

Quadrilaterals

11

Look at the figures given below. Which of them are triangles?

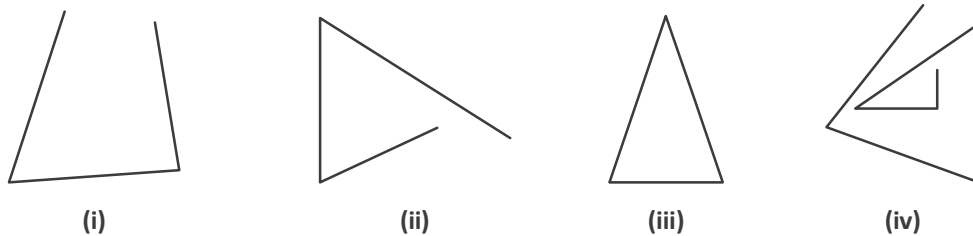


Fig. 1

A figure formed by joining three non-collinear points is called a triangle. A triangle is a figure enclosed by three line segments. It has three sides, three angles and three vertices. What are the other features of a triangle? Discuss.

Look at *Fig.2(i-iv)*. In each of them, four points are joined together.

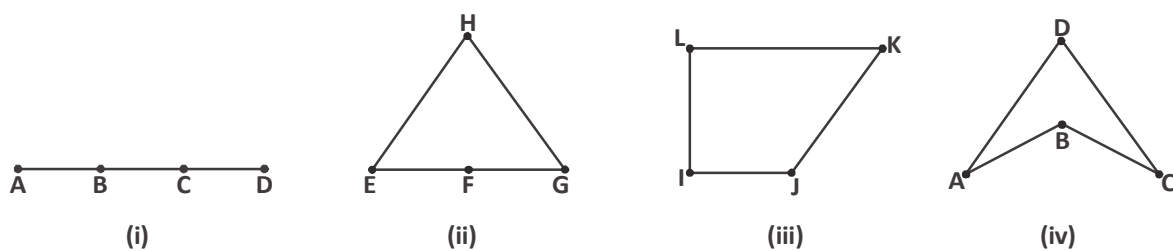


Fig. 2

In *Fig.2(i)*, all four points lie on the same line (are collinear) so we get a line segment when we join them. In *Fig.2(ii)*, three of the points are collinear but not the fourth. Here, we get a triangle when we join the points.

Is a quadrilateral formed in *Fig.2(iii)* or *Fig.2(iv)*? It is clear that to form a quadrilateral, three points out of four have to be non-collinear.

If three out of four points on the same plane are not lying on the same line (i.e. are non-collinear) then they will form a quadrilateral when we join them together in an order.

Try This



Define a quadrilateral based on the properties described so far. Discuss your definition with your friends.

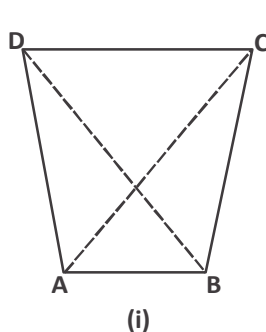
Find different objects in your school or classroom that have surfaces in the shape of quadrilaterals. For example, the blackboard, window panes, each page of a book etc.

Think and Discuss

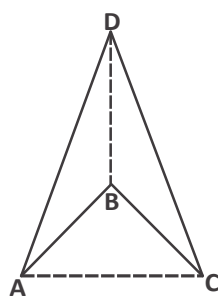


Many of the objects around us are rectangular. A rectangle is also a quadrilateral. Why?

Types of Quadrilateral



(i)



(ii)

Fig. 3

We saw that *Fig.2(iii)* and *Fig.2(iv)* are quadrilaterals. Let us draw the diagonals of these quadrilaterals (*Fig.3*).

We find that both the diagonals in *Fig.3(i)* lie inside but in *Fig.3(ii)*, one diagonal lies inside and the other is outside the quadrilateral. What is different in the two quadrilaterals?

All angles of the quadrilateral in which both diagonals are inside are less than 180° . Such quadrilaterals are called convex quadrilaterals. For

example, quadrilateral PQRS (*Fig.4*).

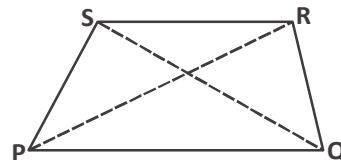


Fig. 4

A quadrilateral in which one of the angles is more than 180° will have one diagonal inside and the other outside of it. Such a quadrilateral is called a concave quadrilateral.

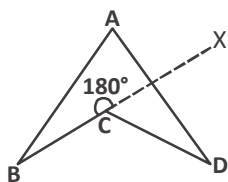


Fig. 5

In *Fig.5* $\angle BCX = 180^\circ$. Therefore, the interior angle $\angle BCD$ of quadrilateral ABCD is more than 180° .

In this chapter, we will only study convex quadrilaterals like those shown in *Fig.4*.

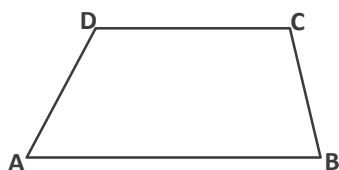
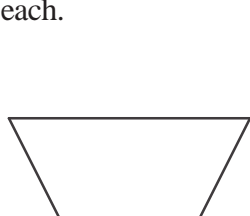


Fig. 6

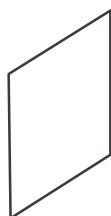
In quadrilateral ABCD, the sides AB and DC are parallel to each other. It is a trapezium. We can say that a quadrilateral in which only one pair of sides is parallel, is a trapezium.

Try This

Look at the figures given below. Which of them are **not** trapeziums? Give reasons for each.



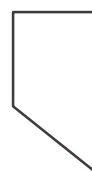
(i)



(ii)



(iii)



(iv)



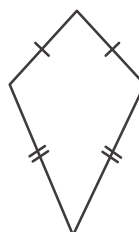
(v)



(vi)



(vii)



(viii)



(ix)

Parallelogram

We know that if one pair of opposite sides of a quadrilateral are parallel then it is called a trapezium. If both pairs of opposite sides of a quadrilateral are parallel then it is called a parallelogram.



Fig. 7

Rhombus

When all sides of a parallelogram are equal it is called a rhombus.

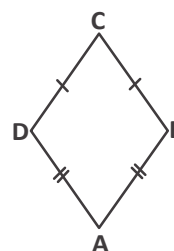


Fig. 8

Think and Discuss

1. You have read about many types of quadrilaterals (trapezium, rhombus, square etc.). Identify which of them are parallelograms.
2. Are all parallelograms also trapeziums? Discuss.



Now draw a square and a rectangle. Are they examples of parallelograms? Yes, a square is a special type of parallelogram and each of its interior angle measures 90° .

Try This



1. Is a rectangle also a square?
2. Try to draw a parallelogram where three of the angles are right angles but which is not a rectangle. Is such a parallelogram possible? Discuss.

Suppose all sides of a rectangle are equal. Then, what will be? Such a rectangle is a square (*Fig.9*).

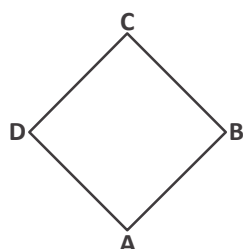


Fig. 9

- | | | | |
|-------|--------------------------|------|------------------------------------|
| (i) | Is a square a rectangle? | (ii) | Are squares parallelograms? |
| (iii) | Is a square a rhombus? | (iv) | Is a rhombus also a parallelogram? |

Now, we will learn how to prove some properties and theorems related to quadrilaterals.

We know that each diagonal of a quadrilateral divides it into two triangles.

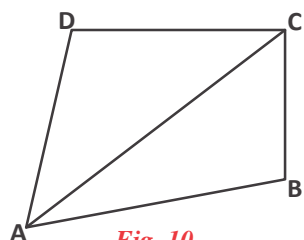


Fig. 10

Assume that ABCD is a quadrilateral and AC is its diagonal. Then, diagonal AC divides the quadrilateral into two triangles $\triangle ABC$ and $\triangle ADC$ (*Fig.10*).

From the angle sum property of triangles we know that the sum of the three interior angles of a triangle is 180° .

$$\text{In } \triangle ADC, \angle ADC + \angle DCA + \angle CAD = 180^\circ \quad \dots(1)$$

$$\text{Similarly in } \triangle ABC, \angle ABC + \angle BCA + \angle CAB = 180^\circ \quad \dots(2)$$

By adding equations (1) and (2)

$$\angle ADC + \angle DCA + \angle CAD + \angle ABC + \angle BCA + \angle CAB = 180^\circ + 180^\circ$$

$$\angle ADC + (\angle DCA + \angle BCA) + (\angle CAD + \angle CAB) + \angle ABC = 360^\circ$$

$$\angle ADC + \angle BCD + \angle BAD + \angle ABC = 360^\circ$$

Therefore, the sum of the four interior angles of quadrilateral ABCD is equal to 360° .

Similarly, the sum of the interior angles of any quadrilateral is equal to 360° .

Try This



1. The interior angles of a quadrilateral are in the ratio 3:5:7:9. Find the measure of each interior angle.
2. If all the angles of the quadrilateral are equal then what is the measure of each angle?

Take a piece of paper and draw a parallelogram on it. Draw any one of its diagonals. Take a pair of scissors and first cut out the parallelogram and then cut along the diagonal as shown in Fig. 11. Place the cut parts on each other. Do they overlap? Turn the cut parts around if needed.

Are the parts overlapping because of some special property of the parallelogram? Which property is it?

We will now study the properties of a parallelogram and logically verify them.

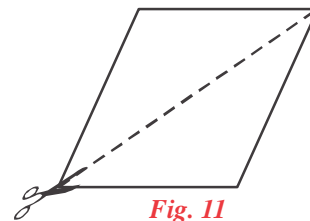


Fig. 11

THEOREM-11.1 : A diagonal of a parallelogram divides it into two congruent triangles.

PROOF : Let ABCD be a parallelogram and AC be one of its diagonals (Fig. 12).

In parallelogram ABCD,

$AB \parallel DC$ and AC is a transversal

$$\angle DCA = \angle CAB \text{ (pair of alternate angles)}$$

Similarly, $DA \parallel CB$ where AC is a transversal

$$\angle DAC = \angle BCA$$

Now, in $\triangle ACD$ and $\triangle CAB$

$$\angle DCA = \angle CAB$$

$$AC = CA \text{ (common side)}$$

$$\angle DAC = \angle BCA$$

$$\therefore \triangle ABC \cong \triangle CDA \text{ (by ASA congruency)}$$

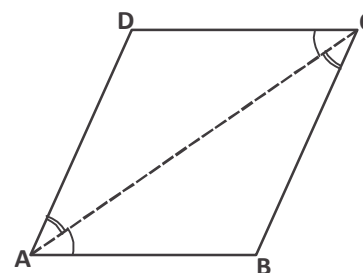


Fig. 12

That is, diagonal AC divides the parallelogram ABCD into two congruent triangles.

Clearly, a diagonal of a parallelogram divides it into two congruent triangles.

THEOREM-11.2 : In a parallelogram, opposite sides are equal.

PROOF : Let ABCD be a parallelogram. Join the vertex A to C. This is diagonal AC. Diagonal AC divides the quadrilateral ABCD into two triangles ABC and ACD (Fig. 13).

Now, in $\triangle ABC$ and $\triangle ACD$

$$\angle DAC = \angle BCA \text{ (alternate interior angles)}$$

$$(AD \parallel BC)$$

Similarly, by alternate interior angles

$$\angle DCA = \angle BAC \text{ (AB } \parallel \text{ DC)}$$

Also $AC = CA$ is a common side

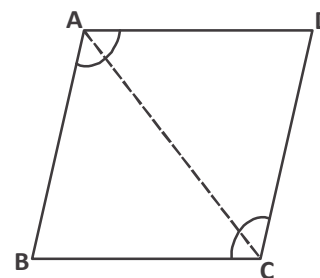


Fig. 13

$\therefore \triangle ABC \cong \triangle CDA$ (by A-S-A congruency)

Therefore, $AD = BC$ and $AB = CD$ (by C.P.C.T.)

That is, opposite sides of a parallelogram are equal (hence proved).

THEOREM-11.3 (CONVERSE) : If each pair of opposite sides of a quadrilateral is equal then it is a parallelogram.

PROOF : Given, quadrilateral ABCD in which $AB = CD$ and $BC = AD$. Now, draw diagonal AC (Fig. 14).

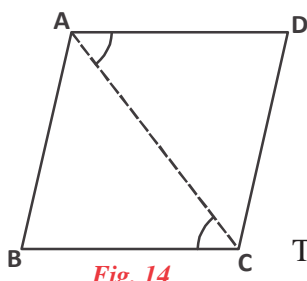


Fig. 14

In $\triangle ABC$ and $\triangle CDA$,

$BC = AD$ (given)

$AB = DC$ (given)

$AC = CA$ (common side)

$\therefore \triangle ABC \cong \triangle CDA$ (by S-S-S congruency)

Therefore, $\angle BCA = \angle DAC$ (by C.P.C.T.)

and $AD \parallel BC$ (1)

where AC is a transversal.

Since $\angle ACD = \angle CAB$,

because CA is a transversal

therefore, $AB \parallel CD$ (2)

Therefore, by (1) and (2), ABCD is a parallelogram.

We have seen that each pair of opposite sides of a parallelogram is equal and conversely, if each pair of opposite sides of a quadrilateral is equal then it will be a parallelogram.

Now, we will prove such a result for those quadrilaterals in which pairs of opposite angles are equal.

THEOREM-11.4 : Opposite angles of a parallelogram are equal.

PROOF : Quadrilateral ABCD is a parallelogram (Fig. 15)

in which $AB \parallel DC$

\therefore Line segment AD intersects the parallel lines AB and DC.

$\angle A + \angle D = 180^\circ$ (interior angles on the same side of the transversal)

and DC intersects the lines AD and BC.

$\angle D + \angle C = 180^\circ$ (interior angles on the same side of the transversal)

therefore, $\angle A + \angle D = \angle D + \angle C$

that is, $\angle A = \angle C$

similarly $\angle B = \angle D$ can also be proven.

It is clear that opposite angles in a parallelogram are equal.

Now, what will happen if opposite angles of a quadrilateral are equal. We will find the logical possibility of such a quadrilateral being a parallelogram.

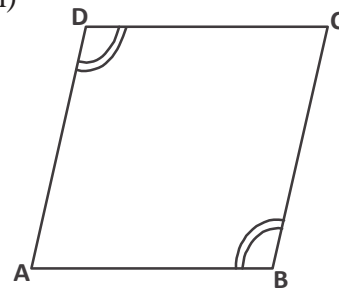


Fig. 15

THEOREM-11.5 (Converse of Theorem-11.4) : If in a quadrilateral each pair of opposite angles are equal then it is a parallelogram.

PROOF : In quadrilateral ABCD, $\angle A = \angle C$ and $\angle B = \angle D$ (Fig.16)(1)

We know that sum of all interior angles of a quadrilateral is 360° .

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\angle A + \angle B + \angle A + \angle B = 360^\circ \text{ (by -1)}$$

$$2\angle A + 2\angle B = 360^\circ$$

$$\angle A + \angle B = \frac{360^\circ}{2}$$

$$\angle A + \angle B = \angle C + \angle D = \frac{360^\circ}{2}$$

$$\therefore \angle C + \angle D = 180^\circ \text{(2)}$$

Now, extend DC upto E-

We see that $\angle C + \angle BCE = 180^\circ$ (3)

Therefore, $\angle BCE = \angle ADC$ by equation (2) and (3)

Since, $\angle BCE = \angle D$ and DC is a transversal

therefore, $AD \parallel BC$

Similarly, $AB \parallel DC$ and so ABCD is a parallelogram

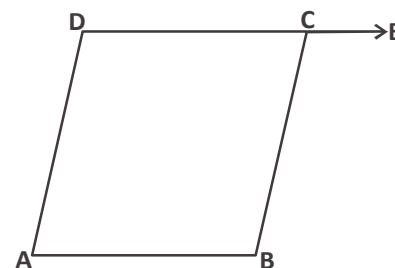


Fig. 16

Properties of Diagonals of a Parallelogram

Draw a parallelogram on paper and draw both its diagonals. Cut the parallelogram into four parts along the diagonals as shown in Fig. 17.



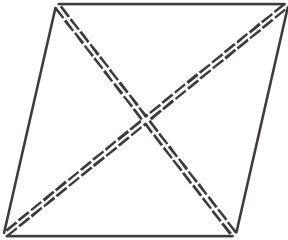


Fig. 17

Do the parts appear similar to each other or different from each other?

We are getting four triangles and they are actually two pairs of congruent triangles. Do the diagonals bisect each other?

Let us check the validity of the statement given in theorem 11.6.

THEOREM-11.6 : Diagonals of a parallelogram bisect each other.

PROOF : ABCD is a parallelogram in which $AB = DC$ and $AB \parallel DC$

Also $AD = BC$ and $AD \parallel BC$ (Fig. 18)

When we join A to C and B to D then AC and BD intersect each other at point O.

In $\triangle AOB$ and $\triangle COD$

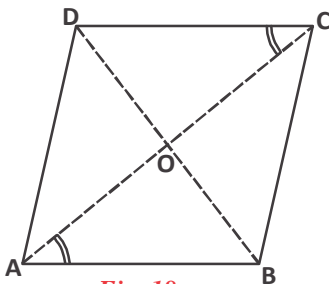


Fig. 18

$$\angle OAB = \angle OCD \quad \dots(1)$$

($AB \parallel DC$ and are cut by the transversal AC)

$$\angle ABO = \angle ODC \quad \dots(2)$$

($AB \parallel DC$ and are cut by the transversal BD)

$$AB = DC$$

$$\therefore \triangle AOB \cong \triangle COD \quad (\text{A-S-A congruency})$$

$$\therefore \text{Side } AO = OC \text{ and } BO = OD \text{ (by C.P.C.T.)}$$

So, we can say that diagonals of a parallelogram bisect each other.

EXAMPLE-1. Prove that if diagonals of a parallelogram are equal then it is a rectangle.

PROOF : Let ABCD be a parallelogram in which AC and BD are diagonals and $AC = BD$ (Fig. 19)

Now in $\triangle ABC$ and $\triangle DCB$

$$AB = DC \text{ (opposite sides of a parallelogram)}$$

$$BC = CB \text{ (common side)}$$

$$AC = BD \text{ (given)}$$

$$\triangle ABC \cong \triangle DCB \text{ (S-S-S congruency)}$$

$$\text{therefore, } \angle ABC = \angle DCB \quad \dots(1)$$

Since $\angle ABC$ and $\angle DCB$ are situated on the same side of the transversal BC of parallel lines AB and CD, therefore,

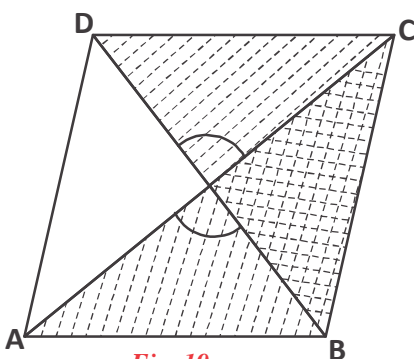


Fig. 19

$$\angle ABC + \angle DCB = 180^\circ \quad \dots(2)$$

by (1) and (2)

$$\angle ABC + \angle ABC = 180^\circ$$

$$2 \angle ABC = 180^\circ$$

$$\angle ABC = 90^\circ$$

$$\text{that is, } \angle DCB = 90^\circ$$

Similarly, we can prove that $\angle A = \angle D$.

$$\angle A = \angle D = 90^\circ$$

therefore, $\angle A = \angle B = \angle C = \angle D = 90^\circ$

Hence, parallelogram ABCD is a rectangle.

Clearly, if diagonals of a parallelogram are equal then it is a rectangle.

Hence proved.



Try This

Similarly, try to prove that if diagonals of a rhombus are equal then it is a square.



EXAMPLE-2. If diagonals of a parallelogram are perpendicular to each other then it is a rhombus.

PROOF : Let ABCD be a parallelogram in which diagonals AC and BD are perpendicular to each other. We need to prove that ABCD is a rhombus (Fig.20).

Now, in $\triangle AOD$ and $\triangle COD$

$AO = CO$ (diagonals of a parallelogram bisect each other)

$\angle AOD = \angle COD$ (each angle is right angle)

$OD = OD$ (common side)

$\triangle AOD \cong \triangle COD$ (SAS congruency)

Therefore, $AD = CD$ (by C.P.C.T.)

Also, $AB = CD$ and $AD = BC$ (\because opposite sides of a parallelogram are equal)

$\therefore AB = BC = CD = AD$

Clearly, parallelogram ABCD is a rhombus. Therefore, it can be said that if the diagonals of a parallelogram are perpendicular to each other then it is a rhombus.

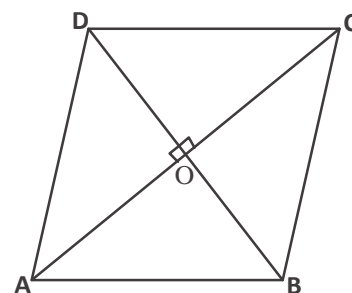


Fig. 20

Try This



Can you show that the diagonals of a rhombus are perpendicular to each other?

EXAMPLE-3. Prove that the diagonals of a rhombus are perpendicular to each other.

PROOF : A rhombus is a parallelogram in which all sides are equal. Consider the rhombus ABCD (Fig.21). We see that in rhombus ABCD, the diagonals AC and BD intersect each other at O. We need to prove that AC is perpendicular to BD.

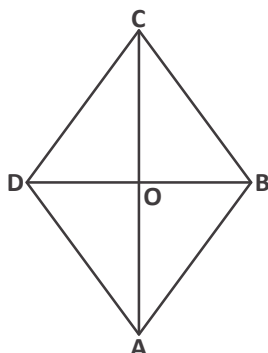


Fig. 21

In $\triangle AOB$ and $\triangle BOC$

$AO = OC$ (diagonals of a parallelogram bisect each other)

$OB = OB$ (common side)

$AB = BC$ (sides of a rhombus)

$\therefore \triangle AOB \cong \triangle BOC$ (SSS congruency)

Therefore, $\angle AOB = \angle BOC$

Now, because $\angle AOB + \angle BOC = 180^\circ$ (Linear pair)

$\therefore \angle AOB + \angle AOB = 180^\circ$

or $2\angle AOB = 180^\circ$

or $\angle AOB = \frac{180^\circ}{2}$

or $\angle AOB = 90^\circ$

Similarly, we can prove that $\angle BOC = \angle COD = \angle AOD = 90^\circ$. That is, the diagonals of a rhombus are perpendicular to each other. Hence proved.

EXAMPLE-4. Prove that the angle bisectors of a rhombus make a rectangle.

PROOF : ABCD is a parallelogram as shown in Fig.22. Bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ intersect at P, Q, R and S forming quadrilateral PQRS (Fig.22).

In $\triangle ASD$,

Since DS bisects $\angle D$ and AS bisects $\angle A$, therefore

$$\begin{aligned}\angle DAS + \angle ADS &= \frac{1}{2} \angle BAD + \frac{1}{2} \angle ADC \\ &= \frac{1}{2} (\angle A + \angle D)\end{aligned}$$

$$= \frac{1}{2} \times 180^\circ = 90^\circ \text{ } (\angle A \text{ and } \angle D \text{ are interior angles}$$

on the same side of the transversal)

.....(1)

In $\triangle ASD$,

$$\angle DAS + \angle ADS + \angle DSA = 180^\circ \text{ (Why?) } \dots\dots(2)$$

From, equation (1) and (2)

$$90^\circ + \angle DSA = 180^\circ$$

$$\angle DSA = 90^\circ$$

Therefore, $\angle PSR = 90^\circ$ (Being vertically opposite to $\angle DSA$)

Similarly, $\angle BQC = \angle PQR$

In $\triangle APB$,

$$\angle PAB + \angle APB + \angle PBA = 180^\circ \text{ (Sum of angles of a triangle)}$$

but $\angle PAB + \angle PBA = 90^\circ$ ($\angle A$ and $\angle B$ are interior angles on the same side of the transversal)

$$\therefore \angle APB = 90^\circ$$

Similarly, $\angle SRQ = 90^\circ$. Thus, PQRS is a quadrilateral in which all the angles are right angles. Therefore, quadrilateral PQRS is a rectangle.

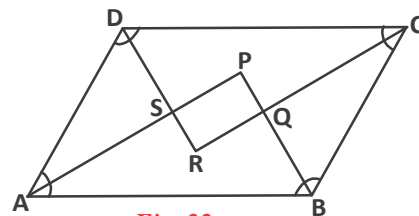


Fig. 22

Think and Discuss

1. Diagonals of a rectangle are of equal length (*Hint*- a rectangle is a parallelogram)
2. Diagonals of a square are equal and they bisect each other at right angles.



EXAMPLE-5. If the diagonals of a parallelogram ABCD intersect at point O and if $OA = 3 \text{ cm}$ and $OB = 4 \text{ cm}$, then find the lengths of the line segments OC, OD, AC and BD.

SOLUTION : ABCD is a parallelogram where AC and BD intersect at O (Fig.23).

$$OA = 3 \text{ cm}$$

$$OB = 4 \text{ cm}$$

because diagonal of the parallelogram AC and BD bisects each other.

$$OC = OA$$

$$\therefore OC = 3 \text{ cm}$$

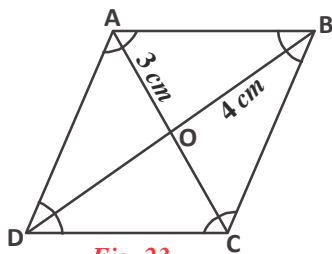


Fig. 23

and $OD = OB$

$\therefore OD = 4 \text{ cm}$

Now, $AC = AO + OC = 3 + 3 = 6 \text{ cm}$

$BD = OB + OD = 4 + 4 = 8 \text{ cm}$

Therefore, it is clear that $AC = 6 \text{ cm}$ and $BD = 8 \text{ cm}$.

THEOREM-11.7 : If in a quadrilateral, a pair of opposite sides is equal and parallel then it is a parallelogram. (Prove with the help of teacher)

EXAMPLE-6. In triangle ABC, median AD was drawn on side BC and extended to E such that $AD = ED$. Prove that ABEC is a parallelogram.

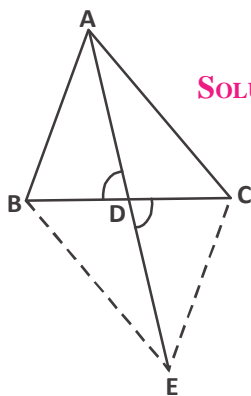


Fig. 24

SOLUTION : Let ABC be the triangle and AD be its median on BC (Fig.24).

Extend AD to E such that $AD = ED$

Now join BE and CE

In $\triangle ABD$ and $\triangle ECD$

$BD = DC$ (since D is the mid point of BC)

$\angle ADB = \angle EDC$ (vertically opposite angles)

$AD = ED$ (given)

$\therefore \triangle ABD \cong \triangle ECD$ (by SAS congruency)

Now, $AB = CE$ (sides of congruent triangles)

and $\angle ABD = \angle ECD$

Both are a pair of alternate interior angles made between the lines AB and CE by the transversal line BC.

$\therefore AB \parallel CE$

So, in quadrilateral ABEC

$AB \parallel CE$ and $AB = CE$

Therefore, ABEC is a parallelogram.

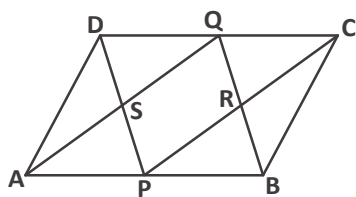


Fig. 25

EXAMPLE-7. ABCD is parallelogram in which P and Q are the mid points of opposite sides AB and CD respectively (Fig.25). If AQ intersects DP at point S and BQ intersects CP at point R then, show that:-

(i) APCQ is a parallelogram.

(ii) DPBQ is a parallelogram.

(iii) PSQR is a parallelogram

SOLUTION :

(i) In quadrilateral APCQ

$$AP \parallel QC \quad (\text{because } AB \parallel CD) \quad \dots(1)$$

$$AP = \frac{1}{2} AB$$

$$CQ = \frac{1}{2} CD \quad (\text{given})$$

Since $AB = CD$

therefore, $AP = QC \quad \dots(2)$

by equation (1) and (2) APCQ is a parallelogram.

(ii) Similarly, quadrilateral DPBQ is a parallelogram because $DQ \parallel PB$ and $DQ = PB$

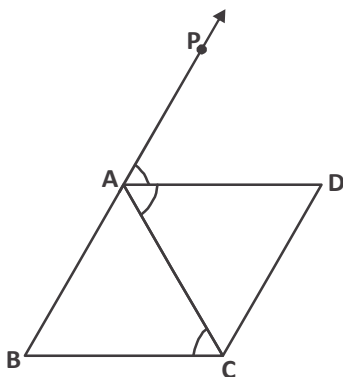
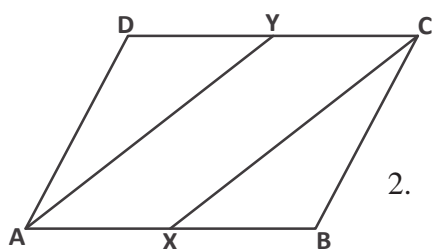
(iii) In quadrilateral PSQR

$$SP \parallel QR$$

(where SP is a part of DP and QR is a part of QB)

$$\text{Similarly } SQ \parallel PR$$

Therefore, PSQR is a parallelogram

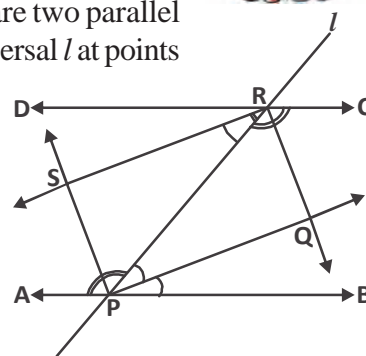


Exercise - 11.1

1. X and Y are the mid points of opposite sides AB and CD of parallelogram ABCD. Prove that AXCY is a parallelogram.



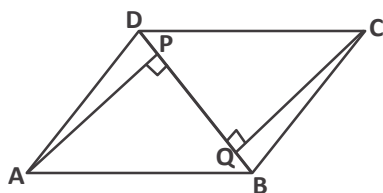
2. In the adjacent figure, AB and DC are two parallel lines which are intersected by transversal l at points P and R respectively. Prove that the bisectors of the interior angles make a rectangle.



3. ABC is an isosceles triangle in which $AB = AC$, AD bisects the exterior angle PAC and $CD \parallel BA$.

Prove that:-

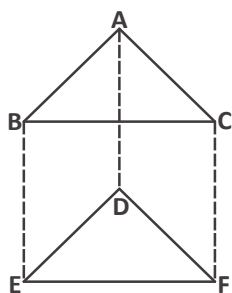
- (i) $\angle DAC = \angle BCA$
- (ii) Quadrilateral ABCD is a parallelogram.



4. ABCD is a parallelogram and BD is one of its diagonals. AP and CQ are perpendiculars on BD from the vertices A and C respectively. Prove that:-

(i) $\triangle APB \cong \triangle CQD$

(ii) $AP = CQ$



5. ABCD is a rectangle in which diagonal AC bisects both the angles A and C. Then prove that:-

(i) ABCD is a square.

(ii) Diagonal BD bisects both angles B and D

6. $\triangle ABC$ and $\triangle DEF$ are such that AB and BC are equal and parallel to DE and EF respectively. Prove that AC and DF are equal and parallel.

The Mid-Point Theorem

You have studied many properties of triangles and quadrilaterals. Let us study a property of triangles which is related to the mid-point of its sides. Let us look at the theorem:-

THEOREM-11.8 : A line segment joining the mid-points of two sides of a triangle is always parallel to and half of the third side.

PROOF : Let us take $\triangle ABC$ to prove this statement. In $\triangle ABC$, D and E are the mid points of AB and AC respectively. Draw a line segment DE by joining mid points D and E such that E is the mid-point of DF. Join C and F (Fig.26).

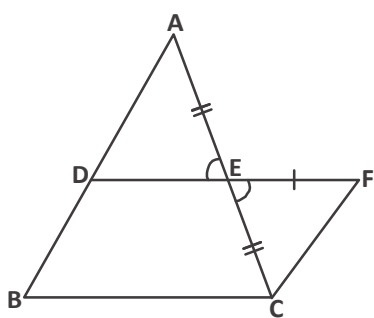


Fig. 26

Now, you can see that in $\triangle ADE$ and $\triangle CFE$

$AE = CE$ (E is the mid-point of AC)

$\angle AED = \angle CEF$ (vertically opposite angles)

$DE = EF$ (by construction)

$\therefore \triangle ADE \cong \triangle CFE$ (by SAS congruency)

$AD = CF$ and $\angle ADE = \angle CFE$ (by C.P.C.T.)

now $BD = AD$ and $AD = CF$

$\therefore BD = CF$ (1)

Also $\angle ADE = \angle CFE$ are equal (proved above). But these are alternate interior angles for AD and CF intersected by DF.

$\therefore AD \parallel CF$

or $BD \parallel CF$ (2)

By (1) and (2), BD and CF in quadrilateral BDFC are both equal and parallel. You know that if a pair of opposite sides is both equal and parallel in a quadrilateral, then it is a parallelogram. Therefore, DBCF is a parallelogram.

Since opposite sides of a parallelogram are equal so $DF = BC$. Also, $DE + EF = DF$, $DE = EF$, so $DF = 2DE$

$$\therefore BC = 2DE \quad \text{and} \quad DE = \frac{1}{2} BC$$



Try This

Now write the converse of theorem 11.8 and verify it.

THEOREM-11.9 : *l, m and n are the three parallel lines intersected by transversals p and q such that l, m and n cut off equal intercepts DE and EF on p . Show that l, m and n cut off equal intercepts AB and BC on q also.*

PROOF : Parallel lines l, m and n are intersected by transversal lines p at point D, E and F such that $DE = EF$

If transversal line q intersects parallel lines l, m and n at points A, B and C respectively, then we need to prove that $AB = BC$.

Now, to prove this we will draw a line which is parallel to q , passes through the point E and intersects l and n at G and H respectively.

Clearly $AG \parallel BE$ (because $l \parallel m$ and A, G and B, E lie on l and m respectively)

$GE \parallel AB$ (by construction)

Then, AGEB is a parallelogram

$$\therefore AG = BE \text{ and } GE = AB \quad \dots(1)$$

Similarly, $BE \parallel CH$ (because $m \parallel n$ and B, E and C, H lie on m and n respectively)

$EH \parallel BC$ (by construction)

Then, BEHC is a parallelogram

$$\therefore BE = CH \text{ and } EH = BC \quad \dots(2)$$

Now, in $\triangle GED$ and $\triangle HEF$

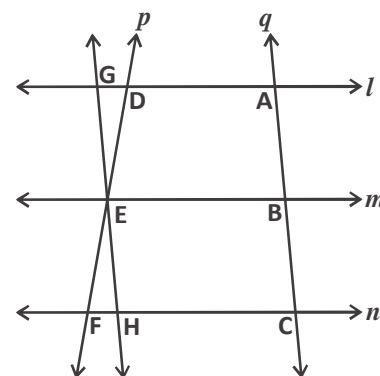


Fig. 27





$$\angle DGE = \angle EHF \quad (\text{alternate angles})$$

$$DE = EF \quad (\text{given})$$

$$\angle DEG = \angle HEF \quad (\text{vertically opposite angles})$$

$$\therefore \triangle GED \cong \triangle HEF \quad (\text{by ASA congruency})$$

therefore, $GE = EH$

$$\therefore GE = AB, EH = BC \quad \text{by (1) and (2)}$$

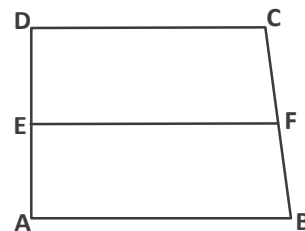
$$\therefore AB = BC$$

Hence, proved.

Exercise - 11.2



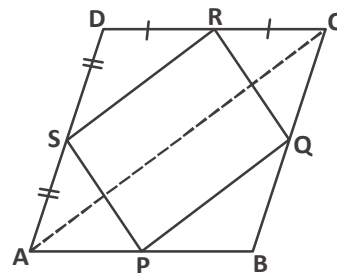
1. ABCD is a trapezium where $AB \parallel DC$. E is the midpoint of AD. A line drawn from E, parallel to AB, meets BC at point F. Prove that F is the midpoint of BC.
2. ABCD is rhombus and P, Q, R, S are the mid points of the sides AB, BC, CD and DA respectively. Show that quadrilateral PQRS is a rectangle.
3. ABCD is a rectangle in which P, Q, R, S are the mid points of sides AB, BC, CD and DA respectively. Show that quadrilateral PQRS is a rhombus.
4. ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA. AC is a diagonal. Show that :



$$(1) SR \parallel AC \text{ and } SR = \frac{1}{2} AC$$

$$(2) PQ = SR$$

$$(3) PQRS \text{ is a parallelogram}$$



What have We Learnt



1. Sum of the interior angles of a quadrilateral is 360° .
2. A diagonal of a parallelogram divides it into two congruent triangles.

3. There are many types of quadrilaterals. Some types of quadrilaterals are:-
 - (i) Parallelograms (ii) Rhombus (iii) Trapezium
 - (iv) Rectangle (v) Square
 4. A quadrilateral is a parallelogram, if
 - (i) Both pairs of opposite sides are equal;
 - (ii) Both pairs of opposite angles are equal;
 - (iii) diagonals bisect each other;
 - (iv) a pair of opposite sides is both equal and parallel.
 5. Diagonals of a rectangle bisect each other and are equal and vice-versa.
 6. Diagonals of a square bisect each other at right angles and are equal and vice-versa.
 7. Diagonals of a rhombus bisect each other at right angles and vice-versa.
 8. The line-segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.
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