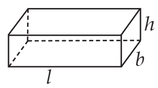
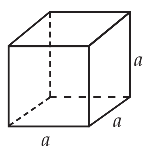
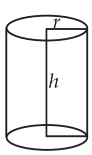
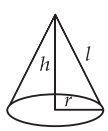
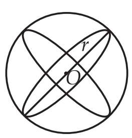
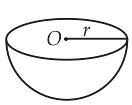
CHAPTER 5

Surface Areas and Volumes

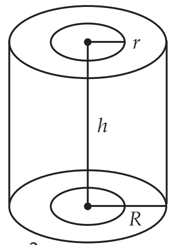
Recap Notes

SURFACE AREA AND VOLUME OF DIFFERENT SHAPES

Sr. no.	Name	Figure	Lateral/curved surface area	Total surface area	Volume	Nomenclature
1.	Cuboid		$2(l + b) \times h$	$2(lb + bh + hl)$	$l \times b \times h$	l = length b = breadth h = height
2.	Cube		$4a^2$	$6a^2$	a^3	a = edge of cube
3.	Right circular cylinder		$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = radius h = height
4.	Right circular cone		πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$	r = radius of base h = height l = slant height $= \sqrt{r^2 + h^2}$
5.	Sphere		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$	r = radius of the sphere
6.	Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$	r = radius of the hemisphere

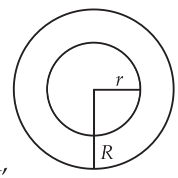
SOME SPECIAL TYPES OF SOLIDS

Hollow Cylinder



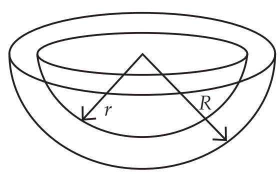
- A solid enclosed between two co-axial cylinders of same height and different radii is called hollow cylinder. Let R and r be the external and internal radii of hollow cylinder of height h. Then,
  - Surface area of each base = $\pi(R^2 - r^2)$
  - Curved surface area = $2\pi h\ (R + r)$
  - Total surface area = $2\pi(R + r)\ (h + R - r)$
  - Volume = $\pi(R^2 - r^2)h$

Spherical Shell



- It is a region or solid enclosed between two concentric spheres. Let R and r be the external and internal radii of spherical shell. Then,
  - Total surface area = $4\pi(R^2 + r^2)$
  - Volume = $\frac{4}{3}\pi(R^3 - r^3)$

Hemispherical Shell



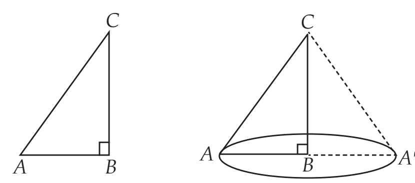
- A solid enclosed between two concentric hemispheres is called hemispherical shell. Let R and r be the external and internal radii of hemispherical shell.

Then,
  - Inner curved surface area = $2\pi r^2$
  - Outer curved surface area = $2\pi R^2$
  - Total surface area = $\pi(3R^2 + r^2)$
  - Volume = $\frac{2}{3}\pi(R^3 - r^3)$

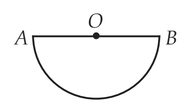
IMPORTANT POINTS

- Diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$, where l, b and h are length, breadth and height respectively.
- Diagonal of the cube = $\sqrt{3}a$, where a is the length of each edge.
- Length of all 12 edges of the cuboid = $4(l + b + h)$
- Length of all 12 edges of the cube = $12a$

Note : (i)
We have a right angle triangle ABC, right angled at B. When we revolve the triangle about its perpendicular BC, then we get a cone, whose radius, $r = AB$, height, $h = BC$ and slant height, $l = AC$.



(ii) When a semicircle is folded to form a cone then, slant height, l = radius, circumference of semicircle = circumference of base of cone and height, $h = \sqrt{l^2 - r^2}$.



Practice Time

OBJECTIVE TYPE QUESTIONS

Multiple Choice Questions (MCQs)

1. The surface area of a cube whose side is 5 cm is

(a) 125 cm^2 (b) 28 cm^2
(c) 100 cm^2 (d) 150 cm^2

2. The total surface area of a cube is 216 cm^2, its each side is

(a) 4 cm (b) 5 cm (c) 6 cm (d) 7 cm

3. The total surface area of a cuboid of dimensions x units, y units and z units is _______ sq. units.

(a) $4\{xy + yz + zx\}$ (b) $2\{xy + yz + zx\}$
(c) $3\{xy + yz + zx\}$ (d) none of these

4. The lateral surface area of a cube of side a units is _______ sq. units.

(a) $4a^2$ (b) $6a^2$ (c) a^2 (d) a^3

5. Number of pair of surfaces of same area in a cuboid are

(a) 6 (b) 4 (c) 2 (d) 3

6. The lateral surface area of a cube of side 6 units is

(a) 144 sq. units (b) 154 sq. units
(c) 134 sq. units (d) 216 sq. units

7. A cuboid is 12 cm long, 9 cm broad and 8 cm high. Its total surface area is

(a) 864 cm^2 (b) 552 cm^2
(c) 432 cm^2 (d) 276 cm^2

8. The side of a cube is 1 cm. The total surface area of the figure formed by joining two such cubes is

(a) $2(2 + 1 + 2)$ cm^2 (b) $2(2 + 2 + 2)$ cm^2
(c) $2(1 + 1 + 1)$ cm^2 (d) $2(1 + 1 + 2)$ cm^2

9. If each edge of a cuboid of total surface area S is doubled, then total surface area of the resulting cuboid is

(a) $2S$ (b) $4S$ (c) $6S$ (d) $8S$

10. If 6 cubes of side 2 cm are joined, then find the total surface area (in cm^2) of resulting cuboid.

(a) 144 (b) 48 (c) 24 (d) 104

11. If each side of a cube is increased by 50%, then the surface area of the cube increases by

(a) 50% (b) 100%
(c) 125% (d) 150%

12. Find the radius of the base of a right circular cylinder whose curved surface area is 2/3 of the sum of the surface areas of two circular faces. The height of the cylinder is given to be 15 cm.

(a) 22 cm (b) 22.5 cm
(c) 20 cm (d) 20.5 cm

13. If the total surface area of a cylinder is 550 cm^2 and its base circumference is 50 cm, then the sum of its height and radius is

(a) 11 cm (b) 50 cm
(c) 45 cm (d) 55 cm

14. The altitude of a circular cylinder is increased by six times and the base area is decreased by one-ninth of its value. The factor by which the lateral surface area of the cylinder increases, is

(a) 2/3 (b) 1/2 (c) 3/2 (d) 2

15. The curved surface area of the cone of slant height $x/2$ is $2\pi x$. Then area of its base is

(a) 4π sq. units (b) $4\pi x^2$ sq. units
(c) πx^2sq. units (d) 16π sq. units

16. The diameters of two cones are equal. If their slant heights are in the ratio 5 : 4, then the ratio of their curved surface areas is

(a) 4 : 5 (b) 25 : 16
(c) 16 : 25 (d) 5 : 4

17. The total surface area of a cone of radius '$2r$' and slant height '$l/2$' is

(a) $2\pi r\ (l + r)$ (b) $\pi r\left(l+\frac{r}{4}\right)$
(c) $\pi r\ (4r + l)$ (d) $2\pi r$

18. The internal and external radii of a hemispherical container are r_1 and r_2 respectively. The curved surface area of the container is

(a) $\pi(r_1^2 + r_2^2)$ (b) $2\pi(r_1^2 + r_2^2)$

(c) $2\pi(r_2^2 - r_1^2)$ (d) $\pi(r_2^2 - r_1^2)$

19. The ratio of the radii of two spheres is 4 : 5. The ratio of their surface areas is

(a) 4 : 5 (b) $2 : \sqrt{5}$

(c) 5 : 4 (d) 16 : 25

20. The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. The ratio of the surface areas of the balloon in the two cases is

(a) 1 : 4 (b) 1 : 3 (c) 2 : 3 (d) 2 : 1

21. A tank measures 6 m × 5 m × 4 m. Its $(2/3)^{rd}$ portion is filled with water. The volume of water is

(a) 80 m^3 (b) 60 m^3 (c) 50 m^3 (d) 40 m^3

22. If A, B and C are the areas of three adjacent faces of a cuboid, then its volume is

(a) ABC (b) $2ABC$

(c) $\sqrt{ABC}$ (d) $A + B + C$

23. If the areas of the adjacent faces of a rectangular block are in the ratio 2 : 3 : 4 and its volume is 9000 cm^3, then the length of the shortest edge is

(a) 30 cm (b) 20 cm

(c) 15 cm (d) 10 cm

24. The heights of two cylinders are in the ratio 5 : 3 and their volumes are in the ratio 20 : 27. Then the ratio of their radii is

(a) 25 : 9 (b) 5 : 3

(c) 4 : 9 (d) 2 : 3

25. The altitude of a circular cylinder is increased three times and the base area is decreased to one-ninth of its value. The ratio of volume of the cylinder is

(a) 2 : 3 (b) 1 : 2

(c) 3 : 1 (d) 2 : 1

26. The curved surface area of a right circular cylinder is 1520 cm^2 and diameter of its base is 30 cm. Then the volume of the cylinder is

(a) 11400 cm^3 (b) 11560 cm^3

(c) 12700 cm^3 (d) 11600 cm^3

27. The radius of a wire is decreased by one-third. The volume remains same, if the length will increase

(a) 3 times (b) 6 times

(c) 9 times (d) 27 times

28. If the radius and height of a cone are both increased by 10%, then the volume of the cone is approximately increased by

(a) 10% (b) 21% (c) 33% (d) 100%

29. If h, S and V denote respectively the height, curved surface area and volume of a right circular cone, then $3\pi Vh^3 - S^2h^2 + 9V^2$ is equal to

(a) 8 (b) 0 (c) 4π (d) $32\pi^2$

30. The diameter of a sphere is decreased by 25%. By what percentage its volume will decrease?

(a) 25% (b) 57.81%

(c) 43.50% (d) 50%

31. A cone, a hemisphere and a cylinder stand on equal bases and have the same height. The ratio of their volumes is

(a) 1 : 2 : 3 (b) 2 : 1 : 3

(c) 2 : 3 : 1 (d) 3 : 2 : 1

32. Volume of a hemisphere is 19404 cubic cm. The total surface area is

(a) 2772 sq. cm (b) 4158 sq. cm

(c) 5544 sq. cm (d) 1386 sq. cm

33. The largest sphere is cut off from a cube of side 5 cm. The volume of the sphere will be

(a) 27π cm^3 (b) 30π cm^3

(c) 108π cm^3 (d) $\frac{125\pi}{6} cm^3$

34. A cylindrical vessel 60 cm in diameter is partially filled with water. A sphere of 60 cm in diameter is gently dropped into the vessel. To what further height will water rise in the cylinder?

(a) 15 cm (b) 30 cm

(c) 40 cm (d) 25 cm

35. The length of diagonal of cube is $(14 \times \sqrt{3})$ cm. The volume of the cube is

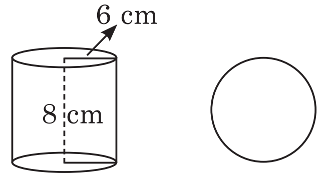
(a) $2744\sqrt{3}\,cm^3$ (b) 2744 cm^3

(c) 588 cm^3 (d) 3528 cm^3

Case Based MCQs

Case I : Read the following passage and answer the questions from 36 to 40.

A class teacher brings some clay in the classroom to teach the topic mensuration. First she forms a cylinder of radius 6 cm and height 8 cm and then she moulds that cylinder into sphere.



Cylinder Sphere

36. Find the volume of the cylindrical shape.

(a) $288\,\pi$ cm^3 (b) $244\,\pi$ cm^3
(c) $240\,\pi$ cm^3 (d) $216\,\pi$ cm^3

37. The formula for 'volume of sphere' is

(a) $\frac{4}{3}\pi r^2$ (b) $\pi r^2 h$ (c) $\frac{4}{3}\pi r^3$ (d) $\frac{2}{3}\pi r^3$

38. When clay changes into one shape to other, which of the following remains same?

(a) Area (b) C.S.A (c) Radius (d) Volume

39. The radius of the sphere is

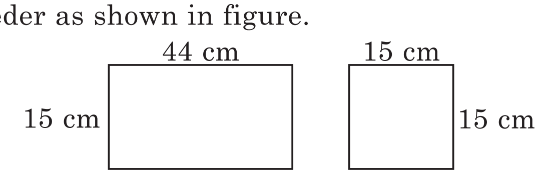
(a) 2 cm (b) 4 cm (c) 5 cm (d) 6 cm

40. Find the volume of sphere, the teacher made.

(a) 288π cm^3 (b) 184π cm^3
(c) 240π cm^3 (d) 216π cm^3

Case II : Read the following passage and answer the questions from 41 to 45.

Ankita realised the need of food for birds on her terrace and decided to make a bird feeder. She got a flexible plastic rectangular sheet of size 44 cm × 15 cm. She rolled it along its length and joined the two opposite ends using a tape for circular base of cylinder. She found a square sheet of size 15 cm × 15 cm by cutting it into required circular shape she prepared the bird feeder as shown in figure.



41. The curved surface area of the cylinder formed is

(a) 550 cm^2 (b) 660 cm^2
(c) 430 cm^2 (d) 840 cm^2

42. The radius of the base of the cylinder is

(a) 5 cm (b) 6 cm (c) 7 cm (d) 8 cm

43. The area of the circular base required for the cylinder is

(a) 154 cm^2 (b) 164 cm^2
(c) 240 cm^2 (d) 184 cm^2

44. How much will be the area of square sheet left unused after removing the circular base of the cylinder from it?

(a) 78 cm^2 (b) 62 cm^2
(c) 75 cm^2 (d) 71 cm^2

45. Volume of the seeds that can be filled in the cylinder, for birds is

(a) 2310 cm^3 (b) 2425 cm^3
(c) 2623 cm^3 (d) 2810 cm^3

Case III : Read the following passage and answer the questions from 46 to 50.

Nakul was doing an experiment to find the radius r of a ball. For this he took a cylindrical container with radius R = 7 cm and height 10 cm. He filled the container almost half by water as shown in the figure-1. Now he dropped the ball into the container as in figure-2.

He observed that in figure-2, the water level in the container raised from P to Q *i.e,* to 3.4 cm.

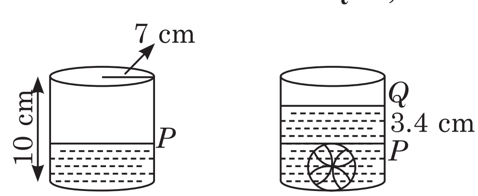


Figure - 1 Figure - 2

46. What is the approximate radius of the ball?

(a) 3 cm (b) 5 cm (c) 7 cm (d) 9 cm

47. What is the volume of the cylinder?

(a) 1260 cm^2 (b) 540 cm^3

(c) 1620 cm^3 (d) 1540 cm^3

48. What is the volume of the spherical ball?

(a) 620 cm^3 (b) 824.26 cm^3

(c) 523.81 cm^3 (d) 430.1 cm^3

49. How many litres of water can be filled in the full container?

(a) 1.54 litres (b) 2 litres

(c) 5 litres (d) 7.5 litres

50. What is the total surface area of the spherical ball?

(a) 441.34 cm^2 (b) 314.29 cm^2

(c) 620 cm^2 (d) 816 cm^2

Assertion & Reasoning Based MCQs

Directions (Q.51 to 55) : In these questions, a statement of Assertion is followed by a statement of Reason is given. Choose the correct answer out of the following choices :

(a) Assertion and Reason both are correct statements and Reason is the correct explanation of Assertion.

(b) Assertion and Reason both are correct statements but Reason is not the correct explanation of Assertion.

(c) Assertion is correct statement but Reason is wrong statement.

(d) Assertion is wrong statement but Reason is correct statement.

51. Assertion : The external dimensions of a wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of the wood is 5 mm, then the internal volume is 765 cm^3.

Reason : If external dimensions of a rectangular box be l, b and h and the thickness of its sides be x, then its internal volume is $(l - 2x)(b - 2x)(h - 2x)$.

52. Assertion : A cone is a solid figure.

Reason : A cone is generated when rectangular sheet is rotated about its axis.

53. Assertion : If curved surface area of a cone is 550 cm^2 and its diameter is 14 cm. Then, its slant height will be 20 cm.

Reason : Curved surface area of a cone having base radius r and slant height l is πrl.

54. Assertion : If the radius of a sphere is tripled, then the ratio of the volume of the original sphere to that of the new is 1 : 27.

Reason : Volume of a sphere with radius r is $4\pi r^3$.

55. Assertion : If a cone and a hemisphere have equal base and volume, then the ratio of their heights is 2 : 1.

Reason : Volume of a cylinder of height h and base radius r is $\pi r^2 h$.

SUBJECTIVE TYPE QUESTIONS

Very Short Answer Type Questions (VSA)

1. A metal sheet 27 cm long, 8 cm broad and 1 cm thick is melted and recast into a cube. Then find the volume of cube formed.

2. The dimensions of an open box are 52 cm, 40 cm and 29 cm. Its thickness is 2 cm. If 1 cm^3 of metal used in the box weighs 0.5 gms, then find the weight of the box.

3. The radius of a circular cylinder is same as that of a sphere. Their volumes are equal. Then find height of the cylinder.

4. A solid cube is cut into two cuboids of equal volumes. Find the total surface area of one of the cuboids.

5. If the radius of a sphere is doubled, what is the ratio of the volume of the first sphere to that of the second sphere.

6. The dimensions of a cinema hall are 120 m, 50 m, 30 m. Then find the volume of hall.

7. If a sphere is inscribed in a cube, find the ratio of the volume of cube to the volume of the sphere.

8. A teak wood log is in the form of a cuboid of volume 76800 m^3. How many rectangular planks of size 40 m × 12 m × 20 m can be cut from the cuboid?

9. The radii of the bases of a cylinder and a cone are in the ratio 3 : 4 and their heights are in the ratio 2 : 3. Then, find the ratio of their volumes.

10. Area of base of a solid hemisphere is 81π sq. cm. Then find its volume.

Short Answer Type Questions (SA-I)

11. Three cubes each of edge 5 cm are joined end to end. Find the surface area of the resulting cuboid.

12. Find the diameter of the sphere, whose total surface area is 616 cm^2.

13. A cube of 8 cm edge is immersed completely in a rectangular vessel containing water. If the dimensions of its base are 17 cm and 14 cm. Find the rise in water level in the vessel.

14. The length of a cold storage is three times its breadth. Its height is 5 m. The area of its four walls (including doors) is 256 m^2. Find its volume.

15. Two cones have their heights in the ratio 1 : 4 and radii of their bases in the ratio 4 : 1. Find the ratio of their volumes.

16. The radius of the circular part of a hemispherical bowl is 9 cm. Find the total capacity of 21 such bowls.

17. Find the length of cloth used for making a cone of height 2 times the base radius, which is square of 2, if the cloth is 100π m wide.

18. The dome of a building is in the form of a hemisphere. If its radius is 14 cm, find the cost of painting it at the rate of ₹ 3 per sq. cm.

19. The curved surface area of a cone is 154 cm^2. If its radius is x cm and slant height is 7 cm. Find the value of $20x$.

20. If the height of a cylinder is 11 cm and area of curved surface is 968 sq. cm. Find the radius of the cylinder.

Short Answer Type Questions (SA-II)

21. The base of a cubical box has a perimeter 250 m. Find the cost of painting its lateral surface area at the rate of ₹ 10 per m^2.

22. Find the total surface area, lateral surface area and the length of diagonal of a cube, each of whose edges measues 20 cm.

(Take $\sqrt{3} = 1.732$)

23. Three equal cubes are placed adjacently in a row. Find the ratio of the total surface area of the resulting cuboid to that of the sum of the surface areas of three cubes.

24. A room is 16 m long, 9 m wide and 3 m high. It has two doors, each of dimensions (2 m × 2.5 m) and three windows, each of dimensions (1.6 m × 75 cm). Find the cost of distempering the walls of the room from inside at the rate of ₹ 8 per sq. metre.

25. A cuboidal oil tin box is 4 m by 2 m by 0.75 m. Find the cost of the tin sheet required for making 20 such tin boxes, if the cost of tin sheet is ₹ 20 per square metre.

26. The external diameter of an iron pipe is 35 cm and its length is 30 cm. If the thickness of the pipe is 2.5 cm, find the curved surface area of the pipe.

27. A right triangle, with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm and 5 cm respectively. Find the ratio of the total surface areas of two cones so formed.

28. On a construction site, a deep pit is barricaded from the remaining portion by using 100 hollow cones. Each one has a base diameter 20 cm and height half a meter. What is the cost of painting the outer surface of all the cones, if cost of painting is ₹ 30 per m^2? [Use, $\pi = 3.14, \sqrt{26} = 5.1$]

29. Find the volume of a lead pipe 3.5 m long, if the external diameter of the pipe is 2.4 cm, thickness of lead is 3 mm and 1 cm^3 of lead weighs 12 g.

30. The total cost of making a solid spherical ball is ₹67914 at the rate of ₹ 14 per cubic metre. Find the radius of this ball.

31. The length and breadth of a rectangular solid are respectively 35 cm and 20 cm. If its volume is 7000 cm^3, then find its height (in cm).

32. If volume and surface area of a sphere is numerically equal then find its radius (in units).

33. How many spherical balls of diameter 1 cm can be made from an iron ball of diameter 6 cm?

34. A closed cuboidal tank can store 5040 litres of water. The external dimensions of the tank are 2.2 m × 1.7 m × 1.7 m. If the walls of the tank are 5 cm thick, then what is the thickness of the bottom (top) of the tank if they are same?

35. The slant height of a cone is 25 cm and the vertical height is 24 cm. Find the radius and the total surface area of the cone.

Long Answer Type Questions (LA)

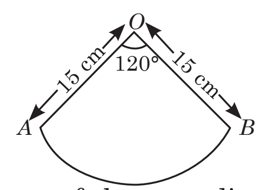
36. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost of painting one sq. cm of the surface is 7 paise. Find the total cost to paint the vessel all over. (Ignore the area of edge)

37. Coins of same size (say 10 rupee coin) are placed one above the other and a cylindrical block is obtained. The volume of this block is 67.76 cm^3. Find the number of coins arranged in the block, if thickness of each coin is 2 mm and radius of each coin is 1.4 cm.

38. A sector of a circle of radius 15 cm and central angle of 120°. It is rolled up and the two bounding radii are joined together to form a cone of radius 5 cm. Find :

(i) the volume of the cone.

(ii) the total surface area of the cone.



39. Three identical cylinders of base radius r units and height h units is placed one above the other to form a new cylinder. Find the ratio (in terms of r and h) of the curved surface area to the total surface area of the cylinder thus formed.

40. A plot of land is in the form of rectangle has dimension 240 m × 180 m. A drainlet 10 m wideis dug around it (on the outside) and the earth dug out is evenly spread out over the plot increasing its surface level by 25 cm. Find the depth of the drainlet.

ANSWERS

OBJECTIVE TYPE QUESTIONS

1. **(d) :** Side of cube (a) = 5 cm

∴ Surface area of cube = $6a^2 = 6(5)^2 = 6 \times 25 = 150$ cm^2

2. **(c) :** TSA of a cube = 216 cm^2 [Given]

⇒ $6 \times (\text{Side})^2 = 216 \Rightarrow (\text{Side})^2 = 36$

⇒ $(\text{Side})^2 = 6^2 \Rightarrow \text{Side} = 6$ cm

3. **(b) :** TSA of cuboid with dimensions x units, y units and z units is $2\{xy + yz + zx\}$ sq. units

4. **(a) :** The lateral surface area of a cube of side a units is $4a^2$ sq. units.

5. **(d) :** 3 such pairs exist.

6. **(a) :** Lateral surface area of cube with side 6 units = 144 sq. units

7. **(b) :** Total surface area of a cuboid = $2(lb + bh + lh)$

$= 2[(12 \times 9) + (9 \times 8) + (12 \times 8)]$

$= 2(108 + 72 + 96) = (2 \times 276) = 552$ cm^2

8. **(a) :** If two cubes of side 1 cm are joined, then dimensions of the resulting cuboid are

$l = 2$ cm, $b = 1$ cm, $h = 1$ cm

∴ Total surface area $= 2(lb + bh + hl)$

$= 2(2 \times 1 + 1 \times 1 + 1 \times 2)$

$= 2(2 + 1 + 2)$ cm^2

9. **(b) :** Let l, b and h be the length, breadth and height of the original cuboid respectively.

$\therefore$ $2(lb + bh + hl) = S$ ...(i)

According to question,

New length = $2l$, new breadth = $2b$ and new height = $2h$

$\therefore$ Total surface area of resulting cuboid

$= 2(4lb + 4bh + 4lh) = 2 \times 4(lb + bh + lh) = 4S$

[Using (i)]

10. **(d) :** If six cubes of side 2 cm are joined, then dimensions of the resulting cuboid are

$l = 2 + 2 + 2 + 2 + 2 + 2 = 12$ cm,

$b = 2$ cm and $h = 2$ cm

$\therefore$ Total surface area of resulting cuboid

$= 2(lb + bh + hl)$

$= 2[(12 \times 2) + (2 \times 2) + (2 \times 12)] = 104\ \text{cm}^2$

11. **(c) :** Let S_1 and S_2 be the surface areas of the original cube and the new cube respectively. Let a be the side of the original cube.

$\therefore$ $S_1 = 6a^2$

Now, side of the new cube = a + 50% of a = $\left(a + \frac{a}{2}\right) = \frac{3}{2}a$

$\therefore$ $S_2 = 6\left(\frac{3}{2}a\right)^2 = \frac{9}{4}(6a^2)$

$\therefore$ Percentage increase in surface area

$$= \frac{\text{Chagne in surface area}}{\text{Original surface area}} \times 100\% = \frac{S_2 - S_1}{S_1} \times 100\%$$

$$= \frac{\frac{9}{4}(6a^2) - (6a^2)}{(6a^2)} \times 100\% = 125\%$$

12. **(b) :** Given, height of cylinder, $h = 15$ cm

C.S.A. of cylinders

$= \frac{2}{3}$ (Sum of surface area of circular faces)

$\Rightarrow$ $2\pi rh = \frac{2}{3}(2\pi r^2) \Rightarrow 15 = \frac{2}{3}r$

$\Rightarrow$ $\frac{45}{2} = r \Rightarrow r = 22.5$ cm.

13. **(a) :** Let h and r be the height and radius of the cylinder respectively. Then,

Total surface area = $2\pi r(h + r) = 550$ cm (i)

Circumference of base = $2\pi r = 50$ cm (ii)

Dividing (i) by (ii), we get

$\frac{2\pi r(r+h)}{2\pi r} = \frac{550}{50} \Rightarrow (h + r) = 11$ cm

14. **(d) :** Let r and h be the radius and height of the cylinder.

$\therefore$ Original lateral surface area = $2\pi rh$

New height = $6h$

New base area = $\frac{1}{9}\pi r^2 = \pi\left(\frac{r}{3}\right)^2 \Rightarrow$ New radius = $\frac{r}{3}$

Now, new lateral surface area = $2\pi \times \frac{r}{3} \times 6h = 4\pi rh$

$\therefore$ Required factor by which lateral surface area increases = $\frac{4\pi rh}{2\pi rh} = 2$

15. **(d) :** Let R be the radius of base of the cone.

$\therefore$ Curved surface area of cone, $\pi R\left(\frac{x}{2}\right) = 2\pi x$

$\Rightarrow$ $R = 4$

$\therefore$ Area of its base = $\pi R^2 = \pi(4)^2 = 16\pi$ sq. units

16. **(d) :** Let r be the radius of two cones and $5x$ and $4x$ be their slant heights.

$\therefore$ The ratio of their curved surface areas

$= \frac{\pi r(5x)}{\pi r(4x)} = \frac{5}{4}$ or $5:4$

17. **(c) :** Total surface area of cone = $\pi R(L + R)$, where R and L are radius and slant height respectively.

$= \pi(2r)\left(\frac{l}{2} + 2r\right) = \pi r(l + 4r)$

18. **(b) :** Inner curved surface area of hemi-spherical container = $2\pi r_1^2$

Outer curved surface area of hemi-spherical container = $2\pi r_2^2$

$\therefore$ Total curved surface area = $2\pi r_1^2 + 2\pi r_2^2 = 2\pi(r_1^2 + r_2^2)$

19. **(d) :** Let r_1 and r_2 be the radii and S_1 and S_2 be the surface areas of two spheres respectively.

$\therefore$ $\frac{r_1}{r_2} = \frac{4}{5}$ Now, $\frac{S_1}{S_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

20. **(a) :** Let S_1 and S_2 be the surface areas of the spherical balloons with radii 7 cm and 14 cm respectively.

$\therefore$ $S_1 = 4\pi(7)^2 = 196\pi\ \text{cm}^2$ and

$S_2 = 4\pi(14)^2 = 784\pi\ \text{cm}^2$

$\therefore$ $S_1 : S_2 = 196\pi : 784\pi = 196 : 784 = 1 : 4$

21. **(a) :** Volume of water in tank = $\frac{2}{3}$ of volume of tank

$= \frac{2}{3} \times (6 \times 5 \times 4) = 80\ \text{m}^3$

22. **(c) :** Let $A = lb$, $B = bh$ and $C = hl$ …(i)

[Where l, b and h be the length, breadth and height of cuboid respectively.]

Volume of cuboid = lbh

$= \sqrt{(lbh)^2} = \sqrt{lb \times bh \times hl} = \sqrt{ABC}$ [Using (i)]

23. **(c) :** Let l, b and h be the length, breadth and height of the block respectively.

$\therefore$ Volume of the block = lbh

$= 9000\ \text{cm}^3$ [Given] …(i)

Area of their adjacent faces are

$lb = 2x$ ...(ii)

$bh = 3x$ ...(iii)

$lh = 4x$ ...(iv)

Multiplying (ii), (iii) and (iv), we get

$l^2b^2h^2 = (2x)(3x)(4x) = 24x^3$

$\Rightarrow (lbh)^2 = 24x^3$

$\Rightarrow (9000)^2 = 24x^3$ [Using (i)]

$\Rightarrow x^3 = \frac{9000 \times 9000}{24} = 3375000 \Rightarrow x^3 = 150^3 \Rightarrow x = 150$

$\therefore$ From (ii), $lb = 300$

$\Rightarrow \frac{lbh}{h} = 300 \Rightarrow h = \frac{9000}{300}$ [Using (i)]

$\Rightarrow h = 30$ cm

Similarly, from (iii) and (iv), we get

$l = 20$ cm and $b = 15$ cm

$\therefore$ The length of the shortest edge of the rectangular block is 15 cm.

24. **(d) :** Let r_1, h_1, V_1 and r_2, h_2, V_2 be the radii, heights and volumes of two cylinders respectively.

$\therefore \frac{h_1}{h_2} = \frac{5}{3}$ and $\frac{V_1}{V_2} = \frac{20}{27}$ [Given]

Now, $\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{r_1^2 h_1}{r_2^2 h_2} \Rightarrow \frac{20}{27} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{5}{3}$

$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{20}{27} \times \frac{3}{5} = \frac{4}{9} = \left(\frac{2}{3}\right)^2 \Rightarrow \frac{r_1}{r_2} = \frac{2}{3}$ or 2 : 3

25. **(c) :** Let r and h be the radius and height of the original cylinder respectively.

$\therefore$ New height = $3h$

New base area = $\frac{1}{9}\pi r^2 = \pi\left(\frac{r}{3}\right)^2 \Rightarrow$ New radius = $\frac{r}{3}$

Now, volume of original cylinder, $V_1 = \pi r^2 h$

Volume of new cylinder, $V_2 = \pi\left(\frac{r}{3}\right)^2 \times 3h = \frac{\pi r^2 h}{3}$

$\therefore \frac{V_1}{V_2} = \frac{\pi r^2 h}{\frac{\pi r^2 h}{3}} = \frac{3}{1}$ or $3:1$

26. **(a) :** Let h be the height of the cylinder.

Now, curved surface area = $2\pi rh = 1520 \text{ cm}^2$ [Given]

$\Rightarrow \pi(15)h = 760$ [$\because$ Diameter = 30 cm (Given)]

$\Rightarrow h = \frac{760}{15\pi}$

$\therefore$ Volume of cylinder $= \pi r^2 h = \pi(15)^2 \times \frac{760}{15\pi}$

$= 11400 \text{ cm}^3$

27. **(c) :** Let r_1 and h_1 be the radius and length of original wire respectively. Also, r_2 and h_2 be the radius and length of new wire respectively.

$\therefore r_2 = \frac{r_1}{3} \Rightarrow r_1 = 3r_2$

Now, $\pi r_1^2 h_1 = \pi r_2^2 h_2 \Rightarrow (3r_2)^2 h_1 = r_2^2 h_2$

$\Rightarrow 9r_2^2 h_1 = r_2^2 h_2 \Rightarrow 9h_1 = h_2$

$\therefore$ The length of wire increases by 9 times.

28. **(c) :** Let V_1 and V_2 be the volumes of original and new cones respectively. r and h are radius and height of original cone.

$\therefore V_1 = \frac{1}{3}\pi r^2 h, V_2 = \frac{1}{3}\pi\left(r + \frac{10r}{100}\right)^2\left(h + \frac{10h}{100}\right)$

$= \frac{1331}{1000}\left(\frac{1}{3}\pi r^2 h\right)$

$\therefore$ Required increased percentage in volume

$= \left(\frac{\text{New volume} - \text{Original volume}}{\text{Original volume}}\right) \times 100\%$

$= \frac{V_2 - V_1}{V_1} \times 100\% = \left(\frac{1331}{1000} - 1\right) \times 100\%$

$= 33.1\% = 33\%$ (approx.)

29. **(b) :** Let r and l be the radius and slant height of the cone.

$\therefore S = \pi rl = \pi r\sqrt{r^2 + h^2}$ and $V = \frac{1}{3}\pi r^2 h$

Now, $3\pi Vh^3 - S^2h^2 + 9V^2$

$= 3\pi\left(\frac{1}{3}\pi r^2 h\right)h^3 - (\pi r\sqrt{r^2 + h^2})^2 h^2 + 9\left(\frac{1}{3}\pi r^2 h\right)^2$

$= \pi^2 r^2 h^4 - \pi^2 r^2 h^2 (r^2 + h^2) + \pi^2 r^4 h^2$

$= \pi^2 r^2 h^4 - \pi^2 r^4 h^2 - \pi^2 r^2 h^4 + \pi^2 r^4 h^2 = 0$

30. **(b) :** Let $2d$ and V be the diameter and volume of the sphere respectively.

$\therefore$ Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(d)^3$

New diameter = ($2d$ – 25% of $2d$) = $\left(2d - \frac{25}{100} \times 2d\right)$

$= \frac{3d}{2}$

$\therefore$ New radius = $\frac{3d}{4}$

Now, new volume = $\frac{4}{3}\pi\left(\frac{3d}{4}\right)^3 = \frac{9d^3}{16}\pi$

$\therefore$ Required percentage decrease

$= \left(\frac{\text{Original volume} - \text{New volume}}{\text{Original volume}}\right) \times 100\%$

$= \left(\frac{\frac{4}{3}\pi d^3 - \frac{9}{16}\pi d^3}{\frac{4}{3}\pi d^3}\right) \times 100\% = \left(\frac{\frac{4}{3} - \frac{9}{16}}{\frac{4}{3}}\right) \times 100\% = 57.81\%$

31. **(a) :** Let r be the radius of cone, hemisphere and cylinder.

Height of hemisphere = Radius of hemisphere = r

$\therefore$ Height of cylinder, h = Height of hemisphere = r
Similarly, height of cone = r

Now, volume of cone = $\frac{1}{3}\pi r^2(r) = \frac{1}{3}\pi r^3$

Volume of hemisphere = $\frac{2}{3}\pi r^3$

Volume of cylinder = $\pi r^2(r) = \pi r^3$

$\therefore$ Required ratio = $\frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 : \pi r^3 = \frac{1}{3} : \frac{2}{3} : 1$
= 1 : 2 : 3.

32. (b) : Volume of the hemisphere = 19404 cm^3

$\Rightarrow \frac{2}{3}\pi r^3 = 19404 \Rightarrow r^3 = \frac{3 \times 19404 \times 7}{22 \times 2}$

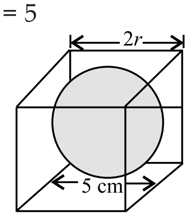
$\Rightarrow r^3 = (21)^3 \Rightarrow r = 21$ cm

$\therefore$ Total surface area = $3\pi r^2$

$= 3 \times \frac{22}{7} \times (21)^2 = 4158$ sq. cm.

33. (d) : Let r be the radius of the largest sphere that can be cut off from the cube.
Then, diameter of sphere = 5 cm $\Rightarrow 2r = 5$

$\Rightarrow r = \frac{5}{2}$ cm



$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times \frac{5 \times 5 \times 5}{2 \times 2 \times 2} = \frac{125\pi}{6}$ cm^3

34. (c) : Let h be the height of displaced water.
Radius of both cylindrical vessel and sphere is $\frac{60}{2} = 30$ cm

According to question,
Volume of displaced water = Volume of sphere

$\Rightarrow \pi(30)^2 h = \frac{4}{3}\pi(30)^3 \Rightarrow h = \frac{4 \times 30}{3} = 40$ cm

35. (b) : Let 'a' be the side of cube.
Length of diagonal = $\sqrt{3}a = 14 \times \sqrt{3} \Rightarrow a = 14$ cm.

$\therefore$ Volume of cube = (14 × 14 × 14) cm^3 = 2744 cm^3

36. (a) : Volume of cylinder = $\pi r^2 h = \pi(6)^2 \times 8$
= 288 π cm^3

37. (c) : Volume of sphere = $\frac{4}{3}\pi r^3$

38. (d) : Its volume will not change.

39. (d) : Let R be the radius of the sphere.
Since volume of sphere = volume of cylinder

$\Rightarrow \frac{4}{3}\pi R^3 = \pi r^2 h$

$\Rightarrow R^3 = \frac{3}{4}(6)^2 \times 8 = 6^3 \Rightarrow R = 6$ cm

40. (a) : Volume of sphere = volume of cylinder
= 288π cm^3

41. (b) : Curved surface area of cylinder formed
= Area of plastic rectangular sheet
= (44 × 15) cm^2 = 660 cm^2

42. (c) : Circumference of base
= Length of rectangular sheet

$\Rightarrow 2\pi r = 44$ cm $\Rightarrow r = 44 \times \frac{7}{22} \times \frac{1}{2} = 7$ cm

43. (a) : Required area = πr^2

$= \left(\frac{22}{7} \times 7 \times 7\right)$ cm^2 = 154 cm^2

44. (d) : Area of square sheet = (15 × 15) cm^2
= 225 cm^2

$\therefore$ Area of square sheet left unused
= Area of square – Area of base of cylinder
= (225 – 154) cm^2 = 71 cm^2

45. (a) : Volume of seeds that can be filled in the cylinder = Volume of cylinder = $\pi r^2 h$

$= \left(\frac{22}{7} \times 7 \times 7 \times 15\right)$ cm^3 = 2310 cm^3

46. (b) : Volume of ball = Increased volume of water

$\Rightarrow \frac{4}{3}\pi r^3 = \pi(7)^2 \times 3.4$

$\Rightarrow r^3 = \frac{7 \times 7 \times 3.4 \times 3}{4} = 124.95 \Rightarrow r \approx 5$ cm.

47. (d) : Volume of cylinder = $\pi R^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 10 = 1540$ cm^3

48. (c) : Volume of spherical ball = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 5 \times 5 \times 5 = \frac{11000}{21} = 523.81$ cm^3

49. (a) : Volume of cylinder = 1540 cm^3

$= \frac{1540}{1000}$ litres [$\because 1l = 1000$ cm^3]

= 1.54 litres

50. (b) : Total surface area of spherical ball = $4\pi r^2$

$= 4 \times \frac{22}{7} \times 5 \times 5 = 314.29$ cm^2

51. (a) : Length of box = 18 cm
Width of box = 10 cm
Height of box = 6 cm
Thickness of box = 5 mm $= \frac{1}{2}$ cm
Internal length, width, height of the box is

$\left(18 - \frac{2 \times 1}{2}\right), \left(10 - \frac{2 \times 1}{2}\right), \left(6 - \frac{2 \times 1}{2}\right)$

$\therefore$ Internal volume of box = 17 × 9 × 5 = 765 cm^3
Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

52. (c) : Clearly, Assertion is correct. But when a rectangular sheet is rotated about its axis, then a cylinder is generated.

∴ Assertion is correct but Reason is wrong.

53. (d) : Reason is clearly correct.

Curved surface area of cone = 550 cm^2

$\Rightarrow \pi rl = 550 \Rightarrow \frac{22}{7}\times 7\times l = 550$ [∴ d = 14 cm (Given)]

$\Rightarrow l = \frac{550}{22} = 25$ cm

So, slant height of the cone is 25 cm.

∴ Assertion is wrong but Reason is correct.

54. (c) : Let r be the radius of original sphere.

∴ Radius of new sphere = $3r$

∴ Required ratio $= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(3r)^3} = \frac{1}{27}$ or 1 : 27.

Here, Reason is clearly wrong.

∴ Assertion is correct but Reason is wrong.

55. (b) : Let r be the radius of base of both cone and hemisphere and h be the height of cone.

∵ Volume of cone = Volume of hemisphere

∴ $\frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3 \Rightarrow \frac{h}{r} = \frac{2}{1}$ or 2 : 1

∴ Both Assertion and Reason are correct but Reason is not the correct explanation of Assertion.

SUBJECTIVE TYPE QUESTIONS

1. Volume of sheet = (27 × 8 × 1) cm^3

= 216 cm^3

Volume of cube formed = 216 cm^3

2. Volume of metal

= (52 × 40 × 29 – 48 × 36 × 27) cm^3 = 13664 cm^3

∴ Weight of the box $= \left(\frac{13664\times 0.5}{1000}\right)$kg = 6.832 kg

3. Let h be the height of cylinder.

Radius of cylinder = Radius of sphere = r

Then, we have $\frac{4}{3}\pi r^3 = \pi r^2 h \Rightarrow h = \frac{4}{3}r$

∴ Height of cylinder $= \frac{4}{3}$ times its radius.

4. Let edge of the solid cube be a cm.

Then, dimensions of each of the cuboids will be a cm, a cm and $\frac{a}{2}$ cm.

∴ Total surface area of one of the cuboids

$= 2\left(a\times a + a\times\frac{a}{2} + \frac{a}{2}\times a\right) = 2\left(a^2 + \frac{a^2}{2} + \frac{a^2}{2}\right) = 4a^2$ cm^2

5. Let the old radius be r.

Then, new radius = $2r$

Volume of old sphere $= \frac{4}{3}\pi r^3$

Volume of new sphere $= \frac{4}{3}\pi(2r)^3$

$= \frac{4}{3}\pi\times 8r^3 = \frac{32}{3}\pi r^3$

∴ Required ratio $= \frac{4}{3}\pi r^3 : \frac{32}{3}\pi r^3 = 1:8$

6. Length(l) = 120 m, Breadth(b) = 50 m, Height (h) = 30 m

Since the shape of cinema hall is of cuboid

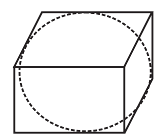
∴ Volume of hall = $l \times b \times h$

= 120 × 50 × 30 m^3 = 180000 m^3

7. If sphere is completely inscribed in a cube then edge of cube is the diameter of sphere.

Let a be the edge of cube.

So a is the diameter of sphere



⇒ Volume of cube = (a^3)

Volume of sphere $= \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$

∴ Required ratio $= \frac{a^3}{\frac{4}{3}\pi\left(\frac{a^3}{8}\right)} = a^3 : \frac{\pi}{6}a^3 = 6:\pi$

8. Number of rectangular planks

$= \frac{\text{Volume of cuboid of wood log}}{\text{Volume of one plank}} = \frac{76800}{40\times 12\times 20} = 8$

9. Let the radii of the bases of a cylinder and a cone be $3x$ and $4x$ respectively and let their heights be $2y$ and $3y$ respectively.

∴ Ratio of their volumes $= \frac{\pi\times(3x)^2\times 2y}{\frac{1}{3}\pi\times(4x)^2\times 3y} = \frac{9}{8}$ or 9 : 8.

10. Let r be the radius of the hemisphere.

∴ $\pi r^2 = 81\pi \Rightarrow r^2 = 81 \Rightarrow r = 9$ cm.

Volume of hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(9)^3 = 486\pi$ cm^3

11. When three cubes are joined end to end, we get a cuboid such that

Length of the resulting cuboid, l

= 5 cm + 5 cm + 5 cm = 15 cm

Breadth of resulting cuboid, b = 5 cm

Height of the resulting cuboid, h = 5 cm

Surface area of the cuboid = 2 ($lb + bh + hl$)

= 2 (15 × 5 + 5 × 5 + 5 × 15) cm^2

= 2 (75 + 25 + 75) cm^2 = 350 cm^2

12. Let r be the radius of the sphere.
Total surface area of sphere = $4\pi r^2$

$\Rightarrow \; 616 = 4 \times \frac{22}{7} \times r^2 \Rightarrow r^2 = \frac{616 \times 7}{4 \times 22} = 49 \Rightarrow r = 7$ cm

$\therefore$ Diameter = $2r = 2 \times 7 = 14$ cm

13. Edge of the cube = 8 cm

$\therefore$ Volume of the cube = $a^3 = (8)^3 = 512 \text{ cm}^3$

If the cube is immersed in the vessel, then the water level rises. Let the rise in water level be x.

Clearly, Volume of the displaced water = Volume of the cube

$\Rightarrow$ Volume of the cube = 17 cm × 14 cm × x cm

$\Rightarrow$ $512 = 17 \times 14 \times x$

$\Rightarrow \; x = \frac{512}{17 \times 14} \Rightarrow x = \frac{512}{238} = 2.15$ cm

14. Let length, breadth and height of the cold storage be l, b and h respectively.

Then, $l = 3b$ and $h = 5$ m.

Now, area of four walls = 256 m^2

$\Rightarrow$ $2(l + b)h = 256 \Rightarrow 2(3b + b) \times 5 = 256$

$\Rightarrow$ $40b = 256 \Rightarrow b = 6.4$ metres

$\therefore$ $l = 3b = 3 \times 6.4 = 19.2$ m

Volume of the cold storage = $l \times b \times h$

$= (19.2 \times 6.4 \times 5) \text{ m}^3 = 614.4 \text{ m}^3$

15. Let the heights of the cones be h and $4h$ and radii of their bases be $4r$ and r and V_1 and V_2 are their respective volumes.

$\therefore \; V_1 = \frac{1}{3}\pi(4r)^2 h = \frac{16}{3}\pi r^2 h, \; V_2 = \frac{1}{3}\pi(r)^2 4h = \frac{4}{3}\pi r^2 h$

$\Rightarrow \; \frac{V_1}{V_2} = \frac{\frac{16}{3}\pi r^2 h}{\frac{4}{3}\pi r^2 h} = \frac{4}{1} \Rightarrow V_1 : V_2 = 4 : 1$

16. Volume of one bowl = Volume of hemisphere

$= \frac{2}{3}\pi r^3 = \left(\frac{2}{3} \times \frac{22}{7} \times 9^3\right) \text{cm}^3$

Capacity of 21 such bowls $= 21 \times \frac{2}{3} \times \frac{22}{7} \times 9^3$

$= 32076 \text{ cm}^3$

17. Let r and h be the radius and height of cone respectively.

$\Rightarrow$ $h = 2r$ and $r = 4$ m $\Rightarrow h = 8$ m

$l = \sqrt{r^2 + h^2} = \sqrt{(4)^2 + (8)^2} = \sqrt{16 + 64} = \sqrt{80}$ cm

Lateral or curved surface area = πrl

$= \pi \times 4 \times \sqrt{80} = 16\sqrt{5}\,\pi \text{ m}^2$

$\therefore$ Length of cloth $= \frac{16\sqrt{5}\,\pi}{100\,\pi}$ m = 0.36 m = 36 cm

18. Since the dome of building is in the shape of hemisphere

$\Rightarrow$ Curved surface area of dome = $2\pi r^2$

$= 2 \times \frac{22}{7} \times 14 \times 14 = 2 \times 22 \times 2 \times 14 = 1232 \text{ cm}^2$

Cost of painting the dome at the rate of 1 sq. cm = ₹ 3

$\therefore$ Cost of painting of 1232 cm^2 dome

= ₹ 3 × 1232 = ₹ 3696

19. We have, curved surface area = 154 cm^2

$\Rightarrow \; \pi rl = 154 \Rightarrow r = \frac{154 \times 7}{22 \times 7} = 7$

Now, $r = x$ cm = 7 cm.

$\therefore$ $x = 7 \Rightarrow 20x = 20 \times 7 = 140$

20. Height of cylinder = 11 cm

Curved surface area = 968 cm^2

$\Rightarrow$ $2\pi rh = 968$

$\Rightarrow \; 2 \times \frac{22}{7} \times \pi \times 11 = 968 \Rightarrow r = 14$ cm

21. Let the side of cubical box be a m.

Perimeter of base of a cubical box = $4a$ m

$\Rightarrow$ $250 = 4a \Rightarrow a = 62.5$ m

Now, LSA of cubical box = $4a^2 = 4 \times (62.5)^2 = 15625 \text{ m}^2$

Cost of painting 1 m^2 = ₹ 10

$\therefore$ Total cost of painting = 15625 × 10 = ₹ 156250.

22. Here, side (a) = 20 cm

$\therefore$ Total surface area of the cube = $6a^2 = 6(20)^2$

$= 2400 \text{ cm}^2$

and, lateral surface area of the cube = $4a^2 = 4(20)^2$

$= 1600 \text{ cm}^2$

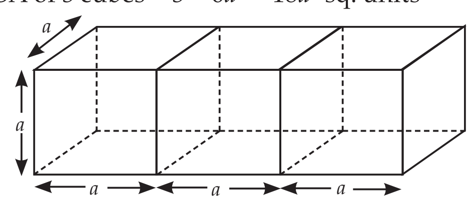
Also, length of diagonal of a cube $= \sqrt{3}a$

$= \sqrt{3} \times 20 = (1.732 \times 20) = 34.64$ cm.

23. Let the side of each cube be a units.

$\therefore$ TSA of 1 cube = $6a^2$ sq. units

$\Rightarrow$ TSA of 3 cubes = $3 \times 6a^2 = 18a^2$ sq. units



Now length, breadth and height of the resulting cuboid is $3a$ units, a units and a units respectively.

$\therefore$ TSA of cuboid = $2(lb + bh + hl)$

$= 2(3a \times a + a \times a + a \times 3a)$

$= 2(3a^2 + a^2 + 3a^2) = 2 \times 7a^2 = 14a^2$ sq. units.

So, required ratio $= \frac{14a^2}{18a^2} = \frac{7}{9}$ or $7 : 9$

24. Given, length (l) = 16 m, breadth (b) = 9 m and height (h) = 3 m

$\because$ Area of 4 walls of the room = $2(l + b) \times h$

$= 2(16 + 9) \times 3 = 150 \text{ m}^2$.

Area of 2 doors = $2 \times (2 \times 2.5) = 10 \text{ m}^2$

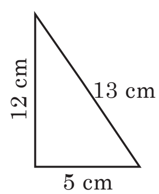
Area of 3 windows $= 3 \times \left(1.6 \times \frac{75}{100}\right) = 3.6 \text{ m}^2$.

Area not to be distempered = 10 + 3.6 = 13.6 m^2
Area to be distempered = 150 – 13.6 = 136.4 m^2
Cost of distempering the walls = ₹ (136.4 × 8) = ₹ 1091.20

25. Length, l = 4 m, breadth, b = 2 m and height, h = 0.75 m
Surface area of one tin box = $2(lb + bh + hl)$
= 2 (4 × 2 + 2 × 0.75 + 0.75 × 4)
= 2 (8 + 1.5 + 3) = 2 × 12.5 = 25 m^2
∴ Surface area of 20 such tin boxes = (20 × 25) m^2 = 500 m^2
Now, cost of 1 square metre of tin sheet = ₹ 20
∴ Cost of 500 m^2 of tin sheet = ₹(20 × 500) = ₹ 10000

26. Length of the pipe, h = 30 cm
External radius of the pipe, $R = \frac{35}{2}$ cm = 17.5 cm
∵ Thickness of the pipe = 2.5 cm
∴ Internal radius of the pipe, r = (17.5 – 2.5) cm = 15 cm
Now, curved surface area of the pipe = External curved surface area + Internal curved surface area
$= 2\pi Rh + 2\pi rh = 2\pi h\,(R + r)$
$= 2 \times \frac{22}{7} \times 30(17.5 + 15) = 2 \times \frac{22}{7} \times 30 \times 32.5$
$= \frac{42900}{7} = 6128.57 \text{ cm}^2$

27. Let S_1 and S_2 be the total surface areas of two cones so formed by revolving about side 12 cm and 5 cm respectively.



Total surface area of a cone = $\pi r(l + r)$
∴ $S_1 = \pi(5)\,(13 + 5) = 90\pi \text{ cm}^2$
$S_2 = \pi(12)\,(12 + 13) = 300\pi \text{ cm}^2$
∴ Required ratio $= \frac{S_1}{S_2} = \frac{90\pi}{300\pi} = 3:10$

28. Radius of cone, $r = \frac{20}{2}$ = 10 cm = 0.1 m
Height of cone, $h = \frac{1}{2}$ m
Slant height of cone, $l = \sqrt{h^2 + r^2}$
$\Rightarrow \quad l = \sqrt{\left(\frac{1}{2}\right)^2 + (0.1)^2} \;\Rightarrow\; l = \sqrt{\frac{26}{100}} = \frac{5.1}{10} = 0.51 \text{ m}$
Curved surface area of 100 cones = $(100 \times \pi rl)\text{m}^2$
= $(100 \times 3.14 \times 0.1 \times 0.51)\text{m}^2 = 16.014 \text{ m}^2$
Total cost of painting = 30 × 16.014 = ₹ 480.42

29. External diameter of the pipe = 2.4 cm
External radius of the pipe, (R) = $\frac{2.4}{2}$ cm = 1.2 cm
Thickness of the pipe = 3 mm = 0.3 cm
Internal radius, (r) = External radius – thickness
= 1.2 cm – 0.3 cm = 0.9 cm
Length of the pipe (h) = 3.5 m = 350 cm
Volume of lead = $\pi(R^2 - r^2)h$
$= \frac{22}{7} \times [(1.2)^2 - (0.9)^2] \times 350 = \frac{22}{7} \times 0.63 \times 350 = 693 \text{ cm}^3$

30. Volume of spherical ball = $\frac{\text{Total cost}}{\text{Cost of 1 m}^3}$
$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{67914}{14} \;\Rightarrow\; \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{67914}{14}$
$\Rightarrow \quad r^3 = \frac{101871}{88} \Rightarrow r^3 = 1157.625 \Rightarrow r = 10.5 \text{ m}$

31. Let h be the height of the solid.
∴ Volume of cuboid = $l \times b \times h$
$\Rightarrow \quad 7000 = 35 \times 20 \times h \Rightarrow h = \frac{7000}{700} = 10 \text{ cm}$

32. Let r be the radius of the sphere.
∵ Volume of sphere = Surface area of sphere
∴ $\frac{4}{3}\pi r^3 = 4\pi r^2 \;\Rightarrow\; \frac{r^3}{r^2} = 3 \;\Rightarrow\; r = 3$ units

33. Volume of iron ball having diameter, 6 cm
$= \frac{4}{3}\pi\left(\frac{6}{2}\right)^3 = \frac{4}{3}\pi(3)^3$
Volume of small ball of diameter, 1 cm $= \frac{4}{3}\pi\left(\frac{1}{2}\right)^3$
∴ Number of balls $= \dfrac{\frac{4}{3}\pi(3)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\right)^3} = 27 \times 8 = 216$

34. Capacity of tank = 5040 litres
$= \frac{5040}{1000} \text{ m}^3 = 5.040 \text{ m}^3$
Internal length of tank = $\left(2.2 - 2 \times \frac{5}{100}\right)$m = 2.1 m
Internal breadth of tank = $\left(1.7 - 2 \times \frac{5}{100}\right)$m = 1.6 m
Let the thickness of tank in the bottom be x m.
∴ Internal height of tank = $(1.7 - 2x)$ m
Now, Internal volume of tank = Capacity of tank
$\Rightarrow \quad 2.1 \times 1.6 \times (1.7 - 2x) = 5.040$
$\Rightarrow \quad 1.7 - 2x = \frac{5.040}{2.1 \times 1.6} = 1.5$
$\Rightarrow \quad 2x = 1.7 - 1.5 = 0.2$
$\Rightarrow \quad x = 0.1$ m = 0.1 × 100 cm = 10 cm
So, required thickness = 10 cm

35. Here, h = 24 cm and slant height (l) = 25 cm
Let r be the radius of cone. Then,
$l^2 = h^2 + r^2$
$\Rightarrow \quad r^2 = 25^2 - 24^2 = 49 \Rightarrow r = 7$
Total surface area of the cone = $\pi r(l + r)$
$= \left[\frac{22}{7} \times 7 \times (25 + 7)\right]\text{cm}^2 = 704 \text{ cm}^2.$

36. Let R cm and r cm be respectively the external and internal radii of the hemispherical vessel.
Then, $R = 12.5$ cm, $r = 12$ cm.
Now, External surface area of the vessel

$= 2\pi R^2 = 2 \times \frac{22}{7} \times (12.5)^2$

Internal surface area of the vessel $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times (12)^2$

$\therefore$ Total area to be painted

$= 2 \times \frac{22}{7} \times (12.5)^2 + 2 \times \frac{22}{7} \times 12^2$

$= 2 \times \frac{22}{7} \times \left\{\left(\frac{25}{2}\right)^2 + 12^2\right\}$

$= 2 \times \frac{22}{7} \times \left(\frac{625}{4} + 144\right) = \frac{13211}{7} \text{cm}^2$

Cost of painting at the rate of 7 paise per sq. cm

$= ₹ \left(\frac{13211}{7} \times \frac{7}{100}\right) = ₹ 132.11$

37. Let h be the height of cylindrical block and n be the number of coins used to obtain it.

Volume of block $= \pi r^2 h \Rightarrow 67.76 = \frac{22}{7} \times 1.4 \times 1.4 \times h$

$\Rightarrow \quad h = \frac{67.76 \times 7}{22 \times 1.4 \times 1.4} = 11 \text{ cm} = 110 \text{ mm}$ [$\because$ 1 cm = 10 mm]

Now, $n \times$ thickness of a coin = height of block

$\Rightarrow \quad n \times 2 = 110 \Rightarrow n = 55$

38. (i) The slant height of the cone = Radius of the given sector of a circle = 15 cm.
Now, let h be the height of the cone. Then,

$h = \sqrt{l^2 - r^2}$ [where l = slant height, r = radius of cone]

$= \sqrt{(15)^2 - (5)^2} = \sqrt{225 - 25} = \sqrt{200} = 10\sqrt{2}$ cm

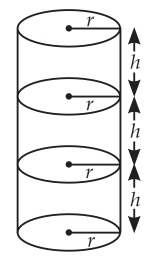
$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(5)^2 \times 10\sqrt{2} = \frac{1}{3} \times \frac{22}{7} \times 25 \times 10 \times \sqrt{2} = 369.29$ cm

(ii) Total surface area of the cone $= \pi r(r + l)$

$= \frac{22}{7} \times 5(5 + 15) = \frac{22}{7} \times 5 \times 20 = 314.29 \text{ cm}^2$

39. Since, three identical cylinders of radius r units and height h units are place one above the other.



$\therefore$ Height of cylinder thus formed
$= 3 \times h = 3h$ units

And radius of new cylinder = r units

$\therefore$ CSA of new cylinder $= 2\pi r(3h)$
$= 6\pi rh$ sq. units

And TSA of new cylinder $= 2\pi r(r + 3h)$ sq. units

Now, required ratio $= \frac{6\pi rh}{2\pi r(r + 3h)} = \frac{3h}{r + 3h}$

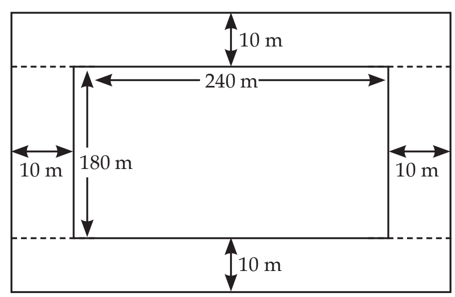
or $3h : (r + 3h)$.

40. Volume of earth dug out $= l \times b \times h$

$= 240 \times 180 \times \frac{25}{100} \text{ m}^3 = 10800 \text{ m}^3$

Let the depth of the drainlet be x m.

$\therefore$ Volume of drainlet $= 2[260 \times 10 \times x] + 2[180 \times 10 \times x]$
$= 8800\, x \text{ m}^3$



Now, volume of earth dug out = Volume of drainlet

$\Rightarrow \quad 10800 = 8800x \Rightarrow x = \frac{10800}{8800} = 1.23$ m (approx.)

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