UNIT 4: GEOMETRY

CHAPTER 7

TRIANGLES

Syllabus

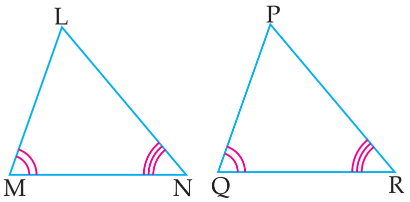
- Definitions, examples, counter examples of similar triangles.
  1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.
  2. (Motivate) If a line divides two sides of a triangle in the same ratio, then the line is parallel to the third side.
  3. (Motivate) If in two triangles, the corresponding angles are equal, then their corresponding sides are proportional and the triangles are similar.
  4. (Motivate) If the corresponding sides of two triangles are proportional, then their corresponding angles are equal and the two triangles are similar.
  5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, then the two triangles are similar.
  6. (Motivate) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on each side of the perpendicular are similar to each other and to the whole triangle.
  7. (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
  8. (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of squares of the other two sides.
  9. (Prove) In a triangle, if the square of one side is equal to the sum of the squares on the other two sides, then the angles opposite to the first side is a right angle.

Trend Analysis

List of Concepts	2018		2019		2020	
	Delhi	Outside Delhi	Delhi	Outside Delhi	Delhi	Outside Delhi
Question based on Area of Triangle	1 Q (1 M) 2 Q (3 M)		1 Q (1 M)		2 Q (1 M)	1 Q (1 M) 1 Q (2 M)
Question based on Proving Properties of Triangle	1 Q (3 M) 1 Q (4 M)		2 Q (3 M)	2 Q (1 M) 1 Q (3 M) 1 Q (4 M)	2 Q (1 M) 1 Q (2 M) 2 Q (3 M)	2 Q (1 M) 1 Q (2 M) 1 Q (3 M)

Revision Notes

- A triangle is one of the basic shapes of geometry. It is a polygon with 3 sides and 3 vertices/corners.
- Two figures are said to be congruent if they have the same shape and the same size.
- Those figures which have the same shape but not necessarily the same size are called **similar figures.**
  Hence, we can say that **all congruent figures are similar but all similar figures are not congruent.**
- **Similarity of Triangles:** Two triangles are similar, if:
  (i) their corresponding sides are proportional.
  (ii) their corresponding angles are equal.
  If ΔABC and ΔDEF are similar, then this similarity can be written as $\Delta ABC \sim \Delta DEF$.
- **Criteria for Similarity of Triangles:**

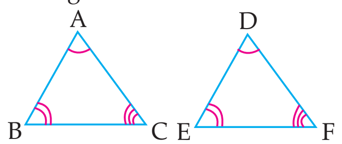


In ΔLMN and ΔPQR, if

(a) $\angle L = \angle P$, $\angle M = \angle Q$ and $\angle N = \angle R$ **(b)** $\frac{LM}{PQ} = \frac{MN}{QR} = \frac{LN}{PR}$,

then $\Delta LMN \sim \Delta PQR$.

(i) AAA-Criterion: In two triangles, if corresponding angles are equal, then the triangles are similar and hence their corresponding sides are in the same ratio.



If ΔABC and ΔDEF are similar

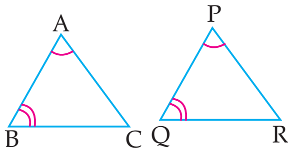
$\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$.

Then,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

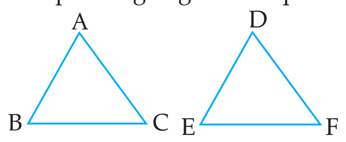
Remark: If two angles of a triangle are respectively equal to the two angles of another triangle, then by the angle sum property of a triangle their third angles will also be equal. Therefore, AAA similarity criterion can also be stated as follows:

AA-Criterion: If two angles of one triangle are respectively equal to two angles of another triangle, then the two triangles are similar.



As we know that the sum of all angles in a triangle is 180° so if two angles in ΔABC and ΔPQR are same *i.e.*, $\angle A = \angle P$, $\angle B = \angle Q$.

(ii) SSS-Criterion: In two triangles if the sides of one triangle are proportional to the sides of another triangle, then the two triangles are similar and hence corresponding angles are equal.



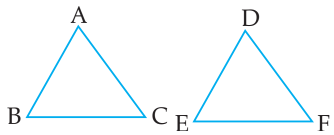
If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

∴ $\Delta ABC \sim \Delta DEF$

then $\angle A = \angle D$, $\angle B = \angle E$

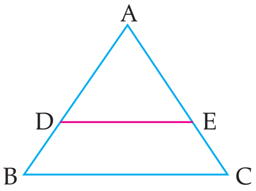
and $\angle C = \angle F$

(iii) SAS-Criterion: If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.



If $\frac{AB}{DE} = \frac{AC}{DF}$ and $\angle A = \angle D$, then $\Delta ABC \sim \Delta DEF$.

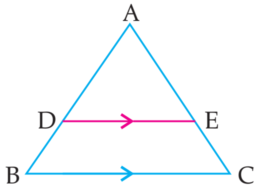
Some theorems based on similarity of triangles:



(i) If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio. It is known as **'Basic Proportionality Theorem'** or **'Thales Theorem'.**

In ΔABC, let $DE \,||\, BC$, then

(a) $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ **(b)** $\dfrac{AB}{DB} = \dfrac{AC}{EC}$ **(c)** $\dfrac{AD}{AB} = \dfrac{AE}{AC}$.



(ii) If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. It is the '**Converse of Basic Proportionality Theorem**'.

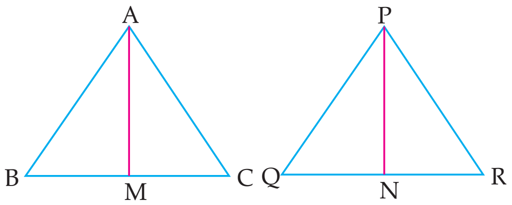
If $\dfrac{AD}{DB} = \dfrac{AE}{EC}$,

then $DE \,||\, BC$

(iii) If two triangles are similar, then the ratio of areas of these triangles is equal to the ratio of squares of their corresponding sides.

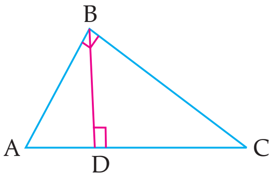
Let $\Delta ABC \sim \Delta PQR$,

then $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{BC}{QR}\right)^2 = \left(\dfrac{CA}{RP}\right)^2 = \left(\dfrac{AM}{PN}\right)^2$



➢ **Theorems Based on Right Angled Triangles:**

(i) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.



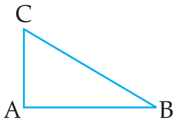
In right ΔABC, $B \perp AC$,

then $\Delta ADB \sim \Delta ABC$

$\Delta BDC \sim \Delta ABC$

and $\Delta ADB \sim \Delta BDC$

(ii) In a right angled triangle, the square of hypotenuse is equal to the sum of the squares of the other two sides. It is known as **Pythagoras Theorem.**



In right ΔABC,

$$BC^2 = AB^2 + AC^2.$$

Some Important Notes:

➢ In any triangle, the sum of the squares of any two sides is equal to twice the square of half of the third side together with the twice of the square of the median which bisects the third side.

➢ Three times the sum of the squares of the sides of a triangle is equal to four times the sum of the squares of the median of the triangle.

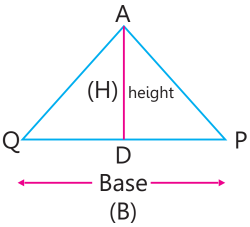
➢ Three times of the square of any side of an equilateral triangle is equal to four times the square of the altitude.

Mnemonics

Area of Right angled Triangle

Audi is the product of half of BMW and Honda

Concept: Area of triangle = $\frac{1}{2}$ × Base × height



Interpretation:

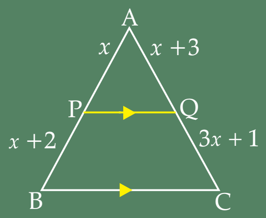
A = Area

B = Base

H = Height

How is it done on the GREENBOARD?

Q.1. In the given triangle, find x if PQ || BC.



Solution:

Step I : Given that PQ || BC

Hence by using Basic Proportionality theorem

Step II :

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

$$\frac{x}{x+2} = \frac{x+3}{3x+1}$$

$\Rightarrow \quad x(3x+1) = (x+2)(x+3)$

$\Rightarrow \quad 3x^2 + x = x^2 + 5x + 6$

$\Rightarrow \quad 2x^2 - 4x - 6 = 0$

$\Rightarrow \quad x^2 - 2x - 3 = 0$

$\Rightarrow \quad (x-3)(x+1) = 0$

$\Rightarrow \quad x = 3$ or $x = -1$

Since, side of a triangle is not negative.

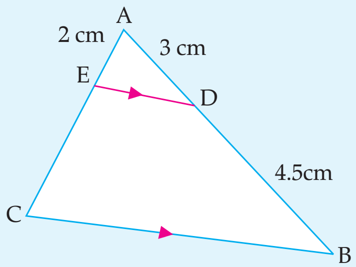
$\therefore \quad x = 3.$

Very Short Answer Type Questions — 1 mark each

AI Q. 1. In the ΔABC, D and E are points on side AB and *AC* respectively such that *DE* || *BC*. If *AE* = 2 cm, *AD* = 3 cm and *BD* = 4.5 cm, then find CE.

A [CBSE SQP, 2020-21]

Sol.



Using basic proportionality theorem

$$\frac{AD}{BD} = \frac{AE}{CE} \quad \text{[By using BPT]} \ \tfrac{1}{2}$$

$$\frac{3}{4.5} = \frac{2}{CE}$$

$$CE = 3 \text{ cm} \quad \tfrac{1}{2}$$

[CBSE Marking Scheme, 2020-21]

AI Q. 2. Given Δ *ABC* ~ Δ *PQR*, if $\frac{AB}{PQ} = \frac{1}{3}$, then $\frac{ar(\Delta ABC)}{ar(\Delta PQR)}$.

U [CBSE Delhi Set-I, 2020]

[CBSE Delhi/OD, 2018] [CBSE SQP, 2017]

Sol. 1 : 9

Explanation: Since, $\Delta ABC \sim \Delta PQR$, we have

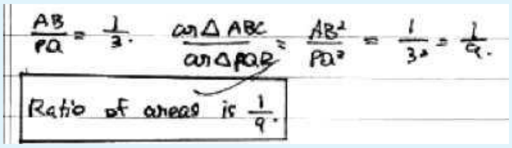
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{1}{3}\right)^2 \quad [\because \text{ By using property of similar triangles}]$$

$$= \frac{1}{9}. \qquad \mathbf{1}$$

[CBSE Marking Scheme, 2020]

Detailed Solution:

Topper Answer, 2018



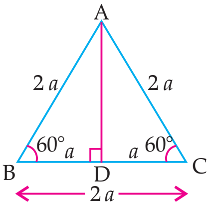
$\frac{AB}{PQ} = \frac{1}{3}$. $\frac{ar\Delta ABC}{ar\Delta PQR} = \frac{AB^2}{PQ^2} = \frac{1}{3^2} = \frac{1}{9}$.

Ratio of areas is $\frac{1}{9}$.

AI **Q. 3. In an equilateral triangle of side 2*a*, find the length of the altitude.** U **[CBSE Delhi Set-I, 2020]**

Sol. ABC is an equilateral triangle in which $AD \perp BC$.

From ΔABC,



$$AB^2 = (AD)^2 + (BD)^2 \qquad \tfrac{1}{2}$$

(By using Pythagoras Theorem)

$$\Rightarrow \quad (2a)^2 = (AD)^2 + (a)^2$$

$$\Rightarrow \quad 4a^2 - a^2 = (AD)^2$$

$$\Rightarrow \quad (AD)^2 = 3a^2$$

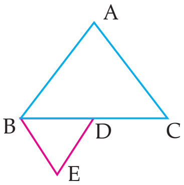
$$\Rightarrow \quad AD = a\sqrt{3}$$

Hence, the length of attitude is $a\sqrt{3}$. ½

Q. 4. ΔABC and ΔBDE are two equilateral triangles such that D is the mid-point of BC. Find the ratio of the areas of triangles ABC and BDE.

A **[CBSE Delhi Set-II, 2020]**

Sol.
$$\frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{\frac{\sqrt{3}}{4}(BC)^2}{\frac{\sqrt{3}}{4}(BD)^2}$$



½

$$= \frac{(BC)^2}{\left(\frac{BC}{2}\right)^2}$$

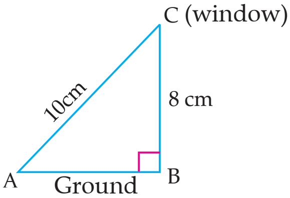
$$= \frac{4BC^2}{BC^2} = \frac{4}{1}$$

$= 4 : 1.$ ½

AI **Q. 5. If a ladder 10 m long reaches a window 8 m above the ground, then find the distance of the foot of the ladder from the base of the wall.**

C **[CBSE Delhi Set-III, 2020]**

Sol. Let BC be the height of the window above the ground and AC be a ladder.



Here, $BC = 8$ cm and $AC = 10$ cm ½

$\therefore$ In right angled triangle ABC,

$$AC^2 = AB^2 + BC^2$$

(By using Pythagoras Theorem)

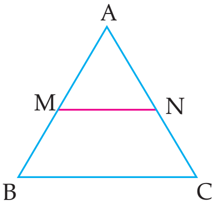
$$\Rightarrow \quad (10)^2 = AB^2 + (8)^2$$

$$\Rightarrow \quad AB^2 = 100 - 64$$

$$= 36$$

$$\Rightarrow \quad AB = 6 \text{ m.} \qquad \tfrac{1}{2}$$

Q. 6. In fig., $MN \parallel BC$ and $AM : MB = 1 : 2$, then find $\frac{ar(\Delta AMN)}{ar(\Delta ABC)}$. A **[CBSE OD Set-I, 2020]**

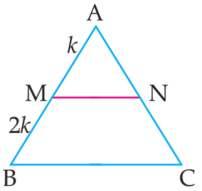


Sol. Here, $AM = k$, $MB = 2k$.

and $MN \parallel BC$

$\therefore \quad AB = AM + MB$

$= k + 2k = 3k$



½

$$\therefore \quad \frac{\text{ar}(\Delta AMN)}{\text{ar}(\Delta ABC)} = \frac{AM^2}{AB^2} = \frac{k^2}{9k^2} = \frac{1}{9}. \quad \text{½}$$

AI **Q. 7. In ΔABC, $AB = 6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm, then find ∠B.** U [CBSE OD Set-I, 2020]

Sol. Given, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.

It can be observed that

AB^2= 108 cm, AC^2 = 144 cm and BC^2 = 36 cm

Now $AB^2 + BC^2$ = 108 + 36 = 144 cm and AC^2 = 144 cm

i.e., $AB^2 + BC^2 = AC^2$, Which is satisfies Pythagoras theorem.

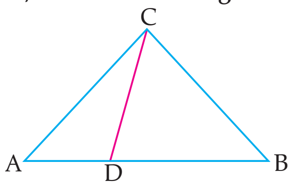
So, $\angle B = 90°$. **1**

Q. 8. If two triangles are similar, then find the relation of their corresponding sides.

A [CBSE OD Set-I, 2020]

Sol. If two triangles are similar, then their corresponding sides are in the same ratio. **1**

Q. 9. In the figure,, if ∠ACB = ∠CDA, AC = 6 cm and AD = 3 cm, then find the length of AB



[CBSE SQP, 2020]

Sol. $\Delta ACB \sim \Delta ADC$ (AA criterion) ½

$$\Rightarrow \quad \frac{AC}{AD} = \frac{AB}{AC}$$

$$\frac{6}{3} = \frac{AB}{6}$$

$\therefore \quad AB = 12$ cm. ½

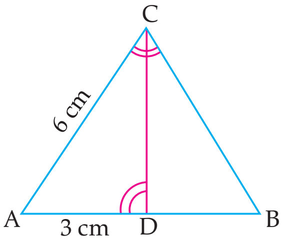
[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Since $\angle CDA = \angle ACB$ (given)

$\angle CAD = \angle CAB$ (common)

$\therefore \quad \Delta ADC \sim \Delta ACB$ (AA Similarity)



½

$$\frac{AC}{AB} = \frac{AD}{AC}$$

$$\frac{6}{AB} = \frac{3}{6}$$

$\therefore \quad AB = 12$ cm. ½

Q. 10. The perimeters of two similar triangles ΔABC and ΔPQR are 35 cm and 45 cm respectively, then find the ratio of the areas of the two triangles.

[CBSE SQP, 2020]

Sol. 49 : 81

[CBSE SQP Marking Scheme, 2020] **1**

Detailed Solution:

Given, Perimeter of ΔABC and ΔPQR are 35 cm and 45 cm,

Since, the ratio of area of the similar triangles is square of the scalar factor of similarity,

$$\text{Scalar factor} = \frac{\text{Perimeter of triangle } ABC}{\text{Perimeter of triangle } PQR}$$

$$= \frac{35}{45} = \frac{7}{9} \quad \text{½}$$

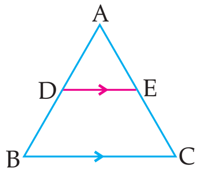
Hence,

$$\frac{\text{Area of triangle } ABC}{\text{Area of triangle } PQR} = (\text{scalar factor})^2$$

$$= \left(\frac{7}{9}\right)^2$$

$$= \frac{49}{81}. \quad \text{½}$$

AI **Q. 11. In Figure, $DE \parallel BC$, AD = 1 cm and BD = 2 cm. What is the ratio of the ar (ΔABC) to the ar (ΔADE) ?**



A [CBSE Delhi Set-1, 2019]

Sol. $AB = 1 + 2 = 3$ cm

$\Delta ABC \sim \Delta ADE$ ½

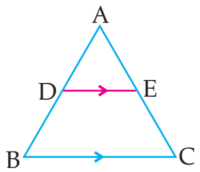
$$\therefore \quad \frac{ar(ABC)}{ar(ADE)} = \frac{AB^2}{AD^2} = \frac{9}{1}$$

$\therefore$ ar(ΔABC) : ar(ΔADE) = 9 : 1 ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given, AD = 1 cm, BD = 2 cm and $DE \parallel BC$



In ΔADE and ΔABC,

$\angle ADE = \angle ABC$ (corresponding angles)

$\angle A = \angle A$ (common)

Therefore, by *AA* criterion corollary condition

$$\Delta ADE \sim \Delta ABC \qquad ½$$

Ratio of areas of similar triangles is equal to the square of the ratio of the corresponding sides,

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{AB^2}{AD^2}$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{(3)^2}{(1)^2} \qquad [AB = AD + BD = 3]$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{9}{1}$$

Hence, the ratio of the ar(ΔABC) to the ar(ΔADE) is 9 : 1. ½

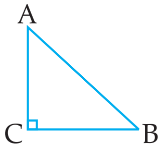
COMMONLY MADE ERROR

- Some candidates took the ratio of area $\frac{\Delta ABC}{\Delta ADE} = \frac{AB}{AD}$ instead of $\frac{AB^2}{AD^2}$.

ANSWERING TIP

- It is necessary to explain how the ratio of the areas of similar triangles is proportional to the square to the corresponding.

AI **Q. 12. In Figure, ABC is an isosceles triangle right angled at C with $AC = 4$ cm. Find the length of AB.**



A **[CBSE OD Set-1, 2019]**

Sol. ΔABC : Isosceles Δ ⇒ $AC = BC = 4$ cm. ½

$$AB = \sqrt{4^2 + 4^2} = 4\sqrt{2} \text{ cm} \qquad ½$$

[CBSE Marking Scheme, 2019]

Detailed Solution:

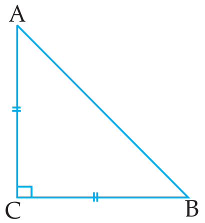
Given, ABC is an isosceles triangle right angled at C

i.e. $AC = BC = 4$ cm

$\angle C = 90°$ ½

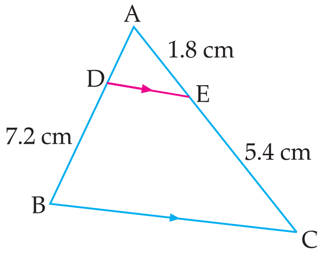
Applying pythagoras theorem in ΔABC,

$$AB^2 = BC^2 + AC^2$$
$$= 4^2 + 4^2$$
$$= 16 + 16$$
$$= 32$$



$\therefore \quad AB = 4\sqrt{2}$ cm. ½

Q. 13. In Figure, $DE \parallel BC$. Find the length of side AD, given that $AE = 1.8$ cm, $BD = 7.2$ cm and $CE = 5.4$ cm.



A **[CBSE OD Set-1, 2019]**

[CBSE Term-2, 2016]

Sol. $\frac{AD}{BD} = \frac{AE}{CE} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$ ½

$\therefore \quad AD = \frac{7.2 \times 1.8}{5.4} = 2.4$ cm. ½

[CBSE Marking Scheme, 2019]

Detailed Solution:

It is given that $DE \parallel BC$

Hence, $\frac{AD}{DB} = \frac{AE}{EC}$ ½

Putting the values, we get

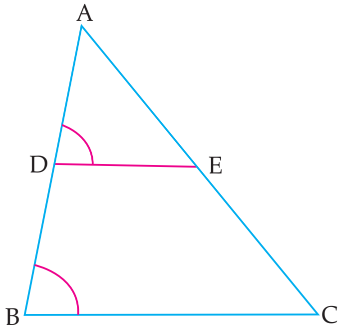
$$\frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\Rightarrow \quad AD = \frac{1.8 \times 7.2}{5.4}$$

$$\Rightarrow \quad AD = \frac{12.96}{5.4}$$

$$\Rightarrow \quad AD = 2.4 \text{ cm.} \qquad ½$$

Q. 14. In figure, if $AD = 6$ cm, $DB = 9$ cm, $AE = 8$ cm and $EC = 12$ cm and $\angle ADE = 48°$. Find $\angle ABC$.



U **[CBSE SQP, 2018-19]**

Sol. $\frac{AD}{DB} = \frac{AE}{EC}$

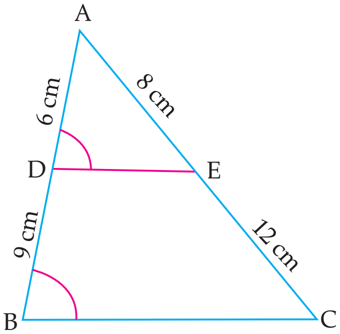
$\therefore \quad DE \parallel BC$ ½

$\Rightarrow \quad \angle ADE = \angle ABC = 48°$ ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

It is given that,



Here, in ΔABC and ΔADE,

$$AB = (9 + 6) \text{ cm} = 15 \text{ cm}$$

Similarly,

$$AC = AE + EC = (8 + 12) \text{ cm} = 20 \text{ cm} \quad ½$$

Now, $\frac{AD}{AB} = \frac{6}{15} = \frac{2}{5}$

and $\frac{AE}{AC} = \frac{8}{20} = \frac{2}{5}$

Then, $\frac{AD}{AB} = \frac{AE}{AC}$

i.e., $DE \,||\, BC$

∴ $\angle ABC = \angle ADE$ (Corresponding angles)

Hence, $\angle ABC = 48°$. ½

AI **Q. 15. If Δ $ABC \sim$ Δ QRP, $\frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{9}{4}$, and $BC = 15$ cm, then find PR.** [CBSE Comptt. Set I, II, III, 2018] [CBSE, Term-I, 2015]

Sol. $\frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \left(\frac{BC}{RP}\right)^2$

⇒ $\frac{9}{4} = \left(\frac{15}{PR}\right)^2 \Rightarrow PR = 10$ cm 1

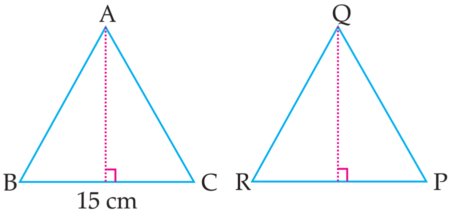
[CBSE Marking Scheme, 2018]

Detailed Solution:

We have, $\Delta ABC \sim \Delta QRP$

∴ $\frac{AB}{QR} = \frac{BC}{RP} = \frac{AC}{QP}$

[corresponding sides of similar triangles]



Now, $\frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \left(\frac{BC}{RP}\right)^2$

[Areas of two similar triangles is equal to the square of the ratio of their corresponding sides]

⇒ $\frac{9}{4} = \left(\frac{15}{RP}\right)^2$

⇒ $\frac{3}{2} = \frac{15}{RP}$

⇒ $RP = \frac{30}{3} = 10 \text{ cm}$. 1

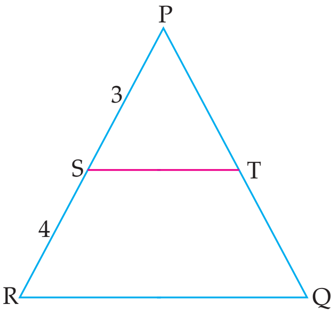
COMMONLY MADE ERROR

- Some candidates take the ratio of area $\frac{\Delta ABC}{\Delta QRP} = \frac{BC}{RP}$ instead of $\frac{BC^2}{RP^2}$.

ANSWERING TIP

- Candidates should more practice to solve such problems.

Q. 16. In the given figure, $ST \parallel RQ$, $PS = 3$ cm and $SR = 4$ cm. Find the ratio of the area of ΔPST to the area of ΔPRQ. A [CBSE, SQP, 2017-18]



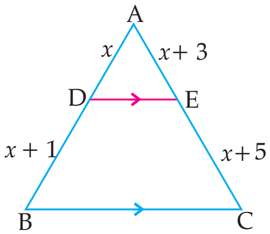
Sol. $PS = 3$ cm, $SR = 4$ cm and $ST \parallel RQ$.

$$PR = PS + SR = 3 + 4 = 7 \text{ cm}$$

$$\frac{\text{ar } \Delta PST}{\text{ar } \Delta PRQ} = \frac{PS^2}{PR^2} = \frac{3^2}{7^2} = \frac{9}{49}$$

Hence, required ratio = 9 : 49. 1

AI **Q. 17. In ΔABC, $DE \,||\, BC$, find the value of x.**



U [CBSE, Term-1, 2016]

Sol. As $DE \,||\, BC$

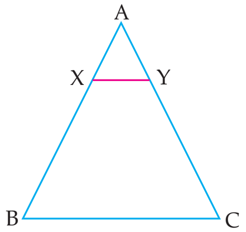
∴ $\frac{AD}{DB} = \frac{AE}{EC}$

or,	$\frac{x}{x+1} = \frac{x+3}{x+5}$	
or,	$x^2 + 5x = x^2 + 4x + 3$	
or,	$x = 3$	1

[CBSE Marking Scheme, 2016]

Q. 18. In ΔABC, if X and Y are points on AB and AC respectively such that $\frac{AX}{XB} = \frac{3}{4}$, $AY = 5$ and $YC = 9$, then state whether XY and BC parallel or not. U [CBSE Term-1, 2015, 2016]

Sol.



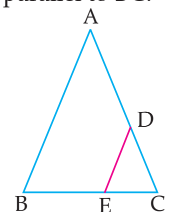
$$\frac{AX}{XB} = \frac{3}{4}, AY = 5 \text{ and } YC = 9 \quad \text{(Given)}$$

Then, $\frac{AX}{XB} = \frac{3}{4}$ and $\frac{AY}{YC} = \frac{5}{9}$

$$\frac{AX}{XB} \neq \frac{AY}{YC}$$

Hence XY is not parallel to BC. **1**

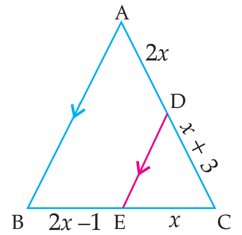
Q. 19.



In the figure of ΔABC, the points D and E are on the sides CA, CB respectively such that DE || AB, $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$. Then, find x. A [CBSE, Term-1, 2016]

OR

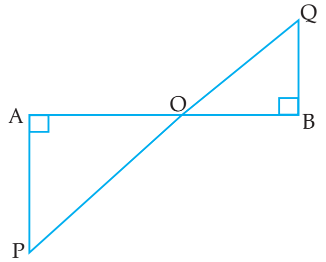
In the figure of ΔABC, DE || AB. If $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$, then find the value of x. [CBSE, Term-1, 2015]



Sol. $\frac{CD}{AD} = \frac{CE}{BE}$ or, $\frac{x+3}{2x} = \frac{x}{2x-1}$ ½

or, $5x = 3$ or, $x = \frac{3}{5}$ ½

Q. 20. In the given figure, if $\angle A = 90°$, $\angle B = 90°$, $OB = 4.5$ cm, $OA = 6$ cm and $AP = 4$ cm, then find QB. U [CBSE Term-1, 2015]



Sol. In ΔPAO and ΔQBO,

$\angle A = \angle B = 90°$ (Given)

$\angle POA = \angle QOB$ (Vertically Opposite Angles)

Since, $\Delta PAO \sim \Delta QBO$, (by AA)

Then, $\frac{OA}{OB} = \frac{PA}{QB}$

or, $\frac{6}{4.5} = \frac{4}{QB}$

or, $QB = \frac{4 \times 4.5}{6}$

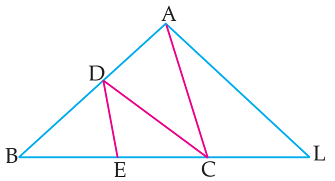
$\therefore$ $QB = 3$ cm **1**

Q. 21. Are two triangles having corresponding sides equal, similar. R [CBSE Term-1, 2015]

Sol. Yes, Two triangles having equal corresponding sides are congruent and all congruent Δs have equal angles, hence they are similar too. **1**

Short Answer Type Questions-I — 2 marks each

AI **Q. 1. In the adjoining figure, DE || AC and DC || AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.**

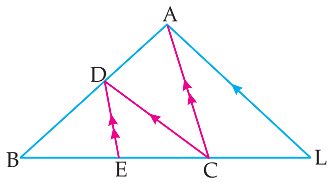


A [CBSE Delhi Set-I, 2020]

Sol. In Δ ABP,

DC || AP (Given)

$\therefore$ $\frac{BD}{DA} = \frac{BC}{CP}$ (From BPT) ....(i) ½



In ΔABC,

$$DE \parallel AC \quad \text{(Given)}$$

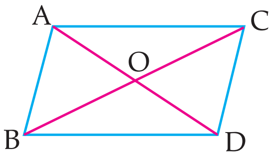
$$\therefore \quad \frac{BD}{DA} = \frac{BE}{EC} \quad \text{(From BPT) ....(ii) ½}$$

From equations (i) and (ii), we have

$$\frac{BE}{EC} = \frac{BC}{CP}$$ **Hence Proved.** ½

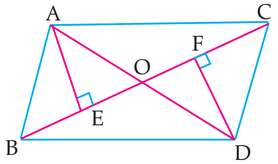
AI Q. 2. In the given figure, ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

U **[CBSE Outside Delhi Set-I, 2020]**
[CBSE Term-1, 2016]



Sol. To prove: $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Construction : Draw $AE \perp BC$ and $DF \perp BC$.



Proof: ½

In ΔAOE and ΔDOF,

$$\angle AOE = \angle DOF$$
(Vertically opposite angles)

$$\angle AEO = \angle DFO = 90° \text{ (Construction)}$$

or, $\Delta AOE \sim \Delta DOF$ (By AA Similarity)

$$\therefore \quad \frac{AO}{DO} = \frac{AE}{DF} \quad ...(i) ½$$

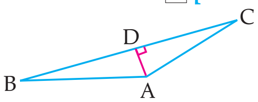
Now, $$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF}$$

$$= \frac{AE}{DF} \quad ½$$

$$= \frac{AO}{DO} \quad \text{[From equation (i)] ½}$$

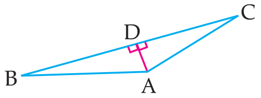
Hence Proved.

AI Q. 3. In fig. 6, if $AD \perp BC$, then prove that $AB^2 + CD^2 = BD^2 + AC^2$. A **[CBSE OD Set-I, 2020]**



Sol. In right ΔADC,

$$AC^2 = AD^2 + CD^2 \quad ...(i) ½$$



In right ΔADB,

$$AB^2 = AD^2 + BD^2 \quad ...(ii) ½$$

Subtracting eq. (i) from eq. (ii),

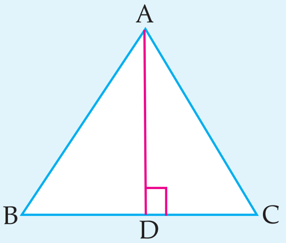
$$AB^2 - AC^2 = BD^2 - CD^2$$

Hence, $AB^2 + CD^2 = AC^2 + BD^2$

Hence Proved. 1

AI Q. 4. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes. C **[CBSE SQP, 2020]**

Sol.



$AD \perp BC \therefore$ In ΔABD, $AB^2 = AD^2 + BD^2$ **1**

$\Rightarrow \quad AB^2 = AD^2 + \frac{BC^2}{4}$ or $4AB^2 = 4AD^2 + BC^2$

$\Rightarrow \quad 3AB^2 = 4AD^2$ **1**

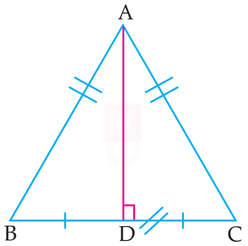
[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Let ABC be an equilateral triangle, in which AD is the perpendicular bisector on BC.

$$\therefore \quad BD = \frac{BC}{2}$$

and $AB = BC = CA.$ **1**



In right angled ΔABD,

$$AB^2 = AD^2 + BD^2$$
(Using Pythagoras Theorem)

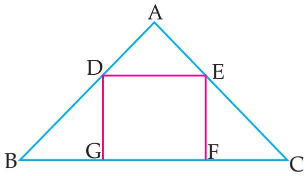
$$\Rightarrow \quad AB^2 = AD^2 + \frac{BC^2}{4}$$

$$\Rightarrow \quad 4AB^2 = 4AD^2 + AC^2$$

$$\Rightarrow \quad 4AB^2 = 4AD^2 + AB^2 \quad [\because AB = AC]$$

Hence, $3AB^2 = 4AD^2$. **Hence Proved. 1**

AI Q. 5. In the given figure, DEFG is a square and $\angle BAC = 90°$. Show that $FG^2 = BG \times FC$



[CBSE SQP, 2020]

Sol. $\Delta ADE \sim \Delta GBD$ and $\Delta ADE \sim \Delta FEC$

$\Rightarrow \quad GBD \sim FEC$ (AA Criterion)

$\Rightarrow \quad \dfrac{GD}{FC} = \dfrac{GB}{FE}$

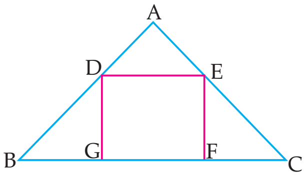
$\Rightarrow GD \times FE = GB \times FC$

or $\quad FG^2 = BG \times FG$

[CBSE SQP Marking Scheme, 2020]

Detailed Solution:

Given, DEFG is a square and



$\angle BAC = 90°$

To prove, $\quad FG^2 = BG \times FC$

Since, DEFG is a square, therefore we can write,

$DE = EF = FG = GD$

In ΔAGE and ΔGDB,

$\therefore \quad \angle A = \angle DGB = 90°$

$\angle ADE = \angle GBD$

(Corresponding angles)

$\therefore$ By AA similarity, $\Delta EAD \sim \Delta GDB$ ...(i)

Now, In ΔAGF and FCE,

$\angle A = \angle EFC = 90°$

$\angle AED = \angle FCE$

(Corresponding angles)

$\therefore$ By AA similarity, $\Delta AED \sim \Delta FCE$ ...(ii)

From (i) and (ii), we get

$\Delta GDB \sim \Delta FCE$

Since, corresponding sides of two similar triangles are proportional.

$\therefore \quad \dfrac{GD}{FC} = \dfrac{BG}{EF}$

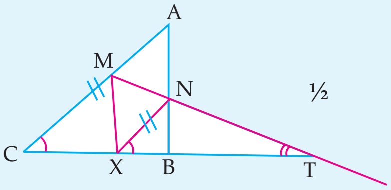
$\Rightarrow \quad GD \times EF = BG \times FC$ **2**

$FG^2 = BG \times FC$ **Hence Proved.**

Q. 6.X is a point on the side BC of ΔABC. XM and XN are drawn parallel to AB and AC respectively meeting AB in N and AC in M. MN produced meets CB produced at T. Prove that $TX^2 = TB \times TC$.

[CBSE Comptt. Set-I, II, III, 2018]

Sol. $\quad \Delta TXN \sim \Delta TCM$



½

$\Rightarrow \quad \dfrac{TX}{TC} = \dfrac{XN}{CM} = \dfrac{TN}{TM}$

$\Rightarrow \quad TX \times TM = TC \times TN$ ...(i) ½

Again, $\Delta TBN \sim \Delta TXM$

$\Rightarrow \quad \dfrac{TB}{TX} = \dfrac{BN}{XM} = \dfrac{TN}{TM}$

$\Rightarrow \quad TM = \dfrac{TN \times TX}{TB}$ ...(ii) ½

Using (ii) in (i), we get

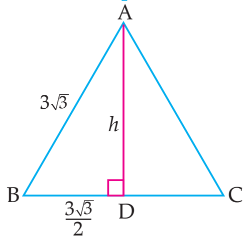
$\Rightarrow \quad TX^2 = \dfrac{TN}{TB} = TC \times TN$

$\Rightarrow \quad TX^2 = TC \times TB$ ½

[CBSE Marking Scheme, 2018]

Q. 7. In an equilateral triangle of side $3\sqrt{3}$ cm, find the length of the altitude. U [CBSE Term-1, 2016, 2015]

Sol.



1

$\Delta ABD, \quad \angle D = 90°$

$\therefore \quad (3\sqrt{3})^2 = h^2 + \left(\dfrac{3\sqrt{3}}{2}\right)^2$

or, $\quad 27 = h^2 + \dfrac{27}{4}$

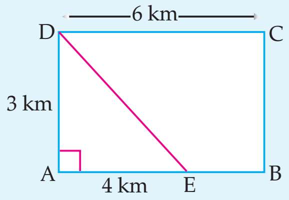
or, $\quad h^2 = 27 - \dfrac{27}{4}$

or, $\quad h^2 = \dfrac{81}{4}$

$\therefore \quad h = \dfrac{9}{2} = 4.5$ cm **1**

Q. 8. In a rectangle *ABCD*, *E* is a point on *AB* such that $AE = \dfrac{2}{3}AB$. If $AB = 6$ km and $AD = 3$ km, then find *DE*. A [CBSE, Term-1, 2016]

Sol.



1

Given, $AE = \dfrac{2}{3}AB = \dfrac{2}{3} \times 6 = 4$ km

In right triangle ADE,

$DE^2 = (3)^2 + (4)^2$

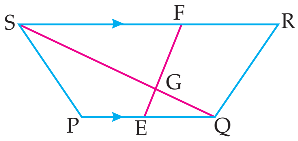
or, $\quad DE^2 = 25$

$\therefore \quad DE = 5$ km. **1**

[CBSE Marking Scheme, 2016]

Q. 9. In the figure, PQRS is a trapezium in which $PQ \parallel RS$. On PQ and RS, there are points E and F respectively such that EF intersects SQ at G. Prove that

$$EQ \times GS = GQ \times FS.$$



U [CBSE Term-1, 2016]

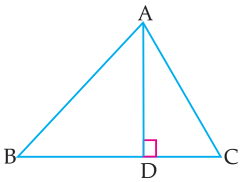
Sol. In ΔGEQ and ΔGFS

$$\angle EGQ = \angle FGS \quad \text{(vert. opp. angles)}$$
$$\angle EQG = \angle FSG \quad \text{(alt. angles)}$$
$$\therefore \quad \Delta GEQ \sim \Delta GFS \quad (AA \text{ similarity}) \quad \mathbf{1}$$
$$\text{or,} \quad \frac{EQ}{FS} = \frac{GQ}{GS}$$
$$\text{or,} \quad EQ \times GS = GQ \times FS. \quad \mathbf{1}$$

[CBSE Marking Scheme, 2016]

Q. 10. In ΔABC, $AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A.

A [CBSE, Term-1, 2015]



Sol. Given, $AD^2 = BD \times CD$

$$\text{or,} \quad \frac{AD}{CD} = \frac{BD}{AD} \quad \tfrac{1}{2}$$
$$\therefore \quad \Delta ADC \sim \Delta BDA \text{ (by } SAS;\ \because \angle D = 90°)$$
$$\text{or,} \quad \angle BAD = \angle ACD;$$
$$\angle DAC = \angle DBA$$

(Corresponding angles of similar triangles) ½

$$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180°$$
$$\text{or,} \quad 2\angle BAD + 2\angle DAC = 180° \quad \tfrac{1}{2}$$
$$\text{or,} \quad \angle BAD + \angle DAC = 90°$$
$$\therefore \quad \angle A = 90° \quad \textbf{Hence proved.} \quad \tfrac{1}{2}$$

Short Answer Type Questions-II — 3 marks each

AI **Q. 1. The perimeter of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm. Find the length of the corresponding side of the second triangle.**

C + A [CBSE SQP, 2020-21]

Sol. $\Delta ABC \sim \Delta DEF$

$$\frac{\text{Perimeter }(\Delta ABC)}{\text{Perimeter }(\Delta DEF)} = \frac{AB+BC+CA}{DE+EF+FD} = \frac{AB}{DE} \quad \mathbf{1}$$
$$\Rightarrow \quad \frac{25}{15} = \frac{9}{x} \quad \tfrac{1}{2}$$

[Using proportionality theorem]

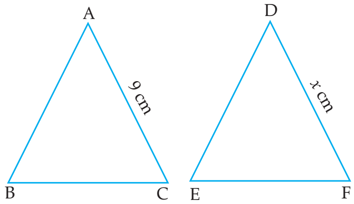
$$\Rightarrow \quad x = 5.4 \text{ cm} \quad \tfrac{1}{2}$$
$$\Rightarrow \quad DE = 5.4 \text{ cm} \quad \mathbf{1}$$

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

Given, perimeters of both triangles are 25 cm, 15 cm
Let both triangles be ABC and DEF.

$$\therefore \quad AB + BC + CA = 25 \text{ cm}$$
$$\text{and} \quad DE + EF + FD = 15 \text{ cm}$$



1

Also given, $CA = 9$ cm
Let $FD = x$ cm

$$\text{Then} \quad \frac{AB+BC+CA}{DE+EF+FD} = \frac{CA}{FD} \quad \mathbf{1}$$

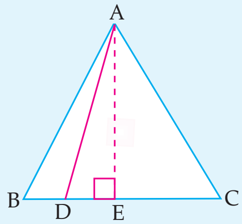
[Given, $\Delta ABC \sim \Delta DEF$]

$$\Rightarrow \quad \frac{25}{15} = \frac{9}{x}$$
$$\Rightarrow \quad 25x = 135$$
$$\Rightarrow \quad x = \frac{135}{25} = 5.4$$

Hence, the length of the corresponding side of second triangle is 5.4 cm. **1**

AI **Q. 2. In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3}$ BC. Prove that $9AD^2 = 7\,AB^2$.** [CBSE SQP, 2020-21]

Sol.



½

Construction: Draw $AM \perp BC$ ½

$$BD = \frac{1}{3}BC$$
$$BM = \frac{1}{2}BC$$

In ΔABM,

$$AB^2 = AM^2 + BM^2 \quad \tfrac{1}{2}$$
$$= AM^2 + (BD + DM)^2$$
$$= AM^2 + DM^2 + BD^2 + 2BD.DM$$
$$= AD^2 + BD^2 + 2BD\,(BM - BD) \quad \tfrac{1}{2}$$
$$= AD^2 + \left(\frac{BC}{3}\right)^2 + 2\frac{BC}{3}\left(\frac{BC}{2} - \frac{BC}{3}\right)$$
$$= AD^2 + 2\frac{BC^2}{9} \quad \tfrac{1}{2}$$
$$= AD^2 + 2\frac{AB^2}{9}$$

Hence, $7AB^2 = 9AD^2$ ½

[CBSE Marking Scheme, 2020-21]

Detailed Solution:

Topper Answer, 2018

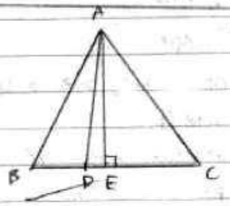
(4) Given: $\triangle ABC$ is equilateral.

$\rightarrow AB = BC = CA$, $\angle A = \angle B = \angle C = 60°$

D is a point on BC such that $BD = \frac{1}{2} BC$.

To prove: $9(AD)^2 = 7(AB)^2$.

Construction: Draw $AE \perp BC$.



Proof: Let $BD = x$.

$\Rightarrow BC = 3x = AB = AC$ [$\because \triangle ABC$ is equilateral] [Given $BD = \frac{1}{3} BC$].

Also, we know that $BE = \frac{1}{2} BC$ [Altitude in equilateral $\triangle$ bisects base].

As $\angle AEB = 90°$,

In ~~triangle~~ $\triangle ABE$, by Pythagoras Theorem,

$BE^2 + AE^2 = AB^2$ $\rightarrow AB^2 = 9x^2$. $\rightarrow$ (a).

$\left(\frac{3x}{2}\right)^2 + AE^2 = (3x)^2$.

$\frac{9x^2}{4} + AE^2 = 9x^2$ $\rightarrow AE^2 = \frac{3 \times 9x^2}{4}$ $\rightarrow$ (1)

Now, in $\triangle ADE$, $\angle E = 90°$. $DE = BE - BD$

By Pythagoras Theorem, $= \frac{3x}{2} - x$.

$DE^2 + AE^2 = AD^2$

$\left(\frac{3x}{2} - x\right)^2 + \frac{27x^2}{4} = AD^2$ [From (1)].

$\left(\frac{x}{2}\right)^2 + \frac{27x^2}{4} = AD^2$.

$\frac{x^2 + 27x^2}{4} = AD^2$

$\Rightarrow AD^2 = \frac{28x^2}{4} = 7x^2 \rightarrow$ (b).

From (a) and (b).

$AB^2 = 9x^2$, $AD^2 = 7x^2$.

$7AB^2 = 63x^2$, $9AD^2 = 63x^2$.

$\Rightarrow 7AB^2 = 9AD^2$.

hence proved.

3

AI **Q. 3. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite the first side is a right angle.** A **[CBSE OD Set-III, 2019; Delhi Set-I, 2020]**

Sol. For correct given, to prove construction. and figure $3 \times ½ = 1½$

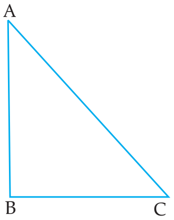
For correct Proof 1½

[CBSE Marking Scheme, 2020]

Detailed Solution:

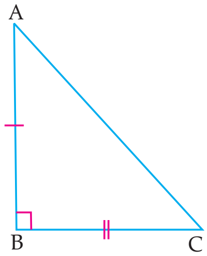
Given: A triangle ABC in which

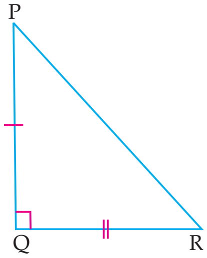
$$AC^2 = AB^2 + BC^2$$



To prove: $\angle B = 90°$

Construction: Draw ΔPQR right angled at Q, such that $PQ = AB$ and $QR = BC$





Proof:

In ΔPQR, $\angle Q = 90°$

$\therefore$ By Pythagoras theorem,

$$PR^2 = PQ^2 + QR^2$$

Since, $PQ = AB$ and $QR = BC$

(By construction)

$\therefore \quad PR^2 = AB^2 + BC^2 \quad$ ...(i)

Also, given

$AC^2 = AB^2 + BC^2 \quad$ ...(ii)

From eq (i) & (ii),

$PR^2 = AC^2$

$\Rightarrow \quad PR = AC \quad$ ...(iii)

Now, in ΔABC and ΔPQR

$AC = PR$ [From (iii)]

$AB = PQ$ (By construction)

$BC = QR$ (By construction)

$\therefore \quad \Delta ABC \cong \Delta PQR$

(By SSS congruence rule)

$\Rightarrow \quad \angle B = \angle Q$ (By cpct)

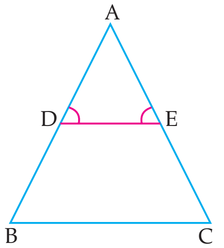
Since, $\angle Q = 90°$ (By construction)

$\therefore \quad \angle B = 90°$ **Hence Proved.**

AI **Q. 4. In the adjoining figure,**

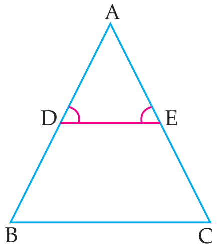
$$\angle D = \angle E \text{ and } \frac{AD}{DB} = \frac{AE}{EC},$$

Prove that ΔBAC is an isosceles triangle.



A **[CBSE Delhi Set-I, 2020]**

Sol. Given: $\angle D = \angle E$ and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$



To prove: ΔBAC is an isosceles triangle.

Proof: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

By converse of *BPT*,

$DE \parallel BC$ **1**

$\therefore \quad \angle ADE = \angle ABC$ (Corresponding angles) ½

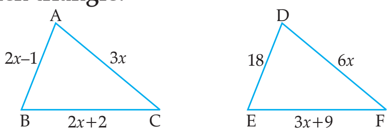
and $\quad \angle AED = \angle ACB$ (Corresponding angles) ½

$\because \quad \angle ADE = \angle AED$ (Given)

$\therefore \quad \angle ABC = \angle ACB$

So, *BAC* is an isosceles triangle. **Hence Proved. 1**

AI **Q. 5. In the given figure, if ΔABC – DEF and their sides of the given figure lengths (in cm) are marked along them, then find the lengths of sides of each triangle.**



C + A **[CBSE OD Set-I, 2020]**

Sol. Given, $\Delta ABC \sim \Delta DEF$

Then according to question,

$$\frac{AB}{BC} = \frac{DE}{EF} \quad \text{[From BPT] ½}$$

$$\Rightarrow \quad \frac{2x-1}{2x+2} = \frac{18}{3x+9} \quad ½$$

$\Rightarrow \quad (2x-1)(3x+9) = 18(2x+2)$

$\Rightarrow \quad (2x-1)(x+3) = 6(2x+2)$

$\Rightarrow \quad 2x^2 - x + 6x - 3 = 12x + 12$

$\Rightarrow \quad 2x^2 + 5x - 12x - 15 = 0$

$\Rightarrow \quad 2x^2 - 7x - 15 = 0 \quad$ ½

$\Rightarrow \quad 2x^2 - 10x + 3x - 15 = 0$

$\Rightarrow \quad 2x(x-5) + 3(x-5) = 0$

$\Rightarrow \quad (x-5)(2x+3) = 0$

Either $x = 5$ or $x = \dfrac{-3}{2}$, which is not possible

So, $\quad x = 5 \quad$ ½

Then in ΔABC, we have

$AB = 2x - 1 = 2 \times 5 - 1 = 9$

$BC = 2x + 2 = 2 \times 5 + 2 = 12$

$AC = 3x = 3 \times 5 = 15 \quad$ ½

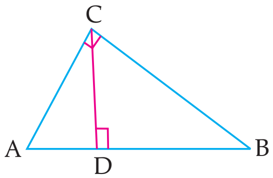
and in ΔDEF, we have

$DE = 18$

$EF = 3x + 9 = 3 \times 5 + 9 = 24$

$DE = 6x = 6 \times 5 = 30. \quad$ ½

Q. 6. In Figure, $\angle ACB = 90°$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



[CBSE Delhi Set-I, 2019]

Sol. $\quad \Delta ACB \sim \Delta ADC$ (AA similarity)

$$\Rightarrow \quad \frac{AC}{BC} = \frac{AD}{CD} \quad \text{...(1) } \mathbf{1}$$

Also $\quad \Delta ACB \sim \Delta CDB$ (AA similarity)

$$\Rightarrow \quad \frac{AC}{BC} = \frac{CD}{BD} \quad \text{...(2) } \mathbf{1}$$

Using equations (1) and (2),

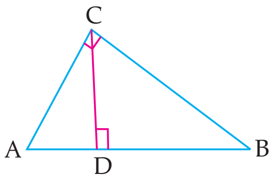
$$\frac{AD}{CD} = \frac{CD}{BD} \quad \mathbf{1}$$

$\Rightarrow \quad CD^2 = AD \times BD$

[CBSE Marking Scheme, 2019]

Alternate Solution:

Given: In ΔACB, $\angle ACB = 90°$ and $CD \perp AB$



To prove: $\quad CD^2 = BD \times AD$

Proof: $\quad$ In ΔCAD, $\angle ADC = 90°$

$CA^2 = CD^2 + AD^2 \quad$ ...(i)

and In ΔCDB, $\angle CDB = 90°$

$CB^2 = CD^2 + BD^2$ ...(ii) **1**

On adding eq's. (i) and (ii), we get

$CA^2 + CB^2 = 2CD^2 + AD^2 + BD^2$

$AB^2 = 2CD^2 + AD^2 + BD^2$

[$\because$ Given $\angle ACB = 90° \Rightarrow AB^2 = CA^2 + CB^2$]

$\Rightarrow AB^2 - AD^2 = BD^2 + 2CD^2$

$\Rightarrow (AB + AD)(AB - AD) = BD^2 + 2CD^2$ **1**

$\Rightarrow (AB + AD)BD - BD^2 = 2CD^2$

$\Rightarrow BD[(AB + AD) - BD] = 2CD^2$

$\Rightarrow BD[AD + (AB - BD)] = 2CD^2$

$\Rightarrow BD[AD + AD] = 2CD^2$

$\Rightarrow BD \times 2AD = 2CD^2$

$\Rightarrow CD^2 = BD \times AD$ **1**

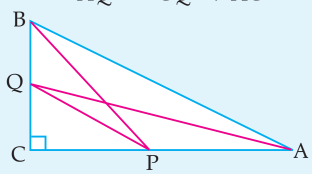
Hence proved.

Q. 7. If P and Q are the points on sides CA and CB respectively of ΔABC, right angled at C. Prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

[CBSE Delhi Set-I, 2019]

Sol. Correct Figure ½

$AQ^2 = CQ^2 + AC^2$ **1**



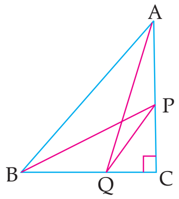
$BP^2 = CP^2 + BC^2$ ½

$\therefore AQ^2 + BP^2 = (CQ^2 + CP^2) + (AC^2 + BC^2)$

$= PQ^2 + AB^2.$ **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:



In right angled triangles ACQ and PCB

$AQ^2 = AC^2 + CQ^2$ ...(i) ½

and $BP^2 = PC^2 + CB^2$ ...(ii) ½

Adding eq (i) and eq (ii), we get

$AQ^2 + BP^2 = (AC^2 + CQ^2) + (PC^2 + CB^2)$ **1**

or $AQ^2 + BP^2 = (AC^2 + CB^2) + (PC^2 + CQ^2)$

or $AQ^2 + BP^2 = AB^2 + PQ^2$ **Hence proved. 1**

Commonly Made Error

- Some candidates make mistakes in applying Pythagoras theorem.

Answering Tip

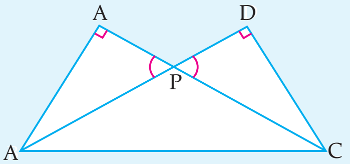
- Students should analyse the diagram carefully to write correct conditions to prove the question.

AI **Q. 8. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times DP$.** **[CBSE OD Set-1, 2019]**

Sol. $\Delta APB \sim \Delta DPC$ [AA similarity] **1**

$\frac{AP}{DP} = \frac{BP}{PC}$ **1**

$\Rightarrow AP \times PC = BP \times DP$ ½

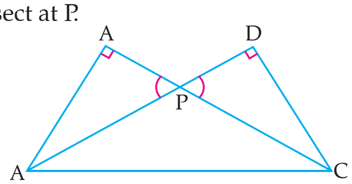


½

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given: ∠BAC and ∠BDC are 90° each AC and BD intersect at P.



To prove: $AP \times PC = BP \times DP$.

Proof: In ΔBAP and ΔCDP,

$\angle A = \angle D$ (Given)

$\angle BPA = \angle DPC$

(vertically opposite angles)

$\therefore \Delta BAP \sim \Delta CDP$ (By AA similarity) **1**

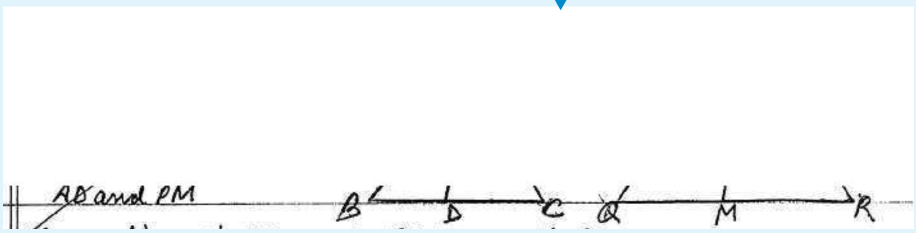
Then $\frac{AP}{DP} = \frac{BP}{PC}$

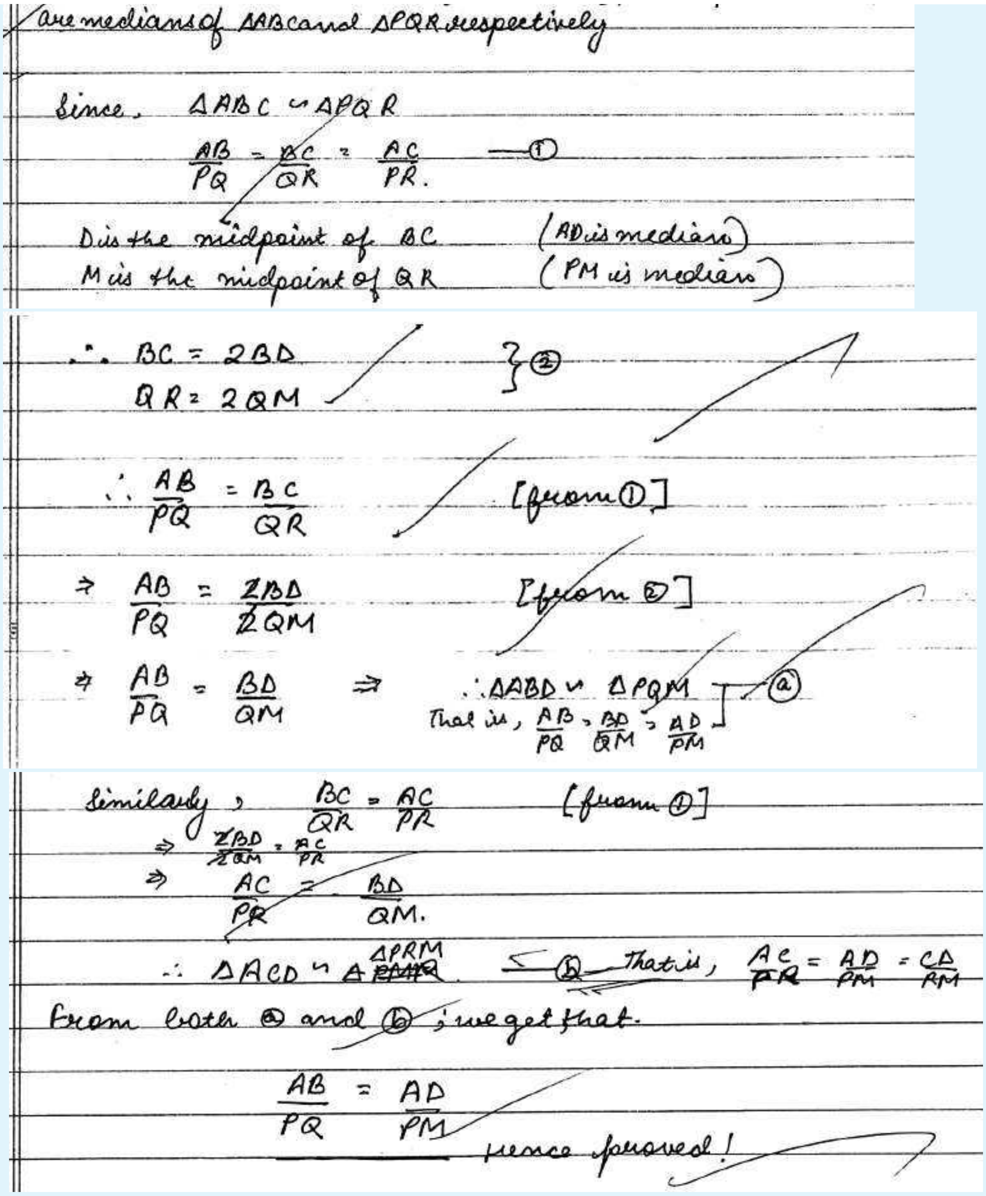
or, $AP \times PC = BP \times DP$. **Hence proved. 1**

Q. 9. In similar triangles, ΔABC and ΔPQR, AD and PM are the medians respectively. Prove that $\frac{AD}{PM} = \frac{AB}{PQ}$.

A **[CBSE Board Term, 2019]**

Topper Answer, 2019





are medians of ΔABC and ΔPQR respectively

Since, $\Delta ABC \sim \Delta PQR$

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ —(1)

D is the midpoint of BC (AD is median)

M is the midpoint of QR (PM is median)

$\therefore$ $BC = 2BD$, $QR = 2QM$ }(2)

$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$ [from (1)]

$\Rightarrow \frac{AB}{PQ} = \frac{2BD}{2QM}$ [from (2)]

$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM}$ $\Rightarrow$ $\therefore \Delta ABD \sim \Delta PQM$ —(a)

That is, $\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$

Similarly, $\frac{BC}{QR} = \frac{AC}{PR}$ [from (1)]

$\Rightarrow \frac{2BD}{2QM} = \frac{AC}{PR}$

$\Rightarrow \frac{AC}{PR} = \frac{BD}{QM}$.

$\therefore \Delta ACD \sim \Delta PRM$ —(b) That is, $\frac{AC}{PR} = \frac{AD}{PM} = \frac{CD}{RM}$

From both (a) and (b), we get that.

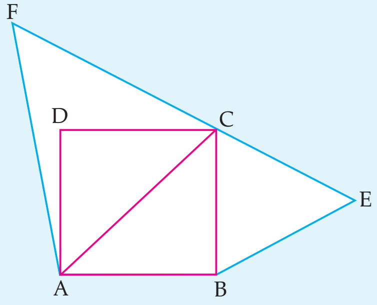
$\frac{AB}{PQ} = \frac{AD}{PM}$

Hence proved!

3

AI Q. 10. Prove that area of the equilateral triangle described on the side of a square is half of the area of the equilateral triangle described on its diagonal. A [CBSE Delhi Set-2018]

Sol.



Let the side of the square be 'a' units

$\therefore \quad AC^2 = a^2 + a^2 = 2a^2$

$\Rightarrow \quad AC = \sqrt{2}a$ units **1**

Area of equilateral triangle $\Delta BCE = \frac{\sqrt{3}}{4}a^2$ sq.u ½

Area of equilateral triangle

$\Delta ACF = \frac{\sqrt{3}}{4}\left(\sqrt{2}a\right)^2 = \frac{\sqrt{3}}{2}a^2$ sq.u **1**

Area of $\Delta BCE = \frac{1}{2}$ Area of ΔACF ½

[CBSE Marking Scheme, 2018]

Detailed Solution:

Topper Answer, 2018

(Choice 1)

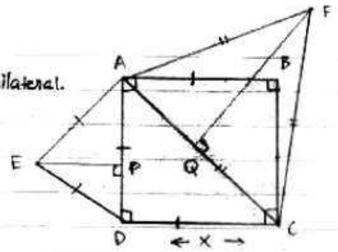
Given: Square ABCD. ΔAED and ΔAFC are equilateral.

To prove: Area $\Delta AFC = 2 \times$ Area ΔAED.

Construction: Draw $EP \perp AD$ and $FQ \perp AC$.

Proof: Let side of square be x.

$\Rightarrow$ sides of $\Delta AED = x$.



In ΔABC, $\angle B = 90°$.

$\Rightarrow$ By Pythagoras Theorem,

$AB^2 + BC^2 = AC^2$

$x^2 + x^2 = AC^2 \Rightarrow AC = \sqrt{2}x$. $\Rightarrow$ Sides of $\Delta AFC = \sqrt{2}x$.

We know, altitude of equilateral Δ bisects the base.

$\rightarrow PD = \frac{x}{2}$, $AQ = \frac{x}{\sqrt{2}}$.

In ΔAEP, $\angle P = 90°$.

By Pythagoras theorem, $AE^2 = EP^2 + AP^2$.

$x^2 = EP^2 + \left(\frac{x}{2}\right)^2$.

$EP^2 = \frac{3x^2}{4} \Rightarrow EP = \frac{\sqrt{3}}{2}x$.

In ΔAFQ, $\angle Q = 90°$

By Pythagoras theorem, $AF^2 = FQ^2 + AQ^2$

$2x^2 = FQ^2 + \frac{x^2}{2}$

$FQ^2 = \frac{3x^2}{2} \Rightarrow FQ = \frac{\sqrt{3}x}{\sqrt{2}}$.

We know, Area of triangle $= \frac{1}{2} \times$ Base $\times$ height sq. units.

$\Rightarrow$ Area of $\Delta AFC = \frac{1}{2} \times \sqrt{2}x \times FQ$

$= \frac{1}{2} \cdot \sqrt{2}x \cdot \frac{\sqrt{3}}{\sqrt{2}}x$

$= \frac{\sqrt{3}}{2}x^2$.

Area of $\Delta AED = \frac{1}{2} \times x \times EP$.

$= \frac{1}{2} \cdot x \cdot \frac{\sqrt{3}}{2}x$

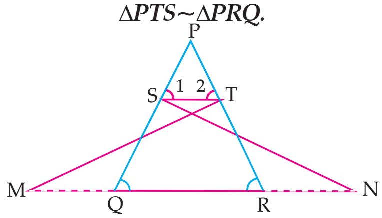
$= \frac{\sqrt{3}}{4}x^2$.

$2 \cdot$ Area of $\Delta AED = \frac{\sqrt{3}}{2}x^2 =$ Area of ΔAFC.

hence proved.

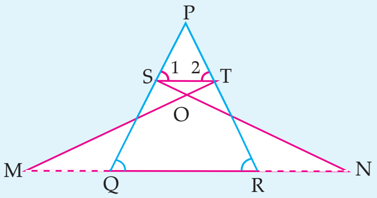
3

Q. 11. In figure $\angle 1 = \angle 2$ and $\Delta NSQ \cong \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRQ$.



A [CBSE SQP, 2018-2019]

Sol.



$\angle 1 = \angle 2$

$\Rightarrow \quad PT = PS$ ....(i) **1**

$\Delta NSQ \cong \Delta MTR$

$\Rightarrow \quad \angle NQS = \angle MRT$

$\Rightarrow \quad \angle PQR = \angle PRQ$

$\Rightarrow \quad PR = PQ$ ....(ii) **1**

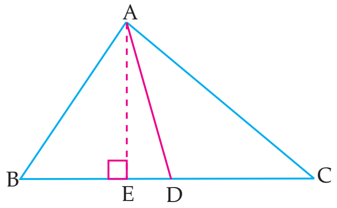
From (i) and (ii), $\frac{PT}{PR} = \frac{PS}{PQ}$

Also, $\angle TPS = \angle RPQ$ (common)

$\Rightarrow \quad \Delta PTS \sim \Delta PRQ$ **1**

[CBSE Marking Scheme, 2018]

Q. 12. In ΔABC, if AD is the median, then show that $AB^2 + AC^2 = 2(AD^2 + BD^2)$.



[A] [CBSE SQP, 2018-19]

Sol. In ΔABE,

$$AB^2 = AE^2 + BE^2 \quad \text{(Pythagoras theorem)}$$

or, $$AB^2 = AD^2 - DE^2 + (BD - DE)^2$$
$$= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$$

$\therefore$ $$AB^2 = AD^2 + BD^2 - 2BD \times DE \quad \text{....(i)} \; \mathbf{1}$$

In ΔAEC,

$$AC^2 = AE^2 + EC^2$$

or, $$AC^2 = (AD^2 - ED^2) + (ED + DC)^2$$
$$= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC$$

or, $$AC^2 = AD^2 + CD^2 + 2ED \times DC$$

or, $$AC^2 = AD^2 + DC^2 + 2DC \times DE \quad \text{....(ii)} \; \mathbf{1}$$

Adding equations (i) and (ii),

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \quad [\because BD = DC] \; \mathbf{1}$$

Hence Proved.

Q. 13. If the area of two similar triangles are equal, then prove that they are congruent.

[A] [CBSE Delhi/OD Set-2018]

Sol. Let, $\Delta ABC \sim \Delta PQR$

$\therefore$ $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} \quad \mathbf{1}$$

Given, ar (ΔABC)= ar (ΔPQR)

$\Rightarrow$ $$\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2} = 1 \quad \mathbf{1}$$

$\Rightarrow$ $AB = PQ$, $BC = QR$ and $AC = PR$

$\Rightarrow$ Therefore, $\Delta ABC \cong \Delta PQR$ **1**

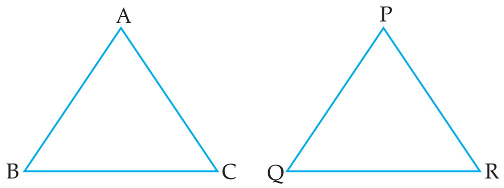
(SSS Congruence Rule)

[CBSE Marking Scheme, 2018]

Detailed Solution:

Given: $\Delta ABC \sim \Delta PQR$,

and $ar\Delta ABC = ar\Delta PQR$



To prove: $\Delta ABC \cong \Delta PQR$

Proof: $\Delta ABC \sim \Delta PQR$ (Given)

or, $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} \quad \text{...(i)}$$

Also $ar(\Delta ABC) = ar(\Delta PQR)$ (Given)

or, $$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1 \quad \mathbf{1}$$

From equation (i), we have

$$\frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2} = 1$$

or, $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = 1 \quad \mathbf{1}$$

or, $AB = PQ$,
$BC = QR$

and $CA = RP$

$\Delta ABC \cong \Delta PQR$ (SSS) **1**

COMMONLY MADE ERROR

- Most candidates are not able to prove $\Delta ABC \cong \Delta PQR$

ANSWERING TIP

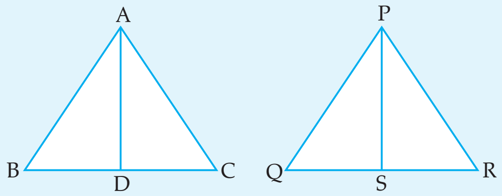
- Candidates should know about SSS-criteria for Congruence of triangles.

Q. 14. If $\Delta ABC \sim \Delta PQR$ and AD and PS are bisectors of corresponding angles A and P, then prove that

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}.$$

[U] [Board Term-1, 2016]

Sol.



$$\Delta ABC \sim \Delta PQR$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \quad \mathbf{1}$$

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} \quad \text{...(i)}$$

$$\angle A = \angle P$$

or, $$\frac{1}{2}\angle A = \frac{1}{2}\angle P$$

or, $$\angle BAD = \angle QPS$$

$$\Delta BAD \sim \Delta QPS \quad (AA \text{ similarity}) \; \mathbf{1}$$

$$\frac{BA}{QP} = \frac{AD}{PS} \quad \text{...(ii)}$$

By eqs. (i) and (ii),

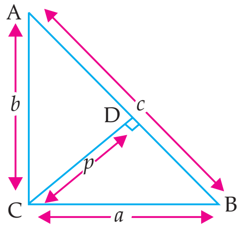
$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}$$ **Hence Proved. 1**

[CBSE Marking Scheme, 2016]

[AI] **Q. 15. ΔABC is a right angled at C. If p is the length of the perpendicular from C to AB and a, b, c are the lengths of the sides opposite to $\angle A$, $\angle B$ and $\angle C$ respectively, then prove that $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.**

[A] [Board Term-1, 2016]

Sol.



In ΔACB and ΔCDB,

$$\angle ACB = \angle CDB = 90°$$

$$\angle B = \angle B \quad \text{(common)}$$

$\therefore$ $\Delta ACB \sim \Delta CDB$ (by AA Similarity) **1**

or, $$\frac{b}{p} = \frac{c}{a}$$

or, $$\frac{1}{p} = \frac{c}{ab}$$ **1**

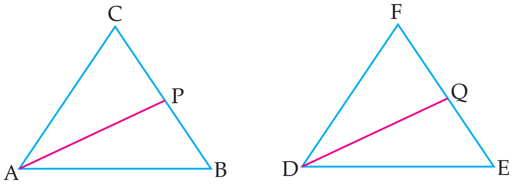
Squaring on both sides,

$$\frac{1}{p^2} = \frac{c^2}{a^2b^2}$$

or, $$\frac{1}{p^2} = \frac{a^2+b^2}{a^2b^2} \quad [\because c^2 = a^2+b^2]$$

$\therefore$ $$\frac{1}{p^2} = \frac{1}{a^2}+\frac{1}{b^2}$$ **Hence Proved. 1**

Q. 16. In given figure $\Delta ABC \sim \Delta DEF$. AP bisects $\angle CAB$ and DQ bisects $\angle FDE$.

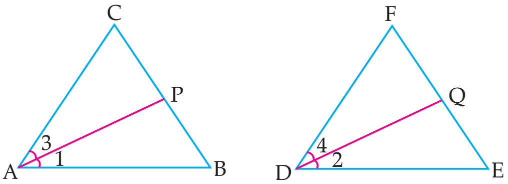


Prove that:

(i) $\dfrac{AP}{DQ} = \dfrac{AB}{DE}$

(ii) $\Delta CAP \sim \Delta FDQ$. U [Board Term-1, 2016]

Sol.



(i) Here, $\Delta ABC \sim \Delta DEF$

$\therefore$ $\angle A = \angle D$ (corresponding angles)

and $2\angle 1 = 2\angle 2$

or, $\angle 1 = \angle 2$

Also $\angle B = \angle E$ (corresponding angles)

Since, $\Delta APB \sim \Delta DQE$ **1**

Hence, $$\frac{AP}{DQ} = \frac{AB}{DE}$$ **1**

(ii) $\because$ $\Delta ABC \sim \Delta DEF$

$\therefore$ $\angle A = \angle D$

and $\angle C = \angle F$

Hence, $2\angle 3 = 2\angle 4$ or, $\angle 3 = \angle 4$ **1**

$\therefore$ $\Delta CAP \sim \Delta FDQ$ (By AA similarity)

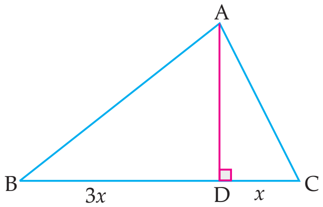
Hence Proved.

AI **Q. 17. The perpendicular AD on the base BC of a ΔABC intersects BC at D so that $DB = 3CD$. Prove that $2(AB)^2 = 2(AC)^2 + BC^2$.**

U [Board Term-1, 2016]

Sol. Given, in ΔADB, $AB^2 = AD^2 + BD^2$ ...(i)

(Pythagoras Theorem)



In ΔADC, $AC^2 = AD^2 + CD^2$ ...(ii)

(Pythagoras theorem)

Subtracting eqn. (ii) from eqn. (i),

$$AB^2 - AC^2 = BD^2 - CD^2$$ **1**

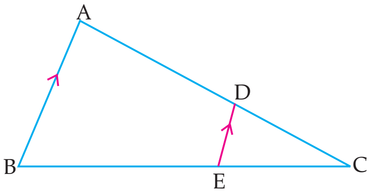
or, $$= \left(\frac{3}{4}BC\right)^2 - \left(\frac{1}{4}BC\right)^2$$

or, $$= \frac{9}{16}BC^2 - \frac{1}{16}BC^2 = \frac{BC^2}{2}$$ **1**

$\therefore$ $2(AB^2 - AC^2) = BC^2$

$\therefore$ $2(AB)^2 = 2AC^2 + BC^2$. **Hence proved. 1**

Q. 18. In given figure, D is a point on AC such that $AD = 2CD$, also $DE || AB$.



Find: $\dfrac{ar(\Delta DCE)}{ar(\Delta ACB)}$ U [Board Term-1, 2015]

Sol. Given $AD = 2CD$

In ΔCDE and ΔCAB

$$\angle C = \angle C \quad \text{(Common)}$$

$$\angle CDE = \angle CAB$$

(Corresponding angles)

$\therefore$ $\Delta CDE \sim \Delta CAB$ (By AA similarity rule)

Now $$\frac{ar(\Delta DCE)}{ar(\Delta ACB)} = \frac{CD^2}{CA^2} = \frac{CD^2}{(AD+DC)^2}$$

or, $$\frac{ar(\Delta DCE)}{ar(\Delta ACB)} = \frac{CD^2}{(3CD)^2} = \frac{1}{9}$$ **3**

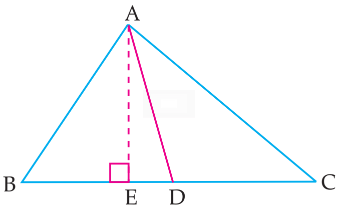
[CBSE Marking Scheme, 2015]

AI Q. 19. If in ΔABC, AD is median and $AE \perp BC$, then prove that $AB^2 + AC^2 = 2AD^2 + \frac{1}{2} BC^2$.

A [Board Term-1, 2015]

Sol. To prove:

$$AB^2 + AC^2 = 2AD^2 + \frac{1}{2}(BC)^2$$



Draw $AE \perp BC$

In ΔABE,

$$AB^2 = AE^2 + BE^2 \quad \text{(Pythagoras theorem)}$$

or, $$AB^2 = AD^2 - DE^2 + (BD - DE)^2$$
$$= AD^2 - DE^2 + BD^2 + DE^2 - 2BD \times DE$$

$\therefore$ $$AB^2 = AD^2 + BD^2 - 2BD \times DE \quad \text{....(i)} \ \mathbf{1}$$

In ΔAEC,

$$AC^2 = AE^2 + EC^2$$

or, $$AC^2 = (AD^2 - ED^2) + (ED + DC)^2$$
$$= AD^2 - ED^2 + ED^2 + DC^2 + 2ED \times DC$$

or, $$AC^2 = AD^2 + CD^2 + 2ED \times CD$$

or, $$AC^2 = AD^2 + DC^2 + 2DC \times DE \quad \text{....(ii)} \ \mathbf{1}$$

Adding eqns. (i) & (ii),

$$AB^2 + AC^2 = 2(AD^2 + BD^2) \qquad (\because BD = DC)$$

$$= \left[2AD^2 + 2\left(\frac{1}{2}BC\right)^2\right]$$

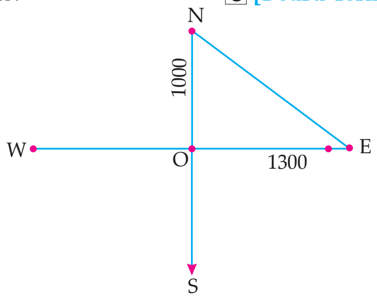
$$= 2AD^2 + \frac{1}{2}BC^2 \quad \left(\text{as } BD = \frac{1}{2}BC\right) \ \mathbf{1}$$

Hence Proved.

Q. 20. From an airport, two aeroplanes start at the same time. If speed of first aeroplane due North is 500 km/h and that of other due East is 650 km/h then find the distance between the two aeroplanes after 2 hours.

C [Board Term-1, 2015]

Sol.



Distance covered by first aeroplane due North after two hours = 500 × 2 = 1000 km. **1**

Distance covered by second aeroplane due East after two hours = 650 × 2 = 1300 km. **1**

Distance between two aeroplanes after 2 hours

$$NE = \sqrt{ON^2 + OE^2}$$
$$= \sqrt{(1000)^2 + (1300)^2}$$
$$= \sqrt{1000000 + 1690000}$$
$$= \sqrt{2690000}$$
$$= 1640.12 \text{ km} \quad \mathbf{1}$$

Long Answer Type Questions

5 marks each

Q. 1. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

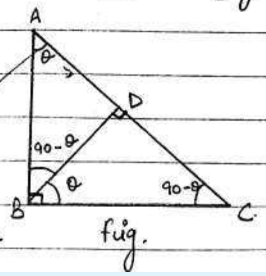
A [CBSE Delhi/OD Set-2018] [SQP, 2018-19]

Topper Answer, 2019

Sol.

To prove : Square of hypotenuse, in a right triangle, is equal to the sum of squares of other two sides. (Pythagoras theorem)

That is, $AC^2 = AB^2 + BC^2$.



Construction : Construct $BD \perp AC$

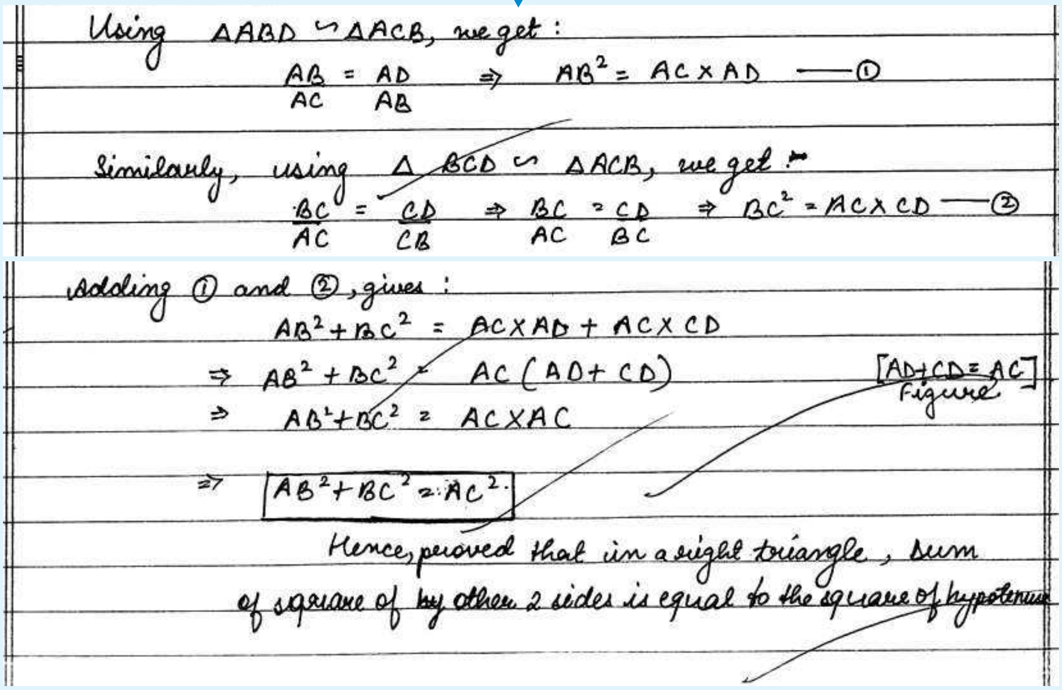
Name $\angle BAC = \theta$.

Then, $\angle BCA = 90 - \theta$, $\angle ABD = 90 - \theta$, $\angle DBC = \theta$

It is clear that,

$\Delta ABD \sim \Delta ACB \sim \Delta BCD$.

Topper Answer, 2019


Using $\Delta ABD \sim \Delta ACB$, we get :

$\frac{AB}{AC} = \frac{AD}{AB} \Rightarrow AB^2 = AC \times AD$ —— ①

Similarly, using $\Delta BCD \sim \Delta ACB$, we get :-

$\frac{BC}{AC} = \frac{CD}{CB} \Rightarrow \frac{BC}{AC} = \frac{CD}{BC} \Rightarrow BC^2 = AC \times CD$ —— ②

Adding ① and ②, gives :

$AB^2 + BC^2 = AC \times AD + AC \times CD$

$\Rightarrow AB^2 + BC^2 = AC(AD + CD)$ [AD+CD=AC] Figure

$\Rightarrow AB^2 + BC^2 = AC \times AC$

$\Rightarrow \boxed{AB^2 + BC^2 = AC^2}$

Hence, proved that in a right triangle, sum of square of the other 2 sides is equal to the square of hypotenuse

5

Q. 2. Prove that in a right triangle, the square of the hypotenuse is equal to sum of squares of other two sides. Using the above result, prove that, in rhombus *ABCD*,

$4\,AB^2 = AC^2 + BD^2$. A [Board Term-1, 2015]

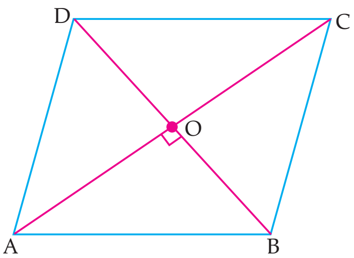
[Sample Question Paper 2017]

Sol. We have already proved $AB^2 + BC^2 = AC^2$ in above Q.1.

Given: *ABCD* is a rhombus. 1

Construction: Draw diagonals *AC* and *BD*

$\therefore \qquad AO = OC = \frac{1}{2}AC$



1

and $\qquad BO = OD = \frac{1}{2}BD$ 1

$AC \perp BD$ [∵ ▭ *ABCD* is rhombus]

To prove: $4\,AB^2 = AC^2 + BD^2$

Proof: $\angle AOB = 90°$ (Diagonal of rhombus bisect each other at right angle)

$AB^2 = OA^2 + OB^2$

or, $\qquad AB^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$ 1

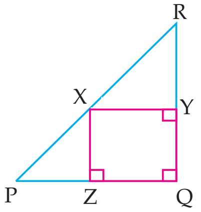
$$AB^2 = \frac{AC^2}{4} + \frac{BD^2}{4}$$

Hence, $\qquad 4AB^2 = AC^2 + BD^2$ 1

Hence proved.

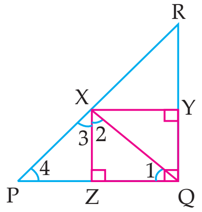
Q. 3. ΔPQR is right angled at *Q*. $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that

$XZ^2 = PZ \times ZQ$.



U [Board Term-1, 2015]

Sol.



1

Here, $RQ \perp PQ$ and $XZ \perp PQ$

or, $\qquad XZ \parallel YQ$

∴ Similarly, $\qquad XY \parallel ZQ$

XYQZ is a rectangle. (∵ $\angle PQR = 90°$) 1

In ΔXZQ, $\quad \angle 1 + \angle 2 = 90°$ ...(i)

and in ΔPZX, $\angle 3 + \angle 4 = 90°$ ...(ii)

$XQ \perp PR$ or, $\quad \angle 2 + \angle 3 = 90°$ ...(iii)

By eqs. (i) and (iii), we get ½

$\angle 1 = \angle 3$

By eqs. (ii) and (iii), we get ½

$$\angle 2 = \angle 4$$

$\therefore$ $\Delta PZX \sim \Delta XZQ$ (*AA* similarity) **1**

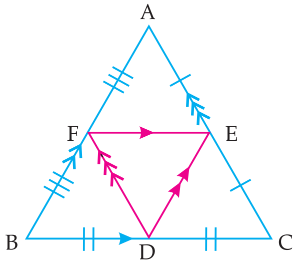
$$\therefore \quad \frac{PZ}{XZ} = \frac{XZ}{ZQ}$$

Thus, $XZ^2 = PZ \times ZQ$ **1**

Hence Proved.

AI Q. 4. In ΔABC, the mid-points of sides BC, CA and AB are D, E and F respectively. Find ratio of ar(ΔDEF) to ar(ΔABC). A [Board Term-1, 2015]

Sol.



1

In ΔABC, given that F, E and D are the mid-points of AB, AC and BC respectively.

Hence, $FE \parallel BC$, $DE \parallel AB$ and $DF \parallel AC$. **1**

By mid-point theorem,

If $DE \parallel BA$

then $DE \parallel BF$

and if $FE \parallel BC$

then $FE \parallel BD$ **1**

$\therefore$ *FEDB* is a parallelogram in which *DF* is diagonal and a diagonal of parallelogram divides it into two equal Areas.

Hence ar(ΔBDF) = ar(ΔDEF) ...(i)

Similarly ar(ΔCDE) = ar(ΔDEF) ...(ii)

or (ΔAFE) = ar(ΔDEF) ...(iii)

or (ΔDEF) = ar(ΔDEF) ...(iv)

On adding eqns. (i), (ii), (iii) and (iv),

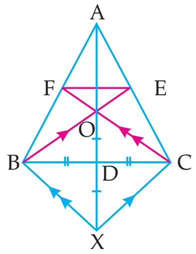
ar(ΔBDF) + ar(ΔCDE) + ar(ΔAFE) + ar(ΔDEF) **1**

= 4ar(ΔDEF)

or, ar(ΔABC) = 4ar(ΔDEF)

$$\frac{\text{ar}(\Delta DEF)}{\text{ar}(\Delta ABC)} = \frac{1}{4}$$ **1**

Q. 5. In ΔABC, AD is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure.



Prove that:

(i) $EF \parallel BC$

(ii) $AO : AX = AF : AB$ A [Board Term-1, 2015]

Sol. (i) Since, BC and OX bisect each other.

So, *BXCO* is a parallelogram then $BE \parallel XC$ and $BX \parallel CF$.

In ΔABX, by B.P.T.,

$$\frac{AF}{FB} = \frac{AO}{OX}$$ ...(i) **1**

In ΔAXC,

$$\frac{AE}{EC} = \frac{AO}{OX}$$ ...(ii) **1**

Eqn. (i) and (ii) gives,

$$\frac{AF}{FB} = \frac{AE}{EC}$$ **1**

So by converse of B.P.T.,

$$EF \parallel BC$$

(ii) Given $\dfrac{OX}{OA} = \dfrac{FB}{AF}$ **1**

Adding 1 on both sides

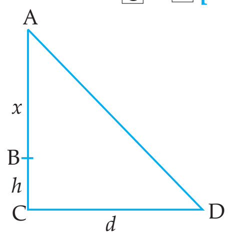
$$\frac{OX}{OA}+1 = \frac{FB}{AF}+1$$

$$\frac{OX+OA}{OA} = \frac{FB+AE}{AF}$$

$$\frac{AX}{OA} = \frac{AB}{AF}$$ (from fig.)

or $OA : AX = AF : AB$ **Hence Proved. 1**

Q. 6. In the right triangle, B is a point on AC such that $AB + AD = BC + CD$. If $AB = x$, $BC = h$ and $CD = d$, then find x (in terms of h and d). C + U [Board Term-1, 2015]



Sol. Given, $AB + AD = BC + CD$

$AD = BC + CD - AB$

or, $AD = h + d - x$ **1½**

In *rt* ΔACD,

$AD^2 = AC^2 + DC^2$

or, $(h + d - x)^2 = (x + h)^2 + d^2$

or, $(h + d - x)^2 - (x + h)^2 = d^2$ **1½**

or, $(h + d - x - x - h)(h + d - x + x + h) = d^2$

or, $(d - 2x)(2h + d) = d^2$

or, $2hd + d^2 - 4hx - 2xd = d^2$

or, $2hd = 4hx + 2xd$

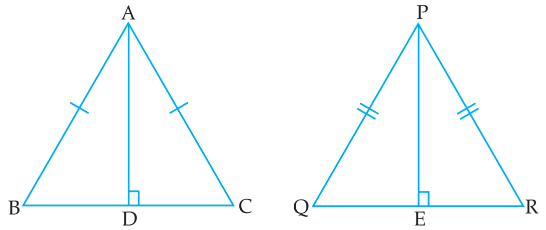
$= 2(2h + d)x$

or, $x = \dfrac{hd}{2h+d}$ **2**

[CBSE Marking Scheme, 2015]

Q. 7. Vertical angles of two isosceles triangles are equal. If their areas are in the ratio 16 : 25, then find the ratio of their altitudes drawn from vertex to the opposite side. U [Board Term-1, 2015]

Sol.



1

Given: $\angle A = \angle P$

$\angle B = \angle Q, \angle C = \angle R$

Proof: Let $\angle A = \angle P$ be x

In ΔABC,

$$\angle A + \angle B + \angle C = 180°$$

or, $x° + \angle B + \angle B = 180°$ (given, $\angle B = \angle C$)

or, $2\angle B = 180° - x$

or, $\angle B = \dfrac{180° - x}{2}$ ...(i) **1**

Now, in ΔPQR

$\angle P + \angle Q + \angle R = 180°$ (Given, $\angle Q = \angle R$)

or, $x° + \angle Q + \angle Q = 180°$

or, $2\angle Q = 180° - x$

or, $\angle Q = \dfrac{180° - x}{2}$ ...(ii) **1**

In ΔABC and ΔPQR,

$\angle A = \angle P$ [Given]

$\angle B = \angle Q$ [from eqs. (i) and (ii)]

$\Delta ABC \sim \Delta PQR$ (AA similarity)

or, $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{AD^2}{PE^2}$ **1**

or, $\dfrac{16}{25} = \dfrac{AD^2}{PE^2}$

or, $\dfrac{4}{5} = \dfrac{AD}{PE}$

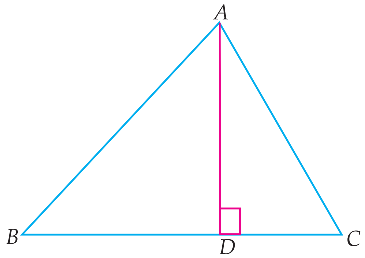
$\therefore$ $\dfrac{AD}{PE} = \dfrac{4}{5}$. **1**

AI Q. 8. In ΔABC, $AD \perp BC$ and point D lies on BC such that $2\,DB = 3\,CD$. Prove that

$$5AB^2 = 5AC^2 + BC^2.$$

U **[Board Term-1, 2015]**

Sol.



Given: $AD \perp BC$

$2DB = 3CD$

To prove: $5\,AB^2 = 5AC^2 + BC^2$

Proof: Since, $2DB = 3CD$

or, $\dfrac{DB}{CD} = \dfrac{3}{2}$ **1**

or, Let DB be $3x$, CD be $2x$, so $BC = 5x$

In ΔADB, $\angle D = 90°$

$AB^2 = AD^2 + DB^2$ **1**

or, $AB^2 = AD^2 + (3x)^2$

or, $AB^2 = AD^2 + 9x^2$

Now $5AB^2 = 5AD^2 + 45\,x^2$

or, $5AD^2 = 5\,AB^2 - 45x^2$ ...(i) **1**

and $AC^2 = AD^2 + CD^2$

or, $AC^2 = AD^2 + (2x)^2$

or, $AC^2 = AD^2 + 4x^2$

or, $5AC^2 = 5AD^2 + 20x^2$

or, $5AD^2 = 5AC^2 - 20x^2$ ...(ii) **1**

From equation (i) & (ii),

$5AB^2 - 45x^2 = 5AC^2 - 20x^2$

or, $5AB^2 = 5AC^2 - 20x^2 + 45x^2$

or, $5AB^2 = 5AC^2 + 25x^2$

or, $5AB^2 = 5AC^2 + (5x)^2$

$\therefore$ $5AB^2 = 5AC^2 + BC^2$ [$\because$ $BC = 5x$]

Hence proved. 1

Visual Case Based Questions — 4 marks each

Note: Attempt any four sub parts from each question. Each sub part carries 1 mark

AI Q. 1. SCALE FACTOR AND SIMILARITY

SCALE FACTOR

A scale drawing of an object is of the same shape as the object but of a different size.

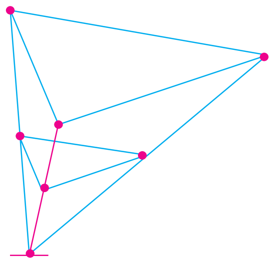
The scale of a drawing is a comparison of the length used on a drawing to the length it represents.

The value of scale is written as a ratio.

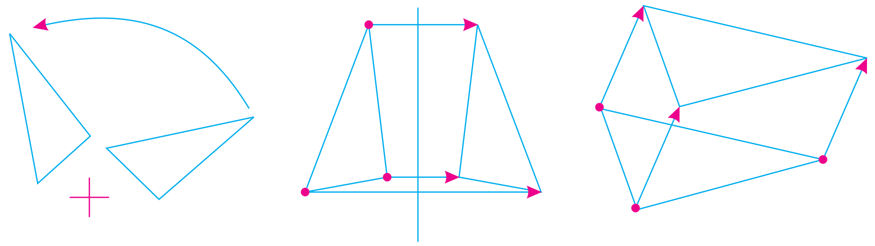
SIMILAR FIGURES

The ratio of two corresponding sides in similar figures is called the scale factor.

Scale factor = $\dfrac{\text{length in image}}{\text{corresponding length in object}}$



If one shape can become another using Resizing then the shapes are Similar.



Rotation or Turn   Reflection or Flip   Translation or Slide

Hence, two shapes are Similar when one can become the other after a resize, flip, slide or turn.

C + A **[CBSE SQP, 2020-21]**

(i) A model of a boat is made on the scale of 1 : 4. The model is 120 cm long. The full size of the boat has a width of 60 cm. What is the width of the scale model ?

(a) 20 cm **(b) 25 cm**
(c) 15 cm **(d) 240 cm**

Sol. Correct option: **(c)**.
Explanation: Width of the scale model = 60/4 = 15 cm.
[CBSE SQP Marking Scheme, 2020-21]

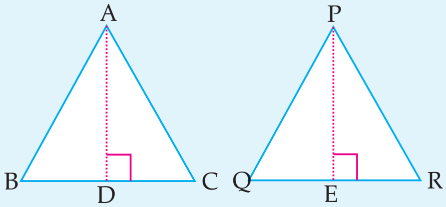
(ii) What will effect the similarity of any two polygons ?
(a) They are flipped horizontally
(b) They are dilated by a scale factor
(c) They are translated down
(d) They are not the mirror image of on another.

Sol. Correct option: **(d)**.
Explanation: They are not the mirror image of one another. **[CBSE SQP Marking Scheme, 2020-21]**

(iii) If two similar triangles have a scale factor of $a : b$, which statement regarding the two triangles is true ?
(a) The ratio of their perimeters is $3a : b$
(b) Their altitudes have a ratio $a : b$
(c) Their medians have a ratio $\frac{a}{2} : b$
(d) Their angle bisectors have a ratio $a^2 : b^2$

Sol. Correct option: **(b)**.
Explanation: Let ABC and PQR be two similar triangles and AD, PE are two altitudes:

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$
(corresponding sides)

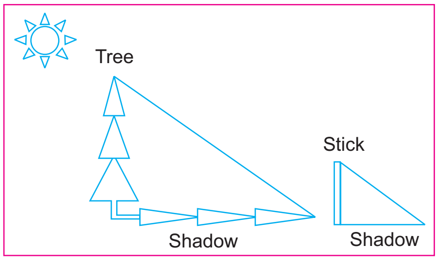


$\therefore \angle B = \angle Q$ and $\angle ADB = \angle PEQ$ (each 90°)

Now, $\frac{AD}{PE} = \frac{AB}{PQ} = \frac{a}{b}$ (corresponding sides)

[CBSE SQP Marking Scheme, 2020-21]

(iv) The shadow of a stick 5 m long is 2 m. At the same time the shadow of a tree 12.5 m high is



(a) 3 m **(b) 3.5 m**
(c) 4. 5 m **(d) 5 m**

Sol. Correct option: **(d)**.
Explanation:
Let shadow of the tree be x.
By the property to similar triangles
we have $\frac{5}{2} = \frac{12.5}{x}$

$$x = \frac{(12.5 \times 2)}{5} = 5 \text{ m}$$

[CBSE SQP Marking Scheme, 2020-21]

(v) Below you see a student's mathematical model of a farmhouse roof with measurements. The attic floor, ABCD in the model, is a square. The beams that support the roof are the edge of a rectangular prism, EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT, and H is the middle of DT. All the edges of the pyramid in the model have length of 12 m.

What is the length of EF, where EF is one of the horizontal edges of the block ?

(a) 24 m **(b) 3 m**

(c) 6 m **(d) 10 m**

Sol. Correct option: **(c)**.

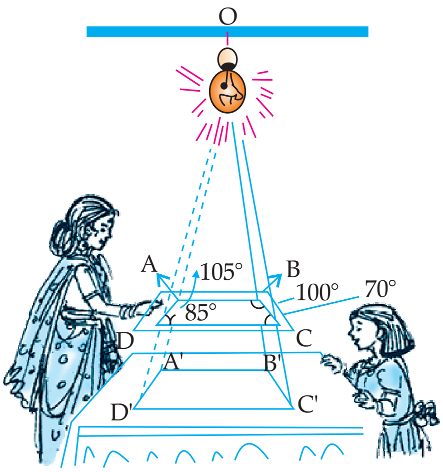
Explanation: Length of the horizontal edge EF

= half of the edge of pyramid

$= \frac{12}{2} = 6$ cm (as E is he mid-point of AT)

[CBSE SQP Marking Scheme, 2020-21]

AI **Q. 2. Seema placed a light bulb at point O on the ceiling and directly below it placed a table. Now, she put a cardboard of shape ABCD between table and lighted bulb. Then a shadow of ABCD is casted on the table as A'B'C'D' (see figure). Quadrilateral A'B'C'D' in an enlargement of ABCD with scale factor 1 : 2, Also, AB = 1.5 cm, BC = 25 cm, CD = 2.4 cm and AD = 2.1 cm; $\angle A$ = 105°, $\angle B$ = 100°, $\angle C$ = 70° and $\angle D$ = 85°.** C + A



(i) What is the measurement of angle A' ?

(a) 105° **(b) 100°**

(c) 70° **(d) 80°**

Sol. Correct Option: **(a)**

Explanation: Quadrilateral A'B'C'D' is similar to *ABCD*.

∴ $\angle A' = \angle A$

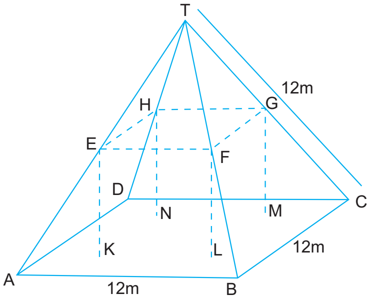
⇒ $\angle A' = 105°$

(ii) What is the length of A'B' ?

(a) 1.5 cm **(b) 3 cm**

(c) 5 cm **(d) 2.5 cm**

Sol. Correct Option: **(b)**



Explanation: Given scale factor is 1 : 2

∴ A'B' = 2AB

⇒ A'B' = 2 × 1.5 = 3 cm

(iii) What is the sum of angles of quadrilateral A'B'C'D' ?

(a) 180° **(b) 360°**

(c) 270° **(d) None of these**

Sol. Correct Option: **(b)**

Explanation: Sum of the angles of quadrilateral A'B'C'D' is 360°

(iv) What is the ratio of sides A'B' and A'D' ?

(a) 5 : 7 **(b) 7 : 5**

(c) 1 : 1 **(d) 1 : 2**

Sol. Correct Option: **(a)**

Explanation:

A'B' = 3 cm

and A'D' = 2AD

= 2 × 2.1 = 4.2 cm

$$\therefore \quad \frac{A'B'}{A'D'} = \frac{3}{4.2} = \frac{30}{42}$$

$$= \frac{5}{7} \text{ or } 5:7$$

(v) What is the sum of angles C' and D' ?

(a) 105° **(b) 100°**

(c) 155° **(d) 140°**

Sol. Correct Option: **(c)**

Explanation:

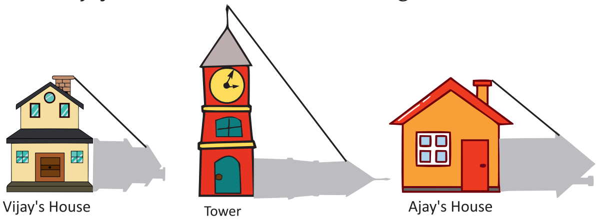
$\angle C' = \angle C = 70°$

and $\angle D' = \angle D = 85°$

∴ $\angle C' + \angle D' = 70° + 85° = 155°$

Q. 3. SIMILAR TRIANGLES

Vijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Vijay's house if 20 m when Vijay's house casts a shadow 10 m long on the ground. At the same time, the tower casts a shadow 50 m long on the ground and the house of Ajay casts 20 m shadow on the ground.



(i) What is the height of the tower?

(a) 20 m **(b) 50 m**

(c) 100 m **(d) 200 m**

Sol. Correct Option: **(c)**

Explanation: When two corresponding angles of two triangles are similar, then ratio of sides are equal.

$$\frac{\text{Height of Vijay's house}}{\text{Length of Shadow}} = \frac{\text{Height of tower}}{\text{length of shadow}}$$

$$\frac{20 \text{ m}}{10 \text{ m}} = \frac{\text{Height of tower}}{50 \text{ m}}$$

$$\text{Height of tower} = \frac{20 \times 50}{10} = \frac{1000}{10}$$

$$= 100 \text{ m.}$$

(ii) What will be the length of the shadow of the tower when Vijay's house casts a shadow of 12 m?

(a) 75 m **(b) 50 m**

(c) 45 m **(d) 60 m**

Sol. Correct Option: **(d)**

(iii) What is the height of Ajay's house?

(a) 30 m **(b) 40 m**

(c) 50 m **(d) 20 m**

Sol. Correct Option: **(b)**

Explanation: ***d***

$$\frac{\text{Height of Vijay's house}}{\text{Length of Shadow}} = \frac{\text{Height of Vijay's house}}{\text{Length of Shadow}}$$

$$\frac{20 \text{ m}}{10 \text{ m}} = \frac{\text{Height of Vijay's house}}{20 \text{ m}}$$

$$\text{Height of Ajay's house} = \frac{20 \text{ m} \times 20 \text{ m}}{10 \text{ m}}$$

$$= 40 \text{ m.}$$

(iv) When the tower casts a shadow of 40 m, same time what will be the length of the shadow of Ajay's house?

(a) 16 m **(b) 32 m**

(c) 20 m **(d) 8 m**

Sol. Correct Option: **(a)**

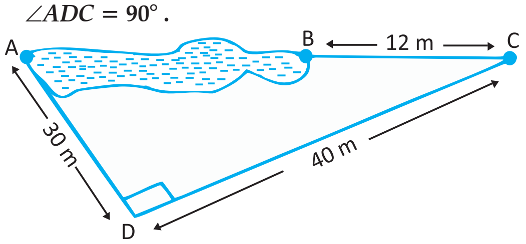
(v) When the tower casts a shadow of 40 m, same time what will be the length of the shadow of Vijay's house?

(a) 15 m **(b) 32 m**

(c) 16 m **(d) 8 m**

Sol. Correct Option: **(d)**

Q. 4. Rohan wants to measure the distance of a pond during the visit to his native. He marks points A and B on the opposite edges of a pond as shown in the figure below. To find the distance between the points, he makes a right-angled triangle using rope connecting B with another point C are a distance of 12 m, connecting C to point D at a distance of 40 m from point C and the connecting D to the point A which is are a distance of 30 m from D such the $\angle ADC = 90°$.



(i) Which property of geometry will be used to find the distance AC?

(a) Similarity of triangles

(b) Thales Theorem

(c) Pythagoras Theorem

(d) Area of similar triangles

Sol. Correct Option: **(c)**

(ii) What is the distance AC?

(a) 50 m **(b) 12 m**

(c) 100 m **(d) 70 m**

Sol. Correct Option: **(a)**

Explanation: According to the pythagoras,

$$AC^2 = AD^2 + CD^2$$
$$AC^2 = (30 \text{ m})^2 + (40 \text{ m})^2$$
$$AC^2 = 900 + 1600$$
$$AC^2 = 2500$$
$$AC = 50 \text{ m}$$

(iii) Which is the following does not form a Pythagoras triplet?

(a) (7, 24, 25) **(b) (15,8,17)**

(c) (5, 12, 13) **(d) (21,20,28)**

Sol. Correct Option: **(d)**

(iv) Find the length AB?

(a) 12 m **(b) 38 m**

(c) 50 m **(d) 100 m**

Sol. Correct Option: **(b)**

Explanation:
$$AC = 50 \text{ m}$$
$$BC = 12 \text{ m}$$
$$AC = AB + BC$$
$$50 \text{ m} = AB + 12 \text{ m}$$
$$AB = 50 \text{ m} - 12 \text{ m}$$
$$AB = 38 \text{ m}$$

(v) Find the length of the rope used.

(a) 120 m **(b) 70 m**

(c) 82 m **(d) 22 m**

Sol. Correct Option: **(c)**

Explanation:

$$\text{Length of Rope} = BC + CD + DA$$
$$= 12 \text{ m} + 40 \text{ m} + 30 \text{ m}$$
$$= 82 \text{ m}$$

Q. 5. SCALE FACTOR

A scale drawing of an object is the same shape at the object but a different size. The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio. The ratio of two corresponding sides in similar figures is called the scale factor

Scale factor= length in image / corresponding length in object

If one shape can become another using revising, then the shapes are similar. Hence, two shapes

are similar when one can become the other after a resize, flip, slide or turn. In the photograph below showing the side view of a train engine. Scale factor is 1:200

This means that a length of 1 cm on the photograph above corresponds to a length of 200 cm or 2 m, of the actual engine. The scale can also be written as the ratio of two lengths.

(i) If the length of the model is 11 cm, then the overall length of the engine in the photograph above, including the couplings(mechanism used to connect) is:

(a) 22 cm **(b) 220 cm**

(c) 220 m **(d) 22 m**

Sol. Correct Option: **(a)**

(ii) What will affect the similarity of any two polygons?

(a) They are flipped horizontally

(b) They are dilated by a scale factor

(c) They are translated down

(d) They are not the mirror image of one another.

Sol. Correct Option: **(d)**

(iii) What is the actual width of the door if the width of the door in photograph is 0.35 cm?

(a) 0.7 m **(b) 0.7 cm**

(c) 0.07 cm **(d) 0.07 m**

Sol. Correct Option: **(a)**

(iv) If two similar triangles have a scale factor 5:3 which statement regarding the two triangles is true?

(a) The ratio of their perimeters is 15:1

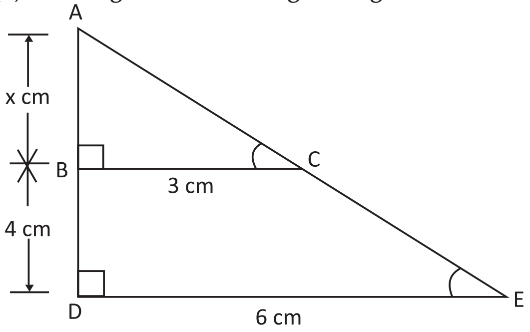
(b) Their altitudes have a ratio 25:15

(c) Their medians have a ratio 10:4

(d) Their angle bisectors have a ratio 11:5

Sol. Correct Option: **(b)**

(v) The length of AB in the given figure:



(a) 8 cm **(b) 6 cm**

(c) 4 cm **(d) 10 cm**

Sol. Correct Option: **(c)**

Explanation: Since, ΔABC and ΔADE are similar, then their ratio of corresponding sides are equal.

$$\frac{AB}{BC} = \frac{AB+BD}{DE}$$

$$\frac{x}{3\text{ cm}} = \frac{(x+4)\text{ cm}}{6\text{ cm}}$$

$$6x = 3(x+4)$$

$$6x = 3x + 12$$

$$6x - 3x = 12$$

$$3x = 12$$

$$x = 4$$

Hence, $AB = 4$ cm.