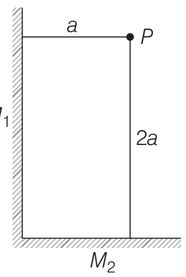
# 23

# Ray Optics and Optical Instruments

## TOPIC 1
Reflection of Light

**01** Two plane mirrors $M_1$ and $M_2$ are at right angle to each other shown. A point source $P$ is placed at $a$ and $2a$ meter away from $M_1$ and $M_2$, respectively. The shortest distance between the images thus formed is (Take $\sqrt{5} = 2.3$) **[2021, 31 Aug Shift-I]**



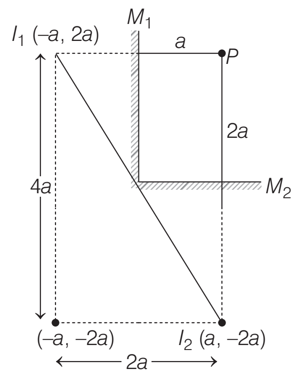

(a) $3a$
(b) $4.6\,a$
(c) $2.3\,a$
(d) $2\sqrt{10}\,a$

**Ans.** *(b)*

According to the given figure, we have two plane mirror, placed at 90º to each other.

Since, image formed by plane mirror are virtual, erect, same size and at same distance behind mirror.

Let $I_1$ is the image of $P$ due to mirror $M_1$ and $I_2$ is the image of $P$ due to mirror $M_2$




Therefore, the shortest distance between images i.e., between $I_1$ and $I_2$

$$I_1I_2 = \sqrt{(4a)^2 + (2a)^2}$$
$$= \sqrt{16a^2 + 4a^2}$$
$$= a\sqrt{20} = a2\sqrt{5}$$
$$= a \times 2 \times 2.3 = 4.6a$$

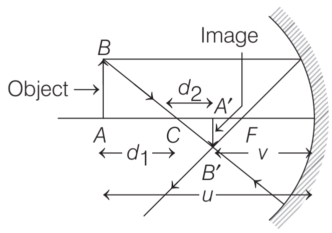
**02** An object is placed beyond the centre of curvature $C$ of the given concave mirror. If the distance of the object is $d_1$ from $C$ and the distance of the image formed is $d_2$ from $C$, the radius of curvature of this mirror is **[2021, 27 Aug Shift-I]**

(a) $\dfrac{2d_1d_2}{d_1 - d_2}$ (b) $\dfrac{2d_1d_2}{d_1 + d_2}$

(c) $\dfrac{d_1d_2}{d_1 + d_2}$ (d) $\dfrac{d_1d_2}{d_1 - d_2}$

**Ans.** *(a)*

The given situation is shown in the following ray diagram.




In concave mirror, when object is placed beyond $C$ the image is formed between $C$ and focus of mirror.

Consider radius of curvature of mirror is $R$.

Object distance, $u = -(R + d_1)$

Image distance, $v = -(R - d_2)$

Focal length of mirror, $f = \dfrac{-R}{2}$

Now, from mirror formula, we have

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{u+v}{uv}$$

$$\Rightarrow \quad uv = f(u+v)$$

Substituting the values in above expression, we get

$$[-(R+d_1)]\,[-(R-d_2)] = -\frac{R}{2}(R + d_1 + R - d_2)$$

$$\Rightarrow \quad 2(R^2 - d_2R + d_1R - d_1d_2)$$
$$= 2R^2 + d_1R - d_2R = 2d_1d_2$$

$$\Rightarrow \quad d_1R - d_2R = 2d_1d_2$$

$$\Rightarrow \quad R = \frac{2d_1d_2}{d_1 - d_2}$$

Thus, radius of curvature of mirror is $\dfrac{2d_1d_2}{d_1 - d_2}$.

**03** Car $B$ overtakes another car $A$ at a relative speed of 40 $ms^{-1}$. How fast will the image of car $B$ appear to move in the mirror of focal length 10 cm fitted in car $A$, when the car $B$ is 1.9 m away from the car $A$?

**[2021, 26 Aug Shift-I]**

(a) 4 $ms^{-1}$ (b) 0.2 $ms^{-1}$
(c) 40 $ms^{-1}$ (d) 0.1 $ms^{-1}$

**Ans.** *(d)*

According to the question,

Velocity of car $B$ w.r.t. mirror of car $A$, $v_{BM} = 40$ m/s

Distance of car $B$ from mirror of car $A$, $u = 1.9$ m $= 190$ cm

Focal length of mirror, $f = 10$ cm

We have to find the velocity of image of car $B$ in the mirror, i.e. $v_{IM}$

We know that,

$$v_{IM} = -m^2 v_{BM} \quad ...(i)$$

where, $m$ is the magnification produced by mirror of car $A$,

$$\Rightarrow \quad m = \frac{f}{f-u} = \frac{10}{10-(-190)}$$

$$= \frac{10}{200} = \frac{1}{20}$$

Substituting the values in Eq. (i), we get

$$v_{IM} = -\left(\frac{1}{20}\right)^2 \times 40$$

$$= -\frac{1}{400} \times 40 = -0.1 \text{ m / s}$$

Here, negative sign shows that, the speed of image w.r.t. mirror of car $A$ is in opposite direction.

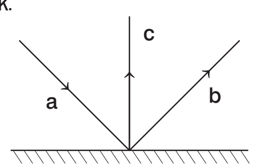
**04** The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors **a**, **b** and **c**, respectively. Then, choose the correct relation for these vectors.

**[2021, 26 Feb Shift-II]**

(a) $\mathbf{b} = \mathbf{a} + 2\mathbf{c}$ (b) $\mathbf{b} = 2\mathbf{a} + \mathbf{c}$
(c) $\mathbf{b} = \mathbf{a} - 2(\mathbf{a} \cdot \mathbf{c})\mathbf{c}$ (d) $\mathbf{b} = \mathbf{a} - \mathbf{c}$

**Ans.** *(c)*

Taking component of **a**, **b** and **c** along $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$.




$$\therefore \quad \mathbf{a} = \sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}}$$

$$\mathbf{b} = \sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}}$$

$$\mathbf{c} = \hat{\mathbf{j}}$$

Now, on solving option (c), we get

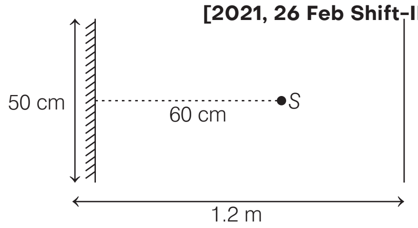
$$\mathbf{a} - 2(\mathbf{a}\cdot\mathbf{c})\,\mathbf{c} = (\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}})$$

$$-2[(\sin\theta\,\hat{\mathbf{i}} - \cos\theta\,\hat{\mathbf{j}})\cdot\hat{\mathbf{j}}]\,\hat{\mathbf{j}}$$

$$= \sin\theta\,\hat{\mathbf{i}} + \cos\theta\,\hat{\mathbf{j}} = \mathbf{b}$$

**05** A point source of light $S$, placed at a distance 60 cm in front of the centre of a plane mirror of width 50 cm, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points, where he can see the image of the light source in the mirror is .......... cm.

**[2021, 26 Feb Shift-II]**



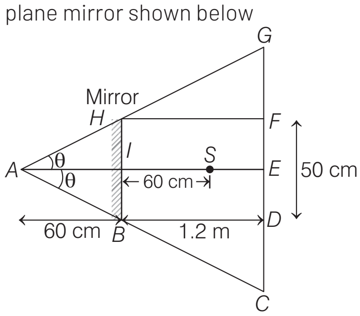

**Ans.** *(150)*

Given, length of mirror, $m = 50$ cm
$= 50 \times 10^{-2}$ m

Distance of source from mirror, $d = 60$ cm
$= 60 \times 10^{-2}$ m

Distance of man from mirror, $d_m = 1.2$ m

By using the concept of ray diagram of plane mirror shown below




Now, using the concept of similar triangle,

$\Delta HAI \sim \Delta GAE$ and $\Delta BAI \sim \Delta CAE$

$$\therefore \quad \frac{AI}{AE} = \frac{HI}{EG}$$

$$\Rightarrow \quad \frac{0.60}{1.8} = \frac{0.25}{EG} \quad (\because AI = IS)$$

$$\Rightarrow \quad EG = 0.25 \times \frac{1.8}{0.6} = 0.25 \times 3 = 0.75 \text{ m}$$

As, $\quad CG = 2EG$

$$\Rightarrow \quad CG = 0.75 \times 2 = 1.50 \text{ m}$$

Hence, distance between the extreme points, where he can see image of light source in mirror is 150 cm.

**06** A short straight object of height 100 cm lies before the central axis of a spherical mirror, whose focal length has absolute value $f = 40$ cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information.

**[2021, 26 Feb Shift-I]**

(a) Image is real, same side of concave mirror
(b) Image is virtual, opposite side of concave mirror
(c) Image is real, same side of convex mirror
(d) Image is virtual, opposite side of convex mirror

**Ans.** *(d)*

Given, height of object, $h_o = 100$ cm

Focal length of mirror, $f = 40$ cm

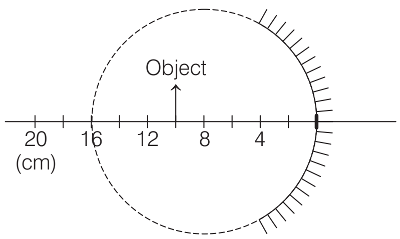
Height of image, $h_i = 25$ cm

Nature of image is erect means virtual.

As, $h_i$ is less than $h_o$, so mirror used is convex mirror.

Hence, (d) is correct option, i.e. image is virtual, opposite and mirror is convex.

**07**




A spherical mirror is obtained as shown in the above figure from a hollow glass sphere. If an object is in front of the mirror, what will be the nature and magnification of the image of the object? (Figure drawn as schematic and not to scale)

**[2020, 2 Sep Shift-I]**

(a) Erect, virtual and unmagnified
(b) Inverted, real and magnified
(c) Erect, virtual and magnified
(d) Inverted, real and unmagnified

**Ans.** *(d)*

Radius of curvature of given concave mirror $\quad R = -8$ cm

So, focal length, $f = \frac{R}{2} = -\frac{8}{2} = -4$ cm

Object distance, $u = -10$ cm

Using mirror formula,

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{4}-\frac{1}{-10}=-\frac{3}{20}$$

$\Rightarrow \quad v=-6.67\text{ cm (real)}$

Also, magnification,

$$m=-\frac{v}{u}=-\frac{(-6.67)}{(-10)}=-0.667$$

As $m$ is negative and less than one, so image is inverted and diminished or unmagnified. Hence, correct option is (d).

**08** When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 $\text{cms}^{-1}$, the speed (in $\text{cms}^{-1}$) with which image moves at that instant is .......... .

**[2020, 3 Sep Shift-II]**

**Ans.** *(1)*

Given, object distance, $u=-30$ cm,

Image distance, $v=-10$ cm

and speed of object, $v_o=9\text{ cms}^{-1}$

Let $v_i$ be the speed of image.

Magnification of concave mirror,

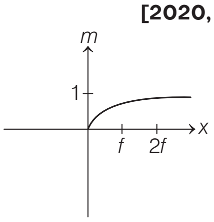
$$m=-\frac{v}{u}=\frac{-10}{30}=-\frac{1}{3}$$

Now, $v_i=v_o(m^2)=9\times\left(\frac{-1}{3}\right)^2=1\text{ cms}^{-1}$

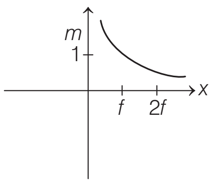
**09** An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification ($m$) *versus* distance of the object from the mirror ($x$) is correctly given by (graphs are drawn schematically and are not to scale)

**[2020, 8 Jan Shift-II]**

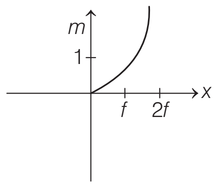
(a)



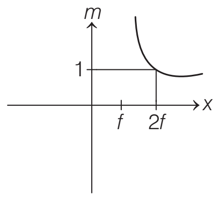

(b)




(c)




(d)

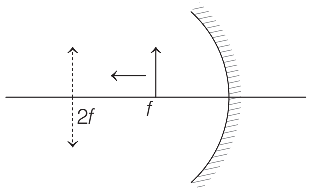



**Ans.** *(d)*

In case of a concave mirror, when object is at $f$, image is formed at infinity.

So, magnification when object is at $f$,

$$m=\frac{-v}{u}=\infty$$




Also, if an object is at of $2f$ from mirror, then the image is also formed at $2f$. So, magnification,

$$m=\frac{-v}{u}=\frac{-(-2f)}{(-2f)}=-1$$

or $\quad |m|=1$

So, these values are correctly depicted in option (d) only.

**10** A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is

**[2019, 9 April Shift-I]**

(a) 0.16 m (b) 1.60 m
(c) 0.32 m (d) 0.24 m

**Ans.** *(c)*

Given, focal length of concave mirror,

$$f=-0.4\text{ m}$$

Magnification = 5

We know that, magnification produced by a mirror, $\quad m=-\frac{\text{image distance}}{\text{object distance}}$

$\Rightarrow \quad \frac{v}{u}=-5$ or $v=-5u$

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

Substituting the given values in the above equation, we get

$$\frac{1}{-5u}+\frac{1}{u}=-\frac{1}{0.4}\Rightarrow\frac{4}{5u}=-\frac{1}{0.4}$$

$\Rightarrow \quad u=-\frac{1.6}{5}=-0.32\text{ m}$

Alternate Solution

Magnification produced by a mirror can also be given as

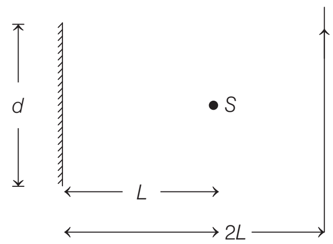
$$m=\frac{f}{f-u}$$

Substituting the given values, we get

$$5=\frac{-0.4}{-0.4-u}\text{ or }u=-0.32\text{ m}$$

**11** A point source of light, $S$ is placed at a distance $L$ in front of the centre of plane mirror of width $d$ which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below.

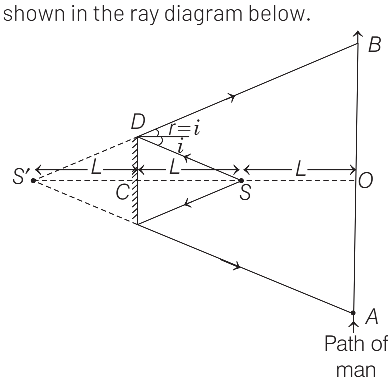
The distance over which the man can see the image of the light source in the mirror is **[2019, 12 Jan Shift-I]**




(a) $\frac{d}{2}$ (b) $d$ (c) $3d$ (d) $2d$

**Ans.** *(c)*

Light from mirror is reflected in a straight line and It is appear to come from its image formed at same distance (as that of source) behind the mirror as shown in the ray diagram below.




From ray diagram in similar triangles

$\Delta S'CD$ and $\Delta S'OB$, we have $\dfrac{S'C}{S'O}=\dfrac{CD}{OB}$

So, $OB=\dfrac{CD\times S'O}{S'C}=\dfrac{\frac{d}{2}\times 3L}{L}=\dfrac{3d}{2}$

Also, $OA=\dfrac{3d}{2}$

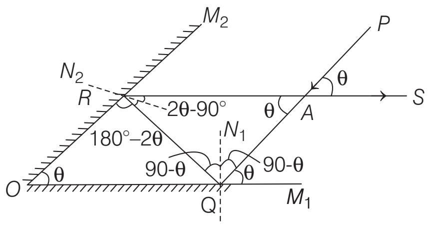
So, distance over which man can see the image $S'$ is $\dfrac{3d}{2}+\dfrac{3d}{2}=3d$

**12** The plane mirrors ($M_1$ and $M_2$) are inclined to each other such that a ray of light incident on mirror $M_1$ and parallel to the mirror $M_2$ is reflected from mirror $M_2$ parallel to the mirror $M_1$. The angle between the two mirror is **[2019, 9 Jan Shift-II]**

(a) 45° (b) 75° (c) 90° (d) 60°

**Ans.** *(d)*

The given condition is shown in the figure given below, where two plane mirror inclined to each other such that a ray of light incident on the first mirror ($M_1$) and parallel to the second mirror ($M_2$) is finally reflected from second mirror ($M_2$) parallel to the first mirror.




where, $PQ=$ incident ray parallel to the mirror $M_2$, $QR=$ reflected ray from the mirror $M_1$,

$RS=$ reflected ray from the mirror $M_2$ which is parallel to the $M_1$ and

$\theta=$ angle between $M_1$ and $M_2$.

According to geometry,

$\angle PAS=\angle PQM_1=\theta$ (angle on same line)

$\angle AQN_1=$ angle of incident $=90-\theta$

$\angle N_1QR=$ angle of reflection $=(90-\theta)$.

Therefore, for triangle $\Delta ORQ$, (according to geometry)

$$\angle\theta+\angle\theta+\angle ORQ=180°$$

$$\angle ORQ=180°-2\theta \quad \text{...(i)}$$

For normal $N_2$, angle of incident $=$ angle of reflection

$$=2\theta-90° \quad \text{...(ii)}$$

Therefore, for the triangle $\Delta RAQ$

$$\Rightarrow \quad 4\theta-180°+180°-2\theta+\theta=180$$

$$3\theta=180°$$

$$\theta=60°$$

**13** A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15m/s. The speed of the image of the second car as seen in the mirror of the first one is **[AIEEE 2011]**

(a) $\dfrac{1}{15}$ m/s (b) 10 m/s

(c) 15 m/s (d) $\dfrac{1}{10}$ m/s

**Ans.** *(a)*

For the mirror, $\dfrac{1}{u}+\dfrac{1}{v}=\dfrac{1}{f}$

Differentiate this equation w.r.t. $t$, we get

$$-\frac{1}{u^2}\frac{du}{dt}-\frac{1}{v^2}\frac{dv}{dt}=0$$

$$\Rightarrow \quad \frac{dv}{dt}=-\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

But $\quad \dfrac{v}{u}=\dfrac{f}{u-f}$

$$\therefore \quad \frac{dv}{dt}=-\left(\frac{f}{u-f}\right)^2\left(\frac{du}{dt}\right)$$

$$=\left(\frac{0.2}{-2.8-0.2}\right)^2\times 15=\frac{1}{15}\,\text{ms}^{-1}$$

**14** To get three images of a single object, one should have two plane mirrors at an angle of **[AIEEE 2003]**

(a) 60° (b) 90°

(c) 120° (d) 30°

**Ans.** *(b)*

Number of images, $n=\dfrac{360°}{\theta}-1$

where, $\theta$ is angle between mirrors.

$$\therefore \quad 3=\frac{360°}{\theta}-1$$

or $\quad \theta=90°$

**15** If two mirrors are kept at 60° to each other, then the number of images formed by them is **[AIEEE 2002]**

(a) 5 (b) 6

(c) 7 (d) 8

**Ans.** *(a)*

Number of images, $n=\dfrac{360°}{\theta}-1$

where, $\theta$ is angle between mirrors.

Thus, $\quad \theta=60°$ [given]

So, number of images, $n=\dfrac{360°}{60°}-1=5$

# TOPIC 2
## Refraction, TIR and Prism

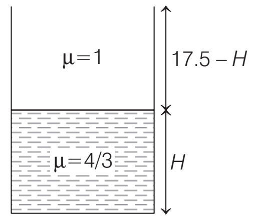
**16** A glass tumbler having inner depth of 17.5 cm is kept on a table. A student starts pouring water ($\mu=4/3$) into it while looking at the surface of water from the above. When he feels that the tumbler is half filled, he stops pouring water. Up to what height, the tumbler is actually filled? **[2021, 1 Sep Shift-II]**

(a) 11.7 cm (b) 10 cm

(c) 7.5 cm (d) 8.75 cm

**Ans.** *(b)*

Let us draw the diagram of glass tumbler.




Consider the actual height of the tumbler be $H$.

The refractive index of the water, $\mu=4/3$

The refractive index of the air, $\mu=1$

As we know that,

$$\mu_{\text{water}}=\frac{H_{\text{real}}}{H_{\text{appearent}}}$$

$$\frac{4}{3}=\frac{H}{H_{\text{appearent}}}$$

$$\Rightarrow \quad H_{\text{appearent}}=\frac{3H}{4}$$

The height of air observed by observer $=17.5-H$

According to the above figure,

$$\frac{3H}{4}=17.5-H$$

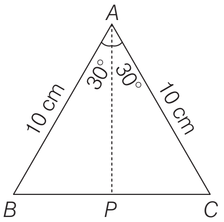
$$\Rightarrow \quad H=10.11\text{ cm}\simeq 10\text{ cm}$$

The actual height of the water in tumbler is 10 cm.

**17** Cross–section view of a prism is the equilateral triangle $ABC$ in the figure. The minimum deviation is observed using this prism when the angle of incidence is equal to the prism angle. The time taken by light to travel from $P$ (mid-point of $BC$) to $A$ is ..... $\times 10^{-10}$ s.

(Given, speed of light in vacuum $= 3 \times 10^8$ m/s and $\cos 30° = \frac{\sqrt{3}}{2}$)

**[2021, 31 Aug Shift-II]**



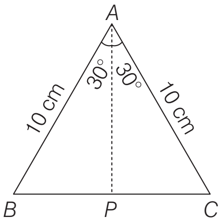

**Ans.** *(5)*

Given, the base of a prism is equilateral triangle of side 10 cm as shown in figure.

At minimum deviation,

Angle of incident = Angle of prism

i.e. $i = A = 60°$




Angle of refraction, $r = \frac{A}{2} = \frac{60°}{2} = 30°$

Let $\mu$ be the refractive index.

Then, by Snell's law of refraction,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Now, in $\Delta ABP$, $AP = AB \cos 30°$

$$= \frac{10\sqrt{3}}{2} = 5\sqrt{3} \text{ cm}$$

$$= 5\sqrt{3} \times 10^{-2} \text{ m}$$

i.e. optical distance travelled by light along AP, $d = \mu \times AP = \sqrt{3} \times 5\sqrt{3} \times 10^{-2}$

$= 15 \times 10^{-2}$ m

Now, the time taken by light ($c = 3 \times 10^8$ m/s) to travel from $P$ to $A$,

$$t = \frac{d}{c} = \frac{15 \times 10^{-2}}{3 \times 10^8} \text{ s}$$

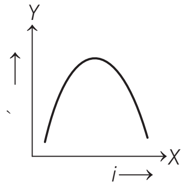
$$= 5 \times 10^{-10} \text{ s}$$

Thus, correct answer is 5.

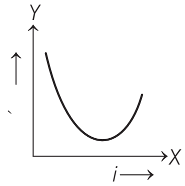
**18** The expected graphical representation of the variation of angle of deviation $\delta$ with angle of incidence $i$ in a prism is

**[2021, 27 July Shift-II]**

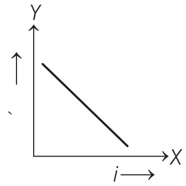
(a)



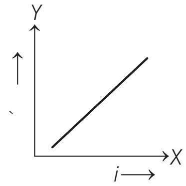

(b)




(c)

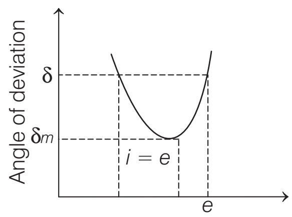



(d)




**Ans.** *(b)*

If the angle of incidence is increased gradually, then the angle of deviation first decreases, attains a minimum value ($\delta_m$) and then again starts increasing.




Hence, the correct option is (b).

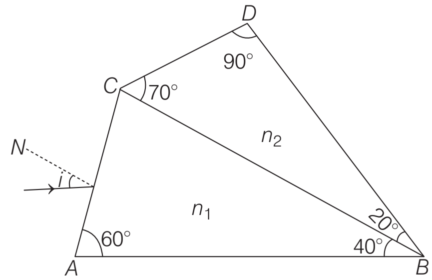
**19** A prism of refractive index $n_1$ and another prism of refractive index $n_2$ are stuck together (as shown in the figure). $n_1$ and $n_2$ depend on $\lambda$, the wavelength of light, according to the relation

$$n_1 = 1.2 + \frac{10.8 \times 10^{-14}}{\lambda^2}$$

and $n_2 = 1.45 + \frac{1.8 \times 10^{-14}}{\lambda^2}$

The wavelength for which rays incident at any angle on the interface $BC$ pass through without bending at that interface will be .............. nm.

**[2021, 27 July Shift-I]**




**Ans.**

(600) ∵ For no deviation, $n_1 = n_2$

$$\Rightarrow \quad 1.2 + \frac{10.8 \times 10^{-14}}{\lambda^2} = 1.45 + \frac{1.8 \times 10^{-14}}{\lambda^2}$$

$$\Rightarrow \quad 0.25 = \frac{9 \times 10^{-14}}{\lambda^2}$$

$$\Rightarrow \quad \lambda^2 = \frac{9 \times 10^{-14}}{0.25}$$

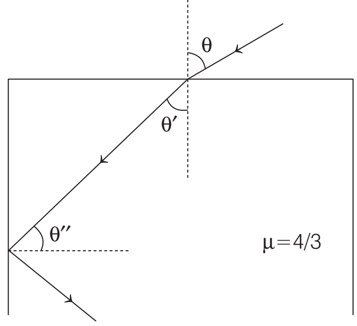
$$\Rightarrow \quad \lambda = \frac{3}{5} \times 10^{-6}$$

$$\Rightarrow \quad \lambda = 6 \times 10^{-7} \text{ m}$$

$$\Rightarrow \quad \lambda = 600 \text{ nm}$$

**20** A ray of light entering from air into a denser medium of refractive index $\frac{4}{3}$, as shown in figure. The light ray suffers total internal reflection at the adjacent surface as shown. The maximum value of angle $\theta$ should be equal to

**[2021, 25 July Shift-II]**




(a) $\sin^{-1} \frac{\sqrt{7}}{3}$ (b) $\sin^{-1} \frac{\sqrt{5}}{4}$

(c) $\sin^{-1} \frac{\sqrt{7}}{4}$ (d) $\sin^{-1} \frac{\sqrt{5}}{3}$

**Ans.** *(a)*

Given, refractive index, $\mu = \frac{4}{3}$

By using Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \frac{4}{3} = \frac{\sin\theta}{\sin\theta'} \quad \text{...(i)}$$

From diagram,

$$\theta'' = 90 - \theta'$$

For total internal reflection,

$$\frac{1}{\mu} = \sin\theta''$$

$$\Rightarrow \quad \sin(90 - \theta') = \frac{3}{4} \Rightarrow \cos\theta' = \frac{3}{4}$$

Since, $\sin\theta' = \sqrt{1 - \cos^2\theta'}$

$$\Rightarrow \quad \sin\theta' = \sqrt{1 - \left(\frac{3}{4}\right)^2} = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

Putting this value in Eq. (i), we get

$$\frac{4}{3} = \frac{\sin\theta \times 4}{\sqrt{7}}$$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{7}}{3} \Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{7}}{3}\right)$$

The maximum value of $\theta$ is $\sin^{-1}\left(\frac{\sqrt{7}}{3}\right)$.

**21** A ray of laser of a wavelength 630 nm is incident at an angle of 30° at the diamond-air interface. It is going from diamond to air. The refractive index of diamond is 2.42 and that of air is 1. Choose the correct option. **[2021, 25 July Shift-I]**

(a) Angle of refraction is 24.41°
(b) Angle of refraction is 30°
(c) Refraction is not possible
(d) Angle of refraction is 53.4°

**Ans.** *(c)*

Given, wavelength of laser light $(\lambda) = 630$ nm

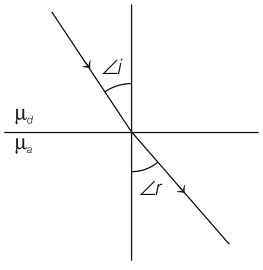
$$= 630 \times 10^{-9} \text{ m}$$

Angle of incidence $(i) = 30°$

Refractive index of diamond, $\mu_d = 2.42$

Refractive index of air, $\mu_r = 1$

The propagation of light can be shown as




By using Snell's law,

$$\frac{\mu_a}{\mu_d} = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \sin r = \frac{\mu_d}{\mu_a}\sin i$$

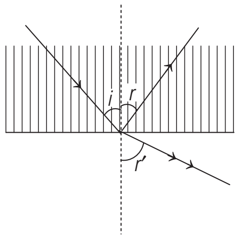
$$= 2.42 \times \sin 30°$$

$$= 2.42 \times \frac{1}{2} = 1.21 > 1$$

But the maximum value of $\sin\theta$ is 1.

$\therefore$ In above case, refraction is not possible.

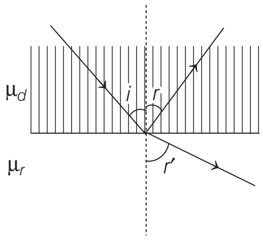
**22** A ray of light passes from a denser medium to a rarer medium at an angle of incidence *i*. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively *r* and *r'*. The critical angle is given by **[2021, 22 July Shift-II]**




(a) $\sin^{-1}(\cot r)$
(b) $\tan^{-1}(\sin i)$
(c) $\sin^{-1}(\tan r')$
(d) $\sin^{-1}(\tan r)$

**Ans.** *(d)*

If refractive index of denser medium and rarer medium be $\mu_d$ and $\mu_r$ respectively, then




By using Snell's law,

$$\mu_{rd} = \frac{\mu_r}{\mu_d} = \frac{\sin i}{\sin r'}$$

$$\Rightarrow \quad \sin i = \frac{\mu_r}{\mu_d}\sin r' \quad \text{...(i)}$$

According to given situation,

$$r + r' = 90°$$

$$\Rightarrow \quad r' = 90° - r \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\sin i = \frac{\mu_r}{\mu_d}\sin(90° - r)$$

$$= \frac{\mu_r}{\mu_d}\cos r$$

By using law of reflection $\angle i = \angle r$, therefore,

$$\frac{\mu_r}{\mu_d} = \frac{\sin r}{\cos r} = \tan r \quad \text{...(ii)}$$

For critical angle, $\frac{\mu_r}{\mu_d} = \frac{\sin C}{\sin 90°}$

$\because \quad \sin 90° = 1$

$\therefore \quad \frac{\mu_r}{\mu_d} = \sin C$

Put in Eq. (ii), we get

$\Rightarrow \quad \sin C = \tan r$

$\Rightarrow \quad C = \sin^{-1}(\tan r)$

**23** A ray of light passing through a prism $(\mu = \sqrt{3})$ suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is ......... (in degrees). **[2021, 22 July Shift-II]**

**Ans.** *(60)*

Given, let refractive index of prism, $\mu = \sqrt{3}$

In the case of minimum deviation,

$$i = 2r = A \quad \text{...(i)}$$

As we know that,

$$i + e = A + \delta_m \quad [\because i = e]$$

$$2i = A + \delta_m$$

$$2A = A + \delta_m$$

$$\Rightarrow \quad A = \delta_m$$

By using prism formula,

$$\mu = \frac{\sin\left(A + \frac{\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \sqrt{3} = \frac{\sin\left(A + \frac{A}{2}\right)}{\sin\frac{A}{2}} = \frac{\sin A}{\sin\frac{A}{2}}$$

$$\Rightarrow \quad \sqrt{3}\sin\frac{A}{2} = \sin A = \sin 2\frac{A}{2}$$

$$= 2\sin\frac{A}{2}\cos\frac{A}{2}$$

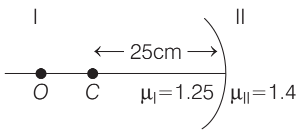
$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \cos\frac{A}{2}$$

$$\Rightarrow \quad \cos 30° = \cos\frac{A}{2}$$

$$\therefore \quad \frac{A}{2} = 30° \Rightarrow A = 60°$$

**24** Region I and II are separated by a spherical surface of radius 25 cm. An object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface is

**[2021, 20 July Shift-I]**




(a) 55.44 cm (b) 9.52 cm
(c) 18.23 cm (d) 37.58 cm

**Ans.** *(d)*

As we know that, the equation of refraction at spherical surface is

$$\frac{\mu_{II}}{v} - \frac{\mu_I}{u} = \frac{\mu_{II} - \mu_I}{R}$$

where, $\mu_{II}$ = refractive index of region II = 1.4

$\mu_I$ = refractive index of region I = 1.25

$R$ = radius of curvature = –25 cm

$u$ = object distance = –40 cm

and $v$ = image distance.

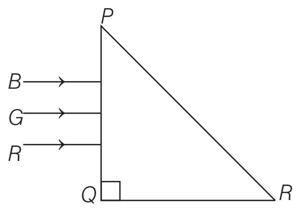
$$\Rightarrow \frac{1.4}{v} - \frac{1.25}{-40} = \frac{1.4 - 1.25}{-25}$$

$$\Rightarrow \quad \frac{1.4}{v} = \frac{-0.15}{25} - \frac{1.25}{40}$$

$$\Rightarrow \quad v = -37.58 \text{ cm}$$

where, negative sign indicate real image.

**25** Three rays of light, namely red ($R$), green ($G$) and blue ($B$) are incident on the face $PQ$ of a right angled prism $PQR$ as shown in figure



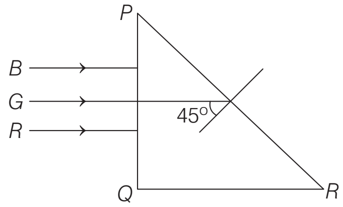

The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49, respectively. The colour of the ray(s) emerging out of the face $PR$ is

**[2021, 18 March Shift-II]**

(a) green (b) red
(c) blue and green (d) blue

**Ans.** *(b)*

From the given figure,




We know that,

$$\mu = \frac{1}{\sin i_C}$$

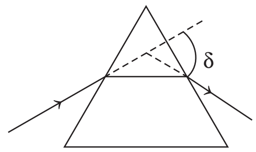
Here, $i_C$ is the critical angle of incidence and $\mu$ is the refractive index.

$$\mu = \frac{1}{\sin 45°} \Rightarrow \mu = 1.414$$

The rays will emerge out when angle of incidence is less than the angle of critical angle of glass-air interface $PR$.

As $\mu_R < \mu$ while $\mu_G$ and $\mu_B > \mu$, so only red colour will be transmitted through face $PR$ while green and blue rays will suffer total internal reflection.

**26** The angle of deviation through a prism is minimum when




A. incident ray and emergent ray are symmetric to the prism
B. the refracted ray inside the prism becomes parallel to its base
C. angle of incidence is equal to that of the angle of emergence
D. angle of emergence is double the angle of incidence

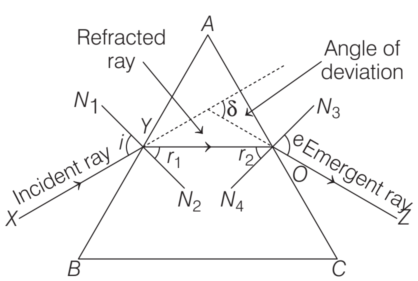
Choose the correct answer from the options given below.

**[2021, 16 March Shift-I]**

(a) Statements (A), (B) and (C) are true.
(b) Only statement (D) is true.
(c) Only statements (A) and (B) are true.
(d) Statements (B) and (C) are true.

**Ans.** *(a)*

The propagation of light ray from a prism is shown below




From the above figure, we can say that minimum value of angle of deviation can only be achieved when

A. incident ray and emergent ray are symmetric to the prism.
B. the refracted ray inside the prism becomes parallel to its base.
C. angle of incidence is equal to angle of emergence.

∴ Statement (A), (B) and (C) are true.

Note Refracted ray inside the prism is parallel to the base only for equilateral and isosceles prism.

**27** The image of an object placed in air formed by a convex refracting surface is at a distance of 10 m behind the surface. The image is real and is at $\frac{2}{3}$ of the distance of the object from the surface .The wavelength of light inside the surface is $\frac{2}{3}$ times the wavelength in air. The radius of the curved surface is $\frac{x}{13}$ m. The value of $x$ is ……………… .

**[2021, 17 March Shift-II]**

**Ans.** *(30)*

Given,

Image distance, $v = 10$ m

Object distance, $u = \left(\frac{3}{2}\right) \times 10 - 15$ m

and $\quad n_2 = \frac{3}{2} n_1 \quad \left(\because n \propto \frac{1}{\lambda}\right)$

Using the lens Maker's formula,

$$\frac{n_2}{v} - \frac{n_1}{u} = \left(\frac{n_2 - n_1}{R}\right)$$

Substituting the values in the above equation, we get

$$\frac{\frac{3}{2}n_1}{10} - \frac{n_1}{-15} = \left(\frac{\frac{3}{2}n_1 - n_1}{R}\right) \Rightarrow R = \frac{30}{13} \text{ m}$$

The radius of the curved surface is 30/13 m. Comparing with $x/13$, we get

$$x = 30$$

**28** Red light differs from blue light as they have **[2021, 16 March Shift-II]**

(a) different frequencies and different wavelengths
(b) different frequencies and same wavelengths

(c) same frequencies and same wavelengths
(d) same frequencies and different wavelengths

**Ans.** *(a)*

Since, $\lambda\nu = c =$ constant

where, $\lambda =$ wavelength of light

and $\nu =$ frequency of light.

Red light and blue light have different wavelengths and different frequencies but same speed.

**29** A deviation of $2^\circ$ is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass are 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6, respectively. The refracting angles for crown glass prism will be ......° (in degree). (Round off to the nearest integer) **[2021, 16 March Shift-II]**

**Ans.** *(12)*

Given,

Dispersive power of crown glass, $\omega_1 = 0.02$

Dispersive power of flint glass, $\omega_2 = 0.03$

Refractive index of yellow light,

for crown glass, $\mu_1 = 1.5$ and

for flint glass, $\mu_2 = 1.6$

This is a case of achromatic combination.

$\therefore \quad \theta_{net} = 0$

$\Rightarrow \quad \theta_1 - \theta_2 = 0 \Rightarrow \theta_1 = \theta_2$

$\Rightarrow \quad \omega_1\delta_1 = \omega_2\delta_2 \quad [\because \theta = \omega\delta] \quad \ldots(i)$

and $\quad \delta_{net} = \delta_1 - \delta_2 = 2^\circ \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$\delta_1 - \frac{\omega_1\delta_1}{\omega_2} = 2^\circ$$

$$\Rightarrow \quad \delta_1\left(1 - \frac{\omega_1}{\omega_2}\right) = 2^\circ \Rightarrow \delta_1\left(1 - \frac{2}{3}\right) = 2^\circ$$

$\Rightarrow \quad \delta_1 = 6^\circ$

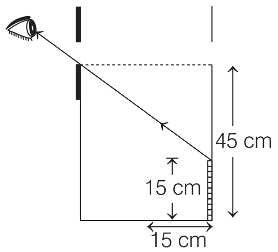
Also, $\quad \delta_1 = (\mu_1 - 1)A_1$

$\Rightarrow \quad 6^\circ = (1.5 - 1)A_1$

$\Rightarrow \quad A_1 = 12^\circ$

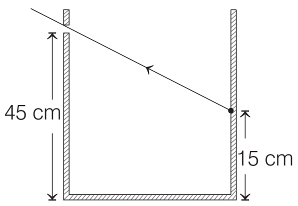
$\therefore$ The refracting angle for crown glass prism will be $12^\circ$.

**30** An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm, the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is $N/100$, where $N$ is an integer, the value of $N$ is .......... . **[2020, 3 Sep Shift-I]**

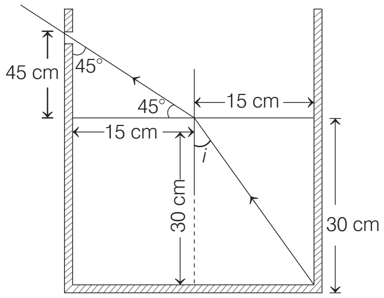



**Ans.** *(158)*

Initially, when the jar is empty.




Finally, when the jar is filled with liquid.




If $i$ be the angle of incidence, then

$$\tan i = \frac{15}{30} = \frac{1}{2}$$

$$\Rightarrow \quad \sin i = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{1}{\sqrt{5}}$$

According to law of refraction,

$$n_1 \sin i = n_2 \sin r$$

$$\Rightarrow \quad \mu\left(\frac{1}{\sqrt{5}}\right) = 1 \times \sin 45^\circ$$

$$\Rightarrow \quad \mu = \frac{\sqrt{5}}{\sqrt{2}} = 1.581$$

Also, given that,

$$\frac{N}{100} = \mu$$

$\Rightarrow \quad N = 100\mu$

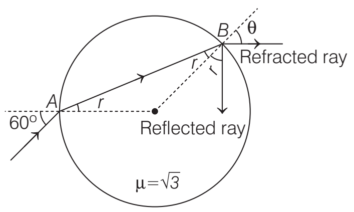
$= 100 \times 1.581 = 158.1$

So, nearest integer value of $N$ is 158.

**31** A light ray enters a solid glass sphere of refractive index $\mu = \sqrt{3}$ at an angle of incidence 60°. The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degree) between the reflected and refracted rays at this surface is ......... . **[2020, 2 Sep Shift-II]**

**Ans.** *(90)*

We are given following situation as shown in figure,




Using Snell's law at point $A$,

$$n_1 \sin i = n_2 \sin r$$

$$1 \times \sin 60^\circ = \sqrt{3} \times \sin r$$

$$\Rightarrow \quad \sin r = \frac{\sqrt{3}/2}{\sqrt{3}} = \frac{1}{2} \Rightarrow r = 30^\circ$$

Now, at point $B$, angle of incidence $= r = 30^\circ$

Again, using Snell's law at point $B$,

$\sqrt{3} \sin 30^\circ = 1 \times \sin\theta$

$$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 60^\circ$$

And, angle of reflection, $r = 30^\circ$

$\therefore$ Angle between reflected ray and refracted ray at point $B = 180^\circ - \theta - r$

$= 180^\circ - 60^\circ - 30^\circ = 90^\circ$

**32** A prism of angle $A = 1^\circ$ has a refractive index $\mu = 1.5$. A good estimate for the minimum angle of deviation (in degree) is close to $N/10$. Value of $N$ is ......... . **[2020, 5 Sep Shift-II]**

**Ans.** *(5)*

Deviation for small-angled prism is given by

$$\delta = (\mu - 1)A$$

Given, $A = 1^\circ$, $\mu = 1.5$

Substituting these values in above equation, we get

$$\delta = (1.5 - 1)1 \; \delta = 0.5$$

According to question, $\delta = \frac{N}{10}$

$$\Rightarrow \quad 0.5 = \frac{N}{10} \Rightarrow N = 5$$

Hence, the value of $N$ is 5.

**33** The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be **[2020, 8 Jan Shift-I]**

(a) 45° (b) 60° (c) 15° (d) 30°

**Ans.** *(d)*

Speed of light in a medium is given by

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \times \frac{1}{\sqrt{\varepsilon_r \mu_r}}$$

$$= \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

Here, $\varepsilon_r = 3$ and $\mu_r = \frac{4}{3}$

$$\therefore \quad \frac{v}{c} = \frac{1}{\sqrt{\varepsilon_r \mu_r}} = \frac{1}{\sqrt{3 \times \frac{4}{3}}} = \frac{1}{2}$$

So, refractive index of medium w.r.t. vacuum or air,

$$n = \frac{c}{v} = 2$$

Hence, critical angle,

$$i_c = \sin^{-1}\left(\frac{1}{n}\right)$$

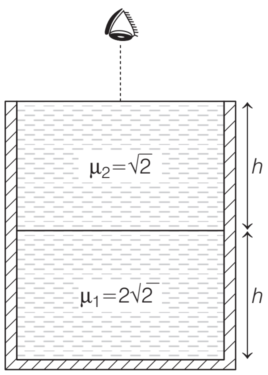
$$= \sin^{-1}\frac{1}{2} = 30°$$

**34** A vessel of depth $2h$ is half filled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid of refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be **[2020, 9 Jan Shift-I]**

(a) $\frac{h}{\sqrt{2}}$ (b) $\frac{3}{4}h\sqrt{2}$

(c) $\frac{h}{3\sqrt{2}}$ (d) $\frac{h}{2(\sqrt{2}+1)}$

**Ans.** *(b)*




When a vessel contains immiscible liquids, then apparent depth will be

$$d_{\text{apparent}} = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} + \ldots$$

In given case,

$$d_{\text{apparent}} = \frac{h}{\mu_1} + \frac{h}{\mu_2}$$

$$= \frac{h}{2\sqrt{2}} + \frac{h}{\sqrt{2}} = \frac{3h}{2\sqrt{2}}$$

or $\quad d_{\text{apparent}} = \frac{3}{4}h\sqrt{2}$

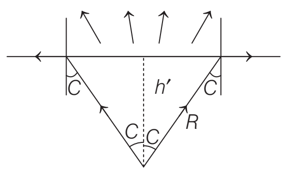
**35** There is a small source of light at some depth below the surface of water (refractive index $= \frac{4}{3}$) in a tank of large cross-sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is (nearly)

[Use the fact that surface area of a spherical cap of height $h$ and radius of curvature $R$ is $2\pi Rh$] **[2020, 9 Jan Shift-II]**

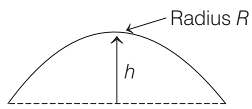
(a) 34% (b) 50% (c) 17% (d) 21%

**Ans.** *(c)*

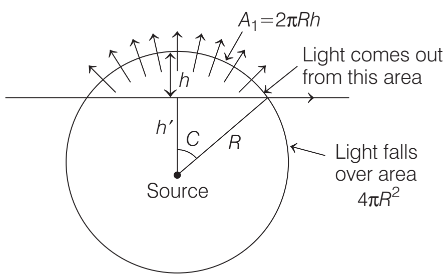
Due to total internal reflection, light comes out from a cone of angle $2C$ as shown in the figure ($C$ = angle of critical incidence).




Now, it is given that surface area of spherical cap of height $h$ and radius $R$ is $2\pi Rh$.




Here, we are given with following situation




So, percentage of light that emerges out of surface is given by,

% of light coming out

$$= \frac{\text{Area through which light comes out}}{\text{Total area over which light falls}} \times 100$$

$$= \frac{2\pi Rh}{4\pi R^2} \times 100 = \frac{1}{2} \times \left(\frac{R-h'}{R}\right) \times 100$$

Now, $\quad \frac{h'}{R} = \frac{\text{base}}{\text{hypotenuse}} = \cos C$

As, $\sin C = \frac{1}{n} = \frac{1}{\frac{4}{3}} = \frac{3}{4} \Rightarrow \cos C = \frac{\sqrt{7}}{4} = \frac{h'}{R}$

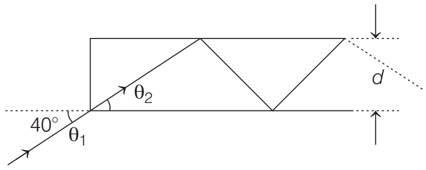
$\therefore$ % of light coming out

$$= \frac{1}{2} \times \left(\frac{R-h'}{R}\right) \times 100 = \frac{1}{2} \times \left(1 - \frac{h'}{R}\right) \times 100$$

$$= \frac{1}{2} \times \frac{\sqrt{7}}{4} \times \quad \simeq 17\%$$

Nearest answer is 17 %.

**36** In figure, the optical fibre is $l = 2$ m long and has a diameter of $d = 20\,\mu$m. If a ray of light is incident on one end of the fibre at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is close to (refractive index of fibre is 1.31 and $\sin 40° = 0.64$) **[2019, 8 April Shift-I]**

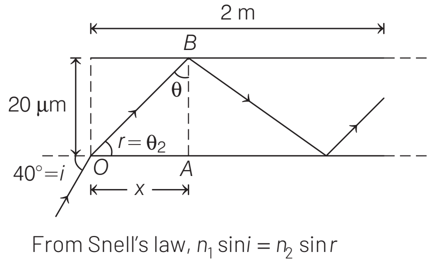



(a) 55000 (b) 66000

(c) 45000 (d) 57000

**Ans.** *(d)*

Total internal reflection occurs through given glass rod as shown in figure.




From Snell's law, $n_1 \sin i = n_2 \sin r$

where, $n_1 = 1$, $n_2 = 1.31$ and $i = 40°$

So, we get

$$1 \sin 40° = 1.31 \sin r$$

$$\Rightarrow \quad \sin r = \frac{0.64}{1.31} = 0.49 \approx 0.5$$

So, $\quad r = 30°$

From $\Delta OAB$, $\theta = 90 - r = 60°$

Now, $\tan\theta = \dfrac{x}{20\,\mu m}$

$\Rightarrow \quad x = 20\sqrt{3}\,\mu m \qquad [\because \tan 60° = \sqrt{3}]$

One reflection occurs in $20\sqrt{3}\,\mu m$.

$\therefore$ Total number of reflections occurring in 2 m

$= n = \dfrac{2m}{20\sqrt{3}\,\mu m} = \dfrac{2}{20\sqrt{3} \times 10^{-6}}$

$= 57735$ reflections

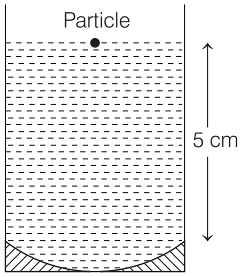
$\approx 57000$ reflections

**37** A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure).

If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance $d$ from the surface of water. The value of $d$ is close to

[Refractive index of water = 1.33]

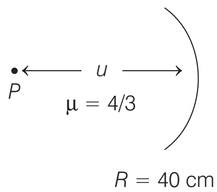
**[2019, 12 April Shift-I]**




(a) 6.7 cm (b) 13.4 cm
(c) 8.8 cm (d) 11.7 cm

**Ans.** *(c)*

In the given case, $u = -5$ cm




Focal length,

$f = \dfrac{-R}{2} = \dfrac{-40}{2} = -20$ cm

Now, using mirror formula,

$\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{v} = \dfrac{1}{f} - \dfrac{1}{u} = -\dfrac{1}{20} + \dfrac{1}{5} = +\dfrac{3}{20}$

$\Rightarrow \quad v = +\dfrac{20}{3}$ cm

For the light getting refracted at water surface, this image will act as an object.

So, distance of object,

$$d = 5\text{ cm} + \frac{20}{3}\text{ cm} = \frac{35}{3}\text{ cm}$$

(below the surface). Let's assume final image at distance $d$ after refraction.

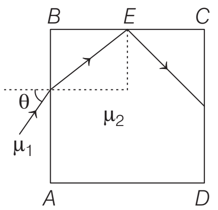
$$\frac{d'}{d} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad d' = d\left(\frac{\mu_2}{\mu_1}\right) = \left(\frac{35}{3}\text{cm}\right)\left(\frac{1}{\frac{4}{3}}\right)$$

$$= \frac{35}{3} \times \frac{3}{4}\text{ cm} = \frac{35}{4}\text{ cm}$$

$$= 8.75\text{ cm} \approx 8.8\text{ cm}$$

**38** A transparent cube of side $d$, made of a material of refractive index $\mu_2$, is immersed in a liquid of refractive index $\mu_1 (\mu_1 < \mu_2)$. A ray is incident on the face $AB$ at an angle $\theta$ (shown in the figure). Total internal reflection takes place at point $E$ on the face $BC$. **[2019, 12 April Shift-II]**




Then, $\theta$ must satisfy

(a) $\theta < \sin^{-1}\dfrac{\mu_1}{\mu_2}$ (b) $\theta > \sin^{-1}\sqrt{\dfrac{\mu_2^2}{\mu_1^2} - 1}$

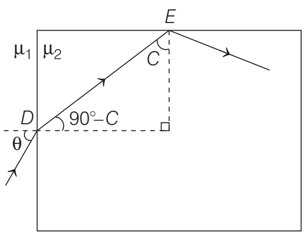
(c) $\theta < \sin^{-1}\sqrt{\dfrac{\mu_2^2}{\mu_1^2} - 1}$ (d) $\theta > \sin^{-1}\dfrac{\mu_1}{\mu_2}$

**Ans.** *(c)*

**Key Idea** The critical angle is defined as the angle of incidence that provides an angle of refraction of 90°.

So, $\quad \theta_c = \sin^{-1}\dfrac{\mu_2}{\mu_1}$

For total internal reflection, angle of incidence($i$) at medium interface must be greater than critical angle ($C$).




where, $\quad \sin C = \dfrac{\mu_1}{\mu_2} \quad$ ...(i)

Now, in given arrangement, at point $D$,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{(Snell's law)}$$

$$\Rightarrow \quad \frac{\sin\theta}{\sin(90° - C)} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos C} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad \sin\theta = \frac{\mu_2}{\mu_1} \cdot \cos C = \frac{\mu_2}{\mu_1}\sqrt{1 - \sin^2 C}$$

[from Eq. (i)]

$$= \frac{\mu_2}{\mu_1}\sqrt{1 - \frac{\mu_1^2}{\mu_2^2}} = \sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$$

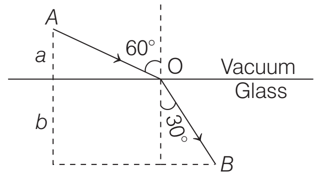
$$\Rightarrow \quad \theta = \sin^{-1}\sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1\right)}$$

For TIR at $E$, $i > C$

$$\Rightarrow \quad \theta < \sin^{-1}\sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1\right)}$$

**39** A ray of light $AO$ in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along $OB$ as shown in the figure.

The optical path length of light ray from $A$ to $B$ is **[2019, 10 April Shift-I]**

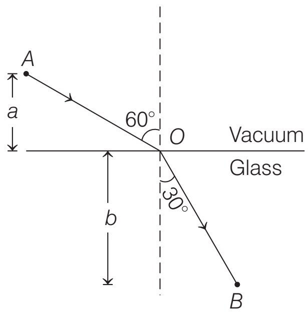



(a) $\dfrac{2\sqrt{3}}{a} + 2b$ (b) $2a + \dfrac{2b}{3}$

(c) $2a + 2b$ (d) $2a + \dfrac{2b}{\sqrt{3}}$

**Ans.** *(c)*

From the figure,




$$\cos 60° = \frac{a}{AO}$$

$$\Rightarrow \quad AO = \frac{a}{\cos 60°} = 2a \quad \text{...(i)}$$

and $\cos 30° = \frac{b}{BO}$

or $BO = \frac{b}{\cos 30°} = \frac{2}{\sqrt{3}} b$ ...(ii)

Optical path length of light ray

$= AO + \mu (BO)$ ...(iii)

Here, $\mu$ can be determined using Snell's law, i.e.

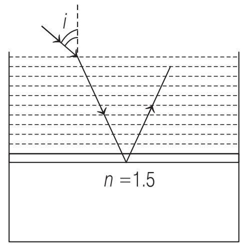
$$\mu = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \quad \text{...(iv)}$$

Substituting the values from Eqs. (i), (ii) and (iv) in Eq. (iii), we get

$\therefore$ Optical path $= 2a + (\sqrt{3} \times \frac{2}{\sqrt{3}} b)$

$= 2a + 2b$

**40** Consider a tank made of glass (refractive index is 1.5) with a thick bottom. It is filled with a liquid of refractive index $\mu$. A student finds that, irrespective of what the incident angle $i$ (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarised.




For this to happen, the minimum value of $\mu$ is **[2019, 9 Jan Shift-I]**

(a) $\frac{3}{\sqrt{5}}$ (b) $\frac{5}{\sqrt{3}}$ (c) $\frac{4}{3}$ (d) $\sqrt{\frac{5}{3}}$

**Ans.** *(a)*

**Key Idea** When a beam of unpolarised light is reflected from a transparent medium of refractive index $\mu$, then the reflected light is completely plane polarised at a certain angle of incidence $i_B$, which is known as Brewster's angle.

In the given condition, the light reflected irrespective of an angle of incidence is never completely polarised. So, $i_C > i_B$ where, $i_C$ is the critical angle.

$\Rightarrow \sin i_C < \sin i_B$ ...(i)

From Brewster's law, we know that

$$\tan i_B = {}^{w}\mu_g = \frac{\mu_{glass}}{\mu_{water}} = \frac{1.5}{\mu} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

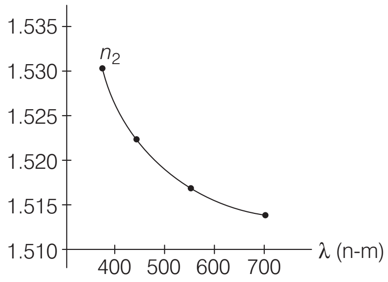
$$\frac{1}{\mu} < \frac{1.5}{\sqrt{(1.5)^2 + (\mu)^2}}$$

$$\Rightarrow \sqrt{(1.5)^2 + \mu^2} < 1.5\,\mu$$

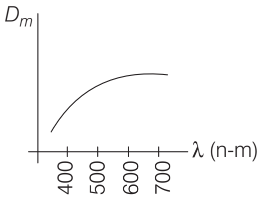
$$\mu^2 + (1.5)^2 < (1.5\mu)^2 \text{ or } \mu < \frac{3}{\sqrt{5}}$$

$\therefore$ The minimum value of $\mu$ should be $\frac{3}{\sqrt{5}}$.

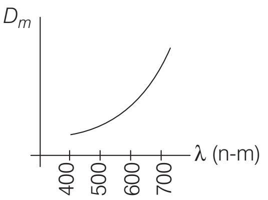
**41** The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if $D_m$ is the angle of minimum deviation? **[2019, 11 Jan Shift-I]**



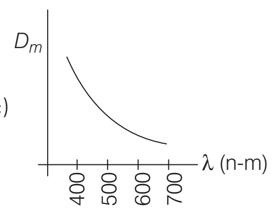

(a)



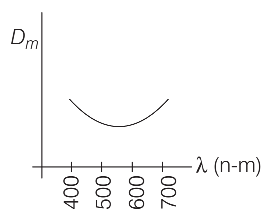

(b)




(c)




(d)




**Ans.** *(c)*

For a crown glass thin prism i.e., prism with small angle, the angle of minimum deviation is given as,

$$D_m = (n-1)A$$

where, $A$ is prism angle and $n$ is refractive index.

$\Rightarrow D_m \propto n$ ...(i)

Since from the given graph, the value of '$n$' decrease with the increase in '$\lambda$'. Thus, from relation (i), we can say that, $D_m$ will also decrease with the increase in '$\lambda$'.

$\therefore$ Hence, option (c) is correct.

**42** A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is **[2019, 11 Jan Shift-II]**

(a) 45º (b) 90º (c) 60º (d) 30º

**Ans.** *(c)*

Given, refractive index of material of prism $n = \sqrt{3}$, prism angle $A = 60°$

**Method 1**

Using prism formula, $n = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$\Rightarrow \sqrt{3} = \frac{\sin\left(\frac{60+\delta}{2}\right)}{\sin 30°}$$

$$\Rightarrow \sin\left(\frac{60+\delta}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin\left(\frac{60+\delta}{2}\right) = \sin 60°$$

or $\frac{60+\delta}{2} = 60$

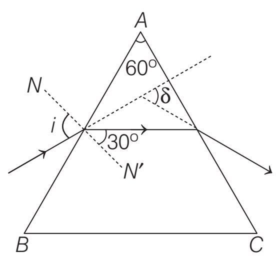
or angle of minimum deviation $\delta = 60°$

Incident angle, $i = \frac{60+\delta}{2} = 60°$

**Method 2** For minimum deviation, ray should pass symmetrically (i.e. parallel to the base of the equilateral prism)

$\Rightarrow$ From geometry of given figure, we have,

$r = 30°$



Using Snell's law, $n = \frac{\sin i}{\sin r}$

$$\sin i = n \sin r = \sqrt{3} \sin 30°$$

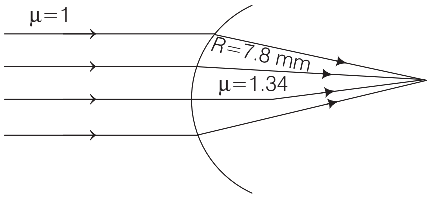
$$\Rightarrow \quad \sin i = \frac{\sqrt{3}}{2} \text{ or } i = 60°$$

**43** The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus. **[2019, 10 Jan Shift-II]**

(a) 4.0 cm (b) 2 cm
(c) 3.1 cm (d) 1 cm

**Ans.** *(c)*

The given condition is shown in the figure below




where, a parallel beam of light is coming from air ($\mu = 1$) to a spherical surface (eye) of refractive index 1.34.

Radius of curvature of this surface is 7.8 mm.

From the image formation formula for spherical surface, i.e. relation between object, image and radius of curvature.

$$\frac{\mu_r}{v} - \frac{\mu_i}{u} = \frac{\mu_r - \mu_i}{R} \quad \text{... (i)}$$

Given, $\mu_r = 1.34$, $\mu_i = 1$, $u = \infty$ (– ve) and $R = 7.8$

Substituting the given values, we get

$$\frac{1.34}{v} + \frac{1}{\infty} = \frac{1.34 - 1}{7.8} \text{ or } \frac{1.34}{v} = \frac{0.34}{7.8}$$

$$\Rightarrow \quad v = \frac{1.34 \times 7.8}{0.34} \text{mm}$$

$$\Rightarrow \quad v = \frac{4}{3} \times 3 \times 7.8 \text{ mm}$$

($\because$ approximately $1.34 = 4/3$ and $0.34 = 1/3$)

$$\Rightarrow \quad v = 31.2 \text{ mm or } 3.12 \text{ cm}$$

**44** In an experiment for determination of refractive index of glass of a prism by $i$-$\delta$, plot, it was found that a ray incident at an angle 35° suffers a deviation of 40° and that it emerges at an angle 79°. In that case, which of the following is closest to the maximum possible value of the refractive index? **[JEE Main 2016]**

(a) 1.5 (b) 1.6
(c) 1.7 (d) 1.8

**Ans.** *(a)*

**Key Idea** If $\mu$ is refractive index of material of prism, then from Snell's law

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin(A + \delta_m)/2}{\sin A/2} \quad \text{...(i)}$$

where, $A$ is angle of prism and $\delta_m$ is minimum deviation through prism.

Given, $i = 35°$, $\delta = 40°$, $e = 79°$.

So, angle of deviation by a glass prism,

$$\delta = i + e - A \Rightarrow 40° = 35° + 79° - A$$

i.e. Angle of prism $\Rightarrow A = 74°$.

Such that, $r_1 + r_2 = A = 74°$.

Let us put $\mu = 1.5$ in eq. (i), we get

$$1.5 = \frac{\sin\left(\frac{A + \delta_{min}}{2}\right)}{\sin A/2}$$

$$\Rightarrow \quad 1.5 = \frac{\sin\left(\frac{74° + \delta_{min}}{2}\right)}{\sin 37°}$$

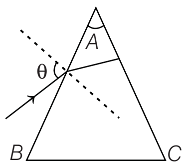
$$\Rightarrow 0.9 = \sin\left(37° + \frac{\delta_{min}}{2}\right) \quad (\because \sin 37° \approx 0.6)$$

$$\text{in } 64° = \sin\left(37° + \frac{\delta_{min}}{2}\right) \quad (\because \sin 64° = 0.9)$$

$$37° + \frac{\delta_{min}}{2} = 64° \Rightarrow \delta_{min} \approx 54°$$

This angle is greater than the 40° deviation angle already given. For greater $\mu$, deviation will be even higher. Hence, $\mu$ of the given prism should be lesser than 1.5. Hence, the closest option will be 1.5.

**45** Monochromatic light is incident on a glass prism of angle $A$. If the refractive index of the material of the prism is $\mu$, a ray incident at an angle $\theta$, on the face $AB$ would get transmitted through the face $AC$ of the prism provided **[JEE Main 2015]**




(a) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(b) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

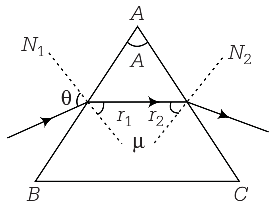
(c) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(d) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

**Ans.** *(a)*

**Key Idea** The ray will get transmitted through face AC if $i_{AC} < i_C$

Consider the ray diagram as shown below.




A ray of light incident on face $AB$ at an angle $\theta$.

$r_1$ = Angle of refraction on face $AB$

$r_2$ = Angle of incidence at face $AC$

For transmission of light through face $AC$

$$i_{AC} < i_C \text{ or } A - r_1 < i_C$$

$$\text{or } \sin(A - r_1) < \sin i_C \text{ or } \sin(A - r_1) < \frac{1}{\mu}$$

$$A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\text{or } \quad \sin r_1 > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

Now, applying Snell's law at the face $AB$

$$1 \times \sin\theta = \mu \sin r_1 \text{ or } \sin r_1 = \frac{\sin\theta}{\mu}$$

$$\Rightarrow \quad \frac{\sin\theta}{\mu} > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

$$\text{or} \quad \theta > \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$$

**46** A green light is incident from the water to the air-water interface at the critical angle ($\theta$). Select the correct statement. **[JEE Main 2014]**

(a) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal
(b) The spectrum of visible light whose frequency is less than that of green light will come out of the air medium
(c) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium
(d) The entire spectrum of visible light will come out of the water at various angles to the normal

**Key Idea** For total internal reflection of light take place, then following conditions must be obeyed.

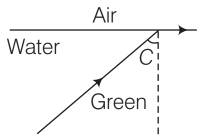
(i) The ray must travel from denser to rarer medium.

(ii) Angle of incidence ($\theta$) must be greater than or equal to critical angle ($C$) *i.e.*, $C = \sin^{-1}\left[\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}\right]$

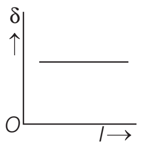
**Ans.** *(b)*

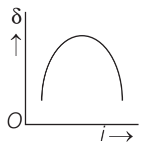
Here, $\sin C = \frac{1}{n_{\text{water}}}$ and $n_{\text{water}} = a + \frac{b}{\lambda^2}$

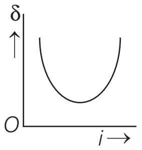
If frequency is less, then $\lambda$ is greater and hence RI $n_{(\text{water})}$ is less and therefore critical angle increases. So, they do not suffer reflection and come out at angle less than 90° of the air medium.

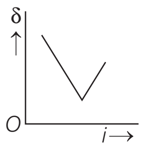



**47** The graph between angle of deviation ($\delta$) and angle of incidence ($i$) for a triangular prism is represented by **[JEE Main 2013]**

(a)


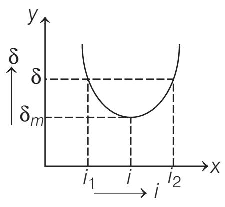

(b)



(c)



(d)



**Ans.** *(c)*

We know that the angle of deviation depends upon the angle of incidence. If we determine experimentally, the angles of deviation corresponding to different angles of incidence and then plot $i$ (on $x$-axis) and $\delta$ (on $y$-axis), we get a curve as shown in figure.




Clearly, if angle of incidence is gradually increased from a small value, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then begins to increase.

**48** A beaker contains water upto a height $h_1$ and kerosene of height $h_2$ above water so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is $\mu_1$ and that of kerosene is $\mu_2$. The apparent shift in the position of the bottom of the beaker when viewed from above is **[AIEEE 2011]**

(a) $\left(1-\frac{1}{\mu_1}\right)h_2 + \left(1-\frac{1}{\mu_2}\right)h_1$

(b) $\left(1+\frac{1}{\mu_1}\right)h_1 + \left(1+\frac{1}{\mu_2}\right)h_2$

(c) $\left(1-\frac{1}{\mu_1}\right)h_1 + \left(1-\frac{1}{\mu_2}\right)h_2$

(d) $\left(1+\frac{1}{\mu_1}\right)h_2 - \left(1+\frac{1}{\mu_2}\right)h_1$

**Ans.** *(c)*

We know, apparent shift, $\Delta h = \left(1-\frac{1}{\mu}\right)h$

$\therefore$ Apparent shift produced by water,

$$\Delta h_1 = \left(1-\frac{1}{\mu_1}\right)h_1$$

and apparent shift produced by kerosene,

$$\Delta h_2 = \left(1-\frac{1}{\mu_2}\right)h_2$$

$$\therefore \quad \Delta h = \Delta h_1 + \Delta h_2 = \left(1-\frac{1}{\mu_1}\right)h_1 + \left(1-\frac{1}{\mu_2}\right)h_2$$

**49** Let the $zx$-plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\mathbf{A} = 6\sqrt{3}\,\hat{\mathbf{i}} + 8\sqrt{3}\,\hat{\mathbf{j}} - 10\hat{\mathbf{k}}$ is incident on the plane of separation. The angle of refraction in medium 2 is **[AIEEE 2011]**

(a) 45° (b) 60°
(c) 75° (d) 30°

**Ans.** *(a)*

As refractive index for $z > 0$ and $z \leq 0$ is different, $xy$-plane should be boundary between two media.

Angle of incidence,

$$\cos i = \left|\frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}\right| = \frac{1}{2}$$

$$\therefore \quad i = 60°$$

From Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{\sqrt{3}}{\sqrt{2}}$

$$\sin r = \frac{\sqrt{2}}{\sqrt{3}} \times \sin 60°$$

$$= \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}} = 45°$$

$$\Rightarrow \quad r = 45°$$

**50** As the beam enters the medium, it will **[AIEEE 2010]**

(a) diverge
(b) converge
(c) diverge near the axis and converge near the periphery
(d) travel as a cylindrical beam

**Ans.** *(b)*

As intensity is maximum at axis. Therefore, $\mu$ will be maximum and speed will be minimum on the axis of the beam. Hence, beam will converge.

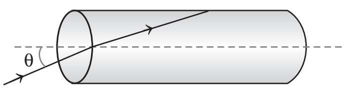
**51** The speed of light in the medium is **[AIEEE 2010]**

(a) minimum on the axis of the beam
(b) the same everywhere in the beam
(c) directly proportional to the intensity $I$
(d) maximum on the axis of the beam

**Ans.** *(c)*

The speed of light in the medium is directly proportional to the intensity $I$.

**52** A transparent solid cylinder rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.




The incident angle $\theta$ for which the light ray grazes along the wall of the rod is **[AIEEE 2009]**

(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

**Ans.** (*d*)

$$\sin C = \frac{\sqrt{3}}{2} \qquad ...(i)$$

$$\sin r = \sin(90° - C) = \cos C = \frac{1}{2}$$

$$\frac{\sin\theta}{\sin r} = \frac{\mu_2}{\mu_1}$$

$$\sin\theta = \frac{2}{\sqrt{3}} \times \frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

**53** The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let $D_1$ and $D_2$ be angles of minimum deviations for red and blue light respectively in a prism of this glass. Then, **[AIEEE 2006]**

(a) $D_1 < D_2$
(b) $D_1 = D_2$
(c) can be less than or greater than $D_2$ depending upon the angle of prism
(d) $D_1 > D_2$

**Ans.** (*a*)

$$D = (\mu - 1)A$$

For blue light, $\mu$ is greater than that for red light, so $D_2 > D_1$.

Aliter

$$D_1 = (1.520 - 1)A$$

$$D_2 = (1.525 - 1)A$$

$$\Rightarrow \quad \frac{D_1}{D_2} < 1$$

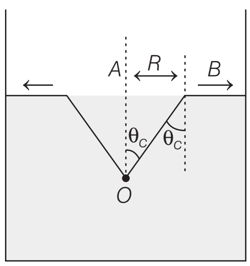
$$\Rightarrow \quad D_2 > D_1$$

**54** A fish looking up through the water sees the outside world, contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the water surface, the radius of this circle (in cm) is **[AIEEE 2005]**

(a) $36\sqrt{7}$ (b) $\frac{36}{\sqrt{7}}$
(c) $36\sqrt{5}$ (d) $4\sqrt{5}$

**Ans.** (*b*)

The situation is shown in figure.




$$\sin\theta_C = \frac{1}{\mu} \Rightarrow \tan\theta_C = \frac{AB}{OA}$$

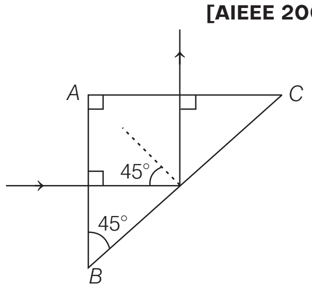
$$\therefore AB = OA\tan\theta_C$$

$$= OA\frac{\sin\theta_c}{\cos\theta_c} = \frac{\sin\theta_c}{\sqrt{1-\sin^2\theta_c}} \times OA$$

$$= \frac{1/\mu}{\sqrt{1-1/\mu^2}} \times OA = \frac{OA}{\sqrt{\mu^2-1}}$$

$$\text{or } AB = \frac{OA}{\sqrt{\mu^2-1}} = \frac{12}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}} = \frac{36}{\sqrt{7}}$$

**55** A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that for the refractive index *n* as **[AIEEE 2004]**




(a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$
(c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$

**Ans.** (*b*)

For total internal reflection from glass-air interface, critical angle *C* must be less than angle of incidence.

$$i.e., \quad C < i$$

$$\text{or} \quad C < 45° \quad [\because \angle i = 45°]$$

$$\text{But } n = \frac{1}{\sin C} \Rightarrow C = \sin^{-1}\left(\frac{1}{n}\right)$$

$$\text{or} \quad \sin^{-1}\left(\frac{1}{n}\right) < 45° \text{ or } \frac{1}{n} < \sin 45°$$

$$\text{or} \quad n > \frac{1}{\sin 45°} \text{ or } n > \frac{1}{(1/\sqrt{2})} \text{ or } n > \sqrt{2}$$

**56** Which of the following is used in optical fibres ? **[AIEEE 2002]**

(a) Total internal reflection
(b) Scattering
(c) Diffraction
(d) Refraction

**Ans.** (*a*)

Optical fibres work on the principle of total internal reflection.

# TOPIC 3
## Lenses

**57** An object is placed at the focus of concave lens having focal length *f*. What is the magnification and distance of the image from the optical centre of the lens? **[2021, 31 Aug Shift-I]**

(a) 1, ∞ (b) Very high, ∞
(c) $\frac{1}{2}, \frac{f}{2}$ (d) $\frac{1}{4}, \frac{f}{4}$

**Ans.** *(c)*

Given, focal length of concave lens, $f' = -f$

Object distance from lens, $u = -f$

Image distance from lens $= v$

Magnification $= m$

By using lens formula,

$$\frac{1}{f'} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f'} + \frac{1}{u}$$

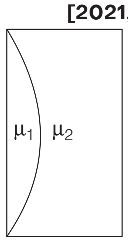
$$= -\frac{1}{f} - \frac{1}{f} = -2/f$$

$$\Rightarrow \quad v = -f/2$$

$$\therefore \quad |v| = \frac{f}{2}$$

$$\therefore \quad m = \frac{v}{u} = \frac{-f/2}{-f} = \frac{1}{2}$$

**58** Curved surfaces of a plano-convex lens of refractive index $\mu_1$ and a plano-concave lens of refractive index $\mu_2$ have equal radius of curvature as shown in figure. Find the ratio of radius of curvature to the focal length of the combined lenses. **[2021, 27 Aug Shift-I]**




(a) $\frac{1}{\mu_2 - \mu_1}$ (b) $\mu_1 - \mu_2$
(c) $\frac{1}{\mu_1 - \mu_2}$ (d) $\mu_2 - \mu_1$

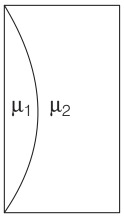
**Ans.** *(b)*

According to given diagram,

Refractive indices are $\mu_1$ and $\mu_2$.

Radius of two lenses are $R_1 = R_2 = R$

By using lens Maker's formula,




$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, $f$ = focal length.

Now, for plano-convex lens $R_1 = \infty$ and $R_2 = -R$

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\Rightarrow \quad \frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R}\right) \quad \text{...(i)}$$

For plano-concave lens, $R_1 = -R$ and $R_2 = \infty$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(-\frac{1}{R}\right) \quad \text{...(ii)}$$

Therefore, combined focal length is given by

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

where, $f$ = net focal length of combination of lenses.

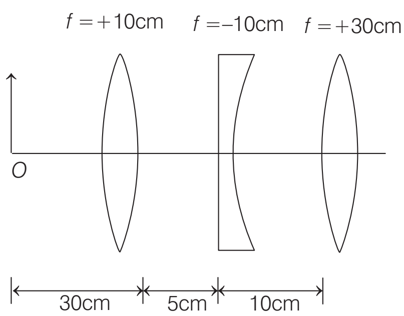
$$= (\mu_1 - 1)\left(\frac{1}{R}\right) + (\mu_2 - 1)\left(-\frac{1}{R}\right)$$

$$= \frac{\mu_1}{R} - \frac{1}{R} - \frac{\mu_2}{R} + \frac{1}{R}$$

$$\frac{1}{f} = \frac{\mu_1 - \mu_2}{R}$$

$$\therefore \quad \frac{R}{f} = \mu_1 - \mu_2$$

**59** Find the distance of the image from object $O$, formed by the combination of lenses in the figure. **[2021, 27 Aug Shift-II]**




(a) 75 cm (b) 10 cm
(c) 20 cm (d) infinity

**Ans.** *(a)*

Given, focal length of first lens,

$f_1 = +10$ cm

Focal length of second lens,

$f_2 = -10$ cm

Focal length of third lens, $f_3 = +30$ cm

Object distance from first lens,

$u_1 = -30$ cm

Using lens formula for first lens, we get

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$$

$$\Rightarrow \quad \frac{1}{-10} = \frac{1}{v_1} - \frac{1}{-30}$$

$$\Rightarrow \quad \frac{1}{v_1} = \frac{1}{10} - \frac{1}{30} = \frac{2}{30}$$

$$\Rightarrow \quad v_1 = 15 \text{ cm}$$

Thus, the image formed by first lens is 15 cm to the right of first lens.

The object distance for second lens will be

$$u_2 = 15 \text{ cm} - 5 \text{ cm}$$
$$= 10 \text{ cm}$$

Using lens formula for first lens, we get

$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2}$$

$$\Rightarrow \quad \frac{1}{-10} = \frac{1}{v_2} - \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{v_2} = \frac{1}{-10} + \frac{1}{10} = 0$$

$$\Rightarrow \quad v_2 = \infty$$

Thus, the object distance for third lens, $u_3 = \infty$.

Using lens formula for third lens, we get

$$\frac{1}{f_3} = \frac{1}{v_3} - \frac{1}{u_3}$$

$$\Rightarrow \quad \frac{1}{30} = \frac{1}{v_3} - \frac{1}{\infty}$$

$$\Rightarrow \quad \frac{1}{v_3} = \frac{1}{30}$$

$$v_3 = 30 \text{ cm}$$

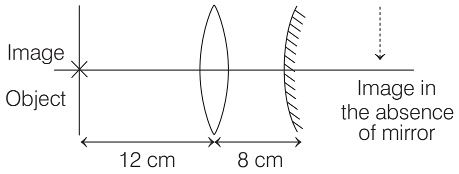
The final image will form 30 cm from the third lens.

The distance of final image from object will be

$$30 + 10 + 5 + 30 = 75 \text{ cm}.$$

Thus, the distance of final image formed from object $O$ is 75 cm.

**60** An object is placed at a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8 cm as shown in the figure. Image of object coincides with the object.

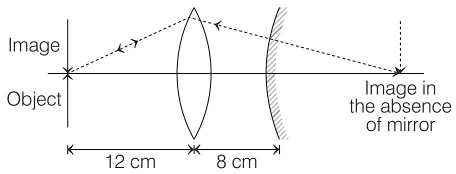



When the convex mirror is removed, a real and inverted image is formed at a position. The distance of the image from the object will be ........ cm.

**[2021, 26 Aug Shift-II]**

**Ans.** *(50)*

It is given that image coincides with the object. In the arrangement given, the image will only coincide with the object when the ray on mirror falls directly perpendicular on the surface of it.




The light will converge at centre of curvature of mirror and due to that after removing the mirror the light will converge at same position.

The focal length of mirror, $f = 15$ cm. So, radius of curvature of mirror, $2f = 30$ cm

The image distance from the object after convex mirror is removed will be calculated as, 12 cm + 8 cm + 30 cm = 50 cm.

Thus, the image will formed at a distance of 50 cm from object.

**61** The thickness at the centre of a plano convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is $2 \times 10^8 \text{ ms}^{-1}$, then the focal length of the lens is **[2021, 17 March Shift-I]**

(a) 0.30 cm (b) 15 cm
(c) 1.5 cm (d) 30 cm

**Ans.** *(d)*

Given, thickness at the centre of plano-convex lens, $t = 3 \text{ mm} = 3 \times 10^{-3}$ m

Diameter of plano-convex lens, $d = 6$ cm

$\therefore$ Radius of plano-convex lens, $r = 3$ cm $= 3 \times 10^{-2}$ m

Speed of light in lens material ,

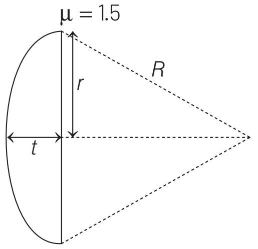
$$v = 2 \times 10^8 \text{ ms}^{-1}$$

Refractive index

$$= \frac{\text{Speed of light in air}}{\text{Speed of light in medium}}$$

$$\mu = \frac{3\times10^8}{2\times10^8} = \frac{3}{2} = 1.5$$

$\Rightarrow \quad \mu = 1.5$




We know that, $R^2 = (R-t)^2 + r^2$

where, $R$ is the radius of curvature of plano convex lens.

$\Rightarrow \quad R^2 = R^2 + t^2 - 2Rt + r^2$

$\Rightarrow \quad 2Rt = r^2 + t^2$

As, $t$ is small, then $t^2$ will be very very small, so it can be neglected.

$\Rightarrow \quad 2Rt = r^2$

$\Rightarrow \quad R = r^2/2t \quad$ ...(i)

From lens Maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here $R_1 = R$ & $R_2 = \infty$

$\Rightarrow \quad \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$

$\Rightarrow \quad \frac{1}{f} = \frac{\mu - 1}{R} \quad$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{1}{f} = \frac{\mu - 1(2t)}{r^2}$$

$\Rightarrow \quad f = \frac{r^2}{(\mu - 1)2t}$

Putting the given values in above equation, we get

$$f = \frac{(3\times10^{-2})^2}{(1.5-1)2\times3\times10^{-3}} \Rightarrow f = 0.3\text{ m}$$

$f = 30$ cm

**62** The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index ? (Assume the radii of curvature of the faces of lens are $R_1$ and $R_2$ respectively)

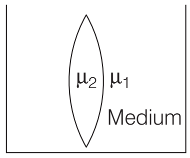
**[2021, 16 March Shift-II]**

(a) 1 (b) Infinite

(c) $\frac{R_1R_2}{R_1 - R_2}$ (d) Zero

**Ans.** ***(b)***

Consider a convex lens of refractive index $\mu_2$ and $\mu_1$ is the refractive index of the medium in which it is placed.




$\Rightarrow \quad \mu_2 = \mu_1 \quad$ ... (i)

According to the lens Maker's formula,

$$\frac{1}{f} = \left[\frac{\mu_1}{\mu_2} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ...(ii)$$

where, $f$ is the focal length of the lens, $R_1$ and $R_2$ are the radii of curvature of respective faces of lens.

From Eqs. (i) and (ii), we can write

$$\frac{1}{f} = [1-1]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = 0$$

$\Rightarrow \quad \frac{1}{f} = 0 \Rightarrow f = \text{Infinite}$

**63** The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is ........... cm.

**[2021, 25 Feb Shift-I]**

**Ans.** ***(15)***

Let $v$ be the position of image, $h_i$ be the height of image and $h_o$ be the height of object.

Given, $\quad h_i = h_o$

Since, magnification, $m = h_i/h_o = h_i/h \quad$ ... (i)

By using lens formula,

$1/f = 1/v - 1/u \Rightarrow 1/v = 1/f + 1/u$

$\Rightarrow \quad v = \frac{fu}{u+f}$

$\therefore \quad m = \frac{v}{u} = \frac{f}{f+u}$

Now, from Eq. (i), $m$ can be $\pm$ 1.

For, $m = +1 = \frac{f}{-10+f} \quad$ ... (ii)

For $m = -1$

$$= \frac{f}{-20+f} \quad ... (iii)$$

On dividing Eq. (iii) by Eq. (ii), we get

$$-1 = \frac{-10+f}{-20+f}$$

$\Rightarrow \quad 20 - f = -10 + f$

$\Rightarrow \quad 30 = 2f$

$\Rightarrow \quad f = 15$ cm

**64** A point like object is placed at a distance of 1 m in front of a convex lens of focal length 0.5 m. A plane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final image formed by the system is **[2020, 6 Sep Shift-I]**

(a) 2.6 m from the mirror, real

(b) 1 m from the mirror, virtual

(c) 1 m from the mirror, real

(d) 2.6 m from the mirror, virtual

**Ans.** ***(a)***

Given that, for convex lens,

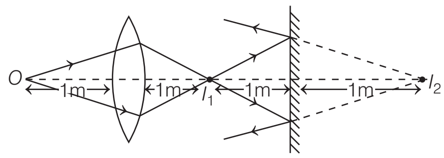
$$u_1 = -1\text{ m}, f = +0.5\text{ m}$$

Applying lens formula,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f}$$

$$\frac{1}{v_1} - \frac{1}{-1} = \frac{1}{0.5}$$

$\Rightarrow \quad v_1 = +1$ m




The image $I_1$ will behave as an object for plane mirror. So, image will be formed 1 m behind the mirror as shown in the above figure.

Now, the virtual image $I_2$ will act as an object at a distance of 3m from lens.

For final image $I_3$,

$$u = -3\text{ m},$$
$$f = +0.5\text{ m}$$

Again, applying lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-3} = \frac{1}{0.5}$$

or $\quad v = \frac{3}{5} = +0.6$ m

So, total distance of final image from mirror $= 2 + 0.6 = 2.6$ m. The image will be real (but in answer it was given virtual).

**65** The distance between on object and a scren is 100 cm. A lens can produce real image of the object on the screen for two different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right)$D, where $N$ is an integer.

Then, the value of $N$ is ......... .

**[2020, 4 Sep Shift-II]**

**Ans.** *(476)*

Using displacement method, the focal length of a lens is given by

$$f = \frac{D^2 - d^2}{4D} \qquad \text{...(i)}$$

Given that, distance between object and screen, $D = 100$ cm

Displacement travelled by object, $d = 40$ cm

Substituting these values in eq. (i), we get

$$f = \frac{(100)^2 - (40)^2}{4(100)} = \frac{10000 - 1600}{400}$$

$$= \frac{8400}{400} = 21\,\text{cm} = 0.21\,\text{m}$$

Now, optical power of lens is

$$P = \frac{1}{f} = \frac{1}{0.21\,\text{m}} = \frac{100}{21\,\text{m}}$$

$$= \frac{100}{21}\,\text{m}^{-1} = \frac{100}{21}\,\text{D} = 4.76\,\text{D} = \left(\frac{476}{100}\right)\text{D}$$

Given that,

$$P = \left(\frac{N}{100}\right)\text{D}$$

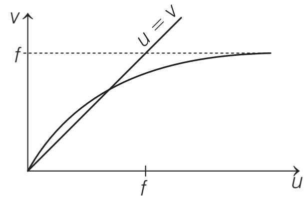
On comparing both the equations, we get

$$N = 476$$

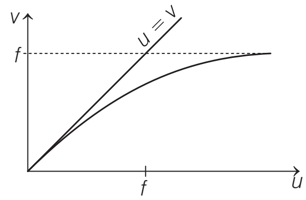
**66** For a concave lens of focal length $f$, the relation between object and image distances $u$ and $v$, respectively, from its pole can best be represented by ($u = v$ is the reference line)

**[2020, 5 Sep Shift-I]**

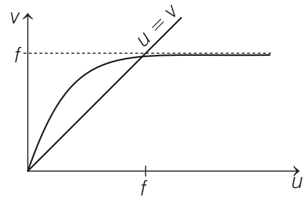
(a)



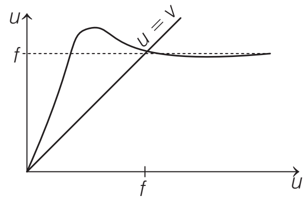

(b)




(c)




(d)




**Ans.** *(b)*

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\Rightarrow \quad v = \frac{uf}{u+f}$$

Now, considering two cases:

Case I If $v = u$,

$$\Rightarrow \quad u = \frac{uf}{u+f} \Rightarrow u + f = u \Rightarrow u = 0$$

So, the curve will pass through origin.

Case II If $u = \infty \Rightarrow v = f$

So, as $u$ tends towards infinity, $v$ tends towards focus and all other distances will be less than focal length $f$ and it will never intersect $u = v$ line.

Hence, correct option is (b).

**67** A double convex lens has power $P$ and same radii of curvature $R$ of both the surfaces. The radius of curvature of a surface of a plano-convex lens made of the same material with power 1.5 $P$ is

**[2020, 6 Sep Shift-II]**

(a) $2R$ (b) $\frac{R}{2}$

(c) $\frac{3R}{2}$ (d) $\frac{R}{3}$

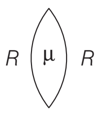
**Ans.** *(d)*

For double convex lens,

$$R_1 = R_2 = R \text{ (given)}$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$$



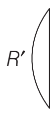

$$\text{Power,} \quad P = \frac{1}{f} = \frac{2(\mu - 1)}{R} \qquad \text{...(i)}$$

For plano-convex lens,

$$R_1 = R', R_2 = \infty$$

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R'} - \frac{1}{\infty}\right)$$

$$\frac{1}{f'} = \frac{\mu - 1}{R'}$$




$$\text{Power,}\ P' = \frac{1}{f'} = \frac{(\mu - 1)}{R'}$$

$$\text{Given,} \quad P' = 1.5P \Rightarrow 1.5P = \frac{\mu - 1}{R'} \qquad \text{...(ii)}$$

Dividing Eqs. (i) by (ii), we get

$$\frac{P}{1.5P} = \frac{\frac{2(\mu - 1)}{R}}{\frac{(\mu - 1)}{R'}} \Rightarrow \frac{2}{3} = \frac{2R'}{R} \text{ or } R' = \frac{R}{3}$$

Hence, correct option is (d).

**68** A thin lens made of glass (refractive index = 1.5) of focal length $f = 16$ cm in immersed in a liquid of refractive index 1.42. If its focal length in liquid is $f_1$, then the ratio $f_1 / f$ is closest to the integer

**[2020, 7 Jan Shift-II]**

(a) 1 (b) 5 (c) 9 (d) 17

**Ans.** *(c)*

Focal length of lens of refractive index $n_2$ placed in a medium of refractive index $n_1$ is

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a glass-lens in air, focal length,

$$\frac{1}{f_{\text{air}}} = \frac{1}{f} = \left(\frac{n_g}{n_a} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \text{... (i)}$$

When lens is dipped in a liquid, its focal length,

$$\frac{1}{f_{\text{liquid}}} = \frac{1}{f_1} = \left(\frac{n_g}{n_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{... (ii)}$$

From Eqs. (i) and (ii), we have

$$\frac{f_1}{f} = \frac{\left(\frac{n_g}{n_a} - 1\right)}{\left(\frac{n_g}{n_l} - 1\right)}$$

As $f = 16$ cm, $n_g = 1.5$, $n_l = 1.42$, we have

$$\frac{f_1}{f} = \frac{\left(\frac{1.5}{1} - 1\right)}{\left(\frac{1.5}{1.42} - 1\right)} \Rightarrow \frac{f_1}{f} = \left(\frac{\frac{3}{2} - 1}{\frac{150}{142} - 1}\right) = \left(\frac{\frac{1}{2}}{\frac{4}{71}}\right)$$

$$= \frac{71}{8} \approx 8.9$$

So, $\frac{f_1}{f}$ is closest to integer 9.

**69** A point object in air is in front of the curved surface of a *plano-convex* lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is ......... .

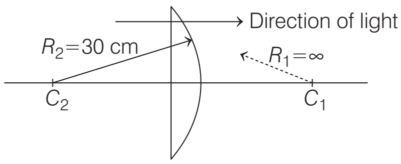
**[2020, 8 Jan Shift-I]**

**Ans.** *(60)*

Focal length of lens by lens maker's formula,

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a plano-convex lens,




$$R_1 = \infty, R_2 = -30 \text{ cm}, n_{21} = 1.5$$

$$\therefore \quad \frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{(-30)}\right)$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{2} \times \frac{1}{30}$$

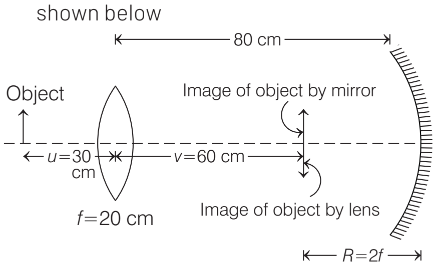
$$\Rightarrow \quad f = 60 \text{ cm}$$

**70** A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be **[2019, 8 April Shift-II]**

(a) 25 cm (b) 20 cm
(c) 10 cm (d) 30 cm

**Ans.** *(c)*

The given situation can be drawn as shown below




For lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Substituting given values, we get

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{20} \Rightarrow v = 60 \text{ cm}$$

So, this image is at a distance of $80 - 60 = 20$ cm from the mirror.

As, the image formed by the mirror coincides with image formed by the lens.

This condition is only possible, if any object that has been placed in front of concave mirror is at centre of curvature, i.e. at $2f$.

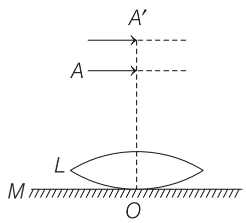
So, radius of curvature of mirror is $R = 20$ cm

$\therefore$ Focal length of mirror, $f = \frac{R}{2} = 10$ cm

As, for virtual image, the object is to kept between pole and focus of the mirror.

$\therefore$ The maximum distance of the object for which this concave mirror by itself produce a virtual image would be 10 cm.

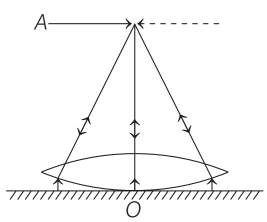
**71** A thin convex lens $L$ (refractive index = 1.5) is placed on a plane mirror $M$. When a pin is placed at $A$, such that $OA = 18$ cm, its real inverted image is formed at $A$ itself, as shown in figure. When a liquid of refractive index $\mu_l$ is put between the lens and the mirror, the pin has to be moved to $A'$, such that $OA' = 27$ cm, to get its inverted real image at $A'$ itself. The value of $\mu_l$ will be **[2019, 9 April Shift-II]**




(a) $\sqrt{3}$ (b) $\sqrt{2}$
(c) $\frac{4}{3}$ (d) $\frac{3}{2}$

**Ans.** *(c)*

Light from plane mirror is reflected back on it's path, so that image of $A$ coincides with $A$ itself.




This would happen when rays refracted by the convex lens falls normally on the plane mirror, i.e. the refracted rays form a beam parallel to principal axis of the lens. Hence, the object would then be considered at the focus of convex lens.

$\therefore$ Focal length of curvature of convex lens is,

$$f_1 = 18 \text{ cm}$$

With liquid between lens and mirror, image is again coincides with object, so the second measurement is focal length of combination of liquid lens and convex lens.

$$\therefore \quad \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{eq}} \Rightarrow \frac{1}{18} + \frac{1}{f_2} = \frac{1}{27}$$

$$\Rightarrow \quad f_2 = -54 \text{ cm}$$

For convex lens by lens maker's formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\Rightarrow \quad \frac{1}{18} = 0.5 \times \frac{2}{R} \Rightarrow R = 18 \text{ cm}$$

and for plano-convex liquid lens, we have

$$\frac{1}{f} = (\mu_l - 1)\left(\frac{-1}{R}\right)$$

$$\Rightarrow \quad -\frac{1}{54} = (\mu_l - 1)\left(\frac{-1}{18}\right)$$

$$\Rightarrow \quad \mu_l = 1 + \frac{1}{3} = \frac{4}{3}$$

**72** A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances $x_1$ and $x_2$ $(x_1 > x_2)$ from the lens. The ratio of $x_1$ and $x_2$ is **[2019, 9 April Shift-II]**

(a) 5 : 3 (b) 2 : 1
(c) 4 : 3 (d) 3 : 1

**Ans.** *(d)*

Since, image formed by a convex lens can be real or virtual in nature.

Thus, $m = +2$ or $-2$.

First let us take image to be real in nature,

then $m = -2 = \frac{v}{u}$, where $v$ is image distance and $u$ is object distance.

$$\Rightarrow \quad v = -2x_1 \quad [\text{Taking } u = x_1]$$

Now, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{-2x_1} - \frac{1}{x_1} = \frac{1}{20} \Rightarrow \frac{-3}{2x_1} = \frac{1}{20}$$

$$\Rightarrow \quad x_1 = -30 \text{ cm}$$

Now, let us take image to be virtual in nature, then

$$m = 2 = \frac{v}{u} \Rightarrow v = 2x_2 \quad [\text{Taking } u = x_2]$$

Again, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{2x_2} - \frac{1}{x_2} = \frac{1}{20}$$

$$\Rightarrow \quad \frac{-1}{2x_2} = \frac{1}{20}$$

$$\Rightarrow \quad x_2 = -10 \text{ cm}$$

So, the ratio of $x_1$ and $x_2$ is

$$\frac{x_1}{x_2} = \frac{-30}{-10} = 3:1$$

Alternate Solution

Magnification for a lens can also be written as

$$m = \left(\frac{f}{f+u}\right)$$

When $m = -2$ (for real image)

$$-2 = \frac{f}{f + x_1}$$

$$\Rightarrow \quad -2f + (-2)\,x_1 = f \text{ or } x_1 = -\frac{3f}{2}$$

Similarly, when $m = +2$ (for virtual image)

$$+2 = \frac{f}{f + x_2} \Rightarrow 2f + 2x_2 = f$$

or $$x_2 = \frac{-f}{2}$$

Now, the ratio of $x_1$ and $x_2$ is

$$\frac{x_1}{x_2} = \frac{\frac{-3f}{2}}{-f/2} = \frac{3}{1}$$

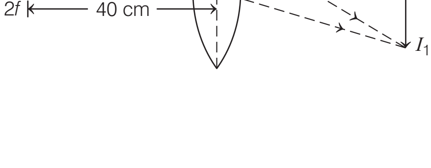
**73** An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be **[2019, 8 April Shift-I]**

(a) 20 cm from the convergent mirror, same size as the object
(b) 40 cm from the convergent mirror, same size as the object
(c) 40 cm from the convergent lens, twice the size of the object
(d) 20 cm from the convergent mirror,twice size of the object

**Ans.** *(*)*

In given system of lens and mirror, position of object *O* in front of lens is at a distance $2f$.

i.e. $u = 2f = 40$ cm

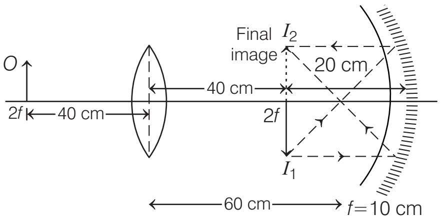



So, image ($I_1$) formed is real, inverted and at a distance, $v = 2f = 2 \times 20 = 40$ cm, (behind lens) magnification,

$$m_1 = \frac{v}{u} = \frac{40}{40} = 1$$

Thus, size of image is same as that of object.

This image ($I_1$) acts like a real object for mirror.




As object distance for mirror is

$$u = C = 2f = -20 \text{ cm}$$

where, $C$ = centre of curvature.

So, image ($I_2$) formed by mirror is at $2f$.

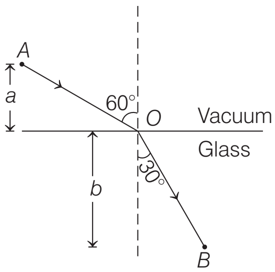
$\therefore$ For mirror $v = 2f = 2(-10) = -20$ cm

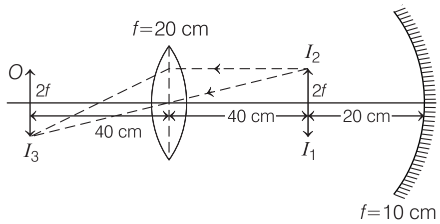
Magnification, $m_2 = -\frac{v}{u} = -\frac{(-20)}{(-20)} = -1$

Thus, image size is same as that of object.

The image $I_2$ formed by the mirror will act like an

object for lens.




As the object is at $2f$ distance from lens, so image ($I_3$) will be formed at a distance $2f$ or 40 cm. Thus, magnification,

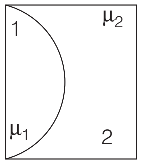
$$m_3 = \frac{v}{u} = \frac{40}{40} = 1$$

So, final magnification,

$$m = m_1 \times m_2 \times m_3 = -1$$

Hence, final image ($I_3$) is real, inverted of same size as that of object and coinciding with object.

**74** One plano-convex and one plano-concave lens of same radius of curvature $R$ but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is $\mu_1$ and that of 2 is $\mu_2$, then the focal length of the combination is **[2019, 10 April Shift-I]**




(a) $\frac{2R}{\mu_1 - \mu_2}$
(b) $\frac{R}{2 - (\mu_1 - \mu_2)}$
(c) $\frac{R}{2(\mu_1 - \mu_2)}$
(d) $\frac{R}{\mu_1 - \mu_2}$

**Ans.** *(d)*

**Key Idea** Focal length of two lenses in contact is given as

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

where, $f_1$ and $f_2$ are the focal lengths of the respective lenses.

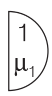
Focal length of a lens is given as

$$\frac{1}{f} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ Focal length of plano-convex lens, i.e. lens 1,

$$R_1 = \infty \text{ and } R = -R$$

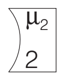
$$\Rightarrow \quad \frac{1}{f_1} = \frac{\mu_1 - 1}{R}$$




or $$f_1 = \frac{R}{(\mu_1 - 1)} \quad \text{...(i)}$$

Similarly, focal length of plano-concave lens, i.e. lens 2,

$$R_1 = -R \text{ and } R_2 = \infty$$




$$\Rightarrow \quad \frac{1}{f_2} = -\frac{(\mu_2 - 1)}{R}$$

or $$f_2 = \frac{-R}{(\mu_2 - 1)} \quad \text{...(ii)}$$

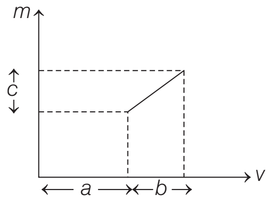
From Eqs. (i) and (ii), net focal length is

$$\frac{1}{f} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R}$$

$$= \frac{\mu_1 - \mu_2}{R}$$

$$\Rightarrow \quad f = \frac{R}{\mu_1 - \mu_2}$$

**75** The graph shows how the magnification $m$ produced by a thin lens varies with image distance $v$. What is the focal length of the lens used? **[2019, 10 April Shift-II]**




(a) $\frac{b^2c}{a}$ (b) $\frac{b^2}{ac}$

(c) $\frac{a}{c}$ (d) $\frac{b}{c}$

**Ans.** *(d)*

First we find relation of magnification $m$ and focal length $f$. By lens equation,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad 1-\frac{v}{u}=\frac{v}{f}$$

$$\Rightarrow \quad 1-m=\frac{v}{f} \quad \left[\because \frac{v}{u}=m\right]$$

$$\Rightarrow \quad m=1-\frac{v}{f} \quad ...(i)$$

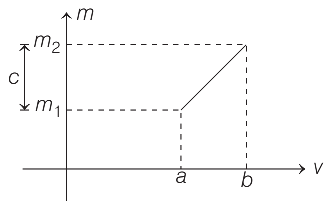
Now, from given graph and from Eq. (i),

At $v=a$, magnification is

$$m_1=1-\frac{a}{f} \quad ...(ii)$$

At $v=a+b$, magnification is

$$m_2=1-\frac{a+b}{f} \quad ...(iii)$$




From graph, we can also say that

$$m_2-m_1=c \quad ...(iv)$$

So, from Eqs. (ii), (iii) and (iv), we have

$$\left(1-\frac{(a+b)}{f}\right)-\left(1-\frac{a}{f}\right)=c$$

$$\Rightarrow \quad \frac{a-(a+b)}{f}=c$$

$$\Rightarrow \quad f=-\frac{b}{c} \text{ or } |f|=\frac{b}{c}$$

**76** A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now, a glass block (refractive index is 1.5) of 1.5 cm thickness is placed in contact with the light source.

To get the sharp image again, the screen is shifted by a distance $d$. Then, $d$ is **[2019, 9 Jan Shift-I]**

(a) 0

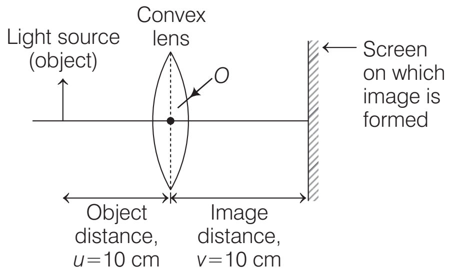
(b) 1.1 cm away from the lens

(c) 0.55 cm away from the lens

(d) 0.55 cm towards the lens

**Ans.** *(c)*

Initially, when a light source (i.e. an object) is placed at 10 cm from the convex mirror and an image is form on the screen as shown in the figure below,




Since, $u=v$ which can only be possible in the situation when the object is place at '$2f$' of the lens.

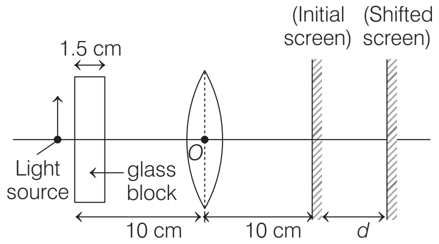
So, that the image can form at '$2f$' only on the other side of the lens.

Thus, the distance from the optical centre ($O$) of the lens and $2f$ is $2\times$ focal length ($f$) = object distance

$$\Rightarrow \quad 2\times f=10$$

or $\quad f=5\,\text{cm}$

Now, when a glass block is placed in contact with the light source i.e., object, then the situation is shown in the figure given below




Then due to the block, the position of the object in front of the lens would now be shifted due to refraction of the light source rays through the block.

The shift in the position of the object is given as

$$x=\left(1-\frac{1}{\mu}\right)t$$

where, $\mu$ is the refractive index of the block and $t$ is its thickness.

$$\Rightarrow \quad x=\left(1-\frac{1}{1.5}\right)1.5$$

$$=\left(1-\frac{2}{3}\right)\frac{3}{2}$$

$$=\frac{1}{3}\times\frac{3}{2}=\frac{1}{2}=0.5\,\text{cm}$$

$\therefore$ The new object distance of the light source in front of the lens will be

$$u'=10-0.5=9.5\,\text{cm}$$

Since, the focal length of the lens is 5 cm.

Therefore, the image distance of the light source now can be given as,

$$\frac{1}{v'}=\frac{1}{f}+\frac{1}{u'} \quad \text{(using lens formula)}$$

Substituting the values, we get

$$\frac{1}{v'}=\frac{1}{5}+\left(\frac{1}{-9.5}\right)=\frac{+9.5-5}{47.5}=\frac{4.5}{47.5}$$

or $\quad v'=10.55\,\text{cm}$

$\therefore$ The value of $d=v'-v=10.55-10$

$=0.55$ cm, away from the lens

Focal length in the above question can be calculated by using lens formula i.e.

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$$

**77** A plano-convex lens of refractive index $\mu_1$ and focal length $f_1$ is kept in contact with another plano-concave lens of refractive index $\mu_2$ and focal length $f_2$. If the radius of curvature of their spherical faces is $R$ each and $f_1=2f_2$, then $\mu_1$ and $\mu_2$ are related as **[2019, 10 Jan Shift-I]**

(a) $3\mu_2-2\mu_1=1$ (b) $2\mu_2-\mu_1=1$

(c) $2\mu_1-\mu_2=1$ (d) $\mu_1+\mu_2=3$

**Ans.** *(c)*

Given, $f_1=2f_2$

$$\Rightarrow \quad \frac{1}{|f_1|}=\frac{1}{|2f_2|} \quad ...(i)$$

Using lens Maker's formula, we get

$$\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{R_1'}-\frac{1}{R_2'}\right)$$

$$\Rightarrow \left|(\mu_1-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)\right|$$

$$=\left|\frac{1}{2}(\mu_2-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)\right| \quad [\because \text{using Eq. (i)}]$$

$$\Rightarrow \quad \frac{\mu_1-1}{R}=\frac{\mu_2-1}{2R}$$

$$\Rightarrow \quad 2\mu_1-\mu_2=1$$

**78** An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be

**[2019, 11 Jan Shift-I]**

(a) $3.22 \times 10^{-3}$ m/s towards the lens
(b) $0.92 \times 10^{-3}$ m/s away from the lens
(c) $2.26 \times 10^{-3}$ m/s away from the lens
(d) $1.16 \times 10^{-3}$ m/s towards the lens

**Ans.** *(d)*

Lens formula is given as

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \quad \text{...(i)}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{f}+\frac{1}{u}$$

$$\Rightarrow \quad \frac{uf}{u+f}=v$$

$$\Rightarrow \quad \frac{v}{u}=\frac{f}{u+f} \quad \text{...(ii)}$$

Now, by differentiating Eq. (i), we get

$$0=-\frac{1}{v^2}\cdot\frac{dv}{dt}+\frac{1}{u^2}\cdot\frac{du}{dt}$$

[$\because f$ (focal length of a lens is constant)]

or $\quad \dfrac{dv}{dt}=\dfrac{v^2}{u^2}du/dt$

$$\Rightarrow \quad \frac{dv}{dt}=\left(\frac{f}{u+f}\right)^2\frac{du}{dt} \quad \text{[using Eq. (ii)]}$$

Given, $f = 0.3$ m, $u = -20$ m, $du/dt = 5$ m/s

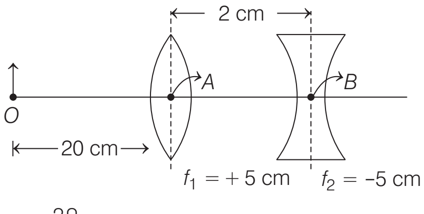
$$\therefore \quad \frac{dv}{dt}=\left(\frac{0.3}{0.3-20}\right)^2\times 5=\left(\frac{3}{197}\right)^2\times 5$$

$$=1.16\times 10^{-3} \text{ m/s}$$

Thus, the image is moved with a speed of $1.16\times 10^{-3}$ m/s towards the lens.

**79** What is the position and nature of image formed by lens combination shown in figure? (where, $f_1$ and $f_2$ are focal lengths)

**[2019, 12 Jan Shift-I]**




(a) $\dfrac{20}{3}$ cm from point $B$ at right, real
(b) 70 cm from point $B$ at right, real
(c) 40 cm from point $B$ at right, real
(d) 70 cm from point $B$ at left, virtual

**Ans.** *(b)*

For lens $A$, object distance, $u = -20$ cm

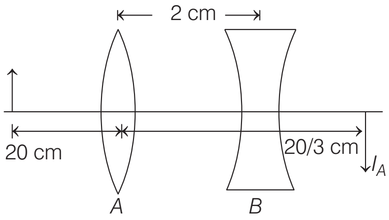
Focal length, $f = +5$ cm

From lens formula, $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$

We have, $\quad \dfrac{1}{v}=\dfrac{1}{5}-\dfrac{1}{20}$

$$\frac{1}{v}=\frac{20-5}{20\times 5}=\frac{15}{100}$$

$$v=\frac{20}{3}\text{ cm}$$




For lens $B$, image of $A$ is object for $B$.

$$\therefore \quad u=\frac{20}{3}-2=+\frac{14}{3}\text{ cm}$$

$$f=-5\text{ cm}$$

Now, from lens formula, we have

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v}=\frac{1}{-5}+\frac{3}{14}$$

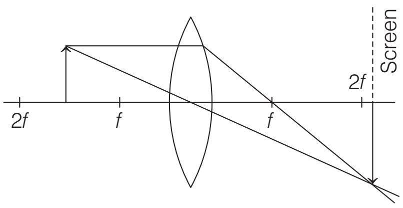
$$\frac{1}{v}=\frac{15-14}{5\times 14}$$

$$\Rightarrow \quad v=70\text{ cm}$$

Hence, image is on right of lens $B$ and is real in nature.

**80** Formation of real image using a biconvex lens is shown below. If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen?

**[2019, 12 Jan Shift-II]**




(a) No change
(b) Magnified image
(c) Image disappears
(d) Erect real image

**Ans.** *(c)*

When given set up is immersed in water, focal length of lens increases,

Note:

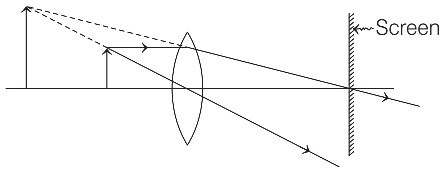
$$\frac{f_{\text{liquid}}}{f_{\text{air}}}=\frac{n_{ga}-1}{n_{gl}-1}=\frac{n_{ga}-1}{\left(\dfrac{n_{ga}}{n_{la}}-1\right)}$$

Now, $n_{ga}=\dfrac{3}{2}$ and $n_{la}=\dfrac{4}{3}$

$$\therefore f_{\text{liquid}}=f_{\text{air}}\left(\frac{\frac{1}{2}}{\frac{1}{8}}\right)=4f_{\text{air}}$$

As focal length increases there is no focussing of image on screen

$\therefore$ image will disappea



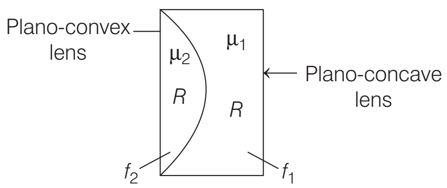

Actually a virtual image is formed on same side of object.

**81** A plano-convex lens (focal length $f_2$, refractive index $\mu_2$, radius of curvature $R$) fits exactly into a plano-concave lens (focal length $f_1$, refractive index $\mu_1$, radius of curvature $R$). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be **[2019, 12 Jan Shift-II]**

(a) $f_1 - f_2$
(b) $\dfrac{R}{\mu_2-\mu_1}$
(c) $f_1 + f_2$
(d) $\dfrac{2f_1 f_2}{f_1+f_2}$

**Ans.** *(b)*

Given combination is as shown below




As lenses are in contact, equivalent focal length of combination is

$$\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

Using lens Maker's formula,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here, $\quad \dfrac{1}{f_1}=(\mu_1-1)\left(\dfrac{1}{-R}-\dfrac{1}{\infty}\right)$

$$=\frac{(1-\mu_1)}{R}$$

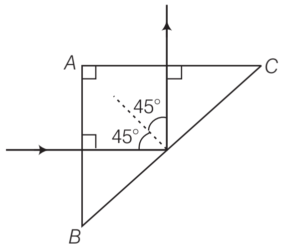
and $\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{\infty} - \frac{1}{(-R)}\right) = \frac{(\mu_2 - 1)}{R}$

∴ $\frac{1}{f_{eq}} = \left(\frac{\mu_2 - 1}{R}\right) + \left(\frac{1-\mu_1}{R}\right)$

$= \frac{\mu_2 - 1 + 1 - \mu_1}{R} = \frac{\mu_2 - \mu_1}{R}$

So, $f_{eq} = \frac{R}{\mu_2 - \mu_1}$.

**82** A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is **[JEE Main 2017]**

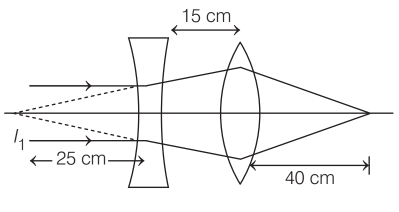



(a) virtual and at a distance of 40 cm from convergent lens
(b) real and at a distance of 40 cm from the divergent lens
(c) real and at a distance of 6 cm from the convergent lens
(d) real and at a distance of 40 cm from convergent lens

**Ans.** *(d)*

Focal length of diverging lens is 25 cm.

As the rays are coming parallel, so the image ($I_1$) will be formed at the focus of diverging lens i.e. at 25 cm towards left of diverging lens.




Now, the image ($I_1$) will work as object for converging lens.

For converging lens, distance of object $u$ (i.e. distance of $I_1$) $= -(25 + 15) = -40$ cm

$f = 20$ cm

∴ From len's formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\frac{1}{20} = \frac{1}{v} - \frac{1}{-40} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40}$

$\frac{1}{v} = \frac{1}{90} \Rightarrow v = 40$ cm

$v$ is positive so image will be real and will form at right side of converging lens at 40 cm.

**83** A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length $f$. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$. It has the focal lengths $f_1$ and $f_2$, respectively. The correct relation between the focal lengths is

(a) $f_1 = f_2 < f$ **[JEE Main 2014]**
(b) $f_1 > f$ and $f_2$ becomes negative
(c) $f_2 > f$ and $f_1$ becomes negative
(d) $f_1$ and $f_2$ both become negative

**Key Idea** It is based on lens maker's formula

*i.e.,* $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

**Ans.** *(b)*

According to lens maker's formula, when the lens in the air

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

⇒ $\frac{1}{f} = \frac{1}{2x} \Rightarrow f = 2x$

Here, $\frac{1}{x} = \frac{1}{R_1} - \frac{1}{R_2}$

In case of liquid,where refractive indices are 4/3 and 5/3, we get

Focal lenght in first liquid,

$$\frac{1}{f_1} = \left(\frac{\mu_s}{\mu_{l_1}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\frac{1}{x}$$

⇒ $F_1$ is positive.

Nature of lens is not change.

$$\frac{1}{f_1} = \frac{1}{8x} = \frac{1}{4(2x)} = \frac{1}{4f} \Rightarrow f_1 = 4f$$

Focal length in second liquid,

$$\frac{1}{f_2} = \left(\frac{\mu_s}{\mu_{l_2}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{x}\right)$$

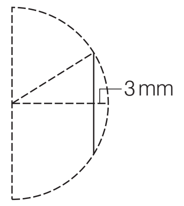
⇒ $f_2$ is negative.

Nature of lens change *i.e.*, convex behave as concave.

**84** Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is $2 \times 10^8$ m / s, the focal length of the lens is **[JEE Main 2013]**

(a) 15 cm (b) 20 cm
(c) 30 cm (d) 10 cm

**Ans.** *(c)*




$n = \frac{3}{2}$ $\left[\because n = \frac{c}{v}\right]$

$3^2 + (R - 3\,\text{mm})^2 = R^2$

$\Rightarrow 3^2 + R^2 - 2R(3\,\text{mm}) + (3\,\text{mm})^2 = R^2$

⇒ $R \approx 15$ cm

∴ $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$

$\left[\because \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ and } R_1 = \infty, R_2 = -15\right]$

⇒ $f = 30$ cm

**85** An object 2.4 m infront of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film? **[AIEEE 2012]**

(a) 7.2 m (b) 2.4 m
(c) 3.2 m (d) 5.6 m

**Ans.** *(d)*

Shift in image position due to glass plate,

$$S = \left(1 - \frac{1}{\mu}\right)t = \left(1 - \frac{1}{1.5}\right) \times 1\,\text{cm} = \frac{1}{3}\,\text{cm}$$

For focal length of the lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{12} - \frac{1}{-240}$$

or $\frac{1}{f} = \frac{20+1}{240}$

⇒ $f = \frac{240}{21}$ cm

Now, to get back image on the film, lens has to form image at $\left(12 - \frac{1}{3}\right)$ cm $= \frac{35}{3}$ cm such that the glass plate will shift the image on the film.

As $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

$\Rightarrow$ $\frac{1}{u}=\frac{1}{v}-\frac{1}{f}=\frac{3}{35}-\frac{21}{240}$

$=\frac{48\times3-7\times21}{1680}=-\frac{1}{560}$

$\Rightarrow$ $u=-5.6\,\text{m}$

**86** When monochromatic red light is used instead of blue light in a convex lens, its focal length will **[AIEEE 2011]**

(a) not depend on colour of light
(b) increase
(c) decrease
(d) remain same

**Ans.** (*b*)

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Also, by Cauchy's formula,

$$\mu=A+\frac{B}{\lambda^2}+\frac{C}{\lambda^4}+\ldots \Rightarrow \mu\propto\frac{1}{\lambda}$$

As $\lambda_{\text{blue}}<\lambda_{\text{red}}\Rightarrow\mu_{\text{blue}}>\mu_{\text{red}}$

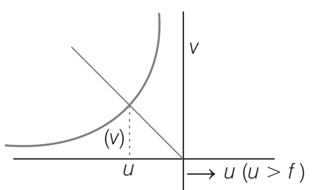
Hence, $f_{\text{red}}>f_{\text{blue}}$

**87** In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance *u* and the image distance *v*, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the *x*-axis meets the experimental curve at *P*. The coordinates of *P* will be **[AIEEE 2009]**

(a) $(2f, 2f)$ (b) $\left(\frac{f}{2},\frac{f}{2}\right)$
(c) $(f, f)$ (d) $(4f, 4f)$

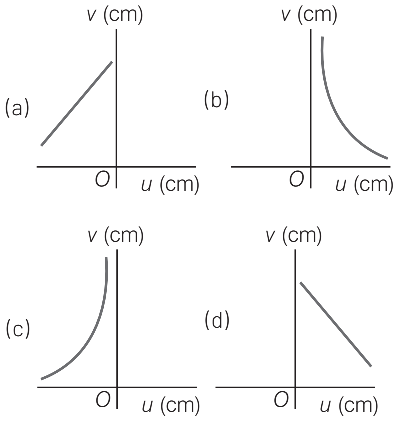
**Ans.** *(a)*

It is possible when object kept at centre of curvature.




For $u=v,\ u=2f,\ v=2f$

**88** A student measures the focal length of a convex lens by putting an object pin at a distance *u* from the lens and measuring the distance *v* of the image pin. The graph between *u* and *v* plotted by the student should look like **[AIEEE 2008]**




**Ans.** (*c*)

Experimental observation at $v=\infty,\ u=-f$

At $u=\infty, v=f$

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=$ constant

So, (c) is the correct graph.

**89** Two lenses of power –15D and +5D are in contact with each other. The focal length of the combination is **[AIEEE 2007]**

(a) –20 cm (b) –10 cm
(c) +20 cm (d) +10 cm

**Ans.** (*b*)

Power of a lens is reciprocal of its focal length. Power of combined lens is

$$P=P_1+P_2$$
$$=-15+5$$
$$=-10\,\text{D}$$

$\therefore$ $f=\frac{1}{P}=\frac{100}{-10}$ cm

or $f=-10$ cm

**90** A thin glass (refractive index 1.5) lens has optical power of –5 D in air. Its optical power in a liquid medium with refractive index 1.6 will be **[AIEEE 2005]**

(a) 1 D (b) –1 D
(c) 25 D (d) –25 D

**Ans.** (*a*)

$$\frac{1}{f_a}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \ldots\text{(i)}$$

$$=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

and $\frac{1}{f_m}=\left(\frac{\mu_g-\mu_m}{\mu_m}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \ldots\text{(ii)}$

$$\frac{1}{f_m}=\left(\frac{1.5}{1.6}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Thus, $\frac{f_m}{f_a}=\frac{(1.5-1)}{\left(\frac{1.5}{1.6}-1\right)}=-8$

or $f_m=-8\times f_a=-8\times\frac{-1}{5}=1.6$ m

$$\left[\because f_a=\frac{1}{P}=-\frac{1}{5}\,\text{m}\right]$$

$\therefore$ $P_m=\frac{\mu}{f_m}=\frac{1.6}{1.6}=1$ D

**91** A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object **[AIEEE 2004]**

(a) 20 cm (b) 30 cm
(c) 60 cm (d) 80 cm

**Ans.** (*a*)

A plano-convex lens behaves as a concave mirror, if its one surface (curved) is silvered. The rays refracted from plane surface are reflected from curved surface and again refract from plane surface. Therefore, in this lens two refractions and one reflection occur.

Let the focal length of silvered lens be *F*.

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}+\frac{1}{f_m}=\frac{2}{f}+\frac{1}{f_m}$$

where, *f* = focal length of lens before silvering,

$f_m$ = focal length of spherical mirror.

$$\frac{1}{F}=\frac{2}{f}+\frac{2}{R} \quad [\because R=2f_m] \ldots\text{(i)}$$

Now, $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \ldots\text{(ii)}$

Here, $R_1=\infty, R_2=30$ cm

$\therefore$ $\frac{1}{f}=(1.5-1)\left(\frac{1}{\infty}-\frac{1}{30}\right)$

or $\frac{1}{f} = -\frac{0.5}{30} = -\frac{1}{60}$ or $f = -60$ cm

Hence, from Eq. (i), we get

$$\frac{1}{F} = \frac{2}{60} + \frac{2}{30} = \frac{6}{60}$$

$$F = 10 \text{ cm}$$

Again given that,

Size of object = Size of image

*i.e.,* $O = I$

$\therefore \quad m = -\frac{v}{u} = \frac{I}{O} \Rightarrow \frac{v}{u} = -1$

or $\quad v = -u$

Thus, from lens formula,

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{10} = \frac{1}{-u} - \frac{1}{u}$$

$$\frac{1}{10} = -\frac{2}{u}$$

$\therefore \quad u = -20$ cm

Hence, to get a real image, object must be placed at a distance 20 cm on the left side of lens.

# TOPIC 4
## Optical Instruments

**92** An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification 6, gives an unresolved image. A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to ......... cm.

**[2021, 20 July Shift-I]**

**Ans.** *(25)*

Given, magnification, $M = 6$

Since we know that magnifying power of a simple microscope is given by

$$M = 1 + \frac{D}{f_0}$$

where, $D$ = least distance of distinct vision = 25 cm

and $f_0$ = focal length of objective lens.

$\Rightarrow \quad 6 = 1 + \frac{D}{f_0}$

$\Rightarrow \quad 6 = 1 + \frac{25}{f_0}$

$\Rightarrow \quad 5 = \frac{25}{f_0} \Rightarrow f_0 = 5$ cm

For compound microscope, magnifying power is given by

$$M = \frac{l \cdot D}{f_0 f_e} = 2M_{\text{simple microscope}}$$

where, $f_0$ and $f_e$ are the focal lengths of the objective lens and eye piece respectively and $l$ = length of the given tube = 0.6 m

$\Rightarrow \quad 12 = \frac{60 \times 25}{5 \cdot f_e}$

[$\because$ magnification is doubled]

$\Rightarrow \quad f_e = 25$ cm

This is the required focal length of eyepiece.

**93** Your friend is having eye sight problem. She is not able to see clearly a distant uniform window mesh and it appears to her as non-uniform and distorted. The doctor diagnosed the problem as

**[2021, 18 March Shift-I]**

(a) astigmatism
(b) myopia with astigmatism
(c) presbyopia with astigmatism
(d) myopia and hypermetropia

**Ans.** *(b)*

A friend is not seen clearly the distant object, then its diagnosis is myopia because in myopia the distant object is blurry and it also appear non-uniform and distorted images of the object, then its diagnosis is astigmatism also.

**94** Given below are two statements: one is labelled as Assertion (A) and the other is labelled as Reason (R).

**Assertion (A)** For a simple microscope, the angular size of the object equals the angular size of the image.

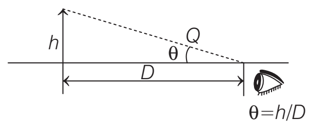
**Reason (R)** Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence, it subtends a large angle.

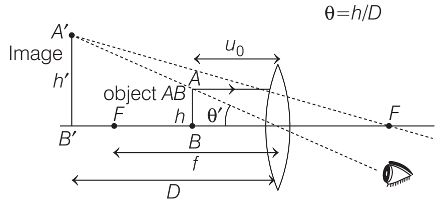
In the light of the above statements, choose the most appropriate answer from the options given below. **[2021, 26 Feb Shift-II]**

(a) A is true but R is false.
(b) Both A and R are true but R is not the correct explanation of A.
(c) Both A and R are true and R is the correct explanation of A.
(d) A is false but R is true.

**Ans.** *(c)*

The formation of image with simple microscope is shown below.




Here, $\theta' = \frac{h}{u_0} = \frac{h'}{D} = \frac{h'}{25}$

where, $D = 25$ cm (least distance of distinct vision)

Here, $\theta'$ is same for both object and image, hence Assertion is true.

Magnification, $m = \frac{\theta'}{\theta} = \frac{D}{u_0}$

Hence, if $u_0 < D$ (25 cm), hence the value of $\theta'$ will obtain large.

So, option (c) is the correct.

**95** In a compound microscope, the magnified virtual image is formed at a distance of 25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is 100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is ......... .

**[2020, 4 Sep Shift-I]**

**Ans.** *(6)*

When the final image is formed at least distance of distinct vision (i.e., at 25 cm) from eye-piece of a compound microscope, then the magnification is given by

$$m = -\frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$$

$\Rightarrow \quad m = \frac{-L}{f_0}\left(1 + \frac{D}{f_e}\right)$

$\Rightarrow \quad -100 = -\frac{20}{1}\left(1 + \frac{25}{f_e}\right)$

$\Rightarrow \quad 5 = 1 + \frac{25}{f_e}$

$$4 = \frac{25}{f_e}$$

$\Rightarrow \quad f_e = \frac{25}{4} = 6.25 \simeq 6$ cm

**96** A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm. The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of $n$ is ...... .

**[2020, 5 Sep Shift-I]**

**Ans.** *(50)*

Strain on the eye is minimum so the intermediate image is formed at focal length of eye-piece.

For objective, $v = 5\,\text{cm}, f = 1\,\text{cm}$

$\therefore$ Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{5} - \frac{1}{u} = \frac{1}{1}$$

$$\Rightarrow \quad u = \frac{-5}{4}\,\text{cm}$$

$$\Rightarrow \quad |u| = \frac{5}{4}\,\text{cm}$$

$$\Rightarrow \quad \frac{n}{40} = \frac{5}{4}$$

$$\Rightarrow \quad n = \frac{5}{4} \times 40$$

$$\Rightarrow \quad n = 50$$

**97** If we need a magnification of 375 from a compound microscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eyepiece should be close to **[2020, 7 Jan Shift-I]**

(a) 22 mm (b) 2 mm
(c) 12 mm (d) 33 mm

**Ans.** *(a)*

In normal setting, magnification obtained by a microscope is given by

$$m = \frac{L \times D}{f_o \times f_e}$$

where,

$L$ = tube length = 150 mm,

$D$ = distance of distinct vision = 25 cm = 250 mm,

$f_o$ = focal length of objective = 5 mm,

$f_e$ = focal length of eyepiece

and $m$ = magnification = 375.

So, we have

$$375 = \frac{150 \times 250}{5 \times f_e}$$

or $\quad f_e = 20$ mm

So, focal length of eyepiece required is closest to 22 mm.

**98** The magnifying power of a telescope with tube length 60 cm is 5. What is the focal length of its eyepiece? **[2020, 8 Jan Shift-I]**

(a) 10 cm (b) 20 cm
(c) 30 cm (d) 40 cm

**Ans.** *(a)*

For a telescope in normal setting,

$$f_o + f_e = L$$

(length of the tube of telescope)

and $\quad \frac{f_o}{f_e} = m \quad$ (magnification)

where, $f_o$ and $f_e$ is the focal length of the objective and eyepiece, respectively.

According to the given values in the question, we have

$$f_o + f_e = 60\,\text{cm and } \frac{f_o}{f_e} = 5$$

$$\Rightarrow \quad f_o = 5 f_e$$

Solving the above equations, we have

$$f_e = 10\,\text{cm}$$

**99** An observer looks at a distance tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears **[JEE Main 2016]**

(a) 10 times taller (b) 10 times nearer
(c) 20 times taller (d) 20 times nearer

**Ans.** *(c)*

Height of image depends upon the magnifying power to see a 20 times taller object, as the angular magnification should be 20 and we observe angular magnification. Option *(c)* would not be very correct as the telescope can be adjusted to form the image anywhere between infinity and least distance for distinct vision.

Suppose that the image is formed at infinity. Then, the observer will have to focus the eyes at infinity to observe the image. Hence, it is incorrect to say that the image will be appear nearer to the observer.

**100** The image formed by an objective of a compound microscope is **[AIEEE 2003]**

(a) virtual and diminished
(b) real and diminished
(c) real and enlarged
(d) virtual and enlarged

**Ans.** *(c)*

Objective of compound microscope is a convex lens. Convex lens form real and enlarged image when an object is placed between its focus and lens.