1. **(b)** $(101)^{50}-(99)^{50}=(100+1)^{50}-(100-1)^{50}$

$= 2[\,^{50}C_1(100)^{49}+\,^{50}C_3(100)^{47}+......+\,^{50}C_{49}(100)]$

$> 2\,.\,^{50}C_1\,.\,(100)^{49}=2\times 50(100)^{49}=(100)^{50}$

$\Rightarrow (101)^{50}>(99)^{50}+(100)^{50}\ \Rightarrow y>x\Rightarrow x<y$.

2. **(c)** Putting the value of C_0, C_2, C_4....., we get

$$=1+\frac{n(n-1)}{3.2!}+\frac{n(n-1)(n-2)(n-3)}{5.4!}+.....\ =\frac{1}{n+1}$$

$$\left[(n+1)+\frac{(n+1)n(n-1)}{3!}+\frac{(n+1)n(n-1)(n-2)(n-3)}{5!}+.....\right]$$

Put $n+1=N$

$$=\frac{1}{N}\left[N+\frac{N(N-1)(N-2)}{3!}+\frac{N(N-1)(N-2)(N-3)(N-4)}{5!}+...\right]$$

$$=\frac{1}{N}\left\{^{N}C_1+\,^{N}C_3+\,^{N}C_5+....\right\}$$

$$=\frac{1}{N}\left\{2^{N-1}\right\}=\frac{2^n}{n+1}\qquad \{\because N=n+1\}$$

3. **(c)** $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$

$$={}^4C_0+{}^4C_1\left(\frac{x}{2}-\frac{2}{x}\right)+{}^4C_2\left(\frac{x}{2}-\frac{2}{x}\right)^2$$

$$+{}^4C_3\left(\frac{x}{2}-\frac{2}{x}\right)^3+{}^4C_4\left(\frac{x}{2}-\frac{2}{x}\right)^4$$

$$={}^4C_0+{}^4C_1\left(\frac{x}{2}-\frac{2}{x}\right)+{}^4C_2\left[\frac{x^2}{4}-2+\frac{4}{x^2}\right]$$

$$+{}^4C_3\left[{}^3C_0\left(\frac{x}{2}\right)^3-{}^3C_1\left(\frac{x}{2}\right)^2\left(\frac{2}{x}\right)+{}^3C_2\left(\frac{x}{2}\right)\left(\frac{2}{x}\right)^2-{}^3C_3\left(\frac{2}{x}\right)^3\right]$$

$$+{}^4C_4\left[{}^4C_0\left(\frac{x}{2}\right)^4-{}^4C_1\left(\frac{x}{2}\right)^3\left(\frac{2}{x}\right)+{}^4C_2\left(\frac{x}{2}\right)^2\left(\frac{2}{x}\right)^2-{}^4C_3\left(\frac{x}{2}\right)\left(\frac{2}{x}\right)^3+{}^4C_4\left(\frac{2}{x}\right)^4\right]$$

The term independent of x in above

$$={}^4C_0+{}^4C_2(-2)+{}^4C_4.{}^4C_2=1-12+6=-5$$

4. **(c)** ✌ x^3 and higher powers of x may be neglected

$$\frac{(1+x)^{\frac{3}{2}}-\left(1+\frac{x}{2}\right)^3}{(1-x)^{\frac{1}{2}}}$$

$$=(1-x)^{\frac{-1}{2}}\left[\left(1+\frac{3}{2}x+\frac{\frac{3}{2}.\frac{1}{2}}{2!}x^2\right)-\left(1+\frac{3x}{2}+\frac{3.2}{2!}\frac{x^2}{4}\right)\right]$$

$$=\left[1+\frac{x}{2}+\frac{\frac{1}{2}.\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right]=\frac{-3}{8}x^2$$

(as x^3 and higher powers of x can be neglected)

5. **(d)** $\sum_{r=0}^{50}{}^{50}C_r\ (2x-3)^r(2-x)^{50-r}$

$= [(2 - x) + (2x - 3)]^{50}$

$= (x - 1)^{50}$

$= (1 - x)^{50}$

$= {}^{50}C_0 - {}^{50}C_1\, x \,....... - {}^{50}C_{25}\, x^{25} + ...$

Coefficient of x^{25} is $-{}^{50}C_{25}$

6. **(d)** $a_0 + a_1 + a_2 + ... = 2^{2n}$ and $a_0 + a_2 + a_4 + ... = 2^{2n-1}$

$a_n = {}^{2n}C_n$ = the greatest coefficient, being the middle coefficient

$a_{n-3} = {}^{2n}C_{n-3} = {}^{2n}C_{2n-(n-3)} = {}^{2n}C_{n+3} = a_{n+3}$

7. **(c)** The number of selection = coefficient of x^8 in

$(1 + x + x^2 + ... + x^8)\,(1 + x + x^2 + ... + x^8).\,(1 + x)^8$

= coefficient of x^8 in $\frac{(1-x^9)^2}{(1-x)^2}(1+x)^8$

$= \text{coefficient of } x^8 \text{ in } (1 + x)^8 \text{ in } (1 + x^8)(1 - x)^{-2}$
$= \text{coefficient of } x^8 \text{ in}$
$({}^8C_0 + {}^8C_1x + {}^8C_2x^2 + ... + {}^8C_8x^8)$
$\times (1 + 2x + 3x^2 + 4x^3 + ... + 9x^8 + ...)$
$= 9.\,{}^8C_0 + 8 \cdot {}^8C_1 + 7.\,{}^8C_2 + ... + 1.\,{}^8C_8$
$= C_0 + 2C_1 + 3C_2 + ... + 9C_8 \quad [C_r = {}^8C_r]$
Now $C_0x + C_1x^2 + ... + C_8x^9 = x(1 + x)^8$
Differentiating with respect to x, we get
$C_0 + 2C_1x + 3C_2x^2 + ... 9C_8x^8 = (1 + x)^8 + 8x(1 + x)^7$
Putting $x = 1$, we get $C_0 + 2C_1 + 3C_2 + ... + 9C_8$
$= 2^8 + 8.2^7. = 2^7(2 + 8) = 10.2^7.$

8. **(b)** $\frac{2}{\sqrt{2x^2+1}+\sqrt{2x^2-1}} = \sqrt{2x^2+1} - \sqrt{2x^2-1}$

given expression

$= (\sqrt{2x^2+1}+\sqrt{2x^2-1})^6 + (\sqrt{2x^2+1}-\sqrt{2x^2-1})^6$

we know that,

$$(a+b)^6 + (a-b)^6 = 2[a^6 + {}^6C_2a^4b^2 + {}^6C_4a^2b^4 + {}^6C_6b^6]$$

$$\therefore (\sqrt{2x^2+1}+\sqrt{2x^2-1})^6 + (\sqrt{2x^2+1}-\sqrt{2x^2-1})^6$$

$$= 2[(2x^2+1)^3 + 15(2x^2+1)^2(2x^2-1)$$

$$+15(2x^2+1)(2x^2-1) + (2x^2-1)^3]$$

Which is a polynomial of degree 6.

9. **(a)** To find

${}^{30}C_0{}^{30}C_{10} - {}^{30}C_1{}^{30}C_{11} + {}^{30}C_2{}^{30}C_{12} - .... + {}^{30}C_{20}{}^{30}C_{30}$

We know that

$(1 + x)^{30} = {}^{30}C_0 + {}^{30}C_1x + {}^{30}C_2x^2$
$+ .... + {}^{30}C_{20}x^{20} + ...{}^{30}C_{30}x^{30} \qquad ....(1)$
$(x - 1)^{30} = {}^{30}C_0x^{30} - {}^{30}C_1x^{29} + ... + {}^{30}C_{10}x^{20}$
$- {}^{30}C_{11}x^{19} + {}^{30}C_{12}x^{18} + ... {}^{30}C_{30}x^0 \qquad ....(2)$

Multiplying eq^n (1) and (2) and equating the coefficients of x^{20} on both sides, we get

$^{30}C_{10} = {}^{30}C_0^{30}C_{10} - {}^{30}C_1^{30}C_{11} + {}^{30}C_2^{30}C_{12} - .... + {}^{30}C_{20}^{30}C_{30}$

$\therefore$ Req. value is $^{30}C_{10}$

10. **(c)** Let the consecutive coefficient of $(1 + x)^n$ are $^nC_{r-1}, {}^nC_r, {}^nC_{r+1}$

From the given condition,

$^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 6 : 33 : 110$

Now $^nC_{r-1} : {}^nC_r = 6 : 33$

$$\frac{n!}{(r-1)!\,(n-r+1)!} \times \frac{r!(n-r)!}{n!} = \frac{6}{33}$$

$$\frac{r}{n-r+1} = \frac{2}{11} \qquad 11r = 2n - 2r + 2$$

$$2n - 13r + 2 = 0 \qquad ....(i)$$

and $^nC_r : {}^nC_{r+1} = 33 : 110$

$$\frac{n!}{r!(n-r)!} \times \frac{(r+1)!\,(n-r-1)!}{n!} = \frac{33}{110} = \frac{3}{10}$$

$$\frac{(r+1)}{n-r} = \frac{3}{10} \qquad 3n - 13r - 10 = 0 \qquad ...(ii)$$

Solving (i) & (ii), we get n = 12

11. **(d)** T_{r+1} in the expansion

$$\left[ax^2 + \frac{1}{bx} \right]^{11} = {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx}\right)^r$$

$= {}^{11}C_r\,(a)^{11-r}\,(b)^{-r}\,(x)^{22-2r-r}$

For the coefficient of x^7, we have

$22 - 3r = 7 \qquad r = 5$

Coefficient of $x^7 = {}^{11}C_5 (a)^6\ (b)^{-5}$ ...(i)

Again T_{r+1} in the expansion

$$\left[ax - \frac{1}{bx^2} \right]^{11} = {}^{11}C_r (ax^2)^{11-r} \left(-\frac{1}{bx^2}\right)^r$$

$= {}^{11}C_r (a)^{11-r} (-1)^r \times (b)^{-r} (x)^{-2r} (x)^{11-r}$

For the coefficient of x^{-7}, we have

$11 - 3r = -7 \qquad 3r = 18 \qquad r = 6$

Coefficient of $x^{-7} = {}^{11}C_6\, a^5 \times 1 \times (b)^{-6}$

Coefficient of x^7 = Coefficient of x^{-7}

${}^{11}C_5\,(a)^6\,(b)^{-5} = {}^{11}C_6\, a^5 \times (b)^{-6} \qquad ab = 1.$

12. **(d)** We have

$$S = C_0 + (C_0 + C_1) + (C_0 + C_1 + C_2) + \ldots + (C_0 + C_1 + \ldots + C_n)$$

$$= (C_0 + C_0 + \ldots n+1 \text{ times}) + (C_1 + C_1 + \ldots n \text{ times})$$

$$(C_2 + C_2 + \ldots\, n-1 \text{ times}) + \ldots + (C_{n-1} + C_{n-1}) + C_n$$

$$= (n+1)C_0 + nC_1 + (n-1)C_2 + \ldots + 2C_{n-1} + C_n$$

$$= C_0 + 2C_1 + 3C_2 + \ldots + (n+1)C_n \qquad [\because C_r = C_{n-r}]$$

General Term $T_{r+1} = (r+1)C_r$

$$T_{r+1} = r\,{}^nC_r + {}^nC_r = n.{}^{n-1}C_{r-1} + {}^nC_r$$

$$\therefore S = \sum_{r=0}^{n} T_{r+1} = n[{}^{n-1}C_0 + {}^{n-1}C_1 + \ldots + {}^{n-1}C_{n-1}]$$

$$+[{}^nC_0 + {}^nC_1 + \ldots + {}^nC_n]$$

$$= n.2^{n-1} + 2^n = (n+2)2^{n-1}$$

13. **(a)** $(1 + x)^{4n} = {}^{4n}C_0 + {}^{4n}C_1\, x + {}^{4n}C_2\, x^2 + {}^{4n}C_3\, x^3$

$+ {}^{4n}C_4\, x^4 + \ldots + {}^{4n}C_{un}\, x^{4n}$

Put $x = 1$ and $x = -1$, then adding.

$2^{4n-1} = {}^{4n}C_0 + {}^{4n}C_2 + {}^{4n}C_4 + \ldots + {}^{4n}C_{4n} \qquad \ldots (i)$

Now put, $x = i$

$(1 + i)^{4n} = {}^{4n}C_0 + {}^{4n}C_1 i - {}^{4n}C_2 - {}^{4n}C_3 i + {}^{4n}C_4 + \ldots + {}^{4n}C_{4n}$

Compare real and imaginary part, we get

$(-1)^n\,(2)^{2n} = {}^{4n}C_0 - {}^{4n}C_2 + {}^{4n}C_4 - {}^{4n}C_6 + \ldots + {}^{4n}C_{4n} \ldots (ii)$

Adding (i) and (ii), we get

$\Rightarrow \quad {}^{4n}C_0 + {}^{4n}C_4 + \ldots + {}^{4n}C_{4n} = (-1)^n\,(2)^{2n-1} + 2^{4n-2}$

14. **(d)** We know that, $(1+x)^{20} = {}^{20}C_0 + {}^{20}C_1 x + {}^{20}C_2\, x^2 + \ldots\ldots\ {}^{20}C_{10}\, x^{10} + \ldots\ldots\ {}^{20}C_{20}\, x^{20}$

Put $x = -1$, $(0) = {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots + {}^{20}C_{10} - {}^{20}C_{11} \ldots\ldots + {}^{20}C_{20}$

$\Rightarrow 0 = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - {}^{20}C_9] + {}^{20}C_{10}$

$\Rightarrow {}^{20}C_{10} = 2[{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - {}^{20}C_9 + {}^{20}C_{10}]$

$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots + {}^{20}C_{10} = \frac{1}{2}\ {}^{20}C_{10}$

15. **(c)** We have, $7^{103} = 7\,(49)^{51} = 7\,(50-1)^{51}$

$= 7\,(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots - 1)$

$= 7\,(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots) - 7 + 18 - 18$

$= 7\,(50^{51} - {}^{51}C_1\, 50^{50} + {}^{51}C_2\, 50^{49} - \ldots) - 25 + 18$

= k + 18 (say) where k is divisible by 25,

$\therefore$ remainder is 18.

16. **(d)** $\left(\frac{a}{a+x}\right)^{\frac{1}{2}} + \left(\frac{a}{a-x}\right)^{\frac{1}{2}} = \left(\frac{a+x}{a}\right)^{-\frac{1}{2}} + \left(\frac{a-x}{a}\right)^{-\frac{1}{2}}$

$$= \left(1+\frac{x}{a}\right)^{-\frac{1}{2}} + \left(1-\frac{x}{a}\right)^{-\frac{1}{2}}$$

$$= \left[1 - \frac{1}{2}\frac{x}{a} + \frac{3}{8}\frac{x^2}{a^2}\right] + \left[1 + \frac{1}{2}\frac{x}{a} + \frac{3}{8}\frac{x^2}{a^2}\right]$$

$$\left[\because x << a, \therefore \frac{x}{a} << 1\right] = 2 + \frac{3}{4}\cdot\frac{x^2}{a^2}$$

17. **(b)** We have,

$$t_{r+1} = \frac{2^{r+2}\ {}^nC_r}{(r+1)(r+2)} = \frac{2^{r+2}}{r+2}\cdot\frac{1}{r+1}\ {}^nC_r$$

$$= \frac{2^{r+2}}{r+2}\cdot\frac{1}{n+1}\ {}^{n+1}C_{r+1}$$

$$= \frac{2^{r+2}}{n+1}\cdot\left(\frac{1}{r+2}\ {}^{n+1}C_{r+1}\right)$$

$$= \frac{2^{r+2}}{n+1} \cdot \frac{1}{n+2} \; {}^{n+2}C_{r+2}$$

$$\left[\because \frac{1}{r+1} \; {}^{n}C_{r} = \frac{1}{n+1} \; {}^{n+1}C_{r+1}\right]$$

Putting r = 0, 1, 2, ..., n and adding we get,

The given expression

$$= \frac{1}{(n+1)(n+2)} \{2^2 . {}^{n+2}C_2 + 2^3 . {}^{n+2}C_3 + ... + 2^{n+2} . {}^{n+2}C_{n+2}\}$$

$$= \frac{1}{(n+1)(n+2)} \{(1+2)^{n+2} - {}^{n+2}C_0 - 2. {}^{n+2}C_1\}$$

$$= \frac{3^{n+2} - 2(n+2) - 1}{(n+1)(n+2)} = \frac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$$

18. (b) $(1 - 9x + 20x^2)^{-1} = [(1 - 4x)(1 - 5x)]^{-1}$

$$= \frac{1}{x}\left[\frac{(1-4x)-(1-5x)}{(1-4x).(1-5x)}\right] = \frac{1}{x} [(1 - 5x)^{-1} - (1 - 4x)^{-1}]$$

$$= \frac{1}{5} [(5 - 4)x + (5^2 - 4^2) x^2 + (5^3 - 4^3) x^3 + ... + (5^n - 4^n) x^n + ...]$$

$\therefore$ coeff. of $x^n = 5^{n+1} - 4^{n+1}$

19. (b) From the given condition, replacing a by ai and $-ai$ respectively, we get

$$(x+ai)^n = (T_0 - T_2 + T_4 - ......) + i(T_1 - T_3 + T_5 - ....) \text{ ....(i)}$$

and $(x-ai)^n = (T_0 - T_2 + T_4 - ......) - i(T_1 - T_3 + T_5 - ....)$ ....(ii)

Multiplying (ii) and (i) we get required result

i.e., $(x^2 + a^2)^n = (T_0 - T_2 + T_4 - ....)^2 + (T_1 - T_3 + T_5 - ....)^2$

20. (d) Since the coefficient of $(r+1)^{th}$ term in the expansion of $(1 + x)^n = {}^nC_r$

$\therefore$ In the expansion of $(1 + x)^{18}$

coefficient of $(2r + 4)^{th}$ term $= {}^{18}C_{2r+3}$,

Similarly, coefficient of $(r-2)^{th}$ term in the expansion of $(1 + x)^{18} = {}^{18}C_{r-3}$

If ${}^nC_r = {}^nC_s$ then $r + s = n$

So, ${}^{18}C_{2r+3} = {}^{18}C_{r-3}$ gives

$2r + 3 + r - 3 = 18$ $3r = 18$ $r = 6$.

21. (120) General term of the given series is

$$r\frac{{}^nC_r}{{}^nC_{r-1}} = n + 1 - r$$

By taking summation over n, we get

$$\sum_1^{15} r\frac{{}^nC_r}{{}^nC_{r-1}} = \sum_{n=1}^{15}(n+1-r) = \sum_1^{15}(16-r)$$

$= 16 \times 15 - \frac{1}{2}\cdot 15\times 16$

By using sum of n natural numbers $= \frac{n(n+1)}{2}$

$= 240 - 120 = 120$

22. (6) The number of subsets of the set which contain at most n elements is

$${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n = K \text{ (say)}$$

We have

$2K = 2\,({}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_n)$

$= ({}^{2n+1}C_0 + {}^{2n+1}C_{2n+1}) + ({}^{2n+1}C_1 + {}^{2n+1}C_{2n})$

$+ ... + ({}^{2n+1}C_n + {}^{2n+1}C_{n+1})$ ($\because {}^nC_r = {}^nC_{n-r}$)

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ... + {}^{2n+1}C_{2n+1}$

$= 2^{2n+1} \Rightarrow K = 2^{2n}$

$\Rightarrow 4096 = 2^{2n}$

$\Rightarrow n = 6$

23. (0) In the expansion of $(1 + \alpha x)^4$

Middle term $= {}^4C_2(\alpha x)^2 = 6\alpha^2x^2$

In the expansion of $(1 - \alpha x)^6$,

Middle term $= {}^6C_3(-\alpha x)^3 = -20\alpha^3x^3$

It is given that

Coefficient of the middle term in $(1 + \alpha x)^4$ = Coefficient of the middle term in $(1 - \alpha x)^6$

$\Rightarrow$ $6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = 0, \alpha = -\frac{3}{10}$

24. **(64)** We have

$7^9 + 9^7 = (8-1)^9 + (8+1)^7 = (1+8)^7 - (1-8)^9$

$$= [1 + {}^7C_1 8 + {}^7C_2 8^2 + ... + {}^7C_7 8^7]$$

$$-[1 - {}^9C_1 8 + {}^9C_2 8^2 - ..... - {}^9C_9 8^9]$$

$$= {}^7C_1 8 + {}^9C_1 8 + [{}^7C_2 + {}^7C_3.8 + ... - {}^9C_2 + {}^9C_3.8 - .....]8^2$$

$= 8(7+9) + 64k = 8..16 + 64k = 64q$, where $q = k + 2$

Thus, $7^9 + 9^7$ is divisible by 64.

25. **(2187)** We know that,

$(a-1)^n = {}^nC_0.a^n - {}^nC_1.a^{n-1} + {}^nC_2 a^{n-2} -$

$$........ + (-1)^{n-1}.{}^nC_{n-1}a + (-1)^n\ {}^nC_n$$

$$\therefore\ \frac{(a-1)^n}{a} = {}^nC_0 a^{n-1} - {}^nC_1 a^{n-2} + {}^nC_2 a^{n-3} -$$

$$........ + (-1)^{n-1}.{}^nC_{n-1} + \frac{(-1)^n}{a}\ {}^nC_n$$

$$\therefore f(n) = \frac{(a-1)^n - (-1)^n}{a}$$

Now, $f(2007) + f(2008) = \frac{(a-1)^{2007}+1}{a} + \frac{(a-1)^{2008}-1}{a}$

$$= \frac{(a-1)^{2007}(1+a-1)}{a} = (a-1)^{2007}$$

$$= \left(3^{\frac{1}{223}}\right)^{2007} = 3^9 = 3^2 \cdot 3^7 = 9\,(2187)\ \therefore k = 2187$$