2 Inverse Trigonometric Functions

Multiple Choice Questions (MCQs)

DIRECTIONS : *This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d) out of which only one is correct.*

1. Principal value of $\text{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is equal to

(a) $-\frac{\pi}{3}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$

2. Principal value of $\sec^{-1}(2)$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{3}$

3. Prinicpal value of $\tan^{-1}\left(\sqrt{3}\right)$ is equal to

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{3}$

4. $-\frac{2\pi}{5}$ is the principal value of

(a) $\cos^{-1}\left(\cos\frac{7\pi}{5}\right)$ (b) $\sin^{-1}\left(\sin\frac{7\pi}{5}\right)$

(c) $\sec^{-1}\left(\sec\frac{7\pi}{5}\right)$ (d) None of these

5. The principal value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is

(a) $-\frac{5\pi}{3}$ (b) $\frac{5\pi}{3}$

(c) $-\frac{\pi}{3}$ (d) $\frac{4\pi}{3}$

6. If $\sin^{-1} x = y$, then

(a) $0 \le y \le \pi$ (b) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(c) $0 < y < \pi$ (d) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

7. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to

(a) π (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

8. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) 1

9. $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$ is equal to

(a) π (b) $-\frac{\pi}{2}$

(c) 0 (d) $2\sqrt{3}$

10. Which of the following is the principal value branch of $\cos^{-1}x$?

(a) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (b) $(0, \pi)$

(c) $[0, \pi]$ (d) $(0, \pi) - \left\{\frac{\pi}{2}\right\}$

11. Which of the following is the principal value branch of $\text{cosec}^{-1}x$?

(a) $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (b) $(0, \pi) - \left[\frac{\pi}{2}\right]$

(c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

12. The domain of the function $\cos^{-1}(2x - 1)$ is

(a) $[0, 1]$ (b) $[-1, 1]$

(c) $(-1, 1)$ (d) $[0, \pi]$

13. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(a) $[1, 2]$ (b) $[-1, 1]$

(c) $[0, 1]$ (d) None of these

Inverse Trigonometric Functions

14. Let $\cos(2\tan^{-1}x) = \frac{1}{2}$, then the value of x will be

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$

(c) $1-\sqrt{3}$ (d) $1-\frac{1}{\sqrt{3}}$

15. $\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}$ is equal to

(a) $\pi/4$ (b) $\pi/6$ (c) $\pi/3$ (d) $2\pi/3$

16. The domain of the function $\cos^{-1}\log_2(x^2+5x+8)$ is-

(a) [2, 3] (b) [–3, –2] (c) [–2, 2] (d) [–3, 1]

17. The domain of the function

$f(x) = \sin^{-1}\left\{\log_2\left(\frac{1}{2}x^2\right)\right\}$ is

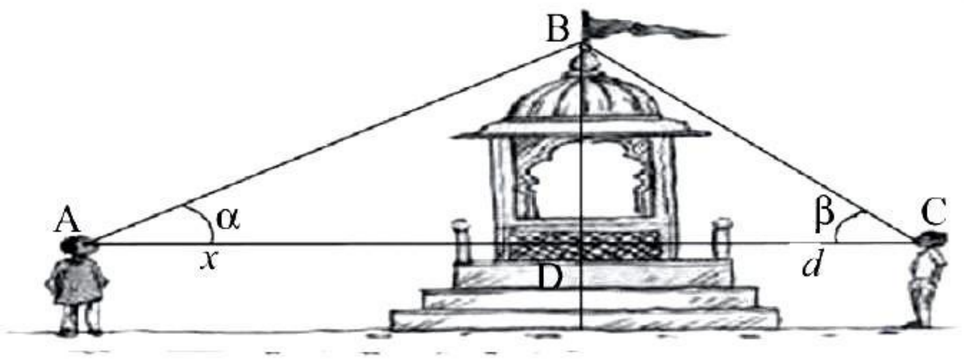
(a) $[-2,-1)\cup[1,2]$ (b) $(-2,-1]\cup[1,2]$

(c) $[-2,-1]\cup[1,2]$ (d) $(-2,-1)\cup(1,2)$

Case/Passage Based Questions

DIRECTIONS : *Study the given Case/Passage and answer the following questions.*

Case/Passage-I



Two men on either side of a temple of 30 meters high observe its top at the angles of elevation α and β respectively. (as shown in the figure above). The distance between the two men is $40\sqrt{3}$ meters and the distance between the first person A and the temple is $30\sqrt{3}$ meters. Based on the above information answer the following: **[From CBSE Question Bank 2021]**

18. $\angle CAB = \alpha =$

(a) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (b) $\sin^{-1}\left(\frac{1}{2}\right)$

(c) $\sin^{-1}(2)$ (d) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

19. $\angle CAB = \alpha =$

(a) $\cos^{-1}\left(\frac{1}{5}\right)$ (b) $\cos^{-1}\left(\frac{2}{5}\right)$

(c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\cos^{-1}\left(\frac{4}{5}\right)$

20. $\angle BCA = \beta =$

(a) $\tan^{-1}\left(\frac{1}{2}\right)$ (b) $\tan^{-1}(2)$

(c) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\tan^{-1}(\sqrt{3})$

21. $\angle ABC =$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{3}$

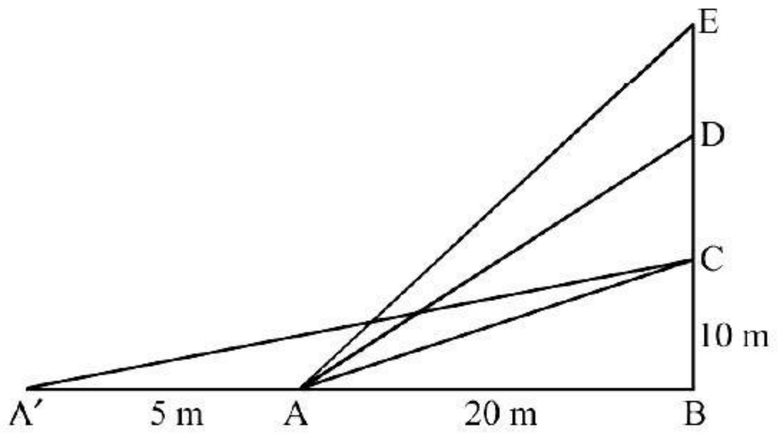
22. Domain and range of $\cos^{-1}x =$

(a) $(-1,1),(0,\pi)$ (b) $[-1,1],(0,\pi)$

(c) $[-1,1],[0,\pi]$ (d) $(-1,1),\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Case/Passage-II

The Government of India is planning to fix a hoarding board at the face of a building on the road of a busy market for awareness on COVID-19 protocol. Ram, Robert and Rahim are the three engineers who are working on this project. "A" is considered to be a person viewing the hoarding board 20 metres away from the building, standing at the edge of a pathway nearby. Ram, Robert and Rahim suggested to the firm to place the hoarding board at three different locations namely C, D and E. "C" is at the height of 10 metres from the ground level. For the viewer A, the angle of elevation of "D" is double the angle of elevation of "C" The angle of elevation of "E" is triple the angle of elevation of "C" for the same viewer. Look at the figure given and based on the above information answer the following:



[From CBSE Question Bank 2021]

23. Measure of $\angle CAB =$

(a) $\tan^{-1}(2)$ (b) $\tan^{-1}\left(\frac{1}{2}\right)$

(c) $\tan^{-1}(1)$ (d) $\tan^{-1}(3)$

24. Measure of $\angle DAB =$

(a) $\tan^{-1}\left(\frac{3}{4}\right)$ (b) $\tan^{-1}(3)$

(c) $\tan^{-1}\left(\frac{4}{3}\right)$ (d) $\tan^{-1}(4)$

25. Measure of $\angle EAB$ =

(a) $\tan^{-1}(11)$ (b) $\tan^{-1} 3$

(c) $\tan^{-1}\left(\frac{2}{11}\right)$ (d) $\tan^{-1}\left(\frac{11}{2}\right)$

26. A′ is another viewer standing on the same line of observation across the road. If the width of the road is 5 meters, then the difference between $\angle CAB$ and $\angle CA'B$ is

(a) $\tan^{-1}(1/2)$ (b) $\tan^{-1}(1/12)$

(c) $\tan^{-1}\left(\frac{2}{5}\right)$ (d) $\tan^{-1}\left(\frac{11}{21}\right)$

27. Domain and range of $\tan^{-1} x$ =

(a) $R^+, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (b) $R^-, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $R, \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $R, \left(0, \frac{\pi}{2}\right)$

Assertion & Reason

DIRECTIONS : *Each of these questions contains an assertion followed by reason. Read them carefully and answer the question on the basis of following options. You have to select the one that best describes the two statements.*

(a) If both **Assertion** and **Reason** are **correct** and the Reason is the **correct explanation** of the Assertion.

(b) If both **Assertion** and **Reason** are correct but Reason is **not the correct explanation** of the Assertion.

(c) If the **Assertion** is **correct** but **Reason** is **incorrect**.

(d) If the **Assertion** is **incorrect** but the **Reason** is **correct**.

28. **Assertion:** The domain of the function $\sec^{-1} x$ is the set of all real numbers.
Reason: For the function $\sec^{-1} x$, x can take all real values except in the interval (–1, 1).

29. **Assertion:** To define the inverse of the function $f(x) = \tan x$ any of the intervals $\left(-\frac{3\pi}{2}, \frac{-\pi}{2}\right), \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ etc. can be chosen.
Reason: The branch having range $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is called principal value branch of the function $g(x) = \tan^{-1} x$.

Match the Following

DIRECTIONS : *Each question contains statements given in two columns which have to be matched. Statements (A, B, C, D) in column-I have to be matched with statements (p, q, r, s) in column-II.*

30.

Column-I	Column-II
(A) Domain of $\cos^{-1}(x)$	(p) $\left[-\frac{1}{5}, \frac{1}{5}\right]$
(B) Domain of $\sin^{-1}(5x)$	(q) [0, 1]
(C) Domain of $\cos^{-1}(2x-1)$	(r) [1, 2]
(D) Domain of $\sin^{-1}\sqrt{x-1}$	(s) [–1, 1]

Fill in the Blanks

DIRECTIONS : *Complete the following statements with an appropriate word / term to be filled in the blank space(s).*

31. Given that $\sin^{-1}\left(\sin\frac{3\pi}{4}\right) = \frac{2\pi}{k}$, then k = _____.

True / False

DIRECTIONS : *Read the following statements and write your answer as true or false.*

32. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is $\frac{-\pi}{3}$.

33. The principal value of $\tan^{-1} 1 + \cos^{-1}\left(\frac{-1}{2}\right) + \sin^{-1}\left(\frac{-1}{2}\right)$ is $\frac{\pi}{12}$.

34. The value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\frac{1}{\sqrt{3}}$ is π.

ANSWER KEY & SOLUTIONS

1. **(a)** Let $\text{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)=\theta$

$\Rightarrow \quad \text{cosec}\,\theta=\frac{-2}{\sqrt{3}}=-\text{cosec}\,\frac{\pi}{3}=\text{cosec}\left(\frac{-\pi}{3}\right)$

$\Rightarrow \quad \theta=\frac{-\pi}{3}\in\left[\frac{-\pi}{3},\frac{\pi}{2}\right]-\{0\}$

$\therefore$ Principal value of $\text{cosec}^{-1}\left(\frac{-2}{\sqrt{3}}\right)$ is $\left(\frac{-\pi}{3}\right)$

2. **(b)** Let $\sec^{-1}(2)=\theta \Rightarrow \sec\theta=2=\sec\frac{\pi}{3}$

$\Rightarrow \theta=\frac{\pi}{3}\in[0,\pi]-\left\{\frac{\pi}{2}\right\}$ $\therefore$ Principal value of $\sec^{-1}(2)$ is $\frac{\pi}{3}$

3. **(b)** Let $\tan^{-1}\left(\sqrt{3}\right)=\theta \Rightarrow \tan\theta=\sqrt{3}=\tan\frac{\pi}{3}$

$\therefore$ Principal value of $\tan^{-1}\sqrt{3}$ is $\frac{\pi}{3}$

4. **(b)** $\cos^{-1}\left(\cos\frac{7\pi}{5}\right)=\cos^{-1}\left\{\cos\left(2\pi-\frac{3\pi}{5}\right)\right\}$

$=\cos^{-1}\cos\left(\frac{3\pi}{5}\right)=\frac{3\pi}{5}.$

also, $\sin^{-1}\left(\sin\frac{7\pi}{5}\right)=\sin^{-1}\left\{\sin\left(\pi+\frac{2\pi}{5}\right)\right\}$

$=\sin^{-1}\left\{-\sin\frac{2\pi}{5}\right\}=\sin^{-1}\left\{\sin\left(-\frac{2\pi}{5}\right)\right\}=-\frac{2\pi}{5}$

and; $\sec^{-1}\left(\sec\frac{7\pi}{5}\right)=\sec^{-1}\left\{\sec\left(2\pi-\frac{3\pi}{5}\right)\right\}$

$=\sec^{-1}\left(\sec\frac{3\pi}{5}\right)=\frac{3\pi}{5}.$

5. **(c)** Let $\theta=\sin^{-1}\left[\sin\frac{5\pi}{3}\right]$

$\Rightarrow \sin\theta=\sin\frac{5\pi}{3}=\sin\left[2\pi-\frac{\pi}{3}\right]$

$\Rightarrow \sin\theta=-\sin\frac{\pi}{3}=\sin\left(\frac{-\pi}{3}\right)(\because \sin(-\theta)=-\sin\theta)$

Therefore, principal value of $\sin^{-1}\left[\sin\frac{5\pi}{3}\right]$ is $\frac{-\pi}{3}$, as principal value of $\sin^{-1}$ x lies between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$.

6. **(b)** The range of principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\therefore$ if $\sin^{-1}x=y$ then $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$

Option (b) is correct.

7. **(b)** $\tan^{-1}\sqrt{3}=\frac{\pi}{3}$, $\sec^{-1}(-2)=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

$\because$ Principal value of $\sec^{-1}$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$

$\therefore \quad \tan^{-1}\sqrt{3}-\sec^{-1}(-2)=\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$

8. **(d)** $\sin^{-1}\left(-\frac{1}{2}\right)=-\sin^{-1}\left(\frac{1}{2}\right)=-\frac{\pi}{6}$

$\therefore \sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]=\sin\left[\frac{\pi}{3}-\left(-\frac{\pi}{6}\right)\right]$

$=\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)=\sin\frac{\pi}{2}=1$

9. **(b)** $\tan^{-1}\sqrt{3}=\frac{\pi}{3}$

$\cot^{-1}\left(-\sqrt{3}\right)$

$\because$ The range of principal value of $\cot^{-1}x$ is $(0,\pi)$

$\therefore \tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})=\frac{\pi}{3}-\left(\frac{5\pi}{6}\right)$

$=\frac{2\pi-5\pi}{6}=\frac{-3\pi}{6}=-\frac{\pi}{2}$

10. **(c)** Principal value branch of $\cos^{-1}x=[0,\pi]$

11. **(d)** Principal value branch of cosec^{-1} x

$=\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$

12. **(a)** Here, $-1\le 2x-1\le 1$ $\quad[\because -1\le\cos\theta\le 1]$

$\Rightarrow \quad 0\le 2x\le 2 \Rightarrow 0\le x\le 1$. So, $x\in[0,1]$

Hence, domain of $\cos^{-1}(2x-1)$ is $[0,1]$.

13. **(a)** $\frac{-\pi}{2}\le\sin^{-1}\sqrt{x-1}\le\frac{\pi}{2} \Rightarrow -1\le\sqrt{x-1}\le 1$

$\Rightarrow \quad 0\le x-1<1 \Rightarrow 1\le x\le 2$

$\therefore$ Domain of $f(x)$ is $[1,2]$

14. **(b)** $\because \cos\left(2\tan^{-1}x\right)=\frac{1}{2}$

$\therefore \cos\left(2\tan^{-1}x\right)=\cos\frac{\pi}{3}$

$\Rightarrow \tan^{-1}x=\frac{\pi}{6} \Rightarrow x=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

15. **(d)** $\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}=\frac{\pi}{3}+\frac{2\pi}{6}=\frac{2\pi}{3}$

16. **(b)** $\cos^{-1}\log_2(x^2+5x+8)$ is real if

$-1\le\log_2(x^2+5x+8)\le 1$

$\Rightarrow\frac{1}{2}\le x^2+5x+8\le 2$

$x^2+5x+8=\left(x+\frac{5}{2}\right)^2+\frac{7}{4}>\frac{1}{2}$ for all x.

$x^2+5x+8\le 2$

$\Rightarrow (x+3)(x+2)\le 0\Rightarrow -3\le x\le -2.$

17. **(c)** For $f(x)$ to be defined, we must have

$-1\le\log_2\left(\frac{1}{2}x^2\right)\le 1$

$\Rightarrow 2^{-1}\le\frac{1}{2}x^2\le 2^1$ [$\because$ the base $=2>1$]

$\Rightarrow 1\le x^2\le 4$ ... (i)

Now, $1\le x^2$

$\Rightarrow x^2-1\ge 0$ i.e. $(x-1)(x+1)\ge 0$

$\Rightarrow x\le -1$ or $x\ge 1$ ... (ii)

Also, $x^2\le 4$

$\Rightarrow x^2-4\le 0$ i.e. $(x-2)(x+2)\le 0$

$\Rightarrow -2\le x\le 2$ ... (iii)

Form (ii) and (iii), we get the domain of f

$=\{(-\infty,-1]\cup[1,\infty)\}\cap[-2,2]$

$=[-2,-1]\cup[1,2]$

18. **(b)** $\angle CAB=\angle DAB=\alpha$

Hypotenuse $(AB)=\sqrt{(30)^2+(30\sqrt{3})^2}=60$

$\therefore\ \sin\alpha=\frac{BD}{AB}=\frac{30}{60}=\frac{1}{2}\Rightarrow\alpha=\sin^{-1}\left(\frac{1}{2}\right)$

$\therefore\ \angle CAB=\alpha=\sin^{-1}\left(\frac{1}{2}\right)$

19. **(c)** $\because\cos\alpha=\frac{AD}{AB}=\frac{30\sqrt{3}}{60}=\frac{\sqrt{3}}{2}\Rightarrow\alpha=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\therefore\ \angle CAB=\alpha=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

20. **(d)** $\tan\beta=\frac{BD}{CD}=\frac{30}{10\sqrt{3}}=\sqrt{3}$

$\Rightarrow\beta=\tan^{-1}\left(\sqrt{3}\right)$

$\therefore\angle BCA=\beta=\tan^{-1}\left(\sqrt{3}\right)$

21. **(c)** $\frac{\pi}{2}$

22. **(c)** $[-1,1],[0,\pi]$

23. **(b)** $\tan(\angle CAB)=\frac{BC}{AB}=\frac{10}{20}\Rightarrow\angle CAB=\tan^{-1}\left(\frac{1}{2}\right)$

24. **(c)** $\because\angle DAB=2\angle CAB$

$\therefore\tan(\angle DAB)=\tan(2\angle CAB)$

$=\frac{2\tan(\angle CAB)}{1-\tan^2(\angle CAB)}=\frac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}=\frac{4}{3}$

$\Rightarrow\angle DAB=\tan^{-1}\left(\frac{4}{3}\right)$

25. **(d)** $\because\angle EAB=3\angle CAB$

$\therefore\tan(\angle EAB)=\tan(3\angle CAB)$

$=\frac{3\tan(\angle CAB)-\tan^3(\angle CAB)}{1-3\tan^2(\angle CAB)}=\frac{\frac{3}{2}-\frac{1}{8}}{1-\frac{3}{4}}=\frac{11}{2}$

$\Rightarrow\angle EAB=\tan^{-1}\left(\frac{11}{2}\right)$

26. **(b)** $\tan(\angle CA'B)=\frac{BC}{A'B}=\frac{10}{25}=\frac{2}{5}$

$\Rightarrow\angle CA'B=\tan^{-1}\left(\frac{2}{5}\right)$

Now, $\angle CAB-\angle CA'B=\tan^{-1}\left(\frac{1}{2}\right)-\tan^{-1}\left(\frac{2}{5}\right)$.

$=\tan^{-1}\left(\frac{\frac{1}{2}-\frac{2}{5}}{1+\frac{1}{2}\times\frac{2}{5}}\right)=\tan^{-1}\left(\frac{1}{12}\right)$

27. **(c)** $R,\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

28. **(d)**

29. **(b)**

30. (A) $\to$ (s), (B) $\to$ (p), (C) $\to$ (q), (D) $\to$ (r)

(C) $-1\le 2x-1\le 1$

$\Rightarrow\ 0\le 2x\le 2$

$\Rightarrow\ 0\le x\le 1\Rightarrow x\in[0,1]$

31. 8

32. (True) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}$

33. (False)

Principal value of $\tan^{-1}(1)+\cos^{-1}\left(\frac{-1}{2}\right)+\sin^{-1}\left(\frac{-1}{2}\right)$

$=\frac{\pi}{4}+\frac{2\pi}{3}-\frac{\pi}{6}=\frac{3\pi}{4}$

34. (True) $\cos^{-1}\left(\frac{1}{2}\right)+\sin^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\frac{1}{\sqrt{3}}$

$=\frac{\pi}{3}+\frac{\pi}{2}+\frac{\pi}{6}=\frac{6\pi}{6}=\pi$