1.5 CHAPTER Ratio, Proportion & Variation

Ratio

Ratio is the relationship between the quantities of same kind. In ratio the quantities are compared as the multiple or parts of other quantities. If ratio is a : b then a is called antecedent and b is called consequent. ratio of the quantities is expressed after removing the common factor between the quantities.

Generally, the ratio is useful in comparision. If the quantities A & B are compared & their ratio comes out as p : q

We can say A/B = p/q

or A & B can be expressed as pK, qK respectively. This provides a scope of comparison in terms of multiples of p & q.

In Equalities in the Ratio

If a ratio is given as a : b & a quantity x is added in both antecedent & consiquent then

$$\rightarrow \frac{a+x}{b+x} > \frac{a}{b} \text{ if } a < b \qquad ...(i)$$

$$\rightarrow \frac{a+x}{b+x} < \frac{a}{b} \text{ if } a > b \qquad ...(ii)$$

$$\rightarrow \frac{a+x}{b+x} = \frac{a}{b} \text{ if } a = b$$

(i) and (ii) was considering x = +ve

If x = –ve (i) and (ii) inequalities will reverse.

Ratio as a part of total value

If two quantities A & B are in the ratio p : q then

$$\frac{A}{B} = \frac{p}{q} \Rightarrow A = pK,\ B = qK$$

So $A + B = (p + q)K$

So $$\frac{A}{A+B} = \frac{p}{p+q}$$

or $$A = \frac{p}{p+q}(A+B)$$

So A will be $\frac{p}{p+q}$ part of total sum of the quantities

similarly B will be $\frac{q}{p+q}$ part of total sum of the quantities.

Example 1.

Given that for two quantities a & b, 2a = 3b. What will be the ratio (4a + 5b) : (2a + 3b)?

Solution:

$$2a = 3b \Rightarrow \frac{a}{b} = \frac{3}{2}$$

$$\Rightarrow \quad a = 3K,\ b = 2K$$

$$\text{so } \frac{4a+5b}{2a+3b} = \frac{4\times 3K + 5\times 2K}{2\times 3K + 3\times 2K}$$

$$= \frac{22K}{12K} = \frac{11}{6}$$

So the required ratio is 11 : 6

Example 2.

If a : b = 3 : 5 & a : c = 6 : 7 then what is a : b : c ?

Solution.

a : b = 3 : 5

a : c = 6 : 7

In this case the common quantity is 'a' so we will make a's value as const.

So we can say

a : b = 6 : 10 (multiplying by 2)

and a : c = 6 : 7

so a : b : c = 6 : 10 : 7

Example 3.

If $\frac{10a^2+ab}{3ab-b^2} = \frac{10}{1}$ what will be a : b

(a) 2 : 3 (b) 2 : 5

(c) 3 : 4 (d) 3 : 7

Solution.

$$\frac{10a^2+ab}{3ab-b^2} = \frac{10}{1}$$

or $10a^2 + ab = 30ab - 10b^2$

$\Rightarrow \frac{10a^2 + ab}{ab} = \frac{30ab - 10b^2}{ab}$

$\Rightarrow 10\left(\frac{a}{b}\right) + 1 = 30 - a\left(\frac{b}{a}\right)$

Assuming $\frac{a}{b} = x$

$\Rightarrow 10x = 29 - 10\left(\frac{1}{x}\right)$

or $10x^2 = 29x - 10$

$\Rightarrow 10x^2 - 29x + 10 = 0$

$\Rightarrow x = \frac{10}{25}$ or $\frac{25}{10}$

$\Rightarrow x = \frac{2}{5}$ or $\frac{5}{2}$

Hence option (b)

Types of Ratio

If two quantities A & B have their ratio as A : B = P : q then

- Duplicate ratio of A : B is $P^2 : q^2$
- Triplicate ratio of A : B is $P^3 : q^3$
- Sub-duplicate ratio of A : B is $\sqrt{P} : \sqrt{q}$
- Sub-triplicate of A : B is $\sqrt[3]{P} : \sqrt[3]{q}$

Example 1.

The ratio of two quantities A & B is 4 : 9 what is the triplicate ratio of sub-duplicate ratio of A & B.

Solution.

The sub-duplicate ratio of A & B is $\sqrt{4} : \sqrt{9}$

$= 2 : 3$

The triplicate ratio of 2 : 3 is $2^3 : 3^3$

$= 8 : 27$

Proportion

Proportion is comparison of two equal ratio when the two ratios are equal then all the quantities comprising the ratios are called in proportion i.e.

$\frac{a}{b} = \frac{c}{d}$ then a, b, c & d are in proportion & 'd' is called as fourth proportion of a, b & c.

If three quantities a, b, c are taken then

$\frac{a}{b} = \frac{b}{c}$, here c is called as third proportion of a & b, b is called as second proportion or mean proportion of a & c.

so $b^2 = ac$

or $b = \sqrt{ac}$

so mean proportion is the geometric mean of the two quantities. Here the quantities b and c will be in 'continued proportion' similarly if a, b, c d are in continued proportion then

$\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

or in other way we can say that if the terms are in G.P. then they will be in continued proportion.

Example 1.

Find out the mean proportion of 3 & 27 also find out their third proportion.

Solution.

For mean proportion 3, x, 27 are in proportion

i.e. $\frac{3}{x} = \frac{x}{27}$

$\Rightarrow x^2 = 3 \times 27$ or $x = 9$

So mean proportion of 3 & 27 is 9

For third proportion 3, 27, x are in proportion

i.e. $\frac{3}{27} = \frac{27}{x}$

$\Rightarrow 3x = 27^2$

$\Rightarrow x = 243$

So third proportion of 3 & 27 is 243.

Applications of Proportion

Proportion is very much useful in removing the complexity of the problems in ratios. There are some basic rules that has to be taken care:

if a, b, c, d are in proportion

i.e. $\frac{a}{b} = \frac{c}{d}$ then

- $\frac{a+b}{b} = \frac{c+d}{d}$ (componendo rule)
- $\frac{a-b}{b} = \frac{c-d}{d}$ (dividendo rule)
- $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (componendo-dividendo rule)
- $\frac{b}{a} = \frac{d}{c}$ (invertendo rule)
- $\frac{a}{c} = \frac{b}{d}$ (alternendo rule)

and if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$

then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$

Variation

The basic meaning of variation is respective change. In arithmetic, use of variation makes problems very easy to approach. It is the relationship bewteen the changes of two quantities.

Let us assume there are two quantities A & B if any change is done 'A' & that change results the change in B then the quantities are called in variation. One thing has to be remembered the change we are counting is multiple change not the differential change i.e. If a quantity is 60 and then it becomes 80 then the change will not be taken as (+20). It will be taken as 80/60 = 4/3.

- Direct variation : For two quantities a & b if the nature of changes are similar then they will be directly varying i.e.

if $a \propto b$

then if 'a' becomes twice then 'b' will also be twice. If a becomes 2/3 times then b will also be 2/3 times.

Example 1.

Two quantities A & B are varying directly. Initially, A was 30 & B was 50 what has to be the value of B if A becomes 24?

Solution.

Here $A \propto B$

Change in A is 24/30 times = 4/5 times

So the change in B will be the same

So B will be 4/5 times of $50 = 4/5 \times 50 = 40$

In other way we can make it as

$$A \propto B$$

or $$A = BK$$

or $$A/B = \text{constant}$$

$$\Rightarrow \quad \frac{A_1}{B_1} = \frac{A_2}{B_2}$$

$$\Rightarrow \quad \frac{30}{50} = \frac{24}{x}$$

So $$x = \frac{24}{30} \times 50$$

$$\Rightarrow \quad x = 40$$

Inverse Variation

When in variation the nature of changes in the two quantities are inverse then the quantities will be in inverse variation?

i.e. $$A \propto \frac{1}{B}$$

then $$A = \frac{K}{B}$$

or $$A.B = K = \text{const.}$$

so $$A_1B_1 = A_2B_2 = A_3B_3 = \ldots A_nb_n = K$$

Example 1.

A car running with some speed increases its speed to 4/3 times & reaches its destination 30 minutes earlier. What time it will take if it runs with original speed?

Solution.

Here we know that speed $\propto$ 1/time

So we can use variation property

Let us assume the initial speed as 'S' & initial time as t

Now by the property of inverse variation

$(\text{speed})_1 \times (\text{time})_1 = (\text{speed})_2 \times (\text{time})_2$

$$S \times t = \frac{4}{3}S \times (\text{time})_2$$

$$\Rightarrow \quad (\text{time})_2 = \frac{3}{4}t$$

given that $(\text{time})_1 - (\text{time})_2 = 30$ mins.

$$t - \frac{3t}{4} = 30$$

$$\Rightarrow \quad \frac{t}{4} = 30 \text{ or } t = 120 \text{ mins.}$$

So the car will take 120 mins with original speed.

Compound Ratio & Concept of Partnership

If there are two ratio a : b and c : d then their compound ratio will be $a \times c : b \times d$.

This leads to the concept of partnerships the main part of the partnership is the division of profit between the partners. The profit among the partners is always divided in the compound ratio of investment ratio and time of investment ratio i.e.

If the partners' investment ratio is $P_1 : P_2 : P_3$ & the ratio of their investment time is $t_1 : t_2 : t_3$ then the ratio of profit = $P_1t_1 : P_2t_2 : P_3t_3$

So there may be three cases of the profit sharing.

Case 1.

If the ratio of investment is unity i.e. the invest equal amount then

$P_1 : P_2 : P_3 = 1 : : 1 : 1$

So ratio of profit sharing will be $t_1 : t_2 : t_3$

Example 1.

It three persons A, B, C invests rupees 20000 in a business for 3 years, 2 years & 5 years respectively then what will be the ratio of their profits.

Solution.

Since the investment is same for all the persons so the profit will be divided in the ratio of

$1 \times 3 : 1 \times 2 : 1 \times 5$

i.e. 3 : 2 : 5 will be the profit ratio.

Case 2.

If the investment time is same for all the persons i.e. $t_1 : t_2 : t_3 = 1 : 1 : 1$, then the profit will be shared in the ratio of their investment i.e. the ratio of profit will be

$P_1 : P_2 : P_3$

Example 1.

A, B, C invest rupees 10000, 15000 & 20000 respectively in a business for 2 years each what will be the ratio of their profit?

Solution.

Since the investment time is constant. So profit will be shared in the ratio of their investment i.e. ratio of profit

$= 10\,K : 15\,K : 20\,K$

$= 2 : 3 : 4$

Case 3.

When the investment is variable we have to look for the effective investment value for the profit sharing.

Example 1.

A, B, C invested rupees 10000, 15000 & 20000 respectively. Next year A & B invested 5000 more while C withdrew 10000 rupees. What will be the ratio of their profit after 3 years?

Solution.

Here we will see the effective investment value of A, B & C respectively.

Effective Investment Value of A

$= 10\,K + 15\,K + 15\,K$

$= 40\,K$

EIV of B $= 15\,K + 20\,K + 20\,K$

$= 55\,K$

EIV of C $= 20\,K + 10\,K + 10\,K$

$= 40\,K$

So ratio of profit will be

$40\,K : 55\,K : 40\,K$

$= 8 : 11 : 8$

Example 2.

A mixture contains alcohol and water in the ratio 4 : 1 by adding 10 litre of water the ratio becomes 2 : 1. Find out the amount of alcohol in the mixture

(a) 20 (b) 30

(c) 40 (d) 50

Ans. (c)

The amount of alcohol = 4 K

The amount of water = K

By adding water $\frac{4K}{K+10} = \frac{2}{1}$

$\Rightarrow \quad 4K = 2K + 20$

$\Rightarrow \quad 2K = 20$ or $K = 10$

So amount of alcohol = $4 \times 10 = 40$ litre

Example 3.

The present ratio of ages of Ram & Shyam is 5 : 4. 18 years ago the ratio of their ages was 16 : 11. The present sum of their ages is

(a) 90 years (b) 105 years

(c) 80 years (d) 110 years

Ans. (a)

Let the ages be 5 K, 4 K

$$\frac{5K-18}{4K-18} = \frac{16}{11} \Rightarrow K = 10$$

So sum of the ages = 50 + 40 = 90

Hence option (a)

Example 4.

The income of Ram and Shyam are in the ratio 2 : 3 and their expenditures are in the ratio 3 : 5. If each saves Rs. 1000 then Ram's income is

(a) 8000 (b) 6000

(c) 5000 (d) 4000

Ans. (d)

Let income be $2x$, $3x$

and expenditure be 3y, 5y

$2x - 3y = 1000$

$3x - 5y = 1000$

So $x = 2000$

So income is 4000

Hence option (d)

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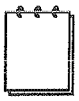

Solved Examples

1. Divide Rs. 500 among A, B, C and D so that A and B together get thrice as much as C and D together, B gets four times of what C gets and C gets 1.5 times as much as D. Now the value of what B gets is

(a) 300 (b) 75

(c) 125 (d) None of these

Ans. (a)

A + B + C + D = 500

Here A + B = 3 (C + D)

So 4 (C + D) = 500

C + D = 125

A + B = 375

also B = 4C and C = 1.5 D

C + D = 125

2.5 D = 125

D = 50, C = 75

$\therefore$ B = 300

2. If 4 examiners can examine a certain number of answer books in 8 days by working 5 hours a day, for how many hours a day would 2 examiners have to work in order to examine twice the number of answer books in 20 days.

(a) 6 (b) 7

(c) 8 (d) 9

Ans. (c)

Man. Day. Hour = constant

= 4 . 8. 5 = 160

Next $\rightarrow$ Man. day. hour = 160 × 2 (twice as earlier)

2. 20. h = 160 × 2

h = 8

3. In a mixture of 40 litres, the ratio of milk and water is 4 : 1. How much water must be added to this mixture so that the ratio of milk and water becomes 2 : 3?

(a) 20 litres (b) 32 litres

(c) 40 litres (d) 30 litres

Ans. (c)

Let water is x

4 x : x = milk : water

32 : 8 = 4x : x (since total 40 liters)

Now $\frac{32}{8+x} = \frac{2}{3}$, x= 40 litre

4. The ratio between two numbers is 3 : 4 and their LCM is 180. The first number is:

(a) 60 (b) 45

(c) 15 (d) 20

Ans. (b)

Let number are 3x and 4x

3x × 4x = HCF × 180

Clearly HCF will be x.

$12x^2 = x \times 180$, x = 15

numbers will be 45 : 60

5. The incomes of A and B are in the ratio 3 : 2 and their expenditures are in the ratio 5 : 3. If each saves Rs. 1000, then, A's income is

(a) Rs. 3000 (b) Rs. 4000

(c) Rs. 6000 (d) Rs. 9000

Ans.(c)

Let incomes are 3x : 2xx

expenditures are 5y : 3y

then

3x – 5y = 1000 (1) also

2x – 3y = 1000 (2)

from (1) and (2) we get

6x – 10y = 2000

+6x – 9y = 3000

$\Rightarrow$ –y = –1000

y = 1000, x= 2000

A's income is 6000

6. Divide Rs. 680 among A, B and C such that A gets 2/3 of what B gets and B gets 1/4th of what C gets. Now the share of C is?

(a) Rs. 480 (b) Rs. 300

(c) Rs. 420 (d) None of these

Ans. (a)

680 = A + B + C

$A = \frac{2}{3}B$ and $B = \frac{1}{4}C$, 4B = C

$A = \frac{2}{3}B$, C = 4B

$680 = \frac{2}{3}B + B + 4B$

$680 = \frac{17}{3}B$, B = 120, C = 480

7. The students in three batches at Made Easy are in the ratio 2 : 3 : 5. If 20 students are increased in each batch, the ratio change to 4 : 5 : 7. The total number of students in the three batches before the increase were.

(a) 10 (b) 90

(c) 100 (d) 150

Ans. (c)

Let students are 2x : 3x : 5x now

According to the given condition

2x + 20 : 3x +20 : 5x + 20 = 4 : 5 : 7

x = 10

20 : 30 : 50 = 2 : 3 : 5 and after adding 20

40 : 50 : 70

8. The speeds of three cars are in the ratio 2 : 3 : 4. The ratio between the times taken by these cars to travel the same distance is

(a) 2 : 3 : 4 (b) 4 : 3 : 2

(c) 4 : 3 : 6 (d) 6 : 4 : 3

Ans. (d)

Speeds are in the ratio 2 : 3 : 4

ratio of time taken will be $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$

$= \frac{6:4:3}{12} = 6:4:3$

9. After an increment of 7 in both the numerator and denominator, a fraction change to 3/4. Find the original fraction.

(a) 5/12 (b) 7/9

(c) 2/5 (d) 3/8

Ans. (c)

$\frac{x+7}{y+7} = \frac{3}{4}$

$4x + 28 = 3y + 21, \quad 4x + 7 = 3y,$

only $x : y = 2/5$ satisfies the given condition.

10. If Rs. 58 is divided among 150 children such that each girl and each boy gets 25 p and 50 p respectively. Then how many girls are there?

(a) 52 (b) 54

(c) 68 (d) 62

Ans. (c)

Let the number of girls and boys are x and y then

$0.25x + 0.5y = 58$ ...(1)

also

$x + y = 150$ ...(2)

from (1) and (2) we get

$0.5x + y = 116$

$x + y = 150$

So, $0.5x = 34, x = 68, y = 82$

11. A mixture contains milk and water in the ratio 5 : 1. On adding 5 litres of water, the ratio of milk to water becomes 5 : 2. The quantity of milk in the mixture is:

(a) 16 litres (b) 25 litres

(c) 32.5 litres (d) 22.75 litres

Ans. (b)

Let milk and water are $5x : x$

now, $\frac{5x+5}{x+5} = \frac{5}{2}$

$= 10x = 5x + 25$

$5x = 25, \quad x = 5$

milk : water = 25 : 10

12. Vijay has coins of the denomination of Re. 1, 50 p and 25 p in the ratio of 12 : 10 : 7. The total worth of the coins he has in Rs. 75. Find the number of 25 p coins that Vijay has

(a) 48 (b) 72

(c) 60 (d) None of these

Ans. (d)

Coins are $12x : 10x : 7x$

It is given that $12x + 10x \times \frac{1}{2} + \frac{7}{4}x = 75$

$\frac{48x + 20x + 7x}{4} = 75$

$75x = 4 \times 75, \quad x = 4$

13. A varies jointly with B and C directly and A = 6 when B = 3, C = 2; find A when B = 5, C = 7.

(a) 17.5 (b) 35

(c) 70 (d) 105

Ans. (b)

Let A = k (BC) then

$6 = k\ (3.2)$

So, k = 1 then

A = k (BC)

$= 1 \times (5 \times 7) = 35$

14. If x varies as y directly, and as z inversely and $x = 14$, when y = 10; find z when $x = 49$, y = 45.

(a) 14/10

(b) 10

(c) 10/14

(d) Cannot be determined

Ans. (d)

$x = k_1 y \qquad x = \frac{k_2}{z}$

According to given values

$14 = k_1.10$

So, $k_1 = \frac{14}{10} = \frac{7}{5}$

also, $14 = \frac{k_2}{z}$

So, $k_2 = 14z,$

But value of k_2 is not given.

Hence Z cannot be determined.

15. A cask contains a mixture of 49 litres of wine and water in the proportion 5 : 2. How much water must be added to it so that the ratio of wine to water may be 7 : 4?

(a) 3.5 (b) 6

(c) 7 (d) None of these

Ans. (b)

Let wine and water are $5x$ and $2x$ litres

$5x + 2x = 49$

$5x = 35, \quad 2x = 14$

Now $\frac{35}{14+w} = \frac{7}{4}$

$98 + 7w = 140$

$7w = 42, w = 6$

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Ratio & Proportion

I Practice Exercise: I

1. Find a fourth proportional to the numbers 60, 48, 30.
   (a) 36 (b) 24
   (c) 48 (d) None of these

2. Find a third proportional to the numbers 4, 42.
   (a) 441 (b) 541
   (c) 641 (d) None of these

3. The prices of a scooter and a television set are in the ratio of 3 : 2. If a scooter costs Rs. 600 more than the television set, then the price of television set is:
   (a) Rs. 1800 (b) Rs. 1200
   (c) Rs. 2400 (d) None of these

4. If A : B = 7 : 5 and B : C = 9 : 11, then A : B : C is equal to
   (a) 55:45:63 (b) 63:45:55
   (c) 45:63:55 (d) None of these

5. The ratio of money with Anju and Sanju is 4 : 5 and that with Sanju and Manju is 5 : 6. If Anju has Rs. 280, then the amount of money Manju has
   (a) Rs. 320 (b) Rs. 420
   (c) Rs. 640 (d) None of these

6. The ratio of present ages of Suresh and Mahesh is 7 : 5. If after 6 years their ages will be in the ratio of 4 : 3, the present age of Mahesh is
   (a) 32 years (b) 36 years
   (c) 30 years (d) None of these

7. Two numbers are in the ratio of 5 : 7. If 25 be subtracted from each, they are in the ratio of 35 : 59. Find the difference of the two numbers.
   (a) 48 (b) 52
   (c) 24 (d) None of these

8. The number that must be added to each of the numbers 8, 21, 13 and 31 to make the ratio of first two numbers equal to the ratio of last two number is
   (a) 5 (b) 7
   (c) 9 (d) None of these

9. A mixture contains alcohol and water in the ratio of 12 : 5. On adding 14 litres of water, the ratio of alcohol to water becomes 4 : 3. The quantity of alcohol in the mixture is
   (a) 18 litres (b) 42 litres
   (c) 26 litres (d) None of these

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Solutions

1. **Ans. (b)**
   Let x be the fourth proportion, then

   $60 : 48 : 38 : x$ or, $\frac{60}{48} = \frac{38}{x}$

   $\therefore \quad x = \frac{30 \times 38}{60} = 24.$

2. **Ans. (a)**
   Let x be the third proportion, then

   $4 : 42 : : 42 : x$ or, $\frac{4}{42} = \frac{42}{x}.$

   $\therefore \quad x = \frac{42 \times 42}{4} = 441.$

3. **Ans. (b)**
   Here, a = 3, b = 2, and $x = 600$.
   $\therefore$ The price of a television set

   $= \frac{bx}{a-b} = \frac{2 \times 600}{3-2} =$ Rs. 1200.

4. **Ans. (b)**
   Here, $n_1 = 7$, $n_2 = 9$, $d_1 = 5$ and $d_2 = 11$.
   $\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$
   $= (7 \times 9) : (5 \times 9) : (5 \times 11)$
   $= 63 : 45 : 55.$

5. **Ans. (b)**
   We have, A : B = 4 : 5 and B : C = 5 : 6.
   Here, $n_1 = 4$, $n_2 = 5$, $d_1 = 5$ and $d_2 = 6$.
   $\therefore$ $A : B : C = (n_1 \times n_2) : (d_1 \times n_2) : (d_1 \times d_2)$
   $= (4 \times 5) : (5 \times 5) : (5 \times 6)$
   $= 20 : 25 : 30$ or, $4 : 5 : 6$.

Thus, ratio of money with Anju, Sanju and Manju is 4 : 5 : 6. Since Anju has Rs. 280, the amount of money Manju has

$$= \frac{280}{4} \times 6 = \text{Rs. } 420.$$

6. Ans. (c)
We have, a : b = 7 : 5, c : d = 4 : 3 and x=6.

$$\therefore \text{ The present age of Mahesh} = \frac{bx(c-d)}{ad-bc}$$

$$= \frac{5 \times 6 \times (4-3)}{(7 \times 3 - 5 \times 4)} = 30 \text{ years.}$$

7. Ans. (c)
We have, a : b = 5 : 7, c : d = 35 : 59 and x = 25.

$$\therefore \text{ The first number} = \frac{ax(d-c)}{ad-bc}$$

$$= \frac{5 \times 25 \times (59-35)}{(5 \times 59 - 7 \times 35)} = 60.$$

$$\text{and, the second number} = \frac{bx(d-c)}{ad-bc}$$

$$= \frac{7 \times 25 \times (59-35)}{(5 \times 59 - 7 \times 35)} = 84$$

$\therefore$ The difference of two numbers
= 84 – 60 = 24

8. Ans. (a)
Here, a = 8, b = 21, c = 13 and d = 31.
$\therefore$ The required number

$$= \frac{bc-ad}{(a+d)-(b+c)}$$

$$= \frac{21 \times 13 - 8 \times 31}{(8+31)-(21+13)} = 5.$$

9. Ans. (b)
Ratio of alcohol and water 12 : 5
Let their qualities be $12x$ and $5x$ respectively.
After adding the litres of water ratio becomes 4 : 3

$$\frac{12x}{5x+14} = \frac{4}{3}$$

$$x = \frac{7}{2}$$

$$\text{Quantity of alcohol} = 12 \times \frac{7}{2} = 42 \text{ litres.}$$

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Partnership

I Practice Exercise: I

1. Amit, Nitin and Ravindra entered into a partnership. Amit invested Rs. 16000 for 9 months. Nitin invested Rs. 12000 for 6 months and Ravindra invested Rs. 8000 for 12 months. At the end of a year there was a profit of Rs. 26000. Find the share of Nitin in the profit.
(a) Rs. 8000 (b) Rs. 7500
(c) Rs. 6000 (d) None of these

2. A starts business with Rs. 3500 and 5 months after B joins A as his partner. After a year the profits are divided in the ratio of 2 : 3. How much did B contribute?
(a) Rs. 7000 (b) Rs. 11000
(c) Rs. 9000 (d) None of these

3. A, B and C invested capitals in the ratio 3 : 5 : 9; the timing of their investments being in the ratio 2 : 3 : 1. In what ratio would their profits be distributed?
(a) 2 : 5 : 3 (b) 3 : 2 : 5
(c) 7 : 5 : 3 (d) None of these

4. A, B and C start a business. If the ratio of their periods of investments are 2 : 3 : 6 and their profits are in the ratio of 4 : 5 : 6, then the ratio of capitals of A, B and C is
(a) 6 : 8 : 10 (b) 12 : 10: 6
(c) 10 : 12 : 6 (d) None of these

5. A and B started a business with initial investments in the ratio 5 : 7. If after one year their profits were in the ratio 1 : 2 and the period for A's investment was 7 months, B invested the money for
(a) 6 months (b) 12
(c) 10 months (d) 4 months

6. A, B, C enter into a partnership with shares in the ratio $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$. After 4 months, A increase his share by 50%. If the total profit at the end of one year be Rs. 21600, then B's share in the profit is
(a) Rs. 2100 (b) Rs. 2400
(c) Rs. 3600 (d) Rs. 4000

7. B is a sleeping partner and A working. A puts in Rs. 5000 and B puts in 6000. A received 12.5% of profit for managing the business and rest is divided in proportion to their capitals. A's share of profit in a total profit of Rs. 880 is
   (a) Rs. 350 (b) Rs. 400
   (c) Rs. 420 (d) Rs. 460

8. A starts business with a capital of Rs. 1200. B and C join with some investments after 3 and 6 months, respectively. If at the end of a year, the profit is divided in the ratio 2 : 3 : 5 respectively, what is B's investment in the business?
   (a) Rs. 2400 (b) Rs. 1800
   (c) Rs. 3600 (d) Rs. 6000

□□□□

Solutions

1. Ans. (c)
Here, $C_1 = 16000$, $C_2 = 12000$, $C_3 = 8000$, $t_1 = 9$, $t_2 = 6$, $t_3 = 12$ and $P = 26000$.
Nitin's share in the profit

$$= \frac{C_2 \times t_2 \times P}{C_1t_1 + C_2t_2 + C_3t_3}$$

$$= \frac{12000 \times 6 \times 26000}{16000 \times 9 + 12000 \times 6 + 8000 \times 12}$$

$$= \frac{1872000000}{312000} = \text{Rs. } 6000.$$

2. Ans. (c)
We have, $C_1 \times t_1 = 3500 \times 12 = 42000$ and $C_2 \times t_2 = x \times 7 = 7x$.

Then, $\frac{\text{Profit for A}}{\text{Profit for B}} = \frac{C_1 \times t_1}{C_2 \times t_2}$

$$\Rightarrow \frac{2}{3} = \frac{42000}{7x}$$

$$\text{or, } x = \frac{42000 \times 3}{2 \times 7} = \text{Rs. } 9000.$$

3. Ans. (a)
Ratio of capitals of A, B and C are 3 : 5 : 9. Let the capitals of A, B and C be $3x$, $5x$ and $9x$, respectively. Ratio of timing of their investments are 2 : 3 : 1. Let A, B and C invest their capitals for 2y, 3y and y months, respectively.
Then, profit of A : profit of B : Profit of C
$= C_1 \times t_1 : C_2 \times t_2 : C_3 \times t_3$
$= 3x \times 2y : 5x \times 3y : 9x \times y$
$= 6 : 15 : 9$ or, $2 : 5 : 3$.

4. Ans. (b)
We have, $P_1 : P_2 : P_3 = 4 : 5 : 6$ and $t_1 : t_2 : t_3 = 2 : 3 : 6$.

$$\therefore \text{ Required ratio } = \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} = \frac{4}{2} : \frac{5}{3} : \frac{6}{6}$$

or, 12 : 10 : 6
Thus, A, B and C invested their capitals in the ratio 12 : 10 : 6.

5. Ans. (c)
Let investments of A and B respectively be $5x$ and $7x$ and period of B's investment be y months.

$$\text{Then, } \frac{(5x) \times 7}{(7x) \times y} = \frac{1}{2} \Rightarrow y = 10.$$

6. Ans. (d)

$$\text{Given ratio } = \frac{7}{2} : \frac{4}{3} : \frac{6}{5} = 105 : 40 : 36$$

Let the initially invest Rs. 105, Rs. 40 and Rs. 36, respectively.
Ratio of investments
= [105 × 4 + (150% of 105) × 8] : (40 × 12) : (36 × 12)
= 1680 : 480 : 432 = 35 : 10 : 9

$$\text{B's share } = \text{Rs.}\left(21600 \times \frac{10}{54}\right) = \text{Rs. } 4000.$$

7. Ans. (d)
A's share for managing the business
= 12.5% of Rs. 880 = Rs. 110
Remaining proift = Rs. 770.
Profit ratio of A and B = 5 : 6

$$\text{A's share} = \frac{5}{11} \text{ of Rs. } 770 = \text{Rs. } 350.$$

A's total profit= Rs. 350 + Rs.110 = Rs. 460.

8. Ans. (a)
Profit ratio of A, B and C is
$(1200 \times 12) : (x \times 9) : (y \times 6) = 2 : 3 : 5$

$$\Rightarrow \frac{1200 \times 12}{2} = \frac{9x}{3}$$

$$\therefore \quad x = \text{Rs. } 2400.$$

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