EXERCISE- 5 (A)

Question 1:
Solve each of the following equations by factorization:

$X^2 – 10x – 24 = 0$

Solution 1:
$X^2 – 10x – 24 = 0$

$\Rightarrow x^2 – 12x + 2x – 24 = 0$

$\Rightarrow x (x – 12) + 2 (x – 12) = 0$

$\Rightarrow (x – 12) (x+2) = 0$

Since $x – 12 = 0$ Or $x + 2 = 0$

Then $x = 12$ Or $x = – 2$

Question 3:
$2x^2 – \frac{1}{2}x = 0$

Solution 3:
$2x^2 – \frac{1}{2}x = 0$

$\Rightarrow x (2x – \frac{1}{2}) = 0$

Since $x = 0$ Or $2x – \frac{1}{2} = 0$

Then $x = 0$ Or $x = \frac{1}{4}$

Question 4:
$x(x – 5) = 24$

Solution 4:
$x(x – 5) = 24$

$\Rightarrow x^2 – 5x – 24 = 0$

$\Rightarrow x^2 – 8x + 3x – 24 = 0$

$\Rightarrow x (x – 8) + 3 (x – 8) = 0$

$\Rightarrow (x – 8) (x + 3) = 0$

Since $x – 8 = 0$ Or $x + 3 = 0$

Then $x = 8$ Or $x = – 3$

Question 5:

$\frac{9}{2}\ x = 5 + x^2$

Solution 5:

$\frac{9}{2}\ x = 5 + x^2$

$\Rightarrow 9x = 10 + 2x^2$

$\Rightarrow 2x^2 – 9x + 10 = 0$

$\Rightarrow 2x^2 – 5x – 4x + 10 = 0$

$\Rightarrow x(2x – 5) –2(2x – 5) = 0$

$\Rightarrow (2x – 5) (x – 2) = 0$

Since $2x – 5 = 0$ Or $x – 2 = 0$

Then $x = \frac{5}{2}$ Or $x = 2$

Question 6:

$\frac{6}{x} = 1 + x$

Solution 6:

$\frac{6}{x} = 1 + x$

$\Rightarrow 6 = x + x^2$

$\Rightarrow x^2 + x – 6 = 0$

$\Rightarrow x^2 + 3x – 2x – 6 = 0$

$\Rightarrow x (x+3) − 2 (x + 3) = 0$

$\Rightarrow (x+3) (x – 2) = 0$

Since $x + 3 = 0$ Or $x – 2 = 0$

Then $x = – 3$ Or $x = 2$

Question 7:

$x = \frac{3x+1}{4x}$

Solution 7:

$x = \frac{3x+1}{4x}$

$\Rightarrow 4x^2 = 3x + 1$

$\Rightarrow 4x^2 – 3x – 1 = 0$

$\Rightarrow 4x^2 – 4x + x – 1 = 0$

$\Rightarrow 4x(x – 1) + 1 (x – 1) = 0$

$\Rightarrow (x – 1) (4x + 1) = 0$

Since $x – 1 = 0$ Or $4x + 1 = 0$

Then x = 1 Or $x = -\frac{1}{4}$

Question 8:

$x + \frac{1}{x} = 2.5$

Solution 8:

$x + \frac{1}{x} = 2.5$

$\Rightarrow \frac{x^2+1}{x} = \frac{5}{2}$

$\Rightarrow 2x^2 + 2 = 5x$

$\Rightarrow 2x^2 - 5x + 2 = 0$

$\Rightarrow 2x^2 - 4x - x + 2 = 0$

$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (x - 2)(2x - 1) = 0$

Since $x - 2 = 0$ Or $2x - 1 = 0$

Then $x = 2$ Or $x = \frac{1}{2}$

Question 9:

$(2x - 3)^2 = 49$

Solution 9:

$(2x - 3)^2 = 49$

Taking square root on both sides

$2x - 3 = \pm 7$

When $2x - 3 = 7 \Rightarrow 2x = 10 \Rightarrow x = 5$

And, when $2x - 3 = -7 \Rightarrow 2x = -4 \Rightarrow x = -2$

Question 10:

$2(x^2 - 6) = 3(x - 4)$

Solution 10:

$2(x^2 - 6) = 3(x - 4)$

$\Rightarrow 2x^2 - 12 = 13x - 12$

$\Rightarrow 2x^2 - 3x = 0$

$\Rightarrow x(2x - 3) = 0$

Since $x = 0$ Or $2x - 3 = 0$

Then $x = 0$ Or $x = \frac{3}{2}$

Question 11:

$(x + 1)(2x + 8) = (x+7)(x+3)$

Solution 11:

$(x + 1)(2x + 8) = (x+7)(x+3)$

$\Rightarrow 2x^2 + 8x + 2x + 8 = x^2 + 3x + 7x + 21$

$\Rightarrow 2x^2 + 10x + 8 = x^2 + 10x + 21$

$\Rightarrow x^2 - 13 = 0$

$\Rightarrow x^2 - \left(\sqrt{13}\right)^2 = 0$

$\Rightarrow (x + \sqrt{13})(x - \sqrt{13}) = 0$

If $x + \sqrt{13} = 0$ Or $x - \sqrt{13} = 0$

$\Rightarrow x = -\sqrt{13}$ Or $x = \sqrt{13}$

Question 12:

$x^2 - (a + b)x + ab = 0$

Solution 12:

$x^2 - (a + b)x + ab = 0$

$\Rightarrow X^2 - ax - bx + ab = 0$

$\Rightarrow x(x - a) - b(x - a) = 0$

$\Rightarrow x(x - a)(x - b) = 0$

Since $x - a = 0$ Or $x - b = 0$

Then $x = a$ Or $x = b$

Question 13:

$(x + 3)^2 - 4(x + 3) - 5 = 0$

Solution 13:

$(x + 3)^2 - 4(x + 3) - 5 = 0$

Let $x + 3 = y$

Then $y^2 - 4y - 5 = 0$

$\Rightarrow y^2 - 5y + y - 5 = 0$

$\Rightarrow y(y - 5) + 1(y - 5) = 0$

$\Rightarrow (y - 5)(y + 1) = 0$

If $y - 5 = 0$ Or $y + 1 = 0$

Then $y = 5$ Or $y = -1$

$\Rightarrow x + 3 = 5$ or $x + 3 = -1$

$\Rightarrow x = 2$ or $x = -4$

Question 14:

$4(2x – 3)^2 – (2x – 3) – 14 = 0$

Solution 14:

$4(2x – 3)^2 – (2x – 3) – 14 = 0$

Let $2x – 3 = y$

Then $4y^2 – y – 14 = 0$

$\Rightarrow 4y^2 – 8y + 7y – 14 = 0$

$\Rightarrow 4y(y – 2) + 7 (y – 2) = 0$

$\Rightarrow (y – 2) (4y + 7) = 0$

If $y – 2 = 0$ Or $4y + 7 = 0$

$\Rightarrow y = 2$ or $y = \frac{-7}{4}$

$\Rightarrow 2x – 3 = 2$ Or $2x – 3 = \frac{-7}{4}$

$\Rightarrow 2x = 5$ Or $2x = \frac{5}{4}$

$\Rightarrow x = \frac{5}{2}$ Or $x = \frac{5}{8}$

Question 15:

$\frac{3x-2}{2x-3} = \frac{3x-8}{x+4}$

Solution 15:

$\frac{3x-2}{2x-3} = \frac{3x-8}{x+4}$

$\Rightarrow (3x – 2) (x + 4) = (2x – 3) (3x – 8)$

$\Rightarrow 3x^2 + 12x – 2x – 8 = 6x^2 – 16x – 9x + 24$

$\Rightarrow 3x^2 + 10x – 8 = 6x^2 – 25x + 24$

$\Rightarrow 3x^2 – 35x + 32 = 0$

$\Rightarrow 3x^2 – 32x – 3x + 32 = 0$

$\Rightarrow x(3x – 32) – 1(3x – 32) = 0$

$\Rightarrow (x – 1) (3x – 32) = 0$

If $x – 1 = 0$ Or $3x – 32 = 0$

$\Rightarrow x = 1$ Or $x = \frac{32}{3} = 10\frac{2}{3}$

Question 16:

$\frac{100}{x} - \frac{100}{x+5} = 1$

Solution 16:

$\frac{100}{x} - \frac{100}{x+5} = 1$

$\Rightarrow \frac{100\,(x+5)-100x}{x\,(x+5)} = 1$
$\Rightarrow \frac{500}{x^2+5x} = 1$
$\Rightarrow x^2 + 5x = 500$
$\Rightarrow x^2 + 5x - 500 = 0$
$\Rightarrow x^2 + 25x - 20(x + 25) = 0$
$\Rightarrow x(x + 25)\,(x - 20) = 0$
If $x + 25 = 0$ Or $x - 20 = 0$
Then $x = -25$ Or $x = 20$

Question 17:

$\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$

Solution 17:

$\frac{x-3}{x+3} + \frac{x+3}{x-3} = 2\frac{1}{2}$
$\Rightarrow \frac{(x-3)^2 + (x+3)^2}{(x+3)(x-3)} = \frac{5}{2}$
$\Rightarrow \frac{x^2-6x+9+x^2+6x+9}{(x^2-9)} = \frac{5}{2}$
$\Rightarrow 2(2x^2 + 18) = 5(x^2 - 9)$
$\Rightarrow 4x^2 + 36 = 5x^2 - 45$
$\Rightarrow x^2 - 81 = 0$
$\Rightarrow x^2 - 9^2 = 0$
$\Rightarrow (x + 9)\,(x - 9) = 0$
If $x + 9 = 0$ Or $x - 9 = 0$
Then $x = -9$ Or $x = 9$

Question 18:

$$\frac{4}{x+2} - \frac{1}{x+3} = \frac{4}{2x+1}$$

Solution 18:

$$\frac{4}{x+2} - \frac{1}{x+3} = \frac{4}{2x+1}$$

$\Rightarrow \frac{4(x+3)-1\,(x+2)}{(x+2)(x+3)} = \frac{4}{2x+1}$
$\Rightarrow \frac{4x+12-x-2}{x^2+2x+3x+6} = \frac{4}{2x+1}$
$\Rightarrow \frac{3x+10}{x^2+5x+6} = \frac{4}{2x+1}$
$\Rightarrow (3x + 10)(2x + 1) = 4(x^2 + 5x + 6)$

$\Rightarrow 6x^2 + 3x + 20x + 10 = 4x^2 + 20x + 24$
$\Rightarrow 2x^2 + 3x - 14 = 0$
$\Rightarrow 2x^2 + 7x - 4x - 14 = 0$
$\Rightarrow x(2x + 7) - 2(2x + 7) = 0$
$\Rightarrow (2x + 7)(x - 2) = 0$
If $2x + 7 = 0$ Or $x - 2 = 0$
Then $x = \frac{-7}{2}$ Or $x = 2$

Question 19:

$$\frac{5}{x-2} - \frac{3}{x+6} = \frac{4}{x}$$

Solution 19:

$$\frac{5}{x-2} - \frac{3}{x+6} = \frac{4}{x}$$
$$\Rightarrow \frac{5(x+6) - 3(x-2)}{(x-2)(x+6)} = \frac{4}{x}$$
$$\Rightarrow \frac{5x + 30 - 3x + 6}{x^2 + 6x - 2x - 12} = \frac{4}{x}$$
$$\Rightarrow \frac{2x + 36}{x^2 + 4x - 12} = \frac{4}{x}$$
$\Rightarrow 4x^2 + 16x - 48 = 2x^2 + 36x$
$\Rightarrow 2x^2 - 20x - 48 = 0$
$\Rightarrow x^2 - 10x - 24 = 0$
$\Rightarrow x^2 - 12x + 2x - 24 = 0$
$\Rightarrow x(x - 12) + 2(x - 12) = 0$
$\Rightarrow (x - 12)(x + 2) = 0$
If $x - 12 = 0$ Or $x + 2 = 0$
Then $x = 12$ Or $x = -2$

Question 20:

$$\left(1 + \frac{1}{x+1}\right)\left(1 - \frac{1}{x-1}\right) = \frac{7}{8}$$

Solution 20:

$$\left(1 + \frac{1}{x+1}\right)\left(1 - \frac{1}{x-1}\right) = \frac{7}{8}$$
$$\Rightarrow \left(\frac{x+1+1}{x+1}\right)\left(\frac{x-1-1}{x-1}\right) = \frac{7}{8}$$
$$\Rightarrow \left(\frac{x+2}{x+1}\right)\left(\frac{x-2}{x-1}\right) = \frac{7}{8}$$
$$\Rightarrow \frac{x^2 - 4}{x^2 - 1} = \frac{7}{8}$$

$\Rightarrow 8x^2 - 32 = 7x^2 - 7$
$\Rightarrow x^2 = 25$
$\Rightarrow x = \pm 5$

Question 21:

Find the quadratic equation, whose solution set is:
(i) {3,5}
(ii) {−2, 3}
(iii) {5, −4,}
(iv) $\{-3, \frac{-2}{5}\}$

Solution 21:

(i) Since solution set is {3,5}

$\Rightarrow x = 3$ Or $x = 5$
$\Rightarrow x - 3 = 0$ Or $x - 5 = 0$
$\Rightarrow (x - 3)(x - 5) = 0$
$\Rightarrow x^2 - 5x - 3x + 15 = 0$
$\Rightarrow x^2 - 8x + 15 = 0$ Which is the required equation.

(ii) Since solution set is {−2, 3}

$\Rightarrow x = -2$ Or $x = 3$
$\Rightarrow x + 2 = 0$ Or $x - 3 = 0$
$\Rightarrow (x + 2)(x - 3) = 0$
$\Rightarrow x^2 - 3x + 2x - 6 = 0$
$\Rightarrow x^2 - x - 6 = 0$ Which is the required equation.

(iii) Since solution set is {5,−4,}

$\Rightarrow x = 5$ Or $x = -4$
$\Rightarrow x - 5 = 0$ Or $x + 4 = 0$
$\Rightarrow (x - 5)(x + 4) = 0$
$\Rightarrow x^2 - 5x + 4x - 20 = 0$
$\Rightarrow x^2 - x - 20 = 0$ Which is the required equation.

(iv) Since solution set is $\{-3, \frac{-2}{5}\}$

$\Rightarrow x = -3$ Or $x = \frac{-2}{5}$
$\Rightarrow x + 3 = 0$ Or $5x + 2 = 0$
$\Rightarrow (x + 3)(5x + 2) = 0$
$\Rightarrow 5x^2 + 2x + 15x + 6 = 0$
$\Rightarrow 5x^2 + 17x + 6 = 0$ Which is the required equation.

Question 22:

Find the value of x, if a+1=0 and $x^2 + ax – 6 = 0$

Solution 22:

If a + 1 = 0, then a = −1
Put this value in the given equation $x^2 + ax − 6 = 0$
$x^2 – x – 6 = 0$
$\Rightarrow x^2 – 3x + 2x − 6 = 0$
$\Rightarrow x(x – 3) + 2 (x – 3) = 0$
$\Rightarrow (x – 3) (x + 2) = 0$
If x – 3 = 0 Or x + 2 = 0
Then x = 3 Or x = −2.

Question 23:

Find the value of x, if a + 1 = 0 and $x^2 + ax – 6 = 0$

Solution 23:

If a + 7 =0, then a = -7
and b + 10 =0, then b = − 10
Put these values of a and b in the given equation
$12x^2 = (−7)x –(−10)$
$\Rightarrow 12x^2 + 7x – 10 = 0$
$\Rightarrow 12x^2 + 15x – 8x − 10 = 0$
$\Rightarrow 3x (4x + 5) – 2(4x + 5) = 0$
$\Rightarrow (4x + 5) (3x – 2) = 0$
If 4x + 5 = 0 Or 3x – 2 = 0
Then $x = \frac{-5}{4}$ Or $x = \frac{2}{3}$

Question 24:

Use the substitution y = 2x + 3 to solve for x, if $4 (2x + 3)^2 − (2x + 3) − 14 = 0$

Solution 24:

$4 (2x+3)^2 − (2x + 3) − 14 = 0$
Put 2x + 3 = y
$4y^2 – y – 14 = 0$
$\Rightarrow 4y^2 – 8y + 7y – 14 = 0$
$\Rightarrow 4y (y – 2) + 7 (y – 2) = 0$
$\Rightarrow (y – 2) (4y + 7) = 0$
If y – 2 = 0 Or 4y + 7 = 0
Then 2x + 3 – 2 = 0 Or 4(2x + 3) + 7 = 0
$\Rightarrow$ 2x = – 1 Or 8x = – 19

$\Rightarrow x = \frac{-1}{2}$ Or $x = \frac{-19}{8}$

Question 25:

Without solving the quadratic equation $6x^2 - x - 2 = 0$, find whether $x = \frac{2}{3}$ is a solution of this equation or not.

Solution 25:

Consider the equation, $6x^2 - x - 2 = 0$

Put $x = \frac{2}{3}$ in L.H.S

$L.H.S = 6\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right) - 2$

$= \frac{24}{9} - \frac{2}{3} - 2$

$= \frac{24 - 6 - 18}{9} = 0 = R.H.S.$

Since L.H.S = R.H.S, then $x = \frac{2}{3}$ is a solution of the given equation.

Question 26:

Determine whether $x = -1$ is a root of the equation $x^2 - 3x + 2 = 0$ or not.

Solution 26:

$x^2 - 3x + 2 = 0$

Put $x = -1$ in L.H.S.

$L.H.S. = (-1)^2 - 3(-1) + 2$

$= 1 + 3 + 2 = 6 \neq R.H.S.$

Then $x = -1$ is not the solution of the given equation.

Question 27:

If $x = \frac{2}{3}$ is a solution of the quadratic equation $7x^2 + mx - 3 = 0$; find the value of m.

Solution 27:

$7x^2 + mx - 3 = 0$

Given $x = \frac{2}{3}$ is the solution of the given equation.

Put given value of x in the given equation

$7\left(\frac{2}{3}\right)^2 + m\left(\frac{2}{3}\right) - 3 = 0$

$\Rightarrow \frac{28}{9} + \frac{2m}{3} - 3 = 0$

$\Rightarrow 28 + 6m - 27 = 0$

$\Rightarrow 6m = -1$

$\Rightarrow m = \frac{-1}{6}$

Question 28:

If $x = -3$ and $x = \frac{2}{3}$ are solution of quadratic equation $mx^2 + 7x + n = 0$, find the values of m and n.

Solution 28:

$mx^2 + 7x + n = 0$

Put $x = -3$ in given equation

$m(-3)^2 + 7(-3) + n = 0$

$\Rightarrow 9m - 21 + n = 0$

$9m + n = 21$ ---------- (1)

Put $x = \frac{2}{3}$ in given equation

$m\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + n = 0$

$\Rightarrow \frac{4m}{9} + \frac{14}{3} + n = 0$

$\Rightarrow 4m + 9n = -42$ ---------(2)

Solving these equations we get

$m = 3$ and $n = -6$

Question 29:

If quadratic equation $x^2 - (m + 1)x + 6 = 0$ has one root as $x = 3$; find the value of m and the other root of the equation.

Solution 29:

$x^2 - (m + 1)x + 6 = 0$

Put $x = 3$ in the given equation

$(3)^2 - (m + 1)(3) + 6 = 0$

$\Rightarrow 9 - 3m - 3 + 6 = 0$

$\Rightarrow -3m = -12$

$\Rightarrow m = 4$

Put this value of m in the given equation, we get

$x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow x(x - 3) - 2(x - 3) = 0$

$\Rightarrow (x - 3)(x - 2) = 0$

If $x - 3 = 0$ Or $x - 2 = 0$

Then $x = 3$ Or $x = 2$

$\therefore$ 2 is the other root of the given equation

EXERCISE 7 (B)

Question 1:

Solve each of the following equations using the formula:

(i) $x^2 - 6x = 27$

(ii) $x^2 - 10x + 21 = 0$

(iii) $x^2 + 6x - 10 = 0$

(iv) $x^2 + 2x - 6 = 0$

(v) $3x^2 + 2x - 1 = 0$

(vi) $2x^2 + 7x + 5 = 0$

(vii) $\frac{2}{3}x = -\frac{1}{6}x^2 - \frac{1}{3}$

(viii) $\frac{1}{15}x^2 + \frac{5}{3} = \frac{2}{3}x$

(ix) $x^2 - 6 = 2\sqrt{2x}$

(x) $\frac{4}{x} - 3 = \frac{5}{2x+3}$

(xi) $\frac{2x+3}{x+3} = \frac{x+4}{x+2}$

(xii) $\sqrt{6x^2} - 4x - 2\sqrt{6} = 0$

(xiii) $\frac{2x}{x-4} + \frac{2x-5}{x-3} = 8\frac{1}{3}$

(xiv) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

Solution 1:

(i) $x^2 - 6x = 27$

$\Rightarrow x^2 - 6x - 27 = 0$

Here a = 1, b = – 6 and c = – 27

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)}$

$= \frac{6 \pm 12}{2} = \frac{6+12}{2}$ and $\frac{6-12}{2} = 9$ and -3

(ii) $x^2 - 10x + 21 = 0$

Here a = 1, b = – 10 and c = 21

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$

$= \frac{10 \pm 4}{2} = \frac{10+4}{2}$ and $\frac{10-4}{2} = 7$ and 3

(iii) $x^2 + 6x - 10 = 0$

Here a = 1, b = 6 and c = – 10

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(6) \pm \sqrt{(-6)^2 - 4(1)(-10)}}{2(1)}$

$= \frac{-6 \pm \sqrt{76}}{2} = \frac{-6 \pm 2\sqrt{19}}{2}$ and $\frac{-6-2\sqrt{19}}{2} = -3 + \sqrt{19}$ and $-3 - \sqrt{19}$

(iv) $x^2 + 2x - 6 = 0$

Here a = 1, b = 2 and c = − 6

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-6)}}{2(1)}$

$= \frac{-2 \pm \sqrt{28}}{2} = \frac{-2 \pm 2\sqrt{7}}{2} = -1 \pm \sqrt{7}$

(v) $3x^2 + 2x - 1 = 0$

Here a = 3, b = 2 and c = −1

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(2) \pm \sqrt{(2)^2 - 4(3)(-1)}}{2(3)}$

$= \frac{-2 \pm 4}{6} = \frac{-2 \pm 4}{6}$ and $\frac{-2-4}{6} = \frac{1}{3}$ and -1

(vi) $2x^2 + 7x + 5 = 0$

Here a = 2, b = 7 and c = 5

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(5)}}{2(2)}$

$= \frac{-7 \pm 3}{4} = \frac{-7+3}{4}$ and $\frac{-7-3}{4} = -1$ and $-\frac{5}{2}$

(vii) $\frac{2}{3}x = -\frac{1}{6}x^2 - \frac{1}{3}$

$\Rightarrow 4x = -x^2 - 2$

$\Rightarrow x^2 + 4x + 2 = 0$

Here a = 1, b = 4 and c = 2

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(2)}}{2(1)}$

$= \frac{-4 \pm \sqrt{8}}{2} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$

(viii) $\frac{1}{15}x^2 + \frac{5}{3} = \frac{2}{3}x$

$\Rightarrow x^2 + 25 = 10x$

$\Rightarrow x^2 - 10x + 25 = 0$

Here a = 1, b = − 10 and c = 25

Then x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= $\frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$

= $\frac{10 \pm \sqrt{0}}{2}$ = 5

(ix) $x^2 - 6 = 2\sqrt{2}x$

$\Rightarrow x^2 - 2\sqrt{2}x - 6 = 0$

Here a = 1, b = $-2\sqrt{2}$ and c = − 6

Then x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= $\frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-6)}}{2(1)}$

= $\frac{2\sqrt{2} \pm \sqrt{32}}{2} = \frac{2\sqrt{2} \pm 4\sqrt{2}}{2} = \frac{2\sqrt{2} + 4\sqrt{2}}{2}$ and $\frac{2\sqrt{2} - 4\sqrt{2}}{2}$

= $\frac{6\sqrt{2}}{2}$ and $\frac{-2\sqrt{2}}{2} = 3\sqrt{2}$ and $-\sqrt{2}$

(x) $\frac{4}{x} - 3 = \frac{5}{2x+3}$

$\Rightarrow \frac{4-3x}{x} = \frac{5}{2x+3}$

$\Rightarrow (4 - 3x)(2x + 3) = 5x$

$\Rightarrow 8x + 12 - 6x^2 - 9x = 5x$

$\Rightarrow 6x^2 + 6x - 12 = 0$

$\Rightarrow x^2 + x - 2 = 0$

Here a = 1, b = 1 and c = − 2

Then x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= $\frac{-(1) \pm \sqrt{(1)^2 - 4(1)(-2)}}{2(1)}$

= $\frac{1 \pm \sqrt{9}}{2} = \frac{-1 \pm 3}{2} = \frac{-1+3}{2}$ and $\frac{-1-3}{2}$ = 1 and – 2

(xi) $\frac{2x+3}{x+3} = \frac{x+4}{x+2}$

$\Rightarrow (2x + 3)(x + 2) = (x + 3)(x + 4)$

$\Rightarrow 2x^2 + 4x + 3x + 6 = x^2 + 4x + 3x + 12$

$\Rightarrow x^2 - 6 = 0$

Here a = 1, b = 0 and c = − 6

Then x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= $\frac{-(0) \pm \sqrt{(0)^2 - 4(1)(-6)}}{2(1)}$

= $\frac{0 \pm \sqrt{24}}{2} = \frac{0 \pm 2\sqrt{6}}{2}$ = $-\sqrt{6}$ and $\sqrt{6}$

(xii) $\sqrt{6}\,x^2 - 4x - 2\sqrt{6} = 0$

Here a = $\sqrt{6}$, b = − 4 and c = − 2 $\sqrt{6}$

Then x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= $\frac{-(-4) \pm \sqrt{(-4))^2 - 4\,(\sqrt{6}\,)\,(-2\sqrt{6})}}{2(\sqrt{6})}$

= $\frac{4 \pm \sqrt{64}}{2\sqrt{6}} = \frac{4 \pm 8}{2\sqrt{6}} = \frac{4+8}{2\sqrt{6}}$ and $\frac{4-8}{2\sqrt{6}}$

= $\frac{6}{\sqrt{6}}$ *and* $\frac{-2}{\sqrt{6}} = \sqrt{6}$ and $\frac{-\sqrt{6}}{3}$

(xiii) $\frac{2x}{x-4} + \frac{2x-5}{x-3} = 8\frac{1}{3}$

$\Longrightarrow \frac{2x\,(x-3) + (x-4)(2x-5)}{(x-4)\,(x-3)} = \frac{25}{3}$

$\Longrightarrow \frac{2x^2 - 6x + 2x^2 - 5x - 8x + 20}{x^2-3x-4x+12} = \frac{25}{3}$

$\Longrightarrow \frac{4x^2 - 19x + 20}{x^2 - 7x + 12} = \frac{25}{3}$

$\Longrightarrow 25x^2 - 175x + 300 = 12x^2 - 57x + 60$

$\Longrightarrow 13x^2 - 118x + 240 = 0$

Here a = 13, b = − 118 and c = 240

Then x = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= $\frac{-(-118) \pm \sqrt{(-118))^2 - 4\,(\sqrt{13}\,)\,(240)}}{2(13)}$

= $\frac{118 \pm \sqrt{1444}}{26} = \frac{118 \pm 38}{26}$

= $\frac{118+38}{26}$ and $\frac{118-38}{26}$ and $\frac{40}{13}$

(xiv) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

$\Longrightarrow \frac{(x-1)(x-4) + (x-2)(x-3)}{(x-2)(x-4)} = \frac{10}{3}$

$\Longrightarrow \frac{x^2 - 4x - x + 4 + x^2 - 3x - 2x + 6}{x^2 - 4x - 2x + 8} = \frac{10}{3}$

$\Longrightarrow \frac{2x^2 - 10x + 10}{x^2 - 6x + 8} = \frac{10}{3}$

$\Longrightarrow 10x^2 - 60x + 80 = 6x^2 - 30x + 30$

$\Longrightarrow 4x^2 - 30x + 50 = 0$

$\Longrightarrow 2x^2 - 15x + 25 = 0$

Here a = 2, b = − 15 and c = 25

Then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-15) \pm \sqrt{(-15)^2 - 4(2)(25)}}{2(2)}$

$= \frac{15 \pm \sqrt{25}}{4} = \frac{15 \pm 5}{4}$

$= \frac{15+5}{4}$ and $\frac{15-5}{4} = 5 \; and \; \frac{5}{2}$

Question 2:

Without solving comment upon the nature of roots of each of the following equations:

(i) $7x^2 - 9x + 2 = 0$

(ii) $6x^2 - 13x + 4 = 0$

(iii) $25x^2 - 10x + 1 = 0$

(iv) $x^2 - ax - b^2 = 0$

(v) $x^2 + 2\sqrt{3x} - 9 = 0$

(vi) $2x^2 + 8x + 9 = 0$

Solution 2:

(i) $7x^2 - 9x + 2 = 0$

$a = 7$, $b = -9$ and $c = 2$

∴ Discriminant $= b^2 - 4ac$

$= (-9)^2 - 4(7)(2)$

$= 81 - 56 = 25$

Since D >0, then equation has two real and unequal roots.

(ii) $6x^2 - 13x + 4 = 0$

$a = 6$, $b = -13$ and $c = 4$

∴ Discriminant $= b^2 - 4ac$

$= (-13)^2 - 4(6)(4)$

$= 169 - 96 = 73$

Since 73 is not a perfect square, roots are irrational

Since D >0, then equation has two real and unequal roots.

(iii) $25x^2 - 10x + 1 = 0$

$a = 25$, $b = -10$ and $c = 1$

∴ Discriminant $= b^2 - 4ac$

$= (-10)^2 - 4(25)(1)$

$= 100 - 100 = 0$

Since D = 0, then equation has two real and unequal roots.

(iv) $x^2 + 2\sqrt{3x} - 9 = 0$

$a = 1$, $b = 2\sqrt{3}$ and $c = -9$

∴ Discriminant $= b^2 - 4ac$

$= (2\sqrt{3})^2 - 4(1)(-9)$
$= 12 + 36 = 48$
Since 48 is not a perfect square, roots are irrational
Since D >0, then equation has two real and unequal roots.

(v) $x^2 - ax - b^2 = 0$
$a = 1, b = -a$ and $c = -b^2$
$\therefore$ Discriminant $= b^2 - 4ac$
$= (-a)^2 - 4(1)(-b)^2$
$= a^2 + 4b^2$ = a positive value
Since $a^2 + 4b^2$ is not a perfect square, roots are irrational
Since D >0, then equation has two real and unequal roots.

(vi) $2x^2 + 8x + 9 = 0$
$a = 2, b = 8$ and $c = 9$
$\therefore$ Discriminant $= b^2 - 4ac$
$= (8)^2 - 4(2)(9)$
$= 64 - 72 = -18$ = a negative value
Since D >0, then equation has no real roots.

Question 3:
Find the value of 'p', if the following quadratic equations have equal roots:
$4x^2 - (p - 2)x + 1 = 0$

Solution 3:
$4x^2 - (p - 2)x + 1 = 0$
Here $a = 4, b = -(p - 2)$ and $c = 1$
Given: equation has equal roots
Then $d = 0$
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow [-(p - 2)]^2 - 4(4)(1) = 0$
$\Rightarrow p^2 + 4 - 4p - 16 = 0$
$\Rightarrow p^2 - 4p - 12 = 0$
$\Rightarrow p^2 - 6p + 2p - 12 = 0$
$\Rightarrow p(p - 6)(p - 6) = 0$
$\Rightarrow (p - 6)(p + 2) = 0$
Then $p - 6 = 0$ Or $p + 2 = 0$
$\Rightarrow p = 6$ Or $p = -2$.

Question 4:
The equation $3x^2 - 12x + (n - 5) = 0$ has equal roots. Find the value of n.

Solution 4:

$3x^2 – 12x + (n – 5) = 0$
Here a = 3, b = − 12 and c = n – 5
Given: equation has equal roots
Then D = 0
$\Rightarrow b^2 – 4ac = 0$
$\Rightarrow [− 12]^2 – 4(3)(n – 5) = 0$
$\Rightarrow 144 – 12n + 60 = 0$
$\Rightarrow − 12n = − 204$
$\Rightarrow n = \frac{-204}{-12} = 17$

Question 5:

Find the value of 'm', if the following equation has equal roots:
$(m – 2)x^2 – (5 + m) x + 16 = 0$

Solution 5:

$(m – 2)x^2 – (5 + m) x + 16 = 0$
Here a = m – 2, b = − (5 + m) and c =16
Given : equation has equal roots
Then D = 0
$\Rightarrow b^2 – 4ac = 0$
$\Rightarrow [− (5 + m)]^2 – 4(m − 2)(16) = 0$
$\Rightarrow 25 + m^2 + 10m – 64m + 128 = 0$
$\Rightarrow m^2 − 54m + 153 = 0$
$\Rightarrow m^2 − 51m – 3m + 153 = 0$
$\Rightarrow m (m – 51) – 3 (m – 51) = 0$
$\Rightarrow (m – 51) (m – 3) = 0$
Then m – 51 = 0 Or m – 3 = 0
$\Rightarrow$ m = 51 Or m = 3

EXERCISE 7 (C)

Question 1:

Solve each of the following equations for x and give, in each case, your answer correct to one decimal place:
(i) $x^2 – 8x + 5 = 0$
(ii) $5x^2 + 10x − 3 = 0$

Solution 1:

(i) $x^2 – 8x + 5 = 0$

Here a = 1, b = − 8 and c = 5

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(5)}}{2(1)}$

$= \frac{8 \pm \sqrt{44}}{2} = \frac{8 \pm 2\sqrt{11}}{4} = 4 \pm \sqrt{11} = 4 \pm 3.3 = 7.3$ and 0.7

(ii) $5x^2 + 10x - 3 = 0$

Here a = 5, b = 10 and c = −3

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-10) \pm \sqrt{(10)^2 - 4(5)(-3)}}{2(5)}$

$= \frac{-10 \pm \sqrt{160}}{10} = \frac{-10 \pm 12.6}{10}$

$= \frac{-10+12.6}{10}$ and $\frac{-10-12.6}{10} = 0.26$ and $-2.26 = 0.3$ and -2.3

Question 2:

Solve each of the following equations for x and give, in each case your answer correct to 2 decimal places:

(i) $2x^2 - 10x + 5 = 0$

(ii) $4x + \frac{6}{x} + 13 = 0$

(iii) $x^2 - 3x - 9 = 0$

(iv) $x^2 - 5x - 10 = 0$

Solution 2:

(i) $2x^2 - 10x + 5 = 0$

Here a = 2, b = −10 and c = 5

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-10) \pm \sqrt{(10)^2 - 4(2)(5)}}{2(2)}$

$= \frac{10 \pm \sqrt{60}}{4} = \frac{10 \pm 7.75}{4}$

$= \frac{10+7.75}{4}$ and $\frac{10-7.75}{4} = 4.44$ and 0.56

(ii) $4x + \frac{6}{x} + 13 = 0$

$\Rightarrow 4x^2 + 6 + 13x = 0$

$\Rightarrow 4x^2 + 13x + 6 = 0$

$\Rightarrow$ Here a = 4, b = 13 and c = 6

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(13) \pm \sqrt{(13)^2 - 4(4)(6)}}{2(4)}$

$= \frac{-13 \pm \sqrt{73}}{8} = \frac{-13 \pm 8.54}{8}$

$= \frac{-13+8.54}{8}$ and $\frac{-13-8.54}{8} = -0.56$ and -2.69

(iii) $x^2 - 3x - 9 = 0$

$\Longrightarrow$ Here a = 1, b = −3 and c = −9

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$

$= \frac{3 \pm \sqrt{45}}{2} = \frac{3 \pm 6.70}{2}$

$= \frac{3+6.70}{2}$ and $\frac{3-6.70}{2} = 4.85$ and -1.85

Question 3:

Solve each of the following equations for x, giving your answer correct to 3 decimal places:

(i) $3x^2 - 12x - 1 = 0$

(ii) $x^2 - 16x + 6 = 0$

(iii) $2x^2 + 11x + 4 = 0$

Solution 3:

(i) **$3x^2 - 12x - 1 = 0$**

$\Longrightarrow$ Here a = 3, b = −12 and c = −1

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-1)}}{2(3)}$

$= \frac{12 \pm \sqrt{156}}{6} = \frac{12 \pm 12.489}{6}$

$= \frac{12+12.489}{6}$ and $\frac{12-12.489}{6} = 4.082$ and -0.082

(ii) $x^2 - 16x + 6 = 0$

$\Longrightarrow$ Here a = 1, b = −16 and c = −1

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(6)}}{2(1)}$

$= \frac{16 \pm \sqrt{232}}{2} = \frac{16 \pm 15.231}{2}$

$= \frac{16 + 15.231}{2}$ and $\frac{16 - 15.231}{2}$ = 15.616 and 0.384

(iii) $2x^2 + 11x + 4 = 0$

$\Longrightarrow$ Here a = 2, b = 11 and c = 4

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{(-11) \pm \sqrt{(11)^2 - 4(2)(4)}}{2(2)}$

$= \frac{-11 \pm \sqrt{89}}{4} = \frac{-11 \pm 9.433}{4}$

$= \frac{-11 + 9.433}{4}$ and $\frac{-11 - 9.433}{4}$ = − 0.392 and − 5.108

Question 4:

Solve:

(i) $x^4 - 2x^2 - 3 = 0$

(ii) $x^4 - 10x^2 + 9 = 0$

Solution 4:

(i) $x^4 - 2x^2 - 3 = 0$

$\Longrightarrow x^4 - 3x^2 + x^2 - 3 = 0$

$\Longrightarrow x^2(x^2 - 3) + 1(x^2 - 3) = 0$

$\Longrightarrow (x^2 - 3)(x^2 + 3) = 0$

If $x^2 - 3 = 0$ Or $x^2 + 1 = 0$

$\Longrightarrow x^2 = 3$ Or $x^2 = -1$ (reject)

$\Longrightarrow x = \pm\sqrt{3}$

(ii) $x^4 - 10x^2 + 9 = 0$

$\Longrightarrow x^4 - 9x^2 - x^2 + 9 = 0$

$\Longrightarrow x^2(x^2 - 9) - 1(x^2 - 9) = 0$

$\Longrightarrow (x^2 - 9)(x^2 - 1) = 0$

If $x^2 - 9 = 0$ Or $x^2 - 1 = 0$

$\Longrightarrow x^2 = 9$ Or $x^2 = 1$

$\Longrightarrow x = \pm 3$ Or $x = \pm 1$

Question 5:

Solve:

(i) $(x^2 - x)^2 + 5(x^2 - x) + 4 = 0$

(ii) $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$

Solution 5:

(i) $(x^2 - x)^2 + 5(x^2 - x) + 4 = 0$

Let $x^2 - x = y$

Then $y^2 + 5y + 4 = 0$

$\Rightarrow y^2 + 4y + y + 4 = 0$

$\Rightarrow y(y + 4) + 1(y + 4) = 0$

$\Rightarrow (y + 4)(y + 1) = 0$

If $y + 4 = 0$ Or $y + 1 = 0$

$\Rightarrow x^2 - x + 4 = 0$ Or $x^2 - x + 1 = 0$

$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(4)}}{2(1)}$ Or $\frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$\Rightarrow 1 = \frac{1 \pm \sqrt{-15}}{2}$ (reject) Or $x = \frac{1 \pm \sqrt{-3}}{2}$ (reject)

$\therefore$ Given equation has no real solution

(ii) $(x^2 - 3x)^2 - 16(x^2 - 3x) - 36 = 0$

Let $x^2 - 3x = y$

Then $y^2 - 16y - 36 = 0$

$\Rightarrow y^2 - 18y + 2y - 36 = 0$

$\Rightarrow y(y - 18) + 2(y - 18) = 0$

$\Rightarrow (y - 18)(y + 2) = 0$

If $y - 18 = 0$ Or $y + 2 = 0$

$\Rightarrow x^2 - 3x - 18 = 0$ Or $x^2 - 3x + 2 = 0$

$\Rightarrow x^2 - 6x + 3x - 18 = 0$ Or $x^2 - 2x - x + 2 = 0$

$\Rightarrow x(x - 6) + 3(x - 6) = 0$ Or $x(x - 2) - 1(x - 2) = 0$

$\Rightarrow (x - 6)(x + 3) = 0$ Or $(x - 2)(x - 1) = 0$

If $x - 6 = 0$ Or $x + 3 = 0$ Or $x - 2 = 0$ Or $x - 1 = 0$

Then $x = 6$ Or $x = -3$ Or $x = 2$ Or $x = 1$

Question 6:

(i) $\sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}$

(ii) $\left(\frac{2x-3}{x-1}\right) - 4\left(\frac{x-1}{2x-3}\right) = 3$

(iii) $\left(\frac{3x+3}{x+1}\right) + \left(\frac{x+1}{3x+1}\right) = \frac{5}{2}$

(iv) $3\sqrt{\frac{x}{5}} + 3\sqrt{\frac{5}{x}} = 10$

Solution 6:

(i) $\sqrt{\frac{x}{x-3}} + \sqrt{\frac{x-3}{x}} = \frac{5}{2}$

Let $\sqrt{\frac{x}{x-3}} = y$

Then $y + \frac{1}{y} = \frac{5}{2}$

$\Rightarrow \frac{y^2+1}{y} = \frac{5}{2}$

$\Rightarrow 2y^2 + 2 = 5y$

$\Rightarrow 2y^2 - 5y + 2 = 0$

$\Rightarrow 2y^2 - 4y - y + 2 = 0$

$\Rightarrow 2y(y-2) - 1(y-2) = 0$

$\Rightarrow (y-2)(2y-1) = 0$

If $y - 2 = 0$ Or $2y - 1 = 0$

Then $y = 2$ Or $y = \frac{1}{2}$

$\Rightarrow \sqrt{\frac{x}{x-3}} = 2 \quad Or \quad \sqrt{\frac{x}{x-3}} = \frac{1}{2}$

$\Rightarrow \frac{x}{x-3} = 4$ Or $\frac{x}{x-3} = \frac{1}{4}$

$\Rightarrow x = 4$ Or $x = -1$

(ii) $\left(\frac{2x-3}{x-1}\right) - 4\left(\frac{x-1}{2x-3}\right) = 3$

Let $\frac{2x-3}{x-1} = y$

Then $y - \frac{4}{y} = 3$

$\Rightarrow \frac{y^2-4}{y} = 3$

$\Rightarrow y^2 - 4 = 3y$

$\Rightarrow y^2 - 3y - 4 = 0$

$\Rightarrow y^2 - 4y + y - 4 = 0$

$\Rightarrow y(y-4) + 1(y-4) = 0$

$\Rightarrow (y-4)(y+1) = 0$

If $y - 4 = 0$ Or $y + 1 = 0$

Then $y = 4$ Or $y = -1$

$\Rightarrow \frac{2x-3}{x-1} = 4$ Or $\frac{2x-3}{x-1} = -1$

$\Rightarrow 4x - 4 = 2x - 3$ Or $2x - 3 = -x + 1$

$\Rightarrow 2x = 1$ Or $3x = 4$

$\Rightarrow x = \frac{1}{2}$ Or $x = \frac{4}{3} = 1\frac{1}{3}$

(iii) $\left(\frac{3x+1}{x+1}\right) + \left(\frac{x+1}{3x+1}\right) = \frac{5}{2}$

Let $\frac{3x+1}{x+1} = y$

Then $y + \frac{1}{y} = \frac{5}{2}$

$\Rightarrow \frac{y^2+1}{y} = \frac{5}{2}$

$\Rightarrow 2y^2 + 2 = 5y$

$\Rightarrow 2y^2 - 5y + 2 = 0$

$\Rightarrow 2y^2 - 4y - y + 2 = 0$

$\Rightarrow 2y(y-2) - 1(y-2) = 0$

$\Rightarrow (y - 2)(2y - 1) = 0$

If $y - 2 = 0$ Or $2y - 1 = 0$

Then $y = 2$ Or $y = \frac{1}{2}$

$\Rightarrow \frac{3x+1}{x+1} = 2$ Or $\frac{3x+3}{x+1} = \frac{1}{2}$

$\Rightarrow 3x + 1 = 2x + 2$ Or $6x + 2 = x + 1$

$\Rightarrow x = 1$ Or $5x = -1$

$\Rightarrow x = 1$ Or $x = \frac{-1}{5}$

(iv) $3\sqrt{\frac{x}{5}} + 3\sqrt{\frac{5}{x}} = 10$

Let $\sqrt{\frac{x}{5}} = y$

Then $3y + \frac{3}{y} = 10$

$\Rightarrow \frac{3y^2+3}{y} = 10$

$\Rightarrow 3y^2 + 3 = 10y$

$\Rightarrow 3y^2 - 10y + 3 = 0$

$\Rightarrow 3y^2 - 9y - y + 3 = 0$

$\Rightarrow 3y(y - 3) - 1(y - 3) = 0$

$\Rightarrow (y - 3)(3y - 1) = 0$

If $y - 3 = 0$ Or $3y - 1 = 0$

Then $y = 3$ Or $y = \frac{1}{3}$

$\Rightarrow \sqrt{\frac{x}{5}} = 3$ or $\sqrt{\frac{x}{5}} = \frac{1}{3}$

$\Rightarrow \frac{x}{5} = 9$ Or $\frac{x}{5} = \frac{1}{9}$

$\Rightarrow x = 45$ Or $9x = 5$

$\Rightarrow x = 45$ Or $x = \frac{5}{9}$

Question 7:

Solve the equation $2x - \frac{1}{x} = 7$. Write your answer correct to two decimal places.

Solution 7:

$2x - \frac{1}{x} = 7$

$\Rightarrow \frac{2x^2-1}{x} = 7$

$\Rightarrow 2x^2 - 1 = 7x$

$\Rightarrow 2x^2 - 7x - 1 = 0$

Here $a = 2, b = -7$ and $c = -1$

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-1)}}{2(2)}$

$= \frac{7 \pm \sqrt{57}}{4} = \frac{7 \pm 7.55}{4}$

$= \frac{7+7.55}{4}$ and $\frac{7-7.55}{4}$ $= 3.64$ and -0.14

Question 8:

Solve the following equation and give your answer correct to 3 significant figures:

$5x^2 - 3x - 4 = 0$

Solution 8:

Consider the given equation:

$5x^2 - 3x - 4 = 0$

Using quadratic formula, we have

$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\Rightarrow x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \times 5 \times (-4)}}{2 \times 5}$

$\Rightarrow x = \frac{3 \pm \sqrt{9+80}}{2 \times 5}$

$\Rightarrow x = \frac{3 \pm \sqrt{89}}{10}$

$\Rightarrow x = \frac{3 \pm 9.434}{10}$

$\Rightarrow x = 1.243$ Or $x = -0.643$

EXERCISE 7 (D)

Question 1:

$\frac{2X}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0;\ x \neq 3, x \neq -\frac{3}{2}$

Solution 1:

$\frac{2X}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0;\ x \neq 3, x \neq -\frac{3}{2}$

$\Rightarrow \frac{2x\,(2x+3)+1\,(x-3)+3x+9}{(x-3)\,(2x+3)} = 0$
$\Rightarrow 4x^2 + 6x + x - 3 + 3x + 9 = 0$
$\Rightarrow 4x^2 + 10x + 6 = 0$
$\Rightarrow 4x^2 + 4x + 6x + 6 = 0$
$\Rightarrow 4x(x + 1) + 6\,(x + 1) = 0$
$\Rightarrow (x + 1)\,(4x + 6) = 0$
$\Rightarrow x + 1 = 0$ Or $4x + 6 = 0$
$\Rightarrow x = -1$ Or $x = \frac{-6}{4} = \frac{-3}{2}$ (reject)

Question 2:

$(2x + 3)^2 = 81$

Solution 2:

$(2x + 3)^2 = 81$
$\Rightarrow 2x + 3 = \pm 9$
$\Rightarrow 2x + 3 = 9$ and $2x + 3 = -9$
$\Rightarrow 2x = 6$ and $2x = -12$
$\Rightarrow x = 3$ and $x = -6$

Question 3:

$a^2x^2 - b^2 = 0$

Solution 3:

$a^2x^2 - b^2 = 0$
$\Rightarrow (ax)^2 - b^2 = 0$
$\Rightarrow (ax + b)\,(ax - b) = 0$
If $ax + b = 0$ and $ax - b = 0$
Then $x = \frac{-b}{a}$ and $x = \frac{b}{a}$

Question 4:

$X^2 - \frac{11}{4}x + \frac{15}{8} = 0$

Solution 4:

$X^2 - \frac{11}{4}x + \frac{15}{8} = 0$
$\Rightarrow \frac{8x^2 - 22x + 15}{8} = 0$
$\Rightarrow 8x^2 - 22x + 15 = 0$
$\Rightarrow 8x^2 - 12x - 10x + 15 = 0$

$\Rightarrow 4x(2x-3)-5(2x-3)=0$
$\Rightarrow (2x-3)(4x-5)=0$
$\Rightarrow 2x-3=0$ Or $4x-5=0$
$\Rightarrow x=\frac{3}{2}$ Or $x=\frac{5}{4}$

Question 5:

$x+\frac{4}{x}=-4;\ x\neq 0$

Solution 5:

$x+\frac{4}{x}=-4$
$\Rightarrow \frac{x^2+4}{x}=-4$
$\Rightarrow x^2+4=-4x$
$\Rightarrow x^2+4x+4=0$
$\Rightarrow (x+2)^2=0$
$\Rightarrow x+2=0$
$\Rightarrow x=-2$

Question 6:

$2x^2-5x^2+3=0$ **Take $x^2=y$**

Solution 6:

$2x^2-5x^2+3=0$
$\Rightarrow 2x^4-3x^2-2x^2+3=0$
$\Rightarrow x^2(2x^2-3)-1(x^2-1)=0$
$(2x^2-3)(x^2-1)=0$
If $2x^2-3=0$ Or $x^2-1=0$
Then $x^2=\frac{3}{2}$ Or $x^2=1$
$\Rightarrow x=\pm\sqrt{\frac{3}{2}}$ Or $x=\pm 1$

Question 7:

$X^4-2x^2-3=0$

Solution 7:

$X^4-2x^2-3=0$
$\Rightarrow x^2-3x^2+x^2-3=0$
$\Rightarrow x^2(x^2-3)+1(x^2-3)=0$
$\Rightarrow (x^2-3)(x^2+1)=0$

If $x^2 - 3 = 0$ Or $x^2 + 1 = 0$
Then $x^2 = 3$ Or $x^2 = -1$ (reject)
$\Rightarrow x = \pm \sqrt{3}$

Question 8:

$9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$

Solution 8:

$9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$
Let $x + \frac{1}{x} = y$
Squaring on both sides
$X^2 + \frac{1}{x^2} + 2 = y^2$
$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$
Putting these values in the given equation
$9(y^2 - 2) - 9y - 52 = 0$
$\Rightarrow 9y^2 - 18 - 9y - 52 = 0$
$\Rightarrow 9y^2 - 9y - 70 = 0$
$\Rightarrow 9y^2 - 30y + 21y - 70 = 0$
$\Rightarrow 3y(3y - 10) + 7(3y - 10) = 0$
$\Rightarrow (3y - 10)(3y + 7) = 0$
$\Rightarrow 3y - 10 = 0$ Or $3y + 7 = 0$
$\Rightarrow y = \frac{10}{3}$ Or $y = \frac{-7}{3}$
$\Rightarrow x + \frac{1}{x} = \frac{10}{3}$ Or $x + \frac{1}{x} = \frac{-7}{3}$
$\Rightarrow \frac{x^2+1}{x} = \frac{10}{3}$ Or $\frac{x^2+1}{x} = \frac{-7}{3}$
$\Rightarrow 3x^2 - 10x + 3 = 0$ Or $3x^2 + 7x + 3 = 0$
$\Rightarrow 3x^2 - 9x - x + 3 = 0$ Or $x = \frac{-7 \pm \sqrt{(-7)^2 - 4(3)(3)}}{2(3)}$
$\Rightarrow 3x(x - 3) - 1(x - 3) = 0$ Or $x = \frac{-7 \pm \sqrt{13}}{6}$
$\Rightarrow (x - 3)(3x - 1) = 0$
$\Rightarrow x = 3$ and $x = \frac{1}{3}$

Question 9:

$2\left(x^2 \frac{1}{x^2}\right) - \left(x + \frac{1}{x}\right) = 11$

Solution 9:

$2\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)=11$

Let $x+\frac{1}{x}=y$

Squaring on both sides

$X^2+\frac{1}{x^2}+2=y^2$

$\Rightarrow x^2+\frac{1}{x^2}=y^2-2$

Putting these values in the given equation

$2(y^2-2)-y=11$

$\Rightarrow 2y^2-4-y-11=0$

$\Rightarrow 2y^2-y-15=0$

$\Rightarrow 2y^2-6y+5y-15=0$

$\Rightarrow 2y(y-3)+5(y-3)=0$

$\Rightarrow (y-3)(2y+5)=0$

$\Rightarrow$ If $y-3=0$ Or $2y+5=0$

$\Rightarrow y=3$ Or $y=\frac{-5}{2}$

$\Rightarrow x+\frac{1}{x}=3$ Or $x+\frac{1}{x}=\frac{-5}{2}$

$\Rightarrow \frac{x^2+1}{x}=3$ Or $\frac{x^2+1}{x}=\frac{-5}{2}$

$\Rightarrow x^2-3x+1=0$ Or $2x^2+5x+2=0$

$\Rightarrow x=\frac{-3\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}$ Or $2x^2+4x+x+2=0$

$x=\frac{-3\pm\sqrt{5}}{2}$ Or $2x(x+2)+1(x+2)=0$

Or $(x+2)(2x+1)=0$

Then $x=-2$ and $x=\frac{-1}{2}$

Question 10:

$\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-2=0$

Let $x-\frac{1}{x}=y \Rightarrow x^2+\frac{1}{x^2}=y^2+2$

Solution 10:

$\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-2=0$

Let $x-\frac{1}{x}=y$

Squaring on both sides

$X^2+\frac{1}{x^2}-2=y^2$

$\Rightarrow x^2+\frac{1}{x^2}=y^2+2$

Putting these values in the given equation

$(y^2 + 2) – 3y – 2 = 11$

$\Rightarrow y^2 – 3y = 0$

$\Rightarrow y (y – 3) = 0$ Or $y – 3 = 0$

If $y = 0$ Or $y = 3$

$\Rightarrow x - \frac{1}{x} = 0$ Or $x - \frac{1}{x} = 3$

$\Rightarrow \frac{x^2 - 1}{x} = 0$ Or $\frac{x^2 - 1}{x} = 3$

$\Rightarrow x^2 – 1 = 0$ Or $x^2 - 3x - 1 = 0$

$\Rightarrow (x + 1) (x – 1) = 0$ Or $\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$

$\Rightarrow x = -1$ and $x = 1$ Or $x = \frac{3 \pm \sqrt{13}}{2}$

Question 11:

$(x^2 + 5x + 4) (x^2 + 5x + 6) = 120$

Take: $x^2 + 5x = y$

Solution 11:

$(x^2 + 5x + 4) (x^2 + 5x + 6) = 120$

Let $x^2 + 5x = y$

Then $(y + 4) (y + 6) = 120$

$\Rightarrow y^2 + 6y + 4y + 24 - 120 = 0$

$\Rightarrow y^2 + 10y – 96 = 0$

$\Rightarrow y^2 + 16y - 6y – 96 = 0$

$\Rightarrow y (y + 16) – 6 (y + 16) = 0$

$\Rightarrow (y + 16) (y - 16) = 0$

Then $y = -16$ Or $y = 6$

$\Rightarrow x^2 + 5x + 16 = 0$ Or $x^2 + 5x - 6 = 0$

$\Rightarrow x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(16)}}{2(1)}$ Or $x^2 + 6x - x - 6 = 0$

$x = \frac{-5 \pm \sqrt{-39}}{2}$ Or $x (x + 6) - 1 (x + 6) = 0$

(reject) Or $(x + 6) (x - 1) = 0$

Then $x = -6$ and $x = 1$

Question 12:

Solve each of the following equations, given answer up to two decimal places.

(i) $x^2 – 5x – 10 = 0$

(ii) $3x^2 – x – 7 = 0$

(i) $x^2 - 5x - 10 = 0$

Solution 12:

Here a = 1, b = − 5 and c = − 10

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)}$

$\Rightarrow x = \frac{5 \pm \sqrt{65}}{2} = \frac{5 \pm 8.06}{2}$

$\Rightarrow x = \frac{13.06}{2}$ and $\frac{-3.06}{2} = 6.53$ and -1.53

(ii) $3x^2 - x - 7 = 0$

Here a = 3, b = − 1 and c = − 7

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-1-7)}}{2(3)}$

$\Rightarrow x = \frac{1 \pm \sqrt{85}}{6} = \frac{1 \pm 9.22}{6}$

$\Rightarrow x = \frac{10.22}{6}$ and $\frac{-8.22}{2} = 1.70$ and -1.37

Question 13:

Solve: $\left(\frac{x}{x+2}\right)^2 - 7\left(\frac{x}{x+2}\right) + 12 = 0;\ x \neq -2$

Solution 13:

$\left(\frac{x}{x+2}\right)^2 - 7\left(\frac{x}{x+2}\right) + 12 = 0;\ x \neq -2$

Let $\frac{x}{x+2} = y$

Then $y^2 - 7y + 12 = 0$

$\Rightarrow y^2 - 4y - 3y + 12 = 0$

$\Rightarrow y(y-4) - 3(y-3) = 0$

$\Rightarrow (y-4)(y-3) = 0$

Then y = 4 and y = 3

$\Rightarrow \frac{x}{x+2} = 4$ and $\frac{x}{x+2} = 3$

$\Rightarrow 4x + 8 = x$ and $3x + 6 = x$

$\Rightarrow x = \frac{-8}{3}$ and $x = -3$

Question 14:

Solve:

(i) $x^2 – 11x – 12 = 0$; when $x \in N$

(ii) $x^2 – 4x – 12 = 0$; when $x \in I$

(iii) $2x^2 – 9x + 10 = 0$; when $x \in Q$

Solution 14:

(i) $x^2 – 11x – 12 = 0$

$\Rightarrow x^2 – 12x + x – 12 = 0$

$\Rightarrow x(x – 12) + 1(x - 12) = 0$

$\Rightarrow (x – 12) (x + 1) = 0$

$\Rightarrow x = 12$ and $x = - 1$

Since $x \in N$, then $x = 12$

(ii) $\mathbf{x^2 – 4x – 12 = 0}$

$\Rightarrow \mathbf{x^2 – 6x + 2x – 12 = 0}$

$\Rightarrow x(x – 6) + 2(x - 6) = 0$

$\Rightarrow (x – 6) (x + 2) = 0$

$\Rightarrow x = 6$ and $x = - 2$

Since $x \in I$, then $x = 6$ and -2

(iii) $\mathbf{2x^2 – 9x + 10 = 0}$

$\Rightarrow \mathbf{2x^2 – 5x - 4x – 10 = 0}$

$\Rightarrow x(2x – 5) - 2(2x - 5) = 0$

$\Rightarrow (2x – 5) (x - 2) = 0$

$\Rightarrow x = \frac{5}{2}$ and $x = 2$

Since $x \in Q$, then $x = \frac{5}{2}$ and 2.

Question 15:

Solve:

$(a + b)^2 x^2 – (a + b)x - 6 = 0$; $a + b \neq 0$

Take : $(a + b) x = y$

Solution 15:

$(a + b)^2 x^2 – (a + b)x - 6 = 0$; $a + b \neq 0$

$\Rightarrow (a + b)^2 x^2 – 3 (a + b) x + 2 (a + b) x – 6 = 0$

$\Rightarrow (a + b)x [(a + b) x – 3] + 2 [(a + b) x – 3] = 0$

$\Rightarrow [(a + b) x – 3] [(a + b) x – 2] = 0$

$\Rightarrow (a + b) x – 3 = 0$ Or $(a + b) x + 2 = 0$

$\Rightarrow x = \frac{3}{a + b}$ Or $x = \frac{-2}{a+b}$

Question 16:

Solve: $\frac{1}{p}+\frac{1}{q}+\frac{1}{x}=\frac{1}{x+p+q}$

Take : $\frac{1}{p}+\frac{1}{q}+\frac{1}{x}-\frac{1}{x+p+q}=0$

Solution 16:

$\frac{1}{p}+\frac{1}{q}+\frac{1}{x}=\frac{1}{x+p+q}$

$\Rightarrow \frac{1}{p}+\frac{1}{q}+\frac{1}{x}-\frac{1}{x+p+q}=0$

$\Rightarrow \frac{q+p}{pq}+\frac{x+p+q-x}{x(x+p+q)}=0$

$\Rightarrow \frac{q+p}{pq}+\frac{p+q}{x(x+p+q)}=0$

$\Rightarrow (p+q)\left[\frac{1}{pq}+\frac{1}{x^2+px+qx}\right]=0$

$\Rightarrow (p+q)\left[\frac{x^2+px+qx+pq}{pq(x^2+px+qx)}\right]=0$

$\Rightarrow x^2+px+qx+pq=0$

$\Rightarrow x(x+p)+q(x+p)=0$

$\Rightarrow (x+p)(x+q)=0$

$\Rightarrow x=-p$ and $x=-q$

Question 17:

Solve:

(i) $x(x+1)+(x+2)(x+3)=42$

(ii) $\frac{1}{x+1}-\frac{2}{x+2}=\frac{3}{x+3}-\frac{4}{x+4}$

Solution 17:

(i) $x(x+1)+(x+2)(x+3)=42$

$\Rightarrow x^2+x+x^2+3x+2x+6-42=0$

$\Rightarrow 2x^2+6x-36=0$

$\Rightarrow 2x^2+12x-6x-36=0$

$\Rightarrow 2x(x+6)-6(x+6)=0$

$\Rightarrow (x+6)(2x-6)=0$

If $x+6=0$ Or $2x-6=0$

Then $x=-6$ Or $x=3$

(ii) $\frac{1}{x+1}-\frac{2}{x+2}=\frac{3}{x+3}-\frac{4}{x+4}$

$\Rightarrow \frac{1(x+2)-2(x+1)}{(x+1)(x+2)}=\frac{3(x+4)-4(x+3)}{(x+3)(x+4)}$

$\Rightarrow \frac{-x}{x^2+3x+2}=\frac{-x}{x^2+7x+12}$

$\Rightarrow -x\ [x^2 + 3x + 2 = x^2 + 7x + 12]$
$\Rightarrow -x\ [-4x = 10]$
X = 0 and $x = \frac{-10}{4} = -2.5$

Question 18:

For each equation, given below find the value of 'm' so that the equation has equal roots also, find the solution of each equation:
(i) $(m - 3)x^2 - 4x + 1 = 0$
(ii) $3x^2 + 12x + (m + 7) = 0$
(iii) $x^2 - (m + 2)x + (m + 5) = 0$

Solution 18:

(i) $(m - 3)x^2 - 4x + 1 = 0$

Here a = (m – 3), b = − 4 and c = 1
Given equation has equal roots
Then D = 0
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow (-4)^2 - 4(m - 3)(1) = 0$
$\Rightarrow 16 - 4m + 12 = 0$
$\Rightarrow -4m = -28$
$\Rightarrow m = 7$
Put value of m in given equation
$4x^2 - 4x + 1 = 0$
$\Rightarrow (2x - 1)^2 = 0$
$\Rightarrow 2x - 1 = 0$
$\Rightarrow X = \frac{1}{2}$

(ii) $3x^2 + 12x + (m + 7) = 0$

Here a = 3, b = 12 and c = m + 7
Given equation has equal roots
Then D = 0
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow (12)^2 - 4(3)(m + 7) = 0$
$\Rightarrow 144 - 12m - 84 = 0$
$\Rightarrow -12m = -60$
$\Rightarrow m = 5$
Put value of m in given equation
$3x^2 - 12x + 12 = 0$
$X^2 + 4x + 4 = 0$
$\Rightarrow (x + 2)^2 = 0$
$\Rightarrow x + 2 = 0$
$\Rightarrow X = -2$

(iii) $x^2 - (m + 2)x + (m + 5) = 0$
Here a = 1, b = − 4 (m + 2) and c = m + 5
Given equation has equal roots
Then D = 0
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow [-(m + 2)]^2 - 4(1)(m + 5) = 0$
$\Rightarrow m^2 + 4m + 4 - 4m - 20 = 0$
$\Rightarrow m^2 - 16 = 0$
$\Rightarrow m^2 = 16$
$\Rightarrow m = \pm 4$
Put value of m in given equation

$x^2 - 6x + 9 = 0$	Or	$x^2 + 2x + 1 = 0$
$\Rightarrow (x - 3)^2 = 0$	Or	$(x + 1)^2 = 0$
$\Rightarrow x - 3 = 0$	Or	$x + 1 = 0$
$\Rightarrow x = 3$	Or	$x = -1$

Question 19:
Without solving the following quadratic equation, find the value of 'p' for which the roots are equal.
$Px^2 - 4x + 3 = 0$
Solution 19:
$Px^2 - 4x + 3 = 0$
Here a = p, b = − 4 and c = 3
Given equation has equal roots
Then D = 0
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow [-4]^2 - 4(p)(3) = 0$
$\Rightarrow 16 - 12p = 0$
$\Rightarrow -12p = -16$
$\Rightarrow p = \frac{-16}{-12} = \frac{4}{3}$

Question 20:
Without solving the following quadratic equation, find the value of 'm' for which the given equation has real and equal roots.
$X^2 + 2(m - 1)x + (m + 5) = 0$
Solution 20:
Consider the given equation:
$X^2 + 2(m - 1)x + (m + 5) = 0$

The nature of the roots of a quadratic equation
$ax^2 + bx + c = 0$, depends entirely on the
value of its discriminant $b^2 – 4ac$.
If a, b and c are real numbers and $a \neq 0$,
Then discriminant:
(i) $b^2 - 4ac = 0 \Rightarrow$ the roots are real and equal
(ii) $b^2 - 4ac > 0 \Rightarrow$ the roots are real and unequal.
(iii) $b^2 - 4ac < 0 \Rightarrow$ the roots are imaginary (not equal)
Since the roots of the given equation are real and equal,
We have,
$b^2 – 4ac = 0$
$\Rightarrow (2 (m – 1))^2 – 4 \times 1 \times (m + 5) = 0$
$\Rightarrow 4 (m^2 + 1 – 2m) – 4 (m + 5) = 0$
$\Rightarrow 4m^2 + 4 – 8m – 4m – 20 = 0$
$\Rightarrow 4m^2 – 12m – 16 = 0$
$\Rightarrow m^2 – 3m – 4 = 0$
$\Rightarrow m^2 – 4m + m – 4 = 0$
$\Rightarrow m (m – 4) +1 (m – 4) = 0$
$\Rightarrow m + 1 = 0$ Or $m – 4 = 0$
$\Rightarrow m = - 1$ Or $m = 4$