : The radical axis of three circles taken in pairs meet at a point, called the radical centre of the circles. Coordinates of radical centre can be found by solving the equations
$$S_1 = S_2 = S_3 = 0.$$
4. The radical centre of three circle described on the sides of a triangle as diameters is the orthocentre of the triangle.
5. If two circles cut a third circle orthogonally, then the radical axis of the two circles pass through the centre of the third circle.
6. The radical axis of the two circles will bisect their common tangents.

RADICAL CENTRE

The radical axes of three circles, taken in pairs, meet in a point, which is called their radical centre. Let the three circles be
$$S_1 = 0 \ \dots \text{(i)}, \ S_2 = 0 \ \dots \text{(ii)} \ \text{and} \ S_3 = 0 \ \dots \text{(iii)}$$



Let the straight lines i.e., AL and AM meet in A.

The equation of any straight line passing through A is
$$(S_1 - S_2) + \lambda(S_3 - S_1) = 0, \ \text{where } \lambda \text{ is any constant.}$$

For $\lambda = 1$, this equation becomes $S_2 - S_3 = 0$, which is, equation of AN.

Thus the third radical axis also passes through the point where the straight lines AL and AM meet.

In the above figure A is the radical centre.

PROPERTIES OF RADICAL CENTRE

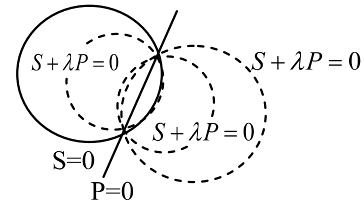
Co-axial system of circles

A system (or a family) of circles, every pair of which have the same radical axis, are called co-axial circles.

1. The equation of a system of co-axial circles, when the equation of the radical axis and of one circle of the system are

$$P \equiv lx + my + n = 0, S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$$

respectively, is $S + \lambda P = 0$ (λ is an arbitrary constant).

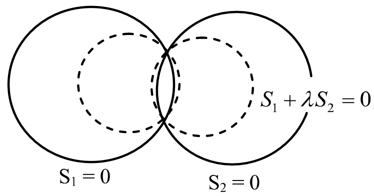


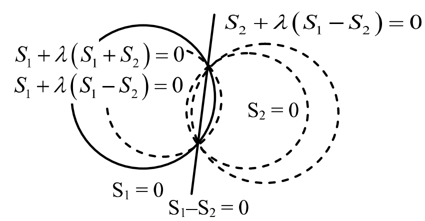
2. The equation of a co-axial system of circles, where the equation of any two circles of the system are

$$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0 \text{ and } S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$$

respectively, is $S_1 + \lambda(S_1 - S_2) = 0$ or $S_2 + \lambda_1(S_1 - S_2) = 0$

Other form $S_1 + \lambda S_2 = 0,\ (\lambda \neq -1)$





3. The equation of a system of co-axial circles in the simplest form is $x^2 + y^2 + 2gx + c = 0$, where g is a variable and c is a constant.

LIMITING POINTS

Limiting points of a system of co-axial circles are the centres of the point circles belonging to the family (Circles whose radii are zero are called point circles).

Let the circle is $x^2 + y^2 + 2gx + c = 0$ … (i)

where g is a variable and c is a constant.

$\therefore$ Centre and the radius of (i) are $(-g, 0)$ and $\sqrt{(g^2 - c)}$ respectively. Let $\sqrt{g^2 - c} = 0 \Rightarrow g = \pm\sqrt{c}$

Thus we get the two limiting points of the given co-axial system as $(\sqrt{c}, 0)$ and $(-\sqrt{c}, 0)$

Clearly the above limiting points are real and distinct, real and coincident or imaginary according as $c >, =, < 0$.

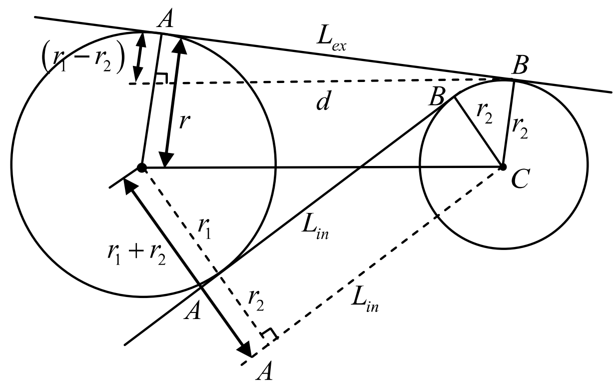
TIPS & TRICKS

Length of an external common tangent and internal common tangent to two circles is given by length of external common tangent

$$L_{ex} = \sqrt{d^2 - (r_1 - r_2)^2}$$

and length of internal common tangent $L_{in} = \sqrt{d^2 - (r_1 + r_2)^2}$ [Applicable only when $d > (r_1 + r_2)$]

where d is the distance between the centres of two circles i.e., $|C_1C_2| = d$ and r_1 and r_2 are the radii of two circles.

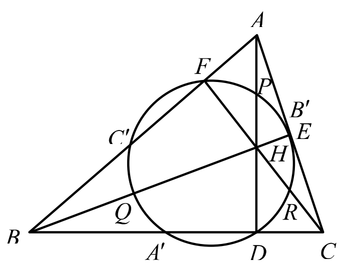


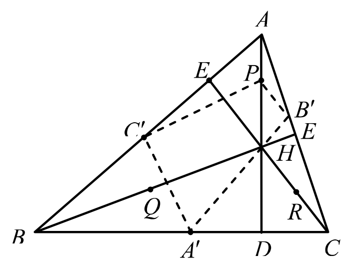
Nine-point circle : The circle through the midpoints of the sides of a triangle passes through the feet of the altitudes and the midpoints of the lines joining the orthocentre to the vertices. This circle is called the nine-point circle of the triangle.

Note :

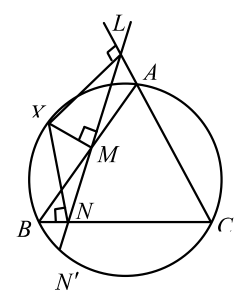
(a) The radius of the nine point circle is half the radius of the circumcircle of the triangle ABC

(b) Centre is midpoint of the line segment joining orthocenter and circumcentre.





Simson's line : The feet L, M, N of the perpendicular on the sides BC, CA, AB of any ΔABC from any point X on the circumcircle of the triangle are collinear. The line LMN is called the ***Simson's line*** or the ***pedal line*** of the point X with respect to ΔABC.



If H is the orthocentre of ΔABC and AH produced meets BC at D and the circumcircle of ΔABC at P, then $HD = DP$.