JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

RELATIONS

1. If $R = \{(x, y) ; x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is :

[*JEE Main-2020* (*September*)]

(*a*) $\{0, 1\}$ (*b*) $\{-2, -1, 1, 2\}$

(*c*) $\{-1, 0, 1\}$ (*d*) $\{-2, -1, 0, 1, 2\}$

2. Let R_1 and R_2 be two relation defined as follows :

[*JEE Main-2020* (*September*)]

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a,b) \in R^2 : a^2 + b^2 \in Q\}$, where Q is the set of the rational numbers. Then :

(*a*) Neither R_1 nor R_2 is transitive.

(*b*) R_2 is transitive but R_1 is not transitive

(*c*) R_1 and R_2 are both transitive.

(*d*) R_1 is transitive but R_2 is not transitive.

FUNCTION

3. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$ and

$S = \{x \in R : f(x) = f(-x)\}$; then S:

[*JEE Main -2016*]

(*a*) is an empty set.

(*b*) contains exactly one element

(*c*) contains exactly two elements

(*d*) contains more than two elements

4. The function $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$ defined as $f(x) = \frac{x}{1+x^2}$, is:

[*JEE Main - 2017*]

(*a*) neither injective nor surjective.

(*b*) invertible.

(*c*) injective but not surjective.

(*d*) surjective but not injective

5. Let S = $\{x \in R : x \geq 0$ and

$2|\sqrt{x} - 3| + \sqrt{x}(\sqrt{x} - 6) + 6 = 0\}$. Then S :

[*JEE Main - 2018*]

(*a*) contains exactly one element.

(*b*) contains exactly two elements.

(*c*) contains exactly four elements.

(*d*) is an empty set.

6. For $x \in R - [0, 1]$, let $f_1(x) = \frac{1}{x}, f_2(x) = 1 - x$ and $f_3(x) = \frac{1}{1-x}$ be three given functions. If a function, J(x) satisfies $(f_2 o J o f_1)(x) = f_3(x)$ then J(x) is equal to :

[*JEE Main - 2019* (*January*)]

(*a*) $f_3(x)$ (*b*) $\frac{1}{x}f_3(x)$

(*c*) $f_2(x)$ (*d*) $f_1(x)$

7. Let A = $\{x \in R : x$ is not a positive integer$\}$ Define a fuction $f : A \to R$ as $f(x) = \frac{2x}{x-1}$ then f is

[*JEE Main - 2019* (*January*)]

(*a*) injection but nor surjective

(*b*) not injective

(*c*) surjective but not injective

(*d*) neither injective nor surjective

8. Let N be the set of natural numbers and two functions f and g be defined as f , g : N → N such that

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

and $g(n) = n - (-1)^n$. Then fog is:

[JEE Main - 2019 (January)]

(*a*) onto but not one-one.
(*b*) one-one but not onto.
(*c*) both one-one and onto.
(*d*) neither one-one nor onto.

9. Let a function f : (0, ∞) → (0, ∞) be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then f is: **[JEE Main - 2019 (January)]**

(*a*) not injective but it is surjective
(*b*) injective only
(*c*) neither injective nor surjective
(*d*) both injective as well as surjective

10. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ for k = 1, 2, 3....Then for all x

11. Let $f(x) = a^x$ (a > 0) be written as $f(x) = f_1(x) + f_2(x)$, where $f_1(x)$ is an even function of $f_2(x)$ is an odd function. Then $f_1(x+y) + f_1(x-y)$ equals **[JEE Main - 2019(April)]**

(*a*) $2f_1(x)f_1(y)$ (*b*) $2f_1(x)f_2(y)$
(*c*) $2f_1(x+y)f_2(x-y)$ (*d*) $2f_1(x+y)f_1(x-y)$

12. Let $\sum_{k=1}^{10} f(a+k) = 16\left(2^{10}-1\right)$, where the function *f* satisfies $f(x+y) = f(x)f(y)$ for all natural numbers x, y and $f(a) = 2$ then the natural number 'a' is

[JEE Main - 2019(April)]

(*a*) 4 (*b*) 3
(*c*) 16 (*d*) 2

13. If the function f : R–{1,–1} → A defined by

$f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to

[JEE Main - 2019(April)]

(*a*) R–[–1,0) (*b*) R–(–1,0)
(*c*) R–{–1} (*d*) [0, ∞)

14. Let $f(x) = x^2$, x ∈ R. For any A ⊆ R, define g(*a*) = {x ∈ R, *f*(x) ∈ A}. If S = [0, 4], then which one of the following statements is not true ? **[JEE Main - 2019(April)]**

(*a*) $f(g(S)) \neq f(S)$ (*b*) $f(g(S)) = S$
(*c*) $g(f(S)) = g(S)$ (*d*) $g(f(S)) \neq S$

15. Let $f(x) = \log_e(\sin x)$, $(0 < x < \pi)$ and $g(x) = \sin^{-1}(e^{-x})$, $(x \geq 0)$. If α is a positive real number such that $a = (fog)'(\alpha)$ and $b = (fog)(\alpha)$, then :

[JEE Main - 2019(April)]

(*a*) $a\alpha^2 - b\alpha - a = 0$ (*b*) $a\alpha^2 + b\alpha - a = -2\alpha^2$
(*c*) $a\alpha^2 + b\alpha + a = 0$ (*d*) $a\alpha^2 - b\alpha - a = 1$

16. For $x \in \left(0, \frac{3}{2}\right)$, let $f(x) = \sqrt{x}$, $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x) = ((hof)og)(x)$, then $\phi = \left(\frac{\pi}{3}\right)$ is equal to : **[JEE Main - 2019(April)]**

(*a*) $\tan\frac{\pi}{12}$ (*b*) $\tan\frac{7\pi}{12}$

(*c*) $\tan\frac{11\pi}{12}$ (*d*) $\tan\frac{5\pi}{12}$

17. If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to **[JEE Main-2020 (January)]**

(*a*) $\frac{1}{2}$ (*b*) $\frac{-3}{2}$

(*c*) $\frac{-1}{2}$ (*d*) $\frac{3}{2}$

18. The inverse function of $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$, x ∈ (– 1, 1) is

[JEE Main-2020 (January)]

(*a*) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

(*b*) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(*c*) $\frac{\log_8 e}{4}\log_e\left(\frac{1+x}{1-x}\right)$

(*d*) $\frac{\log_8 e}{4}\log_e\left(\frac{1-x}{1+x}\right)$

19. Let f : R → R be a function which satisfies $f(x+y) = f(x) + f(y)$ ∀ x, y R. If $f(a) = 2$ and $g(n) = \sum_{k=1}^{(n-1)} f(k), n \in N$ then the value of n, for which g(n) = 20, is :

[JEE Main-2020 (September)]

20. The domain of the function

$f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is **[JEE Main-2020 (September)]**

(– ∞, – a] ∪ [a,∞). Then a is equal to:

21. Let A = {a, b, c} and B = {1, 2, 3, 4}. Then the number of elements in the set C = {f : A → B | 2 ∈ f(*a*) and f is not one-one} is ______.

[JEE Main-2020 (September)]

22. For a suitably chosen real constant a, let a function, f : R – {–a} → R be defined by f(x) = $\frac{a-x}{a+x}$. Further suppose that for any real number x ≠ – a and f(x) ≠ – a, (fof)(x) = x. Then f$\left(-\frac{1}{2}\right)$ is equal to :

[JEE Main-2020 (September)]

(*a*) – 3 (*b*) $\frac{1}{3}$

(*c*) $-\frac{1}{3}$ (*d*) 3

23. If {p} denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to :

[JEE Main-2020 (September)]

(*a*) $\frac{5}{8}$ (*b*) $\frac{1}{8}$

(*c*) $\frac{7}{8}$ (*d*) $\frac{3}{8}$

24. If f(x + y) = f(x) f(y) and $\sum_{x=1}^{\infty} f(x) = 2$, x, y ∈ N, Where N is the set of all natural numbers, then the value of $\frac{f(4)}{f(2)}$ is :

[JEE Main-2020 (September)]

(*a*) $\frac{1}{9}$ (*b*) $\frac{4}{9}$

(*c*) $\frac{1}{3}$ (*d*) $\frac{2}{3}$

25. Let $f : N \to N$ be a function such that $f(m+n) = f(m) + f(n)$ for every $m, n \in N$. If $f(6) = 18$, then $f(2).f(3)$ is equal to: **[JEE Main-2021 (August)]**

(*a*) 6 (*b*) 54

(*c*) 18 (*d*) 36

26. The domain of the function **[JEE Main-2021 (August)]**

$f(x) = \sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right) + \cos^{-1}\left(\frac{x-1}{x+1}\right)$ is :

(*a*) $\left[0, \frac{1}{4}\right]$ (*b*) $[-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

(*c*) $\left[\frac{1}{4}, \frac{1}{2}\right] \cup \{0\}$ (*d*) $\left[0, \frac{1}{2}\right]$

27. The domain of the function $\operatorname{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is:

(*a*) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$ (*b*) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

(*c*) $\left(-\frac{1}{2}, \infty\right) - \{0\}$ (*d*) $\left[-\frac{1}{2}, \infty\right) - \{0\}$

28. Consider function $f : A \to B$ and $g : B \to C (A, B, C \subseteq R)$ such that $(gof)^{-1}$ exists, then :

(*a*) f and g both are one-one

(*b*) f and g both are onto

(*c*) f is one-one and g is onto

(*d*) f is onto and g is one-one

29. Let N be the set of natural numbers and a relation R on N be defined by $R = \left\{\begin{matrix}(x, y) \in N \times N : \\ x^3 - 3x^2y - xy^2 + 3y^3 = 0\end{matrix}\right\}$. Then the relation R is :

(*a*) symmetric but neither reflexive nor transitive

(*b*) reflexive but neither symmetric nor transitive

(*c*) reflexive and symmetric, but not transitive

(*d*) an equivalence relation

30. If the domain of the function $f(x) = \frac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\frac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to:

(*a*) $\frac{3}{2}$ (*b*) 2

(*c*) $\frac{1}{2}$ (*d*) 1

JEE-ADVANCED PREVIOUS YEAR'S

1. Let f(x) = x^2 and g(x) = sin x for all x ∈ R. Then the set of all x satisfying (f o g o g o f) (x) = (g o g o f) (x), where (f o g) (x) = f(g(x)), is **[IIT JEE - 2012]**

(*a*) $\pm\sqrt{n\pi}$, n ∈ {0, 1, 2,....}

(*b*) $\pm\sqrt{n\pi}$, n ∈ {1, 2,....}

(*c*) $\frac{\pi}{2}$ + 2nπ, n ∈ {.....–2, –1, 0, 1, 2,....}

(*d*) 2nπ, n ∈ {...., –2, –1, 0, 1, 2,....}

2. The function $f : [0, 3] \rightarrow [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is **[*IIT JEE-2012*]**

(*a*) one-one and onto

(*b*) onto but not one-one

(*c*) one-one but not onto

(*d*) neither one-one nor onto

3. Let $f : (-1, 1) \rightarrow IR$ be such that $f(\cos 4\theta) = \dfrac{2}{2 - \sec^2 \theta}$ for $\theta \in \left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$. Then the value(s) of $f\left(\dfrac{1}{3}\right)$ iσ (are) **[*IIT JEE-2012*]**

(*a*) $1 - \sqrt{\dfrac{3}{2}}$

(*b*) $1 + \sqrt{\dfrac{3}{2}}$

(*c*) $1 - \sqrt{\dfrac{2}{3}}$

(*d*) $1 + \sqrt{\dfrac{2}{3}}$

4. Let f: $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \rightarrow R$ be given by

$f(x) = (\log(\sec x + \tan x))^3$. Then **[*JEE Advanced-2014*]**

(*a*) f(x) is an odd function

(*b*) f(x) is a one-one function

(*c*) f(x) is an onto function

(*d*) f(x) is an even function

5. If $\alpha = 3\sin^{-1}\left(\dfrac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\dfrac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct option(s) is(are)

[*JEE Advanced- 2015*]

(*a*) $\cos \beta > 0$ (*b*) $\sin \beta < 0$

(*c*) $\cos(\alpha + \beta) > 0$ (*d*) $\cos \alpha < 0$

6. Let the function $f : [0,1] \rightarrow \mathbb{R}$ be defined by $f(x) = \dfrac{4^x}{4^x + 2}$

Then the value of

$$f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + f\left(\frac{3}{40}\right) + ... + f\left(\frac{39}{40}\right) - f\left(\frac{1}{2}\right) \text{ is ....}$$

[*JEE(Advanced) - 2020*]

JEE Mains & Advanced Past Years Questions

JEE-MAIN PREVIOUS YEAR'S

1. (*c*) Given $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \le 8\}$

So $R = \{(1, 1), (2, 1), (1, -1), (0, 1), (1, 0)\}$

So $D_{R^{-1}} = \{-1, 0, 1\}$

2. (*a*) (i) If $(a, b) \in R_1$ and $(b,c) \in R_1$

$\Rightarrow a^2 + b^2 \in Q$ and $b^2 + c^2 \in Q$

then $a^2 + 2b^2 + c^2 \in Q$ but we cannot say anything about $a^2 + c^2$, that it is rational or not. So R_1 is not transitive.

(ii) If $(a, b) \in R_2$ and $(b,c) \in R_2$

$\Rightarrow a^2 + b^2 \notin Q$ and $b^2 + c^2 \notin Q$

but we can't say anything about $a^2 + c^2$ that it is rational or irrational.

So R_2 is not transitive.

3. (*c*) $f(x) + 2f\left(\frac{1}{x}\right) = 3x$ ……..(1)

$f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$ ………(2)

Multiply eq (2) by 2 & subtract from eq (1)

$f(x) = \frac{2}{x} - x$

Now, $f(x) = f(-x)$

$\frac{2}{x} - x = \frac{2}{-x} + x$

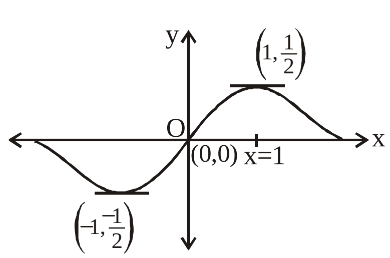
$\Rightarrow 2x = \frac{4}{x} \Rightarrow 2x^2 = 4 \Rightarrow x = \pm\sqrt{2}$

So, S contains exactly two elements.

4. (*d*) $f : R \to \left[-\frac{1}{2}, \frac{1}{2}\right]$,

$f(x) = \frac{x}{1+x^2} \; \forall \; x \in R$

$\Rightarrow f'(x) = \frac{(1+x^2).1 - x.2x}{(1+x^2)^2} = \frac{-(x+1)(x-1)}{(1+x^2)^2}$



singn of $f'(x)$

$\therefore$ From above diagram of f(x), f(x) is surjective but not injective.

5. (*b*) Case - I : $x \in [0,9]$

$2(3-\sqrt{x}) + x - 6\sqrt{x} + 6 = 0$

$\Rightarrow x - 8\sqrt{x} + 12 = 0 \Rightarrow \sqrt{x} = 4, 2$

$x = 16, 4 \Rightarrow x = 4$ ↑

rejected

Case - II : $x \in [9, \infty]$

$2(\sqrt{x} - 3) + x - 6\sqrt{x} + 6 = 0$

$x - 4\sqrt{x} = 0 \Rightarrow x = 16, 0$

↑

rejected

So, $x = 4, 16$

6. (*a*) $x \in R - \{0,1\}$

$f_1(x) = \frac{1}{x}, f_2(x) = 1 - x, f_3(x) = \frac{1}{1-x}$

Given $f_2(J(f_1(x))) = f_3(x)$

$1 - J(f_1(x)) = f_3(x)$

$J(f_1(x)) = 1 - f_3(x) = 1 - \frac{1}{1-x}$

$J(f_1(x)) = \frac{x}{x-1}$

$J\left(\frac{1}{x}\right) = \frac{x}{x-1} = \frac{1}{1-\frac{1}{x}}$

$J(x) = \frac{1}{1-x} = f_3(x)$

7. (*a*) $f(x) = 2\left(1 + \frac{1}{x-1}\right)$

$f'(x) = -\frac{2}{(x-1)^2}$

$\Rightarrow$ f is one – one but not onto

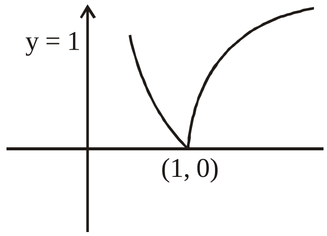
8. (*a*) $\left.\begin{array}{l} f(g(1)) = 1 \\ f(g(2)) = 1 \end{array}\right\}$ Many one

$f(g(2)) = k$

$f(g(2k+1)) = k + 1$

$\therefore$ Onto

9. (*c*) $y = \left|1 - \frac{1}{x}\right|$



Neither one-one nor Onto

10. (*a*) $F_4(x) = \frac{\sin^4 x + \cos^4 x}{4} = \frac{1 - 2\sin^2 x.\cos^2 x}{4}$

$= \frac{1}{4} - \frac{1}{2}\sin^2 x.\cos^2 x$

$F_6(x) = \frac{\sin^6 x + \cos^6 x}{6}$

$= \frac{1 - 3\sin^2 x.\cos^2 x(\sin^2 + \cos^2 x)}{6}$

$= \frac{1}{6} - \frac{1}{2}\sin^2 x.\cos^2 x$

$F_4(x) - f_6(x) = \frac{1}{4} - \frac{1}{6} = \frac{6-4}{24} = \frac{2}{24} = \frac{1}{12}$

11. (*a*) $f(x) = a^x, a > 0$

$f(x) = \frac{a^x + a^{-x} + a^x - a^{-x}}{2}$

$\Rightarrow f_1(x) = \frac{a^x + a^{-x}}{2}$

$\Rightarrow f_2(x) = \frac{a^x - a^{-x}}{2}$

$\Rightarrow f_1(x+y) + f_1(x-y)$

$= \frac{a^{x+y} + a^{-x-y}}{2} + \frac{a^{x-y} + a^{-x+y}}{2}$

$= \frac{\left(a^x + a^{-x}\right)}{2}\left(a^y + a^{-y}\right)$

$= f_1(x) \times 2 f_1(y)$

$= 2 f_1(x)\ f_1(y)$

12. (*b*) From the given functional equation :

$f(x) = 2x \qquad \forall\ x \in N$

$2^{a+1} + 2^{a+2} + \ldots\ldots. + 2^{a+10} = 16(2^{10} - 1)$

$2^a(2 + 2^2 + \ldots\ldots.. + 2^{10})\ 16(2^{10} - 1)$

$2^a.\frac{2.\left(2^{10} - 1\right)}{1} = 16\left(2^{10} - 1\right)$

$2^{a+1} = 16 = 2^4$

$a = 3$

13. (*a*) $y = \frac{x^2}{1 - x^2}$

Range of y : R–[–1,0)

for surjective function, A must be same as above range.

14. (*d*) g(S) = [–2, 2]

So, *f*(g(S)) = [0, 4] = S

And *f*(S) = [0, 16]

$\Rightarrow f(g(S) \neq f(S)$

Also, $g(f(S)) = [-4, 4] \neq g(S)$

So, $g(f(S) \neq S$

15. (*d*) $fog(x) = (-x) \Rightarrow (fg(\alpha)) = -\alpha = b$

$(fg(x))' = -1 \Rightarrow (fg(\alpha))' = -1 = a$

16. (*c*) $f(x) = \sqrt{x}$, $g(x) = \tan x$, $h(x) = \frac{1 - x^2}{1 + x^2}$

$fog\ (x) = \sqrt{\tan x}$

$hofog\ (x) = h(\sqrt{\tan x}) = \frac{1 - \tan x}{1 + \tan x}$

$= -\tan\left(\frac{\pi}{4} - x\right)$

$\phi\ (x) = \tan\left(\frac{\pi}{4} - x\right)$

$\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$

$= \tan\left(\pi - \frac{\pi}{12}\right) = \tan\frac{11\pi}{12}$

17. (*c*) $g((x)) = f^2(x) + f(x) - 1$

$g\left(f\left(\frac{5}{4}\right)\right) = 4\left(\frac{5}{4}\right)^2 - 10.\frac{5}{4} + 5 = -\frac{5}{4}$

$g\left(f\left(\frac{5}{4}\right)\right) = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$

$\frac{-5}{4} = f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) - 1$

$f^2\left(\frac{5}{4}\right) + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$

$\left(f\left(\frac{5}{4}\right) + \frac{1}{2}\right)^2 = 0$

$f\left(\frac{5}{4}\right) = \frac{-1}{2}.$

18. (*c*) $y = \dfrac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$

$\dfrac{1+y}{1-y} = \dfrac{8^{2x}}{8^{-2x}} \Rightarrow 8^{4x} = \dfrac{1+y}{1-y}$

$4x = \log_8\left(\dfrac{1+y}{1-y}\right) \Rightarrow x = \dfrac{1}{4}\log_8\left(\dfrac{1+y}{1-y}\right)$

$f^{-1}(x) = \dfrac{1}{4}\log_8\left(\dfrac{1+x}{1-x}\right) = \dfrac{\log_8 e}{4}\log_e\left(\dfrac{1+x}{1-x}\right).$

19. (*c*) $f(x+y) = f(x) + f(y)\ \forall\ x, y\ R$. If $f(1) = 2$

$\Rightarrow f(x) = 2x$

Now, $g(n) = \sum_{k-1}^{(n-1)} f(k)$

$= f(1) + f(2) + f(3) + ...f(n-1)$

$= 2 + 4 + 6 + ... + 2(n-1)$

$= 2[1 + 2 + 3 + ... + (n-1)]$

$= 2 \times \dfrac{(n-1)(n)}{2} = n^2 - n$

So, $n^2 - n = 20$ (given)

$\Rightarrow n^2 - n - 20 = 0$

$(n-5)(n+4) = 0$

$\Rightarrow n = 5$

22. (*a*) $\because f(x) = \sin^{-1}\left(\dfrac{|x|+5}{x^2+1}\right)$

$\therefore -1 \le \dfrac{|x|+5}{x^2+1} \le 1$

$\therefore x^2 - |x| - 4 \ge 0$

$\left(|x| - \dfrac{1-\sqrt{17}}{2}\right)\left(|x| - \dfrac{1+\sqrt{17}}{2}\right) \ge 0$

$\therefore |x| \ge \dfrac{1-\sqrt{17}}{2}$

$\therefore x \in \left(-\infty, -\dfrac{1+\sqrt{17}}{2}\right] \cup \left(\dfrac{1+\sqrt{17}}{2}, \infty\right)$

21. (19)The desired functions will contain either one element or two elements in its codomain of which '2' always belongs to f(A).

$\therefore$ The set B can be

{2}, {1, 2}, {2, 3}, {2, 4}

Total number of functions

$= 1 + (2^3 - 2)3$

$= 19$

22. (*d*) $f(f(x)) = \dfrac{a - \left(\dfrac{a-x}{a+x}\right)}{a + \left(\dfrac{a-x}{a+x}\right)} = x$

$\Rightarrow \dfrac{a^2 + ax - a + x}{a^2 + ax + a - x} = x$

$\Rightarrow a^2 + (a+1)x - a = a^2x + (a-1)x^2 + ax$

$\Rightarrow (a-1)x^2 + (a^2-1)x + (a - a^2) = 0$

$\forall x \in R - \{-a\}$

Hence $a = 1$

$\therefore f(x) = \dfrac{1-x}{1+x} \Rightarrow f\left(-\dfrac{1}{2}\right) = 3$

23. (*b*) $\dfrac{3^{200}}{8} = \dfrac{1}{8}(9^{100})$

$= \dfrac{1}{8}(1+8)^{100}$

$= \dfrac{1}{8}$ + Integer $\therefore \left\{\dfrac{3^{200}}{8}\right\} = \left\{\dfrac{1}{8} + \text{intger}\right\}$

$= \dfrac{1}{8}$

24. (*b*) Let $f(1) = a$

then $f(1+1) = a^2$

$f(2+1) = a^2$

and so on.

$\sum_{x=1}^{\infty} f(x) = 2 \Rightarrow a + a^2 + a^3 + ...\infty = 2$

$\Rightarrow \dfrac{a}{1-a} = 2$

$\Rightarrow a = \dfrac{2}{3}$

Now, $\dfrac{f(4)}{f(2)} = \dfrac{a^4}{a^2} = a^2 = \dfrac{4}{9}$

25. (*b*) $f(m+n) = f(m) + f(n)$

Put $m = 1, n = 1$

$f(2) = 2f(1)$

Put $m = 2, n = 1$

$f(3) = f(2) + f(1) = 3f(1)$

Put $m = 3, n = 3$

$f(6) = 2f(3) \Rightarrow f(3) = 9$

$\Rightarrow f(1) = 3, f(2) = 6$

$f(2) \cdot f(3) = 6 \times 9 = 54$

26. (*c*) $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$ ……..(1)

$-1\le\frac{x-1}{x+1}\le 1\Rightarrow 0\le x<\infty$

$-1\le\frac{3x^2+x-1}{(x-1)^2}\le 1\Rightarrow x\in\left[\frac{-1}{4},\frac{1}{2}\right]\cup\{0\}$ ………(2)

(1) & (2)

$\Rightarrow$ Domain $=\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$

27. (*d*) $\frac{1+x}{x}\in(-\infty,-1]\cup[1,\infty)$

$\Rightarrow\frac{1}{x}\in(-\infty,-2]\cup[0,\infty)$

$\Rightarrow x\in\left[-\frac{1}{2},0\right)\cup(0,\infty)\Rightarrow x\in\left[-\frac{1}{2},0\right)\cup\{0\}$

28. (*c*) $\therefore$ (gof) $^{-1}$ exist $\Rightarrow$ gof is bijective

$\Rightarrow$ 'f' must be one-one and ' g ' must be ONTO.

29. (*b*) $x^3-3x^2y-xy^2+3y^3=0$

$\Rightarrow x\left(x^2-y^2\right)-3y\left(x^2-y^2\right)=0$

$\Rightarrow(x-3y)(x-y)(x+y)=0$

30. (*a*) $O\le x^2-x+1\le 1$

$\Rightarrow x^2-x\le 0\Rightarrow x\in[0,1]$

Also, $0<\sin^{-1}\left(\frac{2x-1}{2}\right)\le\frac{\pi}{2}$

$\Rightarrow 0<\frac{2x-1}{2}\le 1$

$\Rightarrow 0<2x-1\le 2$

$1<2x\le 3$

$\frac{1}{2}<x\le\frac{3}{2}$

Taking intersection

$x\in\left(\frac{1}{2},1\right]$

$\Rightarrow\alpha=\frac{1}{2},\beta=1$

$\Rightarrow\alpha+\beta=\frac{3}{2}$

JEE-ADVANCED PREVIOUS YEAR'S

1. (*a*) $f(x)=x^2$; $g(x)=\sin x\Rightarrow \text{gof}(x)=\sin x^2$

$\Rightarrow$ gogof (x) = $\sin(\sin x^2)$

$\Rightarrow$ (fogogof) (x) = $(\sigma in(\sin x^2))^2=\sin^2(\sin x^2)$

Now$\sin^2(\sin x^2)=\sin(\sin x^2)$

$\Rightarrow\sin(\sin x^2)=0,1$

$\Rightarrow\sin x^2=n\pi,(4n+1)\frac{\pi}{2}$; $\eta\in I$

$\Rightarrow\sin x^2=0$

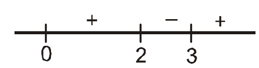
$\Rightarrow x^2=n\pi$

$\Rightarrow x=\pm\sqrt{n\pi}$; $n\in W$

2. (*b*) $F:[0,3]\to[1,29]$

$f(x)=2x^3-15x^2+36x+1$

$f'(x)=6x^2-30x+36=6(x^2-5x+6)=6(x-2)(x-3)$



in given domain function has local maxima, it is many-one

Now at x = 0 f(0) = 1

x = 2f(2) = 16 – 60 + 72 + 1 = 29

x = 3f(3) = 54 – 135 + 108 + 1 = 163 – 135 = 28

Has range = [1, 29]

Hence given funχτιon is onto

3. (*ab*)**NOTE :** Since a functional mapping can't have two images for pre-image 1/3, so this is ambiguity in this question perhaps the answer can be A or B or AB or marks to all.

$\cos4\theta=\frac{1}{3}\Rightarrow 2\cos^2 2\theta-1=\frac{1}{3}\Rightarrow\cos^2 2\theta=\frac{2}{3}$

$\Rightarrow\cos2\theta=\pm\sqrt{\frac{2}{3}}$

Now $f(\cos4\theta)=\frac{2}{2-\sec^2\theta}=\frac{1+\cos2\theta}{\cos2\theta}=1+\frac{1}{\cos2\theta}$

$\Rightarrow f\left(\frac{1}{3}\right)=1\pm\sqrt{\frac{3}{2}}$

4. (a,b,c) (i) $f(-x) = -f(x)$ so it is odd function

(ii) $f'(x) = 3(\log(\sec x + \tan x))^2 \frac{1}{(\sec x + \tan x)}(\sec x \tan x + \sec^2 x) > 0$

(iii) Range of f(x) is R as $f\left(-\frac{\pi}{2}\right) \Rightarrow -\infty$

5. (b,c,d) $\alpha = 3\sin^{-1}\frac{6}{11} > 3\sin^{-1}\frac{6}{12}$ and

$\beta = 3\cos^{-1}\frac{4}{9} > 3\cos^{-1}\frac{4}{8}$

$\Rightarrow \alpha > \frac{\pi}{2}$ & $\beta > \pi$

$\Rightarrow \alpha + \beta > \frac{3\pi}{2}$

6. (19.00) $f(x) + f(1-x) = \frac{4^x}{4^x + 2} + \frac{4^{1-x}}{4^{1-x} + 2}$

$= \frac{4^x}{4^x + 2} + \frac{4/4^x}{\frac{4}{4^x} + 2}$

$= \frac{4^x}{4^x + 2} + \frac{4}{4 + 2.4^x}$

$= \frac{4^x}{4^x + 2} + \frac{2}{2 + 4^x}$

$= 1$

so, $f\left(\frac{1}{40}\right) + f\left(\frac{2}{40}\right) + ... + f\left(\frac{39}{40}\right) - f\left(\frac{1}{2}\right)$

$= 19 + f\left(\frac{1}{2}\right) - f\left(\frac{1}{2}\right) = 19$