CHAPTER 23
INTEGRALS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. $\int x^x (1+\log x)dx$ is equal to

(a) x^x (b) x^{2x}
(c) $x^x \log x$ (d) $1/2\,(1+\log x)^2$

2. $\int x^{51}(\tan^{-1}x+\cot^{-1}x)\,dx$

(a) $\frac{x^{52}}{52}(\tan^{-1}x+\cot^{-1}x)+c$

(b) $\frac{x^{52}}{52}(\tan^{-1}x-\cot^{-1}x)+c$

(c) $\frac{\pi x^{52}}{104}+\frac{\pi}{2}+c$

(d) $\frac{x^{52}}{52}+\frac{\pi}{2}+c$

3. Let $\int \frac{x^{1/2}}{\sqrt{1-x^3}}dx=\frac{2}{3}gof(x)+C$, then

(a) $f(x)=\sqrt{x}$
(b) $f(x)=x^{3/2}$ and $g(x)=\sin^{-1}x$
(c) $f(x)=x^{2/3}$
(d) None of these

4. $\int \sec^{2/3}x\,\mathrm{cosec}^{4/3}\,x\,dx=$

(a) $-3(\tan x)^{1/3}+c$ (b) $-3(\tan x)^{-1/3}+c$
(c) $3(\tan x)^{-1/3}+c$ (d) $(\tan x)^{-1/3}+c$

5. $\int_{\log 1/2}^{\log 2}\sin\left\{\frac{e^x-1}{e^x+1}\right\}dx$ equals

(a) $\cos\frac{1}{3}$ (b) $\sin\frac{1}{2}$
(c) 2 cos 2 (d) 0

6. Evaluate: $\int 2^{2^{2^x}}\,2^{2^x}\,2^x\,dx$

(a) $\frac{1}{(\log 2)^3}2^{2^{2^x}}+C$ (b) $\frac{1}{(\log 2)^3}2^{2^x}+C$

(c) $\frac{1}{(\log 2)^2}2^{2^x}+C$ (d) $\frac{1}{(\log 2)^4}2^{2^{2x}}+C$

7. $\int \frac{10x^9+10^x\log_e 10}{10^x+x^{10}}dx$ is equal to

(a) $10^x-x^{10}+C$ (b) $10^x+x^{10}+C$
(c) $(10^x-x^{10})^{-1}+C$ (d) $\log_e(\log^x+x^{10})+C$

8. If $\int\frac{e^x(1+\sin x)dx}{1+\cos x}=e^x f(x)+C$, then f(x) is equal to

(a) $\sin\frac{x}{2}$ (b) $\cos\frac{x}{2}$

(c) $\tan\frac{x}{2}$ (d) $\log\frac{x}{2}$

9. $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$ is equal to

(a) $-e^x\tan\left(\frac{x}{2}\right)+C$ (b) $-e^x\cot\left(\frac{x}{2}\right)+C$

(c) $-\frac{1}{2}e^x\tan\left(\frac{x}{2}\right)+C$ (d) $\frac{1}{2}e^x\cot\left(\frac{x}{2}\right)+C$

10. Evaluate $\int_1^2 x^2dx$ as limit of sums.

(a) 1 (b) $\frac{7}{3}$

(c) $\frac{1}{3}$ (d) 0

11. Evaluate: $\int_0^{\pi/2}\frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx$

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$
(c) $3+\sqrt{3}$ (d) $3-\sqrt{3}$

12. $\int \frac{x^9}{(4x^2+1)^6}dx$ is equal to

(a) $\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+C$ (b) $\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+C$

(c) $\frac{1}{10x}\left(\frac{1}{x}+4\right)^{-5}+C$ (d) $\frac{1}{10}\left(\frac{1}{x^2}+4\right)^{-5}+C$

13. $\int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}dx$ is equal to

(a) $\frac{1}{8}(x^2-1)+k$ (b) $\frac{1}{2}x^2+k$

(c) $\frac{1}{2}x+k$ (d) None of these

14. $\int e^{3\log x}(x^4+1)^{-1}dx$ is equal to

(a) $\log(x^4+1)+C$ (b) $\frac{1}{4}\log(x^4+1)+C$

(c) $-\log(x^4+1)+C$ (d) None of these

15. The value of integral, $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$ is

(a) 1/2 (b) 3/2 (c) 2 (d) 1

16. The value of $\int \frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$ is

(a) $\frac{1}{2}\log\left|e^{x}+e^{-x}\right|+C$ (b) $2\log\left|e^{2x}+e^{-2x}\right|+C$

(c) $\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$ (d) None of these

17. $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx$ equals

(a) $-\cot(ex^x)+C$ (b) $\tan(xe^x)+C$

(c) $\tan(e^x)+C$ (d) $\cot(e^x)+C$

18. $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$ equal to

(a) $\frac{1}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$ (b) $\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$

(c) $\frac{2}{3}\sin^{-1}\sqrt{\frac{x}{a}}+C$ (d) None of these

19. The value of $\int\sqrt{\frac{a-x}{a+x}}dx$ is

(a) $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{x^2-a^2}+C$

(b) $a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}+C$

(c) $a\sin^{-1}\left(\frac{a}{x}\right)+\sqrt{x^2-a^2}+C$

(d) None of these

20. If $\int \sin^3 x\cos^5 x\,dx = A\sin^4 x+B\sin^6 x+C\sin^8 x+D$. Then

(a) $A=\frac{1}{4},\ B=-\frac{1}{3},\ C=\frac{1}{8},\ D\in R$

(b) $A=\frac{1}{8},\ B=\frac{1}{4},\ C=\frac{1}{3},\ D\in R$

(c) $A=0,\ B=-\frac{1}{6},\ C=\frac{1}{8},\ D\in R$

(d) None of these.

21. $\int\left(x+\frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right)dx$ is equal to :

(a) $\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$ (b) $\left[\frac{x^2+1}{x^2}\right]^{n+6}(n+6)+c$

(c) $\left[\frac{x}{x^2+1}\right]^{n+6}(n+6)+c$ (d) None of these

22. Value of $\int_0^4 \frac{1}{\sqrt{x^2+2x+3}}dx$ is

(a) $\log\left(\frac{1+\sqrt{3}}{5+3\sqrt{3}}\right)$ (b) $\log\left(\frac{5-3\sqrt{3}}{1-\sqrt{3}}\right)$

(c) $\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$ (d) None of these

23. $\int \sin 2x.\log\cos x\,dx$ is equal to

(a) $\cos^2 x\left(\frac{1}{2}+\log\cos x\right)+k$

(b) $\cos^2 x.\log\cos x+k$

(c) $\cos^2 x\left(\frac{1}{2}-\log\cos x\right)+k$

(d) None of these.

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

24. Which of the following is/are correct?

I. $\int \frac{dx}{x\sqrt{x^2-1}} = \operatorname{cosec}^{-1}x+C$

II. $\int e^x dx = \log e^x + C$

III. $\int \frac{1}{x}dx = \log|x|+C$

IV. $\int a^x dx = a^x + C$

(a) I and III are correct (b) All are correct

(c) Only III is correct (d) All are incorrect

25. Consider the following statements

Statement-I: The value of $\int \frac{dx}{\sqrt{16-9x^2}}$ is $\frac{1}{3}\sin^{-1}\frac{3x}{4}+C$

Statement-II: The value of $\int \frac{dt}{\sqrt{3t-2t^2}}$ is $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{3-4t}{3}\right)+C$.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

26. Consider the following statements

Statement-I : The value of $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$ is $\frac{22}{7}-\pi$.

Statement-II : The value of integral $\int_{-1}^{1}\frac{|x+2|}{x+2}dx$ is 2.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

27. Consider the following statements

Statement-I: $\int_0^{\lambda}\frac{y\,dy}{\sqrt{y+\lambda}}$ is equal to $\frac{2}{3}\left(2+\sqrt{2}\right)\lambda\sqrt{\lambda}$.

Statement-II: $3a\int_0^1\left(\frac{ax-1}{a-1}\right)^2 dx$ is equal to $(a-1)+(a-1)^2$.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

28. Consider the following statements

Statement-I: $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$ if f is an odd function i.e., $f(-x) = -f(x)$.

Statement-II: $\int_{-a}^{a} f(x)dx = 0$, if f is an even function i.e., if $f(-x) = f(x)$.

(a) Statement I is true
(b) Statement II is true
(c) Both statements are true
(d) Both statements are false

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

29. Match the following derivatives of the functions in column-I with their respective anti-derivatives in column-II.

Column - I	Column - II
A. $\frac{1}{\sqrt{1-x^2}}$	1. $\tan^{-1}x+C$
B. $\frac{-1}{\sqrt{1-x^2}}$	2. $\cot^{-1}x+C$
C. $\frac{1}{1+x^2}$	3. $\sin^{-1}x+C$
D. $\frac{-1}{1+x^2}$	4. $\cos^{-1}x+C$

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	3	4	2	1
(c)	3	4	1	2
(d)	4	3	2	1

30. Match the following integrals in column-I with their corresponding values in column-II.

Column-I	Column-II
A. $\int\sqrt{ax+b}\,dx$	1. $\frac{2}{5}(x+2)^{5/2}-\frac{4}{3}(x+2)^{3/2}+C$
B. $\int x\sqrt{x+2}\,dx$	2. $\frac{1}{6}\left(1+2x^2\right)^{3/2}+C$
C. $\int x\sqrt{1+2x^2}\,dx$	3. $\frac{4}{3}\left(x^2+x+1\right)^{3/2}+C$
D. $\int(4x+2)\sqrt{x^2+x+1}\,dx$	4. $\frac{2}{3a}(ax+b)^{3/2}+C$

Codes

	A	B	C	D
(a)	4	1	2	3
(b)	3	4	2	1
(c)	1	3	2	4
(d)	3	2	4	1

31. Match the following integrals in column-I with their corresponding solutions in column-II.

Column - I	Column - II
A. $\int \frac{\cos x - \sin x}{1+\sin 2x} dx$	1. $\frac{1}{6}\sec^3 2x - \frac{1}{2}\sec^2 2x + C$
B. $\int \tan^3 2x \sec 2x \, dx$	2. $\tan x + C$
C. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$	3. $\frac{-1}{\sin x + \cos x} + C$
D. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$	4. $\sec x - \operatorname{cosec} x + C$

Codes

	A	B	C	D
(a)	1	2	3	4
(b)	3	1	4	2
(c)	3	4	1	2
(d)	2	1	4	3

32. Match the following definite integrals in column-I with their corresponding values in column-II.

Column - I	Column - II
A. $\int_{-1}^{1} x^{17} \cos^4 x \, dx$	1. $\frac{\pi}{2} - 1$
B. $\int_{0}^{\pi/2} \sin^3 x \, dx$	2. 0
C. $\int_{0}^{\pi/4} 2\tan^3 x \, dx$	3. $\frac{2}{3}$
D. $\int_{0}^{1} \sin^{-1} x \, dx$	4. $1 - \log 2$

Codes

	A	B	C	D
(a)	1	3	2	4
(b)	2	3	4	1
(c)	1	2	3	4
(d)	2	4	3	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

33. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x+x^2}\right) dx$ is

(a) 1 (b) 0 (c) – 1 (d) $\frac{\pi}{4}$

34. $\int_0^{2\pi} \log\left(\frac{a+b\sec x}{a-b\sec x}\right) dx =$

(a) 0 (b) $\pi/2$

(c) $\frac{\pi(a+b)}{a-b}$ (d) $\frac{\pi}{2}(a^2-b^2)$

35. Value of $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}} dx$ is

(a) 2 (b) 3 (c) 4 (d) 5

36. The value of $\int_{-1}^{1} (x-[x]) dx$ (where [.] denotes greatest integer function) is

(a) 0 (b) 1 (c) 2 (d) None of these

37. The value of definite integral $\int_0^{\frac{\pi}{2}} \log(\tan x) \, dx$ is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

38. If m is an integer, then $\int_0^{\pi} \frac{\sin(2mx)}{\sin x} dx$ is equal to :

(a) 1 (b) 2 (c) 0 (d) π

39. The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right) dx$ is

(a) 2 (b) $\frac{3}{4}$ (c) 0 (d) – 2

40. The value of $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right) dx$ is

(a) 1 (b) 0 (c) – 1 (d) $\frac{\pi}{4}$

41. If $\int \cos^n x \sin x \, dx = -\frac{\cos^6 x}{6} + C$, then n =

(a) 0 (b) 1 (c) 2 (d) 5

42. If $\int \frac{3x+1}{(x-3)(x-5)} dx = \int \frac{-5}{(x-3)} dx + \int \frac{B}{(x-5)} dx$, then the value of B is

(a) 3 (b) 4 (c) 6 (d) 8

43. If $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{3} + c$, then a =

(a) 3 (b) 4 (c) 6 (d) 8

44. If $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx = \log(e^{2x}-1) - Ax + C$, then A =

(a) 0 (b) 1 (c) 2 (d) 5

45. $\int_{-a}^{4}(x^8-x^4+x^2+1)\,dx = 2\int_{0}^{4}(x^8-x^4+x^2+1)\,dx$,
then a =
(a) 3 (b) 4 (c) 6 (d) 8

ASSERTION - REASON TYPE QUESTIONS

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.
(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion
(c) Assertion is correct, Reason is incorrect
(d) Assertion is incorrect, Reason is correct.

46. **Assertion :** $I = \int_0^{\pi/2}\sqrt{\tan x}\,dx = \frac{\pi}{\sqrt{2}}$
Reason: $\tan x = t^2$ makes the integrand in I as a rational function.

47. **Assertion :** $\int_{-2}^{2}\log\left(\frac{1+x}{1-x}\right)dx = 0$.
Reason : If f is an odd function, then $\int_{-a}^{a} f(x)dx = 0$.

48. **Assertion :** If the derivative of function x is $\frac{d}{dx}(x)=1$, then its anti-derivatives or integral is $\int(1)dx = x + C$.
Reason : If $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$, then the corresponding integral of the function is $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$.

49. **Assertion:** It is not possible to find $\int e^{-x^2}\,dx$ by inspection method.
Reason : Function is not expressible in terms of elementary functions.

50. **Assertion :** If $\frac{d}{dx}\int f(x)dx = f(x)$, then $\int f(x)\,dx = f'(x) + C$ where C is an arbitrary constant.
Reason : Process of differentiation and integration are inverses of each other.

51. **Assertion :** Geometrically, derivative of a function is the slope of the tangent to the corresponding curve at a point.
Reason : Geometrically, indefinite integral of a function represents a family of curves parallel to each other.

52. **Assertion :** Derivative of a function at a point exists.
Reason : Integral of a function at a point where it is defined, exists.

53. **Assertion :** $\int[\sin(\log x)+\cos(\log x)]dx = x\sin(\log x) + C$
Reason : $\frac{d}{dx}[x\sin(\log x)] = \sin(\log x) + \cos(\log x)$.

54. **Assertion :** The value of $\int_a^b f(t)dt$ and $\int_a^b f(u)du$ are equal
Reason : The value of definite integral of a function over any particular interval depends on the function and the interval not on the variable of integration.

55. **Assertion :** $\int_0^{\pi} x\sin x\cos^2 x\,dx = \frac{\pi}{2}\int_0^{\pi}\sin x\cos^2 x\,dx$
Reason : $\int_a^b x\,f(x)\,dx = \frac{a+b}{2}\int_a^b f(x)\,dx$

56. **Assertion :** The value of the integral $\int e^x[\tan x + \sec^2 x]dx$ is $e^x \tan x + C$
Reason : The value of the integral $e^x\{f(x)+f'(x)\}dx$ is $e^x f(x) + C$.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

57. If $f(a+b-x) = f(x)$, then $\int_a^b x\,f(x)\,dx$ is equal to
(a) $\frac{a+b}{2}\int_a^b f(b-x)\,dx$ (b) $\frac{a+b}{2}\int_a^b f(b+x)\,dx$
(c) $\frac{b-a}{2}\int_a^b f(x)\,dx$ (d) $\frac{a+b}{2}\int_a^b f(x)\,dx$

58. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}\,dx$, a > 0, is
(a) π (b) $a\pi$ (c) $\pi/2$ (d) 2π

59. Evaluate: $\int \sin^3 x\cos^3 x\,dx$
(a) $\frac{1}{32}\left\{\frac{3}{2}\cos 2x - \frac{1}{6}\cos 6x\right\} + C$
(b) $\frac{1}{32}\left\{-\frac{3}{2}\cos 2x + \frac{1}{6}\cos 6x\right\} + C$
(c) $\frac{1}{32}\left\{-\frac{3}{2}\cos 2x - \frac{1}{6}\cos 6x\right\} + C$
(d) None of these

60. Evaluate: $\int \frac{1}{\sqrt{\sin^3 x \cos^5 x}} dx$

(a) $\frac{2}{\sqrt{\tan x}} - \frac{2}{3}(\tan x)^{3/2} + C$

(b) $-\frac{2}{\sqrt{\tan x}} + \frac{2}{3}(\tan x)^{3/2} + C$

(c) $-\frac{2}{\sqrt{\tan x}} - \frac{2}{3}(\tan x)^{3/2} + C$

(d) None of these

61. Evaluate: $\int \frac{1}{\sqrt{9+8x-x^2}} dx$

(a) $-\sin^{-1}\left(\frac{x-4}{5}\right)+C$ (b) $-\sin^{-1}\left(\frac{x+4}{5}\right)+C$

(c) $\sin^{-1}\left(\frac{x-4}{5}\right)+C$ (d) None of these

62. Evaluate: $\int \frac{1}{1+3\sin^2 x+8\cos^2 x} dx$

(a) $\frac{1}{6}\tan^{-1}(2\tan x)+C$ (b) $\tan^{-1}(2\tan x)+C$

(c) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$ (d) None of these

63. Evaluate: $\int \frac{x^3+x}{x^4-9} dx$

(a) $\frac{1}{4}\log|x^4-9|+\frac{1}{12}\log\left|\frac{x^2+3}{x^2-3}\right|+C$

(b) $\frac{1}{4}\log|x^4-9|-\frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|+C$

(c) $\frac{1}{4}\log|x^4-9|+\frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|+C$

(d) None of these

64. If $\int \frac{3x+4}{x^3-2x-4} dx = \log|x-2| + k\log f(x) + c$, then

(a) $f(x) = |x^2 + 2x + 2|$ (b) $f(x) = x^2 + 2x + 2$

(c) $k = -\frac{1}{2}$ (d) All of these

65. Evaluate: $\int \frac{1-\cos x}{\cos x(1+\cos x)} dx$

(a) $\log|\sec x+\tan x|-2\tan(x/2)+C$

(b) $\log|\sec x-\tan x|-2\tan(x/2)+C$

(c) $\log|\sec x+\tan x|+2\tan(x/2)+C$

(d) None of these

66. Evaluate: $\int_0^{\pi} \frac{1}{5+4\cos x} dx$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

67. If $\int_0^{\pi} \ln \sin x\, dx = k$, then value of $\int_0^{\pi/4} \ln(1+\tan x) dx$ is

(a) $-\frac{k}{4}$ (b) $\frac{k}{4}$ (c) $-\frac{k}{8}$ (d) $\frac{k}{8}$

68. $\int \tan^{-1}\sqrt{x}\, dx$ is equal to

(a) $(x+1)\tan^{-1}\sqrt{x}-\sqrt{x}+C$

(b) $x\tan^{-1}\sqrt{x}-\sqrt{x}+C$

(c) $\sqrt{x}-x\tan^{-1}\sqrt{x}+C$

(d) $\sqrt{x}-(x+1)\tan^{-1}\sqrt{x}+C$

69. $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x} dx$ is equal to

(a) $\frac{1}{2}\sin 2x+c$ (b) $-\frac{1}{2}\sin 2x+c$

(c) $-\frac{1}{2}\sin x+c$ (d) $-\sin^2 x+c$

70. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B\log\sin(x-\alpha)+C,$ then value of (A, B) is

(a) $(-\cos\alpha, \sin\alpha)$ (b) $(\cos\alpha, \sin\alpha)$

(c) $(-\sin\alpha, \cos\alpha)$ (d) $(\sin\alpha, \cos\alpha)$

71. If $f(x) = \begin{cases} x^2, & 0 \le x \le 1 \\ \sqrt{x}, & 1 \le x \le 2 \end{cases}$ then $\int_0^2 f(x)dx =$

(a) $\frac{1}{3}$ (b) $4\sqrt{2}$

(c) $4\sqrt{2}-1$ (d) None of these

72. If $g(x) = \int_0^x \cos^4 t\, dt$, then $g(x+\pi)$ equals

(a) $g(x) + g(\pi)$ (b) $g(x) - g(\pi)$

(c) $f(x)\, g(\pi)$ (d) $\frac{g(x)}{g(\pi)}$

73. The integral $\int_0^{\pi/2} |\sin x - \cos x|\, dx$ is equal to :

(a) $2\sqrt{2}$ (b) $2(\sqrt{2}-1)$

(c) $\sqrt{2}+1$ (d) None of these

74. $\int_{\pi/4}^{3\pi/4} \frac{\phi d\phi}{1+\sin\phi}$ is equal to

(a) $\sqrt{2}-1$ (b) $\frac{1}{\sqrt{2}-1}$

(c) $\frac{\pi}{\sqrt{2}+1}$ (d) $\frac{\pi}{\sqrt{2}-1}$

75. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is

(a) $\frac{\pi^2}{4}$ (b) π^2 (c) zero (d) $\frac{\pi}{2}$

76. If $\int \frac{1}{(\sin x+4)(\sin x-1)} dx$

$= A\frac{1}{\left(\tan\frac{x}{2}-1\right)} + B\tan^{-1}\left[f(x)\right] + C_1$, then

(a) $A=-\frac{1}{5}$, $B=\frac{-2}{5\sqrt{15}}$, $f(x)=\frac{4\tan x+3}{\sqrt{15}}$

(b) $A=-\frac{1}{5}$, $B=\frac{1}{\sqrt{15}}$, $f(x)=\frac{4\tan\left(\frac{x}{2}\right)+1}{\sqrt{15}}$

(c) $A=\frac{2}{5}$, $B=-\frac{2}{5}$, $f(x)=\frac{4\tan x+1}{5}$

(d) $A=\frac{2}{5}$, $B=\frac{-2}{5\sqrt{15}}$, $f(x)=\frac{4\tan\left(\frac{x}{2}\right)+f}{\sqrt{15}}$

77. If f and g are defined as f (x) = f (a – x) and

g (x) + g (a – x) = 4, then $\int_0^a f(x)g(x)dx$ is equal to

(a) $\int_0^a f(x)dx$ (b) $2\int_0^a f(x)dx$

(c) $\int_0^a g(x)dx$ (d) $2\int_0^a g(x)dx$

78. Value of $\int \frac{dx}{\sqrt{x(a-x)}}$ is

(a) $2\sin^{-1}\sqrt{\frac{x}{a}}+c$ (b) $-2\sin^{-1}\sqrt{\frac{x}{a}}+c$

(c) $2\sin^{-1}\frac{\sqrt{x}}{a}+c$ (d) None of these

79. Value of $\int_0^{\pi} |\cos x|\, dx$ is

(a) 2 (b) –2 (c) 1 (d) None of these

80. Value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ is

(a) $\frac{\pi}{2}$ (b) $\frac{-\pi}{2}$

(c) $\frac{\pi}{4}$ (d) None of these

81. Value of $\int \frac{(x-x^3)^{1/3}}{x^4} dx$ is

(a) $\frac{3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}+C$ (b) $\frac{-3}{8}\left(\frac{1}{x^2}-1\right)^{\frac{4}{3}}+C$

(c) $\frac{-3}{8}\left(\frac{1}{x^2}+1\right)^{\frac{4}{3}}+C$ (d) None of these

82. Value of $\int \frac{x^2+1}{(x-1)(x-2)} dx$ is

(a) $x+\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$ (b) $x+\log\left[\frac{(x-1)^2}{(x-2)^5}\right]+C$

(c) $x-\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$ (d) None of these

83. Value of $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}$ if $(\beta>\alpha)$ is

(a) $2\sin^{-1}\sqrt{\frac{\beta-\alpha}{x-\alpha}}+C$ (b) $2\sin^{-1}\sqrt{\frac{x-\alpha}{\beta-\alpha}}+C$

(c) $2\sin^{-1}\sqrt{\frac{x+\alpha}{\beta-\alpha}}+C$ (d) None of these

84. Value of $\int \frac{dx}{4\sin^2 x+4\sin x\cos x+5\cos^2 x}$ is

(a) $\frac{-1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$

(b) $\frac{1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$

(c) $\frac{1}{22}\tan^{-1}\left(\frac{\tan x+2}{2}\right)+C$

(d) None of these

85. Value of $\int \frac{x^2+1}{x^4+x^2+1} dx$ is

(a) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{x^2-1}\right)+C$ (b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right)+C$

(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{3}x}\right)+C$ (d) None of these

86. Value of $\int_0^1 \log\left(\frac{1}{x}-1\right) dx$ is

(a) 2I (b) –2I (c) 0 (d) None of these

87. Value of $\int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt{\cot x}} dx$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{12}{\pi}$ (d) None of these

88. Value of $\int_{-1}^{2} \frac{|x|}{x} dx$ is

(a) 0 (b) 1 (c) –1 (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. **(a)** $I=\int x^x(1+\log x)dx$

Put $x^x = t$, then $x^x(1+\log x)\,dx = dt$

$\therefore \quad I=\int dt \Rightarrow I=t+C \Rightarrow I=x^x+C.$

2. **(a)** $\int x^{51}(\tan^{-1}x+\cot^{-1}x)dx$

$=\int x^{51}.\frac{\pi}{2}dx \quad \left\{\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right\}$

$=\frac{\pi x^{52}}{104}+c=\frac{x^{52}}{52}(\tan^{-1}x+\cot^{-1}x)+c.$

3. **(b)** Put $x^{3/2}=t \Rightarrow \frac{3}{2}x^{1/2}dx=dt$

$\therefore$ integral is

$$\int\frac{\frac{2}{3}dt}{\sqrt{1-t^2}}=\frac{2}{3}\sin^{-1}t+C=\frac{2}{3}\sin^{-1}(x^{3/2})+C$$

4. **(b)** $\int\sec^{2/3}x\,\text{cosec}^{4/3}x\,dx=\int\frac{dx}{\sin^{4/3}x\cos^{2/3}x}$

Multiplying N^r and D^r by $\cos^2 x$, we get

{ Putting $\tan x = t \Rightarrow \sec^2 x\,dx = dt$}

$=\int\frac{\sec^2 x\,dx}{\tan^{4/3}x}=\int\frac{dt}{t^{4/3}}=\frac{t^{-1/3}}{(-1/3)}+c=-3(\tan x)^{-1/3}+c.$

5. **(d)** $I=\int_{-\log 2}^{\log 2}\sin\left\{\frac{e^x-1}{e^x+1}\right\}dx$

If $f(x)=\sin\left\{\frac{e^x-1}{e^x+1}\right\}$

$f(-x)=\sin\left\{\frac{1-e^x}{1+e^x}\right\}=-\sin\left\{\frac{e^x-1}{e^x+1}\right\}=-f(x)$

Hence $f(x)$ is an odd function of x $\therefore I=0$

6. **(a)** Let $I=\int 2^{2^{2^x}}2^{2^x}2^x\,dx$

Let $2^{2^{2^x}}=t \Rightarrow 2^{2^{2^x}}2^{2^x}2^x(\log 2)^3\,dx=dt$

$\Rightarrow I=\int\frac{1}{(\log 2)^3}dt=\frac{1}{(\log 2)^3}t+C=\frac{1}{(\log 2)^3}2^{2^{2^x}}+C$

7. **(d)** Put $10^x+x^{10}=t$

$\therefore \quad (10^x\log_e 10+10x^9)\,dx=dt$

$\therefore \quad \int\frac{10x^9+10^x\log_e 10}{10^x+x^{10}}dx=\int\frac{dt}{t}$

$=\log_e t+c=\log_e(10^x+x^{10})+C$

8. **(c)** $\int e^x\frac{(1+\sin x)}{(1+\cos x)}dx=\int e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right]dx$

$=\frac{1}{2}\int e^x\sec^2\frac{x}{2}dx+\int e^x\tan\frac{x}{2}dx$

$=e^x\tan\frac{x}{2}+C$

But $I=e^x f(x)+C$ (given)

$\therefore \ f(x)=\tan\frac{x}{2}$

9. **(b)** $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx=\int e^x\left(\frac{1-\sin x}{2\sin^2\frac{x}{2}}\right)dx$

$=\int e^x\left(\frac{1}{2}\text{cosec}^2\frac{x}{2}-\cot\frac{x}{2}\right)dx$

$=\frac{1}{2}\int e^x\text{cosec}^2\frac{x}{2}dx-e^x\cot\frac{x}{2}-\frac{1}{2}\int e^x\text{cosec}^2\frac{x}{2}dx+C$

$=-e^x\cot\frac{x}{2}+C$

10. **(b)** Here, $a=1$, $b=2$, $f(x)=x^2$, $b-a=1=nh$

$\therefore \quad \int_1^2 x^2dx=\lim_{h\to\infty}\sum_{r=0}^{n-1}f(a+rh)$

$=\lim_{h\to 0}\left[h\left\{1^2+(1+h)^2+(1+2h)^2+...+(1+(n-1)h)^2\right\}\right]$

$=\lim_{h\to 0}\left[h\left\{\begin{matrix}(1^2+1^2+n\text{ times})+h^2(1^2+2^2+......\\+(n-1)^2+2h(1+2+...+(n-1))\end{matrix}\right\}\right]$

$=\lim_{h\to 0}h\left\{n+h^2\frac{(n-1)n(2n-1)}{6}+2h\frac{(n-1)n}{2}\right\}$

$=\lim_{h\to 0}\left\{nh+\frac{(nh-h)nh(2nh-h)}{6}+\frac{2(hn)(nh-h)}{2}\right\}$

$=1+\frac{1}{3}+1=\frac{7}{3}$ (as $n\to\infty$, $h\to 0$)

11. **(a)** We have,

$$I=\int_0^{\pi/2}\frac{\cos x}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx$$

$$=\int_0^{\pi/2}\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^3}dx=\int_0^{\pi/2}\frac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}dx$$

Let $\cos\frac{x}{2}+\sin\frac{x}{2}=t$. Then,

$$\frac{1}{2}\left(-\sin\frac{x}{2}+\cos\frac{x}{2}\right)dx=dt$$

$$\Rightarrow \left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)dx=2dt$$

Also, $x = 0 \Rightarrow t = 1$ and $x=\frac{\pi}{2} \Rightarrow t=\sqrt{2}$

$$\therefore\ I=\int_1^{\sqrt{2}}\frac{2dt}{t^2}=2\int_1^{\sqrt{2}}\frac{1}{t^2}dt$$

$$=2\left[-\frac{1}{t}\right]_1^{\sqrt{2}}=2\left[-\frac{1}{\sqrt{2}}+1\right]=\left(2-\sqrt{2}\right)$$

12. (d) We have, $I=\int\frac{x^9}{\left(4x^2+1\right)^6}dx$

$$=\int\frac{x^9dx}{x^{12}\left(4+\frac{1}{x^2}\right)^6}=\int\frac{dx}{x^3\left(4+\frac{1}{x^2}\right)^6}$$

Put $4+\frac{1}{x^2}=t\Rightarrow\frac{-2}{x^3}dx=dt$

$$I=-\frac{1}{2}\int\frac{dt}{t^6}=\frac{-1}{2}\int t^{-6}dt$$

$$=-\frac{1}{2}\frac{t^{-5}}{-5}+C=\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+C$$

13. (b) Put $x = \cos 2\theta$

$$\therefore\ I=\int\cos\{2\tan^{-1}\tan\theta\}(-2\sin2\theta)d\theta$$

$$=-\int\sin4\theta\, d\theta=\frac{1}{4}\cos4\theta+c$$

$$=\frac{1}{4}(2x^2-1)+c=\frac{1}{2}x^2+k$$

14. (b) $\int e^{3\log x}(x^4+1)^{-1}dx=\int e^{\log x^3}\frac{1}{x^4+1}dx$

$$=\int\frac{x^3}{x^4+1}dx=\frac{1}{4}\log(x^4+1)+C$$

[since $e^{\log_e x^3}=x^3$]

15. (b) $I=\int_3^6\frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}dx$

$$I=\int_3^6\frac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x}}dx$$

$$2I=\int_3^6 dx=3\ \Rightarrow\ I=\frac{3}{2}$$

16. (c) $\int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx$

Let $e^{2x}+e^{-2x}=t$

$$\Rightarrow\ 2e^{2x}-2e^{-2x}=\frac{dt}{dx}\Rightarrow dx=\frac{dt}{2\left(e^{2x}-e^{-2x}\right)}$$

$$\therefore\ \int\frac{e^{2x}-e^{-2x}}{e^{2x}+e^{-2x}}dx=\int\frac{e^{2x}-e^{-2x}}{t}=\frac{dt}{2\left[e^{2x}-e^{-2x}\right]}$$

$$=\frac{1}{2}\int\frac{1}{t}dt=\frac{1}{2}\log|t|+C$$

$$=\frac{1}{2}\log\left|e^{2x}+e^{-2x}\right|+C$$

17. (b) $\int\frac{e^x(1+x)}{\cos^2\left(e^xx\right)}dx$

Let $xe^x = t$

$$\Rightarrow\ \left(xe^x+e^x\right)=\frac{dt}{dx}$$

$$\Rightarrow\ dx=\frac{dt}{e^x(x+1)}$$

$$\therefore\ \int\frac{e^x(1+x)}{\cos^2\left(e^xx\right)}dx=\int\frac{e^x(1+x)}{\cos^2 t}\times\frac{dt}{e^x(1+x)}$$

$$=\int\frac{1}{\cos^2 t}dt=\int\sec^2 t\,dt$$

$$=\tan t+C=\tan\left(xe^x\right)+C$$

18. (b) We have, $I=\int\frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx$

$$I=\int\frac{\sqrt{x}}{\sqrt{\left(a^{3/2}\right)^2-\left(x^{\frac{3}{2}}\right)^2}}dx$$

Put, $x^{\frac{3}{2}}=t$

$$\Rightarrow\ \frac{3}{2}x^{\frac{1}{2}}dx=dt$$

$$\Rightarrow\ \sqrt{x}\,dx=\frac{2}{3}dt$$

$$\therefore\ I=\frac{2}{3}\int\frac{dt}{\sqrt{\left(a^{\frac{3}{2}}\right)^2-t^2}}=\frac{2}{3}\sin^{-1}\left(\frac{t}{a^{\frac{3}{2}}}\right)+C$$

$$=\frac{2}{3}\sin^{-1}\left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}}\right)+C=\frac{2}{3}\sin^{-1}\sqrt{\frac{x^3}{a^3}}+C$$

19. (b) $I=\int\sqrt{\frac{a-x}{a+x}}dx=\int\sqrt{\frac{a-x}{a+x}\times\frac{a-x}{a-x}}dx=\int\frac{a-x}{\sqrt{a^2-x^2}}dx$

$\Rightarrow I=\int\frac{a}{\sqrt{a^2-x^2}}dx-\int\frac{x}{\sqrt{a^2-x^2}}dx$

$\Rightarrow I=a\int\frac{1}{\sqrt{a^2-x^2}}dx+\frac{1}{2}\int\frac{-2x}{\sqrt{a^2-x^2}}dx$

Putting $a^2-x^2=t$, and $-2x\,dx=dt$, we get

$I=a\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}\int\frac{dt}{\sqrt{t}}=a\sin^{-1}\left(\frac{x}{a}\right)+\frac{1}{2}\left(\frac{t^{1/2}}{1/2}\right)+C$

$\Rightarrow I=a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{t}+C=a\sin^{-1}\left(\frac{x}{a}\right)+\sqrt{a^2-x^2}+C$

20. (a) $I=\int\sin^3x.\cos^5x\,dx$

Put $\sin x=t\Rightarrow\cos x\,dx=dt$

$I=\int\sin^3x\,.\cos^4x\,.\cos x\,dx=\int t^3(1-t^2)^2dt$

$=\int(t^3-2t^5+t^7)\,dt=\frac{1}{4}t^4-\frac{2}{6}t^6+\frac{1}{8}t^8+D$

$=\frac{1}{4}\sin^4x-\frac{1}{3}\sin^6x+\frac{1}{8}\sin^8x+D$

21. (a) $I=\int\left(x+\frac{1}{x}\right)^{n+5}\left(\frac{x^2-1}{x^2}\right)dx$

Put $x+\frac{1}{x}=t\Rightarrow\left(1-\frac{1}{x^2}\right)dx=dt$

$\Rightarrow\left(\frac{x^2-1}{x^2}\right)dx=dt$

$\therefore I=\int t^{n+5}\,dt=\frac{t^{n+6}}{n+6}+c=\frac{\left(x+\frac{1}{x}\right)^{n+6}}{n+6}+c$

22. (c) We know that,

$\int_0^4\frac{1}{\sqrt{x^2+2x+3}}dx=\int_0^4\frac{1}{\sqrt{(x+1)^2+(\sqrt{2})^2}}dx$

$=\left[\log\left|x+1+\sqrt{(x+1)^2+(\sqrt{2})^2}\right|\right]_0^4$

$=\left[\log\left|x+1+\sqrt{x^2+2x+3}\right|\right]_0^4$

$=\log(5+\sqrt{16+8+3})-\log(1+\sqrt{3})$

$=\log(5+3\sqrt{3})-\log(1+\sqrt{3})$

$=\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$

23. (c) $I=\int 2\sin x.\cos x.\log\cos x\,dx$

put $\log\cos x=t$

$\therefore-\frac{\sin x}{\cos x}dx=dt$

$I=\int 2\sin x.\cos x.t\frac{\cos x}{-\sin x}dt$

$=-2\int\cos^2x.t\,dt=-2\int te^{2t}dt$

$=-2\left[t.\frac{e^{2t}}{2}-\int\frac{e^{2t}}{2}.dt\right]=-te^{2t}+\frac{1}{2}e^{2t}+k$

$=e^{2t}\left(\frac{1}{2}-t\right)+k=\cos^2x\left\{\frac{1}{2}-\log\cos x\right\}+k$

STATEMENT TYPE QUESTIONS

24. (c) I. $\frac{d}{dx}\left(-\text{cosec}^{-1}x\right)=\frac{1}{x\sqrt{x^2-1}}$

$\int\frac{dx}{x\sqrt{x^2-1}}=-\text{cosec}^{-1}x+C$

II. $\frac{d}{dx}(e^x)=e^x;\int e^xdx=e^x+C$

III. $\frac{d}{dx}(\log|x|)=\frac{1}{x};\int\frac{1}{x}dx=\log|x|+C$

IV. $\frac{d}{dx}\left(\frac{a^x}{\log a}\right)=a^x;\int a^xdx=\frac{a^x}{\log x}+C$

25. (a) I. We have,

$I=\int\frac{dx}{\sqrt{16-9x^2}}$

$=\frac{1}{3}\int\frac{dx}{\sqrt{\left(\frac{4}{3}\right)^2-x^2}}=\frac{1}{3}\sin^{-1}\left(\frac{x}{\frac{4}{3}}\right)+C$

$\left(\because\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\frac{x}{a}+C\right)$

$=\frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right)+C$

II. We have,

$I=\int\frac{dt}{\sqrt{3t-2t^2}}$

$=\frac{1}{\sqrt{2}}\int\frac{dt}{\sqrt{-\left[(t^2)-\frac{3}{2}t+\left(\frac{3}{4}\right)^2-\left(\frac{3}{4}\right)^2\right]}}$

$=\frac{1}{\sqrt{2}}\int\frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2-\left(t-\frac{3}{4}\right)^2}}$

$=\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{t-\frac{3}{4}}{\frac{3}{4}}\right)=\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{4t-3}{3}\right)+C$

26. (c) I. Let, $\int_0^1 \frac{x^4(1-x)^4}{1+x^2}dx$

$= \int_0^1 \frac{(x^4-1)(1-x^4)+(1-x)^4}{(1+x^2)}dx$

$= \int_0^1 (x^2-1)(1-x)^4 dx + \int_0^1 \frac{(1+x^2-2x)^2}{(1+x^2)}dx$

$= \int_0^1 \left\{(x^2-1)(1-x)^4+(1+x^2)-4x+\frac{4x^2}{(1+x^2)}\right\}dx$

$= \int_0^1 \left((x^2-1)(1-x)^4+(1+x^2)-4x+4-\frac{4}{1+x^2}\right)dx$

$= \int_0^1 \left(x^6-4x^5+5x^4-4x^2+4-\frac{4}{1+x^2}\right)dx$

$= \left[\frac{x^7}{7}-4\frac{x^6}{6}+\frac{5x^5}{5}-\frac{4x^3}{3}+4x-4\tan^{-1}x\right]_0^1$

$= \frac{1}{7}-\frac{4}{6}+\frac{5}{5}-\frac{4}{3}+4-4\left(\frac{\pi}{4}-0\right) = \frac{22}{7}-\pi$

II. Let $I = \int_{-1}^{1} \frac{|x+2|}{x+2}dx$

For $-1 \le x \le 1, |x+2| = 2+x$

$\therefore \ I = \int_{-1}^{1} \frac{x+2}{x+2}dx = \int_{-1}^{1} 1\,dx$

$= [x]_{-1}^{1} = 1-(-1) = 2$

27. (d) I. Let, $I = \int_0^\lambda \frac{y\,dy}{\sqrt{y+\lambda}} = \int_0^\lambda \left[\frac{y+\lambda-\lambda}{\sqrt{y+\lambda}}\right]dy$

$= \int_0^\lambda \left[\sqrt{y+\lambda}-\frac{\lambda}{\sqrt{y+\lambda}}\right]dy$

$= \int_0^\lambda (y+\lambda)^{1/2}dy - \int_0^\lambda \frac{\lambda}{\sqrt{y+\lambda}}dy$

$= \left[\frac{(y+\lambda)^{3/2}}{3/2}\right]_0^\lambda - \left[\frac{\lambda\sqrt{y+\lambda}}{1/2}\right]_0^\lambda$

$= \frac{2}{3}\left[(2\lambda)^{3/2}-\lambda^{3/2}\right]-2\lambda\left[(2\lambda)^{1/2}-(\lambda)^{1/2}\right]$

$= 2\lambda\sqrt{\lambda}\left[\frac{2\sqrt{2}-1}{3}-(\sqrt{2}-1)\right] = \frac{2}{3}\lambda\sqrt{\lambda}\left(2-\sqrt{2}\right)$

II. Let $I = 3a\int_0^1 \left(\frac{ax-1}{a-1}\right)^2 dx = \frac{3a}{(a-1)^2}\left[\frac{(ax-1)^3}{3}\times\frac{1}{a}\right]_0^1$

$= \frac{1}{(a-1)^2}\left[(a-1)^3+1\right] = (a-1)+(a-1)^{-2}$

28. (d) We have

$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_0^a f(x)dx$ Then

Let t = –x in the first integral on the right hand side.
dt = – dx. When, x = – a, t = a and
when x = 0, t = 0. Also x = – t

Therefore, $\int_{-a}^{a} f(x)dx = -\int_a^0 f(-t)dt + \int_0^a f(x)dx$

$= \int_0^a f(-x)dx + \int_0^a f(x)dx$ by $\left[\int_0^a f(t)dt = \int_0^a f(x)dx\right]$ ...(i)

I. Now, if f is an even function, then f(– x) = f(x) and so, eq. (i) becomes

$\int_{-a}^{a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(x)dx = 2\int_0^a f(x)dx$

II. If f is an odd function, then f(– x) = – f(x) and so, eq. (i) becomes

$\int_{-a}^{a} f(x)dx = -\int_0^a f(x)dx + \int_0^a f(x)dx = 0$

MATCHING TYPE QUESTIONS

29. (c) The function in column-I are derived functions of column-II, then we say that each function of column-II is an anti-derivative of each function in column-I.

A. $\frac{d}{dx}\left(\sin^{-1}x\right) = \frac{1}{\sqrt{1-x^2}}$; $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C$

B. $\frac{d}{dx}\left(\cos^{-1}x\right) = \frac{-1}{\sqrt{1-x^2}}$; $\int \frac{-dx}{\sqrt{1-x^2}} = +\cos^{-1}x + C$

C. $\frac{d}{dx}\left(\tan^{-1}x\right) = \frac{1}{1+x^2}$; $\int \frac{dx}{1+x^2} = \tan^{-1}x + C$

D. $\frac{d}{dx}\left(\cot^{-1}x\right) = \frac{-1}{1+x^2}$; $\int \frac{-dx}{1+x^2} = +\cot^{-1}x + C$

30. (a) A. $\int \sqrt{ax+b}\,dx = \int (ax+b)^{1/2}dx$

$= \frac{(ax+b)^{(1/2)+1}}{a\left(\frac{1}{2}+1\right)} + C = \frac{(ax+b)^{3/2}}{a\left(\frac{3}{2}\right)} + C$

$= \frac{2}{3a}(ax+b)^{3/2} + C$

B. $\int x\sqrt{x+2}\,dx = \int (x+2-2)\sqrt{x+2}\,dx$

$= \int (x+2)\sqrt{x+2}\,dx - 2\int \sqrt{x+2}\,dx$

$= \int (x+2)^{3/2}dx - 2\int (x+2)^{1/2}dx$

$= \frac{(x+2)^{(3/2)+1}}{(3/2)+1} - 2\frac{(x+2)^{(1/2)+1}}{(1/2)+1} + C$

$= \frac{2}{5}(x+2)^{5/2} - \frac{2\times 2}{3}(x+2)^{3/2} + C$

$= \frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + C$

C. Let $I = \int x\sqrt{1+2x^2}\,dx$

Let $1 + 2x^2 = t$

On differentiating w.r.t.x, we get

$4x = \frac{dt}{dx} \Rightarrow dx = \frac{dt}{4x}$

$\therefore\ I = \int x\sqrt{t}\frac{dt}{4x} = \frac{1}{4}\int\sqrt{t}\,dt = \frac{1}{4}\int t^{1/2}dt$

$= \frac{1}{4}\frac{t^{(1/2)+1}}{(1/2)+1} + C = \frac{1}{6}\left(1+2x^2\right)^{3/2} + C$

D. Let $I = \int(4x+2)\sqrt{x^2+x+1}\,dx$

Let $x^2 + x + 1 = t$

On differentiating w.r.t. x, we get

$2x + 1 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{(2x+1)}$

$\therefore\ I = \int(4x+2)\sqrt{t}\frac{dt}{(2x+1)}$

$= \int 2(2x+1)\sqrt{t}\frac{dt}{(2x+1)} = 2\int\sqrt{t}\,dt$

$= 2\frac{t^{(1/2)+1}}{(1/2)+1} + C = \frac{4}{3}\left(x^2+x+1\right)^{3/2} + C$

31. **(b)** A. Let $I = \int\frac{\cos x - \sin x}{1+\sin 2x}dx$

$= \int\frac{\cos x - \sin x}{\sin^2 x + \cos^2 x + 2\sin x\cos x}dx$

$= \int\frac{\cos x - \sin x}{(\sin x + \cos x)^2}dx$

Let cos x + sin x = t $\Rightarrow$ – sin x + cos x = $\frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{(\cos x - \sin x)}$

$\therefore\ I = \int\frac{\cos x - \sin x}{t^2}\cdot\frac{dt}{(\cos x - \sin x)}$

$= \int\frac{1}{t^2}dt = \int t^{-2}dt = \frac{t^{-2+1}}{-2+1} + C$

$= \frac{-1}{\cos x + \sin x} + C$

B. $\int\tan^3 2x\sec 2x\,dx$

Let sec 2x = t

$\Rightarrow$ 2 sec 2x tan 2x = $\frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2\sec 2x.\tan 2x}$

$\therefore\ \int\tan^3 2x\sec 2x\,dx$

$= \int\tan^3 2x\sec 2x\frac{dt}{2\sec 2x.\tan 2x}$

$= \frac{1}{2}\int\tan^2 2x\,dt = \frac{1}{2}\int\left[\sec^2 2x - 1\right]dt$

$\left(\because \tan^2 x = \sec^2 x - 1\right)$

$= \frac{1}{2}\int\left[\left(t^2-1\right)dt\right] = \frac{1}{2}\left[\frac{t^3}{3} - t\right] + C$

$= \frac{1}{2}\left[\frac{\sec^3 2x}{3} - \sec 2x\right] + C$

$= \frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + C$

C. $\int\frac{\sin^3 x + \cos^3 x}{\sin^2 x\cos^2 x}dx$

$= \int\frac{\sin^3 x}{\sin^2 x\cos^2 x}dx + \int\frac{\cos^3 x}{\sin^2 x\cos^2 x}dx$

$= \int\frac{\sin x}{\cos x\cos x}dx + \int\frac{\cos x}{\sin x\sin x}dx$

$= \int\tan x.\sec x\,dx + \int\cot x.\operatorname{cosec} x\,dx$

= sec x – cosec x + C

D. $\int\frac{\cos 2x + 2\sin^2 x}{\cos^2 x}dx$

$= \int\frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x}dx$

$\left(\because \cos 2x = 1 - 2\sin^2 x\right)$

$= \int\frac{1}{\cos^2 x}dx = \int\sec^2 x\,dx = \tan x + C$

32. **(b)** A. Let $f(x) = x^{17}\cos^4 x$,

$\Rightarrow f(-x) = (-x)^{17}\cos^4(-x) = -x^{17}\cos^4 x = -f(x)$

Therefore, f(x) is an odd function.

We know that, if f(x) is an odd function, then

$\int_{-a}^{a} f(x)dx = 0$

$\therefore\ \int_{-1}^{1} x^{17}\cos^4 x\,dx = 0$

B. Let $I = \int_0^{\pi/2}\sin^3 x\,dx = \int_0^{\pi/2}\sin^2 x.\sin x\,dx$

$= \int_0^{\pi/2}\left(1-\cos^2 x\right)\sin x\,dx$

$\left(\because \sin^2 x = 1 - \cos^2 x\right)$

Put, cos x = t $\Rightarrow$ – sinx dx = dt

When, x = 0 $\Rightarrow$ t = cos 0 = 1, when $x = \frac{\pi}{2}$

$\Rightarrow\ t = \cos\frac{\pi}{2} = 0$

$\therefore\ I = \int_0^{\pi/2}\left(1-\cos^2 x\right)\sin x\,dx = \int_1^0\left(1-t^2\right)(-dt)$

$= -\left[t - \frac{t^3}{3}\right]_1^0 = -\left\{(0-0)-\left(1-\frac{1}{3}\right)\right\} = \frac{2}{3}$

C. Let $I = \int_0^{\pi/4} 2\tan^3 x\,dx = 2\int_0^{\pi/4}\tan^2 x.\tan x\,dx$

$= 2\int_0^{\pi/4}\left(\sec^2 x - 1\right)\tan x\,dx$

$\left[\because 1+\tan^2 x = \sec^2 x\right]$

$= 2\left[\int_0^{\pi/4}\sec^2 x\tan x\,dx - \int_0^{\pi/4}\tan x\,dx\right]$

$= 2\int_0^{\pi/4}(\tan x)\sec^2 x\,dx - 2\left[-\log|\cos x|\right]_0^{\pi/4}$

$\left[\begin{array}{l}\because \text{Let } I_1 = \int(\tan x)\sec^2 x\,dx \text{ put } \tan x = t\\ \Rightarrow \sec^2 x\,dx = dt \therefore I_1 = \int t\,dt = \frac{t^2}{2} = \frac{\tan^2 x}{2}\end{array}\right]$

$= 2\left[\frac{\tan^2 x}{2}\right]_0^{\pi/4} + 2\left[\log\left|\cos\frac{\pi}{4}\right| - \log|\cos 0|\right]$

$= \tan^2\left(\frac{\pi}{4}\right) - 0 + 2\left[\log\left(\frac{1}{\sqrt{2}}\right) - \log 1\right]$

$= 1 + 2\log 2^{-1/2} - 0 \qquad (\because \log 1 = 0)$

$= 1 - 2\times\frac{1}{2}\log 2 = 1 - \log 2$

D. Let $I = \int_0^1 \sin^{-1} x\,dx = \int_0^1 \sin^{-1} x.1\,dx$

Applying rule of integration by parts taking $\sin^{-1}x$ as the first function and 1 as second function,

we get $I = \left[\left(\sin^{-1} x\right)x\right]_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}dx$

Put $1 - x^2 = t \Rightarrow -2x\,dx = dt$

When, $x = 0$

$\Rightarrow t = 1$ and when $x = 1 \Rightarrow t = 0$

$\therefore\ I = \left[x\sin^{-1} x\right]_0^1 + \frac{1}{2}\int_1^0 \frac{dt}{\sqrt{t}}$

$= \left[x\sin^{-1} x\right]_0^1 + \frac{1}{2}\left[\frac{t^{1/2}}{1/2}\right]_1^0$

$= 1\sin^{-1}(1) + \left[-\sqrt{1}\right] = \frac{\pi}{2} - 1$

INTEGER TYPE QUESTIONS

33. (b) $\int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx = \int_0^1 \tan^{-1}\left[\frac{x+(x-1)}{1-x(x-1)}\right]dx$

$I = \int_0^1\left[\tan^{-1} x + \tan^{-1}(x-1)\right]dx$ ... (i)

let $I = \int_0^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$= \int_0^1[\tan^{-1} x + \tan^{-1}(x-1)]\,dx$

$= \int_0^1[\tan^{-1}(1-x) - \tan^{-1}(1-x-1)]\,dx$

$= \int_0^1[-\tan^{-1}(x-1) - \tan^{-1} x]\,dx$,

$I = -\int_0^1[\tan^{-1} x + \tan^{-1}(x-1)]\,dx$ ...(ii)

Adding (i) & (ii) $2I = 0$ or $I = 0$.

34. (a) $\int_0^{2\pi}\log\left(\frac{a+b\sec x}{a-b\sec x}\right)dx = 2\int_0^{\pi}\log\left(\frac{a+b\sec x}{a-b\sec x}\right)dx$

$= 2\int_0^{\pi}\log(a+b\sec x)dx - 2\int_0^{\pi}\log(a-b\sec(\pi-x))dx$

$= 2\int_0^{\pi}\log(a+b\sec x)\,dx - 2\int_0^{\pi}\log(a+b\sec x)\,dx = 0$

35. (b) We have $I = \int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$ ... (i)

$= \int_2^8 \frac{\sqrt{10-(10-x)}}{\sqrt{10-x}+\sqrt{10-(10-x)}}dx$

$\Rightarrow \quad I = \int_2^8 \frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}dx$ ... (ii)

Adding (i) and (ii), we get

$2I = \int_2^8 1dx = 8 - 2 = 6$

Hence I = 3

36. (b) $I = \int_{-1}^1 (x-[x])dx = \int_{-1}^1 x dx - \int_{-1}^1 [x]dx$

$= \left[\frac{x^2}{2}\right]_{-1}^1 - \left[\int_{-1}^0 [x]dx + \int_0^1 [x]dx\right]$

$= \frac{1}{2}[1-1] - \left[\int_{-1}^0 (-1)dx + \int_0^1 0.dx\right]$

$\left[\begin{array}{l}\text{If } -1 \le x < 0, [x] = -1\\ \text{If } 0 \le x < 1, [x] = 0\end{array}\right]$

$= 0 - [-x]_{-1}^0 - 0 = 0 - [-0-(-1)] = 1$

37. **(a)** $I=\int_0^{\frac{\pi}{2}}\log(\tan x)\,dx=\int_0^{\frac{\pi}{2}}\log\left\{\tan\left(\frac{\pi}{2}-x\right)\right\}dx$

$=\int_0^{\frac{\pi}{2}}\log(\cot x)\,dx$

$\therefore\ 2I=\int_0^{\frac{\pi}{2}}\log(\tan x)\,dx+\int_0^{\frac{\pi}{2}}\log(\cot x)\,dx$

$=\int_0^{\frac{\pi}{2}}[\log\tan x+\log\cot x]\,dx$

$=\int_0^{\frac{\pi}{2}}\log(\tan x\,.\cot x)\,dx$

$\int_0^{\frac{\pi}{2}}\log(1)\,dx=\int_0^{\frac{\pi}{2}}0\,dx=0\quad\therefore I=0$

38. **(c)** Use $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$\int_0^{\pi}\frac{\sin 2mx}{\sin x}dx=\int_0^{\pi}\frac{\sin(2m\pi-2mx)}{\sin(\pi-x)}dx$

$=\int_0^{\pi}\frac{-\sin 2mx}{\sin x}dx=-I\Rightarrow 2I=0\Rightarrow I=0$

39. **(c)** Let $I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ ... (i)

$\Rightarrow\ I=\int_0^{\frac{\pi}{2}}\log\left(\frac{4+3\sin(\pi/2-x)}{4+3\cos(\pi/2-x)}\right)dx$

$\left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$\Rightarrow\ I=\int_0^{\pi/2}\log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$ ... (ii)

$\left[\because\sin\left(\frac{\pi}{2}-x\right)=\cos x\text{ and }\cos\left(\frac{\pi}{2}-x\right)=\sin x\right]$

On adding eqs. (i) and (ii), we get

$2I=\int_0^{\pi/2}\left[\log\left(\frac{4+3\sin x}{4+3\cos x}\right)+\log\left(\frac{4+3\cos x}{4+3\sin x}\right)\right]dx$

$\Rightarrow\ 2I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\times\frac{4+3\cos x}{4+3\sin x}\right)dx$

$[\because\log m+\log n=\log mn]$

$\Rightarrow\ 2I=\int_0^{\pi/2}\log 1\,dx$

$\Rightarrow\ 2I=\int_0^{\pi/2}0\,dx\ \ (\because\log 1=0)$

$\Rightarrow\ I=0$

40. **(b)** Let $I=\int_0^1\tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$

$=\int_0^1\tan^{-1}\left\{\frac{x+(x-1)}{1-x(x-1)}\right\}dx$

$=\int_0^1\left\{\tan^{-1}x+\tan^{-1}(x-1)\right\}dx$

$\left[\because\tan^{-1}A+\tan^{-1}B=\tan^{-1}\left(\frac{A+B}{1-AB}\right)\right]$

$\Rightarrow\ I=\int_0^1\left\{\tan^{-1}x-\tan^{-1}(1-x)\right\}dx$ ...(i)

Also, $I=\int_0^1\left\{\tan^{-1}(1-x)-\tan^{-1}(1-(1-x))\right\}dx$

$\left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$

$\Rightarrow\ I=\int_0^1\left[\tan^{-1}(1-x)-\tan^{-1}(x)\right]dx$ ...(ii)

On adding eqs. (i) and (ii), we get

$2I=0\Rightarrow I=0$

41. **(d)** Let $I=\int\cos^n x\sin x\,dx$

Put cos x = t

– sin x dx = dt

$\therefore\quad I=-\int t^n dt=-\frac{t^{n+1}}{n+1}+C$

$=-\frac{\cos^{n+1}x}{n+1}+C=-\frac{\cos^6 x}{6}+C$

$\therefore$ n + 1 = 6 or n = 5

42. **(d)** We have,

$\frac{3x+1}{(x-3)(x-5)}=\frac{-5}{x-3}+\frac{B}{x-5}$

$3x+1=-5(x-5)+B(x-3)$

Put x = 5

$3(5)+1=B(5-3)$

16 = 2B or B = 8

43. **(a)** $\because\ \int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{a}+c$

But it is given that

$\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\frac{x}{3}+c$

$\therefore$ a = 3

44. **(b)** $I = \int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$

Put $e^x + e^{-x} = t$

$(e^x + e^{-x})dx = dt$

$\therefore \quad I = \int \frac{dt}{t} = \log t + C$

$= \log(e^x - e^{-x}) + C$

$= \log\left(e^x - \frac{1}{e^x}\right) + C$

$= \log\left(\frac{e^{2x} - 1}{e^x}\right) + C$

$= \log(e^{2x} - 1) - \log e^x + C$

$= \log(e^{2x} - 1) - x + C$

$\therefore A = 1$

45. **(b)** If f (x) is an even function then

$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

Here $f(x) = x^8 - x^4 + x^2 + 1$ is an even function, therefore a = 4.

ASSERTION - REASON TYPE QUESTIONS

46. **(a)** $I = \int_0^{\frac{\pi}{2}} 2\sqrt{\tan x}\, dx$, Put $\tan x = t^2 \Rightarrow dx = \frac{2t\,dt}{1+t^4}$

If $x = 0 \Rightarrow t = 0$ and $x = \frac{\pi}{2} \Rightarrow t = \infty$

$I = \int_0^{\infty} \frac{2t^2 dt}{1+t^4} = \int_0^{\infty} \frac{t^2 + 1 + t^2 - 1}{1+t^4} dt$

$= \int_0^{\infty} \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2}} dt + \int_0^{\infty} \frac{1 - \frac{1}{t^2}}{t^2 + \frac{1}{t^2}} dt$

$= \int_0^{\infty} \frac{d\left(t - \frac{1}{t}\right)}{\left(t - \frac{1}{t}\right)^2 + 2} + \int_0^{\infty} \frac{d\left(t + \frac{1}{t}\right)}{\left(t + \frac{1}{t}\right)^2 - 2} dt$

$= \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{t - \frac{1}{t}}{\sqrt{2}}\right)\Bigg|_0^{\infty} + \frac{1}{2\sqrt{2}} \ln\left(\frac{t + \frac{1}{t} - \sqrt{2}}{t + \frac{1}{t} + \sqrt{2}}\right)_0^{\infty}$

$= \frac{\pi}{\sqrt{2}}$

47. **(a)** $\int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right) dx = 0$

$f(x) = \log\left(\frac{1+x}{1-x}\right)$

$f(-x) = \log\left(\frac{1-x}{1+x}\right) = -\log\left(\frac{1+x}{1-x}\right) = -f(x)$

f is an odd function $\Rightarrow \int_{-a}^{a} f(x)dx = 0$

Both are true and Reason is correct explanation of Assertion.

48. **(a)**

Derivatives	**Integrals (Anti-derivatives)**
(i) $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$ Particularly, we note that $\frac{d}{dx} x = 1$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C; n \neq -1$ $\int dx = x + C$

49. **(a)** Sometimes, function is not expressible in terms of elementary functions viz., polynomial, logarithmic, exponential, trigonometric functions and their inverses etc. We are therefore blocked for finding $\int f(x)dx$.

Therefore, it is not possible to find $\int e^{-x^2} dx$ by inspection since, we can not find a function whose derivative is e^{-x^2}.

50. **(d)** The process of differentiation and integration are inverses of each other in sense of the following results.

$\frac{d}{dx}\int f(x)dx = f(x)$ and $\int f'(x)dx = f(x) + C$,

where C is any arbitrary constant

Let F be any anti-derivative of f, i.e.,

$\frac{d}{dx} F(x) = f(x)$

Then $\int f(x)dx = F(x) + C$

Therefore, $\frac{d}{dx}\int f(x)dx = \frac{d}{dx}(F(x) + C) = \frac{d}{dx}F(x) = f(x)$

Similarly, we know that

$f'(x) = \frac{d}{dx} f(x)$

and hence $\int f'(x)dx = f(x) + C$

where, C is arbitrary constant called constant of integration.

51. **(b)** The derivative of a function has a geometrical meaning, namely, the slope of the tangent to the corresponding curve at a point. Similarly, the indefinite integral to a function represents geometrically, a family of curves placed parallel to each other having parallel tangents at the points of intersection of the curves of the family with the lines orthogonal (perpendicular) to the axis representing the variable of integration.

52. **(c)** We can speak of the derivative at a point. We never speak of the integral at a point, we speak of the integral of a function over an interval on which the integral is defined.

53. (a) Here, $I = \int [\sin(\log x) + \cos(\log x)]dx$ ... (i)

By using inspection method,

$$\frac{d}{dx}\{x\sin(\log x)\} = x\frac{d}{dx}\sin(\log x) + \sin(\log x)\frac{d}{dx}(x)$$

$$= x\cos(\log x) \times \frac{1}{x} + \sin(\log x)$$

$$= \cos(\log x) + \sin(\log x) \quad ...(ii)$$

From eqs. (i) and (ii), we get

$$I = \int \frac{d}{dx}\{x\sin(\log x)\}dx$$

$$= x\sin(\log x) + C$$

54. (a) The value of definite integral of a function over any particular interval depends on the function and the interval, but not on the variable of integration that we choose to represent the independent variable. If the independent variable is denoted by t or u instead of x, we simply write the integral as $\int_a^b f(t)dt$ or $\int_a^b f(u)du$ instead of $\int_a^b f(x)dx$.

Hence the variable of integration is called a dummy variable.

55. (c) $\int_a^b x f(x)\, dx = \int_a^b (a+b-x) f(a+b-x)\, dx$

$$= (a+b)\int_a^b f(a+b-x)\, dx - \int_a^b xf(a+b-x)\, dx$$

$\therefore$ Reason is true only when $f(a+b-x) = f(x)$ which holds in Assertion.

$\therefore$ Reason is false and Assertion is true.

56. (a) Here $f'(x) = \tan x$.

CRITICALTHINKING TYPE QUESTIONS

57. (d) Let $I = \int_a^b x f(x)dx$

Let $a + b - x = z \Rightarrow -dx = dz$

When $x = a, z = b$ and when $x = b, z = a$

$$\therefore \quad I = -\int_b^a (a+b-z)f(z)dz$$

$$I = (a+b)\int_b^a f(x)dx - \int_a^b x f(x)dx$$

$$I = (a+b)\int_b^a f(x)dx - I\,; \quad 2I = (a+b)\int_a^b f(x)dx$$

Hence, $I = \left(\frac{a+b}{2}\right)\int_a^b f(x)dx$

58. (c) $I = \int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x}dx$ ... (i)

Put $x = -y$ then $dx = -dy$

$$I = \int_{\pi}^{-\pi} \frac{\cos^2 y}{1+a^{-y}}dy = \int_{-\pi}^{\pi} \frac{a^y\cos^2 y}{1+a^y}dy$$

$$I = \int_{-\pi}^{\pi} \frac{a^x\cos^2 x}{1+a^x}dx \quad ... (ii)$$

$$\left[\because \int_a^b f(y)dy = \int_a^b f(x)dx\right]$$

Adding (i) and (ii),

$$2I = \int_{-\pi}^{\pi} \frac{(1+a^x)\cos^2 x}{(1+a^x)}dx = \int_{-\pi}^{\pi} \cos^2 x\, dx$$

$$2I = 2\int_0^{\pi} \cos^2 x\, dx \qquad \text{(even function)}$$

$$I = 2\int_0^{\pi/2} \cos^2 x\, dx \quad ... (iii)$$

$$[\because \int_0^{2a} f(x)dx = 2\int_0^a f(x)dx \text{ if } f(2a-x) = f(x)]$$

$$= 2\int_0^{\pi/2} \sin^2 x dx \quad ... (iv)$$

Adding (iii) and (iv)

$$2I = 2\int_0^{\pi/2} (\cos^2 x + \sin^2 x)dx = 2.\pi/2 = \pi$$

$\therefore I = \pi/2$

59. (b) Let $I = \int \sin^3 x\cos^3 x\, dx$. Then,

$$I = \frac{1}{8}\int (2\sin x\cos x)^3\, dx$$

$$\Rightarrow \quad I = \frac{1}{8}\int \sin^3 2x\, dx \Rightarrow I = \frac{1}{8}\int \frac{3\sin 2x - \sin 6x}{4}dx$$

$$\Rightarrow \quad I = \frac{1}{32}\int (3\sin 2x - \sin 6x)dx$$

$$= \frac{1}{32}\left\{-\frac{3}{2}\cos 2x + \frac{1}{6}\cos 6x\right\} + C$$

60. (b) Let $I = \int \frac{1}{\sqrt{\sin^3 x\cos^5 x}}dx$

$$\Rightarrow \quad I = \int \frac{1}{\sin^{3/2} x\cos^{5/2} x}dx \Rightarrow I = \int \frac{\sec^4 x}{\tan^{3/2} x}dx$$

[Dividing numerator and denominator by $\cos^4 x$]

$$\Rightarrow \quad I = \int \frac{(1+\tan^2 x)}{\tan^{3/2} x}\sec^2 x\, dx$$

Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$$I = \int \frac{1+t^2}{t^{3/2}}dt$$

$$\Rightarrow \quad I = \int (t^{-3/2} + t^{1/2})dt = \frac{-2}{\sqrt{t}} + \frac{t^{3/2}}{3/2} + C$$

$$= -\frac{2}{\sqrt{\tan x}} + \frac{2}{3}(\tan x)^{3/2} + C$$

61. **(c)** Let $I=\int \frac{1}{\sqrt{9+8x-x^2}}dx$. Then,

$$I=\int \frac{1}{\sqrt{-\{x^2-8x-9\}}}dx$$

$$I=\int \frac{1}{\sqrt{-\{x^2-8x+16-25\}}}dx$$

$$\Rightarrow I=\int \frac{1}{\sqrt{-\{(x-4)^2-5^2\}}}dx=\int \frac{1}{\sqrt{5^2-(x-4)^2}}dx$$

$$=\sin^{-1}\left(\frac{x-4}{5}\right)+C$$

62. **(c)** $I=\int \frac{1}{1+3\sin^2 x+8\cos^2 x}dx$

Dividing the numerator and denominator by $\cos^2 x$, we get

$$I=\int \frac{\sec^2 x}{\sec^2 x+3\tan^2 x+8}dx$$

$$\Rightarrow I=\int \frac{\sec^2 x}{1+\tan^2 x+3\tan^2 x+8}dx=\int \frac{\sec^2 x}{4\tan^2 x+9}dx$$

Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$, we get

$$I=\int \frac{dt}{4t^2+9}=\frac{1}{4}\int \frac{dt}{t^2+(3/2)^2}=\frac{1}{4}\times\frac{1}{3/2}\tan^{-1}\left(\frac{t}{3/2}\right)+C$$

$$\Rightarrow I=\frac{1}{6}\tan^{-1}\left(\frac{2t}{3}\right)+C=\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$$

63. **(c)** Let $I=\int \frac{x^3+x}{x^4-9}dx$. Then,

$$I=\int \frac{x^3}{x^4-9}dx+\int \frac{x}{x^4-9}dx = I_1+I_2+C(\text{say}), \text{ where}$$

$$I_1=\int \frac{x^3}{x^4-9}dx \text{ and } I_2=\int \frac{x}{x^4-9}dx$$

Putting $x^4-9=t$ in $I_1 \Rightarrow 4x^3dx=dt$, we get

$$I_1=\frac{1}{4}\int \frac{1}{t}dt=\frac{1}{4}\log|t|=\frac{1}{4}\log\left|x^4-9\right|$$

$$I_2=\int \frac{x}{x^4-9}dx=\int \frac{x}{\left(x^2\right)^2-3^2}dx$$

Putting $x^2=t \Rightarrow 2x\,dx=dt$, we get

$$I_2=\frac{1}{2}\int \frac{dt}{t^2-3^2}=\frac{1}{2}.\frac{1}{2\times 3}\log\left|\frac{t-3}{t+3}\right|=\frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|$$

Hence, $I=\frac{1}{4}\log\left|x^4-9\right|+\frac{1}{12}\log\left|\frac{x^2-3}{x^2+3}\right|+C$

64. **(d)** $\frac{3x+4}{x^3-2x-4}=\frac{3x+4}{(x-2)\left(x^2+2x+2\right)}$

$$=\frac{A}{x-2}+\frac{Bx+C}{x^2+2x+2}$$

$\Rightarrow 3x+4=A(x^2+2x+2)+(Bx+C)(x-2)$

$\therefore A+B=0$

$2A-2B+C=3$

$2A-2C=4$

$\Rightarrow A=1, B=C=-1$

$$\therefore \int \frac{3x+4}{x^3-2x-4}dx=\int \frac{dx}{x-2}-\frac{1}{2}\int \frac{2x+2}{x^2+2x+2}dx$$

$$=\log_e|x-2|-\frac{1}{2}\log\left|x^2+2x+2\right|+C$$

$\Rightarrow k=-\frac{1}{2}$ and $f(x)=|x^2+2x+2|$

65. **(a)** Let $I=\int \frac{1-\cos x}{\cos x(1+\cos x)}dx$

Let $\cos x = y \Rightarrow \frac{1-\cos x}{\cos x(1+\cos x)}=\frac{1-y}{y(1+y)}$

Now $\frac{1-y}{y(1+y)}=\frac{A}{y}+\frac{B}{1+y}$ ...(i)

$\Rightarrow 1-y=A(1+y)+By$

Put $y=0$ in (i), we get $A=1$.

Put $y=-1$ in (i), we get $B=-2$ ...(ii)

Substituting the values of A and B in (i), we obtain

$$\frac{1-y}{y(1+y)}=\frac{1}{y}-\frac{2}{1+y}$$

$$\Rightarrow \frac{1-\cos x}{\cos x(1+\cos x)}=\frac{1}{\cos x}-\frac{2}{1+\cos x} \quad [\because y=\cos x]$$

$$\therefore I=\int \frac{1-\cos x}{\cos x(1+\cos x)}dx=\int \frac{1}{\cos x}dx-\int \frac{2}{1+\cos x}dx$$

$$\Rightarrow I=\int \sec x\,dx-\int \frac{1}{\cos^2(x/2)}dx$$

$$=\int \sec x\,dx-\int \sec^2(x/2)dx$$

$$\Rightarrow I=\log|\sec x+\tan x|-2\tan(x/2)+C$$

66. **(a)** We have, $I=\int_0^{\pi} \frac{1}{5+4\cos x}dx$

$$=\int_0^{\pi} \frac{1}{5+4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$=\int_0^{\pi} \frac{1+\tan^2\frac{x}{2}}{5\left(1+\tan^2\frac{x}{2}\right)+4\left(1-\tan^2\frac{x}{2}\right)}dx$$

$$=\int_0^{\pi} \frac{1+\tan^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx=\int_0^{\pi} \frac{\sec^2\frac{x}{2}}{9+\tan^2\frac{x}{2}}dx$$

Let $\tan\frac{x}{2}=t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx=dt$

Also, $x=0 \Rightarrow t=0$ and $x=\pi \Rightarrow t=\infty$

$$\therefore I=\int_0^{\infty} \frac{dt}{9+t^2}$$

$\therefore \quad I = 2\int_0^{\infty} \frac{dt}{3^2 + t^2}$

$\therefore \quad I = \frac{2}{3}\left[\tan^{-1}\frac{t}{3}\right]_0^{\infty} = \frac{2}{3}\left[\tan^{-1}\infty - \tan^{-1}0\right]$

$= \frac{2}{3}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{3}$

67. (c) Given, $\int_0^{\pi} \ell n \sin x\, dx = k$

$\therefore \quad k = 2\int_0^{\pi/2} \ell n \sin x\, dx = 2\left(-\frac{\pi}{2}\ell n 2\right)$

$\therefore \quad k = \pi\, \ell n\, 2$ … (i)

Then, $\int_0^{\pi/4} \ell n(1 + \tan x)dx = \frac{\pi}{8}\ell n 2$

$= -\frac{k}{8}$ [From eq. (i)]

68. (a) We have, $I = \int 1.\tan^{-1}\sqrt{x}\, dx$

Using by parts,

$I = \tan^{-1}\sqrt{x}.(x) - \int \frac{1}{1+x} \times \frac{1}{2\sqrt{x}} \times x\, dx$

$= x\tan^{-1}\sqrt{x}. - \int \frac{x}{(1+x)2\sqrt{x}}dx$

$= x\tan^{-1}\sqrt{x} - \int\left(\frac{1+x}{(1+x)2\sqrt{x}} - \frac{1}{(1+x)2\sqrt{x}}\right)dx$

$= x\tan^{-1}\sqrt{x} - \int\frac{dx}{2\sqrt{x}} + \int\frac{dx}{2\sqrt{x}(1+x)}$

$= x\tan^{-1}\sqrt{x} - \sqrt{x} + \tan^{-1}\sqrt{x} + C$

$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$

69. (b) $I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x\cos^2 x}dx$

$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cos^4 x)}{1 - 2\sin^2 x\cos^2 x}dx$

$= \int \frac{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x)}{1 - 2\sin^2 x\cos^2 x}dx$

$= \int \frac{1.(\sin^2 x - \cos^2 x)[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x]}{1 - 2\sin^2 x\cos^2 x}$

$= \int \frac{(\sin^2 x - \cos^2 x)(1 - 2\sin^2 x\cos^2 x)}{1 - 2\sin^2 x\cos^2 x}dx$

$= -\int \cos 2x\, dx = -\frac{1}{2}\sin 2x + c$

70. (b) $\int \frac{\sin x}{\sin(x-\alpha)}dx = \int \frac{\sin(x - \alpha + \alpha)}{\sin(x-\alpha)}dx$

$= \int \frac{\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)}dx$

$= \int\{\cos\alpha + \sin\alpha\cot(x-\alpha)\}dx$

$= (\cos\alpha)x + (\sin\alpha)\log\sin(x-\alpha) + C$

$\therefore$ A = cosα, B = sin α

71. (d) $I = \int_0^2 f(x)dx = \int_0^1 f(x)dx - \int_1^2 f(x)dx = \int_0^1 x^2 dx + \int_1^2 \sqrt{x}dx$

$= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^{3/2}}{3/2}\right]_1^2 = \left[\frac{1}{3} - 0\right] + \left[2^{3/2} - 1\right]\frac{2}{3}$

$= \frac{1}{3} + \frac{2}{3}.2\sqrt{2} - \frac{2}{3} = \frac{1}{3}\left(4\sqrt{2} - 1\right)$

72. (a) We have $g(x) = \int_0^x \cos^4 t\, dt$

$\therefore g(x+\pi) = \int_0^{x+\pi} \cos^4 t\, dt = \int_0^{\pi} \cos^4 t\, dt + \int_{\pi}^{x+\pi} \cos^4 t\, dt$

$= g(\pi) + \int_0^x \cos^4 t\, dt$ $\left[\because \cos^4 t \text{ is periodic with period } \pi\right]$

$= g(\pi) + g(x)$

73. (b) We have $\cos x \geq \sin x$ for $0 \leq x \leq \frac{\pi}{4}$

and $\sin x \geq \cos x$ for $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$

$\therefore \int_0^{\frac{\pi}{2}} |\sin x - \cos x|\, dx$

$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)dx$

$= [\sin x + \cos x]_0^{\frac{\pi}{4}} + [-\cos x - \sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 0 - 1\right] - \left[0 + 1 - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right]$

$= \sqrt{2} - 1 - 1 + \sqrt{2} = 2\sqrt{2} - 2$

74. (c) Use $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Here, $a + b = \pi$

$\therefore \quad I = \int_{\pi/4}^{3\pi/4} \frac{\phi d\phi}{1+\sin\phi} = \int_{\pi/4}^{3\pi/4} \frac{(\pi - \phi)d\phi}{1 + \sin(\pi - \phi)}$

$= \int_{\pi/4}^{3\pi/4} \frac{\pi d\phi}{1+\sin\phi} - I$

$\Rightarrow \quad 2I = \pi\int_{\pi/4}^{3\pi/4} \frac{1 - \sin\phi}{\cos^2\phi}d\phi$

$= \pi\int_{\pi/4}^{3\pi/4} (\sec^2\phi - \sec\phi\tan\phi)d\phi$

$= \pi[\tan\phi - \sec\phi]_{\pi/4}^{3\pi/4}$

$I = \frac{\pi}{2}(2\sqrt{2} - 2) = \pi(\sqrt{2} - 1) = \frac{\pi}{\sqrt{2} + 1}$

75. (b) $\int_{-\pi}^{\pi}\frac{2x\,(1+\sin x)}{1+\cos^2 x}dx=\int_{-\pi}^{\pi}\frac{2x\,dx}{1+\cos^2 x}+2\int_{-\pi}^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x}$

$$=0+4\int_0^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x};$$

$$I=4\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$$

$$I=4\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2 x}dx$$

$$\Rightarrow I=4\pi\int_0^{\pi}\frac{\sin x\,dx}{1+\cos^2 x}-4\int_0^{\pi}\frac{x\sin x\,dx}{1+\cos^2 x};$$

$$\Rightarrow 2I=4\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$$

put cos x = t and solve it.

76. (d) We have, $I=\int\frac{1}{(\sin x+4)(\sin x-1)}dx$

$$=\frac{1}{5}\int\frac{(\sin x+4)-(\sin x-1)}{(\sin x+4)(\sin x-1)}dx$$

$$=\frac{1}{5}\int\frac{1}{\sin x-1}dx-\frac{1}{5}\int\frac{1}{\sin x+4}dx$$

$$=\frac{1}{5}\int\frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}-1-\tan^2\frac{x}{2}}dx$$

$$-\frac{1}{5}\int\frac{\sec^2\frac{x}{2}}{2\tan\frac{x}{2}+4+4\tan^2\frac{x}{2}}dx$$

Put, $\tan\frac{x}{2}=t$

$$\Rightarrow \sec^2\frac{x}{2}dx=2\,dt$$

$$\therefore I=\frac{1}{5}\int\frac{2dt}{2t-1-t^2}-\frac{1}{5}\int\frac{2dt}{\left[2t+4\left(1+t^2\right)\right]}$$

$$\therefore I=-\frac{2}{5}\int\frac{dt}{t^2-2t+1}-\frac{1}{10}\int\frac{dt}{t^2+\frac{1}{2}t+1}$$

$$=-\frac{2}{5}\int\frac{1}{(t-1)^2}dt-\frac{1}{10}\int\frac{dt}{\left(t+\frac{1}{4}\right)^2+\left(\frac{\sqrt{15}}{4}\right)^2}$$

$$=\frac{2}{5}\cdot\frac{1}{t-1}-\frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4t+1}{\sqrt{15}}\right)+C$$

$$=\frac{2}{5}\frac{1}{\tan\frac{x}{2}-1}-\frac{2}{5\sqrt{15}}\tan^{-1}\left(\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}\right)+C \quad ...(i)$$

But, given that

$$I=A\frac{1}{\left(\tan\frac{x}{2}-1\right)}+B\tan^{-1}\left[f(x)\right]+C \quad ...(ii)$$

From eqs. (i) and (ii), we get

$$A=\frac{2}{5},\ B=\frac{-2}{5\sqrt{15}},\ f(x)=\frac{4\tan\frac{x}{2}+1}{\sqrt{15}}$$

77. (b) We have, f(x) = f(a – x) and g(x) + g(a – x) = 4

Let $I=\int_0^a f(x)g(x)dx$ … (i)

$$\Rightarrow I=\int_0^a f(a-x)g(a-x)dx$$

$$\left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$\Rightarrow I=\int_0^a f(x)\{4-g(x)\}dx \quad \text{… (ii)}$$

[∵ f(x) = f(a – x) and g(x) + g(a – x) = 4 (given)]

On adding eqs. (i) and (ii), we get

$$2I=\int_0^a 4f(x)dx \Rightarrow I=2\int_0^a f(x)dx$$

78. (a) Let $x=a\sin^2\theta$

then $dx=2a\sin\theta\cos\theta\,d\theta$

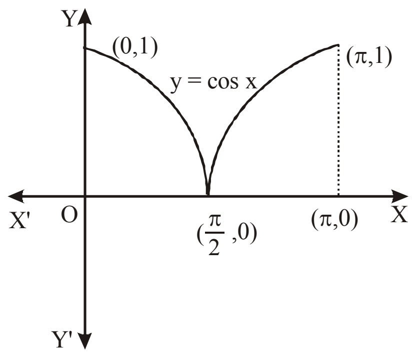
$$\therefore \quad I=\int\frac{2a\sin\theta.\cos\theta}{\sqrt{a\sin^2\theta.a\cos^2\theta}}d\theta$$

$$=2\int d\theta=2\theta+c$$

$$=2\sin^{-1}\left(\sqrt{x/a}\right)+c$$

79. (a) We have

$$|\cos x|=\begin{cases}\cos x & \text{when } 0\le x\le\frac{\pi}{2}\\ -\cos x & \text{when } \frac{\pi}{2}\le x\le\pi\end{cases}$$



$$\therefore \int_0^{\pi}|\cos x|dx=\int_0^{\pi/2}|\cos x|dx+\int_{\pi/2}^{\pi}|\cos x|dx$$

$$=\int_0^{\pi/2}\cos x\,dx+\int_{\pi/2}^{\pi}(-\cos x)dx$$

$$=[\sin x]_0^{\pi/2}-[\sin x]_{\pi/2}^{\pi}=1+1=2$$

80. **(c)** Let $I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ … (i)

Then, $I=\int_0^{\pi/2}\frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}}dx$

$\Rightarrow\ I=\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ … (ii)

Adding (i) and (ii), we get

$2I=\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx+\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

$=\int_0^{\pi/2}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx=\int_0^{\pi/2}1.dx=[x]_0^{\pi/2}=\frac{\pi}{2}-0$

$\Rightarrow\ I=\frac{\pi}{4}\Rightarrow\int_0^{\pi/2}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx=\frac{\pi}{4}$

81. **(b)** $\int\frac{(x-x^3)^{1/3}}{x^4}dx=\int\frac{1}{x^3}\left(\frac{1}{x^2}-1\right)^{1/3}dx$

$=\frac{-1}{2}\int t^{1/3}\,dt\quad\left[\text{Putting}\ \frac{1}{x^2}-1=t\Rightarrow\frac{-2}{x^3}dx=dt\right]$

$=\frac{-1}{2}.\frac{t^{4/3}}{4/3}+C=\frac{-3}{8}\left(\frac{1}{x^2}-1\right)^{4/3}+C$

82. **(a)** Here since the highest powers of x in numerator and denominator are equal and coefficients of x^2 are also equal, therefore

$\int\frac{x^2+1}{(x-1)(x-2)}\equiv 1+\frac{A}{x-1}+\frac{B}{x-2}$

On solving we get A = – 2, B = 5

Thus $\int\frac{x^2+1}{(x-1)(x-2)}\equiv 1-\frac{2}{x-1}+\frac{5}{x-2}$

The above method is used to obtain the value of constant corresponding to non-repeated linear factor in the denominator.

Now, $I=\int\left(1-\frac{2}{x-1}+\frac{5}{x-2}\right)dx$

$=x-2\log(x-1)+5\log(x-2)+C$

$=x+\log\left[\frac{(x-2)^5}{(x-1)^2}\right]+C$

83. **(b)** Put $x-\alpha=t^2\Rightarrow dx=2t\,dt$

$\therefore\quad I=2\int\frac{t\,dt}{\sqrt{t^2(\beta-\alpha-t^2)}}$

$=2\int\frac{dt}{\sqrt{(\beta-\alpha)-t^2}}=2\sin^{-1}\frac{t}{\sqrt{\beta-\alpha}}+C$

$=2\sin^{-1}\sqrt{\frac{x-\alpha}{\beta-\alpha}}+C$

84. **(b)** After dividing by $\cos^2 x$ to numerator and denominator of integration

$I=\int\frac{\sec^2 x\,dx}{4\tan^2 x+4\tan x+5}$

$=\int\frac{\sec^2 x\,dx}{(2\tan x+1)^2+4}$

$=\frac{1}{22}\tan^{-1}\left(\frac{2\tan x+1}{2}\right)+C$

85. **(b)** $I=\int\frac{1+1/x^2}{x^2+1+1/x^2}dx=\int\frac{d\,(x-1/x)}{(x-1/x)^2+3}$

$=\frac{1}{\sqrt{3}}\tan^{-1}\frac{x-1/x}{\sqrt{3}}+C$

$=\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{(x^2-1)}{\sqrt{3}x}\right)+C$

86. **(c)** $I=\int_0^1\log\left(\frac{1-x}{x}\right)dx$ ...(i)

$\Rightarrow I=\int_0^1\log\left[\frac{1-(1-x)}{1-x}\right]dx$

$=\int_0^1\log\left(\frac{x}{1-x}\right)dx=-\int_0^1\log\left(\frac{1-x}{x}\right)dx=I$

$\Rightarrow 2I=0\Rightarrow I=0$

87. **(b)** $I=\int_{\pi/6}^{\pi/3}\frac{1}{1+\sqrt{\cot x}}dx=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ … (i)

Then, $I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$

$\Rightarrow\ I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ … (ii)

Adding (i) and (ii), we get

$2I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$

$\Rightarrow\ 2I=\int_{\pi/6}^{\pi/3}1.dx=[x]_{\pi/6}^{\pi/3}=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}\Rightarrow I=\pi/12$

88. **(b)** $\because\ \frac{|x|}{x}=\begin{cases}-1 & \text{when } -1<x<0\\ 1 & \text{when } 0<x<2\end{cases}$

$\therefore I=\int_{-1}^{0}\frac{|x|}{x}dx+\int_0^2\frac{|x|}{x}dx=\int_{-1}^{0}(-1)\,dx+\int_0^2 1\,dx$

$=-[x]_{-1}^{0}+[x]_0^2=-1+2=1$