Chapter -2

Rational Numbers

Radha asked her friends, “Can you divide the difference of two numbers into three parts?”

Hamid, “Why not? If the numbers are 10 and 9, then 10-9=1 and each of the three equal parts of 1 is $\frac{1}{3}$.”

Suresh, “In Chapter on Fractions we have learnt to divide 1 into three equal parts” But 9-10=-1, how can we divide this in three equal parts?

All of them were trying to divide -1 into 3 equal parts.

Radha suggested, “Look at the number line, 1/3 is on the right side of 0 and we get 1/3 upon dividing 1 into 3 equal parts. Similarly, on the left side of O we can get $-\frac{1}{3}$ upon dividing 1 into 3 equal parts.

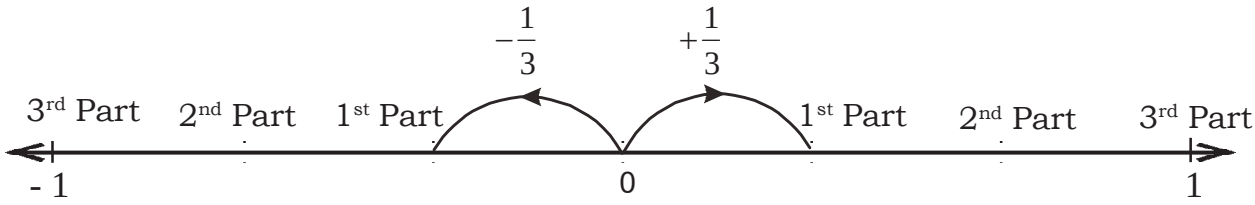

Figure 2.1

Suresh did not understand what Radha had said. He asked, “When we take 2 parts from 3 equal parts, then we get 2/3 but how will we depict $\frac{-2}{3}$?

Hamid, “In the previous class we had read that we can have 5 flowers, 5 goats, 5 leaves and 5 spectacles. There can be any 5 objects. This means that the counting number is not related to any special object. This is an idea which helps us in counting”.

Radha said, “Yes! You are right. Positive numbers are used for counting objects but we never used negative numbers for counting. For example, 2,3,5 etc are used for counting but -2, -3, -5 etc are not used for counting”.

In the previous class we had also learnt that $\frac{2}{3}, \frac{5}{6}, \frac{7}{9}$..... etc can be shown by taking 2 parts from 3 equal parts or 5 parts from 6 equal parts or 7 parts from 9 equal parts of a rectangle or a circle.

But no negative fraction can be depicted like that.

All the students started thinking of negative fractions along with positive fractions. They did not know how to look at negative fractions in comparison to positive fractions? Are these some different kind of numbers?

They put the problem before their mathematics teacher.

The teacher told them, "We learnt about natural numbers first then we added 'zero' to make whole numbers. Then we thought about fractions and then we learnt negative numbers. All these numbers together with negative fractions are called Rational Numbers. Hence $-\frac{3}{4}, -\frac{1}{2}, 0, \frac{2}{8}, \frac{1}{2}, \frac{15}{7}, \frac{3}{1}$ etc are all rational numbers".

Write 10 examples of rational numbers.

Activity 1

In the following table pairs of integers are given, use one integer as the numerator and the other as the denominator and make rational numbers.

Table 1

S. No.	Integer	Numerat-or	Denomi-nator	Rational Number	Numerat-or	Denomi-nator	Rational Number
1	2 &3	2	3	$\frac{2}{3}$	3	2	$\frac{3}{2}$
2	-5& 7						
3	4& -8						
4	-7&-9						
5	1& 6						

We denote Natural Numbers by N, Whole Number by W, Integers by I. Similarly, Rational Numbers are denoted by Q. A rational number is written in the form of p/q.

What will happen if q = 0 ?

What will be the quotient if the number is divided by 0 ?
Any number divided by 0 gives a infinite quotient.

We cannot calculate the value of $\frac{p}{0}$ as it does not indicate any definite number.

Thus, **rational numbers can be expressed in the form $\frac{p}{q}$ where p, q are integers and q # 0.**

Converting Integers into Rational Numbers

Is every integer a rational number too? Examples of integers are 4, 8, 11, -3, -7 etc. Let us think how we can write 4 as a rational number.

We can write 4 in many forms.

$\frac{4}{1}, \frac{8}{2}, \frac{12}{3}$ ..... etc. Do you agree with this? In how many other ways can we write 4?

Similarly – 7 can be written as $\frac{-7}{1}, \frac{-14}{2}, \frac{-21}{3}$ etc. All the numbers are written in fractional form. Since the numerator and denominator are integers therefore these numbers are rational numbers.

Thus, integers can be written as Rational Numbers

Write the following integers as rational numbers.

-3 = ——— = ——— = ——— 13 = ——— = ——— = ———

10 = ——— = ——— = ——— 100 = ——— = ——— = ———

-11 = ——— = ——— = ——— 33 = ——— = ——— = ———

0 = ——— = ——— = ———

Here, for each integer we have provided 3 blank spaces. In how many more ways can you write each integer?

Here, we can write 0 as $\frac{0}{1}, \frac{0}{3}, \frac{0}{15}, \frac{0}{999}$ ..... etc. Can you think of an integer which can not be written as a rational number?

Think and do – What did you do to write an integer as many different rational numbers? Take any 5 integers and convert them into rational numbers.

Equivalent Rational Numbers

We can write each integer in the form of many rational numbers. The value of all these rational numbers is equal to that integer. We had learnt that we can also write a fraction in more than one form with the same value. Does this happen with rational numbers as well? Let us see.

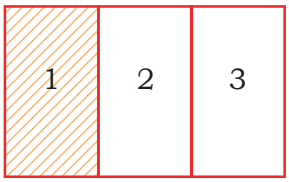

Figure 2.2

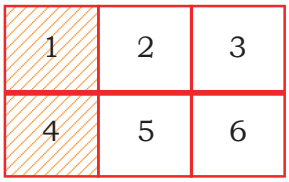

Figure 2.3

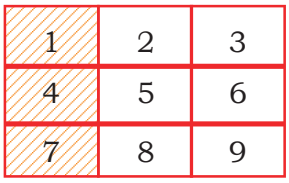

Figure 2.4

In fig 2.2 the coloured part shows $\frac{1}{3}$

If we divide fig. 2.2 into 2 parts then we have 2 equal coloured parts out of a total 6 equal parts. Fig. 2.2 this means that the coloured part is $\frac{2}{6}$, which is $\frac{1}{3}$ of the total part. Now if we divide fig. 2.2 into 3 equal parts. Then its 3 equal parts are coloured out of 9 equal parts or $\frac{3}{9}$ part is coloured which is $\frac{1}{3}$ of the total. (fig. 2.4)

In fact $\frac{2}{6}$ and $\frac{3}{9}$ are equal to $\frac{1}{3}$.

We have read about equivalent fractions in the 6th standard. If the fractional number is a negative number, we can change the form ofnumber by multiplying the numerator and denominator with the same number and its value remains the same e.g.

$-\frac{1}{2}$ Multiply numerator & denominator with 2, $-\frac{1\times2}{2\times2}=-\frac{2}{4}$

$-\frac{1}{2}$ Multiply numerator & denominator with 3, $-\frac{1\times3}{2\times3}=-\frac{3}{6}$

$-\frac{1}{2}$ Multiply numerator & denominator with 4, $-\frac{1\times4}{2\times4}=-\frac{4}{8}$

Here the form of the rational number is changed but the value $-\frac{1}{2}$ is the same.

Thus, $\frac{-2}{4}, \frac{-3}{6}$ and $\frac{-4}{8}$ are equivalent rational numbers.

Can you choose the groups of equivalent rational numbers from the following numbers?

$\frac{6}{18}, \frac{1}{3}, \frac{2}{5}, \frac{2}{6}, \frac{4}{10}, \frac{12}{30}, \frac{6}{15}$ and $\frac{4}{12}$:

Choose and fill up the blanks

$\frac{2}{5}$ = _____ = _____= _____=

$\frac{1}{3}$ = _____ = _____= _____=

Write Equivalent Rational Number in the table as directed.

Table 2

S. No.	Rational No.	Equivalent Rational Number
1.	$\frac{2}{5}$	$\frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}$ ------
2.	$\frac{-3}{7}$	
3.	$\frac{8}{-11}$	
4.	$\frac{-7}{-9}$	
5.	$-\frac{6}{15}$	

Rational numbers are converted into equivalent rational number by multiplying or dividing the numerator and denominator with the same number.

Simplest or lowest form of Rational Number

While filling above table Anu said that $-\frac{6}{15}$ is equivalent to the rational numbers $-\frac{2}{5}$ and $-\frac{4}{10}$. It means equivalent rational number can be obtained by dividing the numerator and the denominator by the same number.

Seema said that in $\frac{2}{5}$, we do not have a common factor. So we could not divide further. Therefore this is the simplest form. Thus the lowest form of the rational number is obtained by dividing numerator and denominator by the highest common factor. H.C.F. By the same method we can get the simplest from of a rational number.

Ramesh said that in $\frac{28}{35}$, Factors of 28 are 2,4 and 7 and those of 35 are 5 and 7. The highest common Factor is 7.

$\therefore$ We can divide the numerator and the denominator by 7 to get the simplest form of

$$\frac{28}{35} \cdot \frac{28}{35} = \frac{28 \div 7}{35 \div 7} = \frac{4}{5}$$

This is the simplest form of rational number. All the children said," This is correct. We can not simplify it further".

Anu and Seema's team suggested the method of simplest form. Do you have any other method for this? Using that method find the simplest form of $\frac{24}{36}$ and $\frac{98}{112}$.

Activity 2

Table 3

S. No.	Rational No.	Fac tors of numerators	Factors of denominator	Highest Common factor	$\frac{\text{Numerator} \div \text{H.C.F.}}{\text{Denominator} \div \text{H.C.F.}}$	Simplest form
1.	$\frac{45}{54}$	1,3,5,9,15, 45	1,2,3,6,9, 29,54	9	$\frac{45 \div 9}{54 \div 9}$	$\frac{5}{6}$
2.	$\frac{57}{76}$					
3.	$\frac{18}{36}$					
4.	$\frac{27}{81}$					
5.	$\frac{-63}{85}$					

Exercise 2.1

1. Find rational numbers from the following numbers

$\frac{4}{1}, \frac{-3}{7}, -27, \frac{24}{0}, \frac{-3}{-5}$

2. Convert the following numbers into rational numbers.

-38, 17, 0, -100, 79

3. Write 3 equivalent rational numbers for each of the following rational numbers.

$\frac{1}{5}, \frac{-3}{4}, \frac{-5}{8}, \frac{6}{11}$ and $\frac{4}{3}$

4. Write the lowest form of the following rational numbers..

$\frac{25}{40}, \frac{-16}{36}, \frac{-15}{-45}, \frac{-48}{96}$ and $\frac{-70}{100}$

5. Choose the equivalent rational numbers from the following rational numers.

(i) $\frac{4}{12}, \frac{8}{24}, \frac{1}{3}, \frac{16}{36}$ and $\frac{25}{75}$

(ii) $\frac{-3}{5}, \frac{-6}{10}, \frac{-15}{25}, \frac{-27}{45}$ and $\frac{-15}{20}$

6. Are the following pair of rational numbers equivalent? Give reasons.

(i) $\frac{9}{11}, \frac{9+3}{11+3}$ (ii) $\frac{5}{7}, \frac{5-2}{7-2}$

7. Express $\frac{-3}{8}$ into equivalent rational numbers, where

(i) numerator is – 6 (ii) numerator is 12

(iii) denominator is -24 (iv) denominators is -32

8. Find the value of 'a' if the following pairs are equivalent rational numbers

(i) $\frac{5}{11}$ *and* $\frac{a}{-33}$ (ii) $\frac{2}{3}$ and $\frac{8}{a}$

(iii) $\frac{3}{7}$ and $\frac{a}{35}$ (iv) $\frac{a}{5}$ and $\frac{18}{30}$

(v) $\frac{-a}{13}$ and $\frac{-24}{39}$

Rational numbers on the number line

You know how to represent integers on the number line. You also know that positive integers are shown on the right side of zero and negative integers on the left side of zero. +2 and -5 as shown in the following figure.

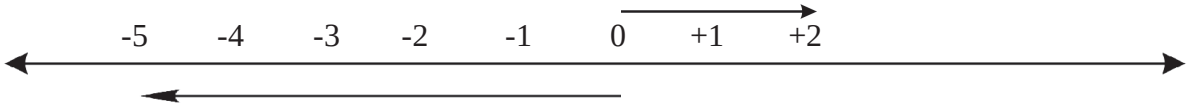

Figure 2.5

Rational number $\frac{2}{1}$ is at the place of +2. All the rational numbers, who have denominator as one, are located at the place of the integers.

Now you can show $\frac{-3}{1}$ and $\frac{+6}{1}$ on the number line. If the denominator is other than 1, the number is shown either on the right side of 0 or on the left side of 0. We divide the number line into as many parts as the denominator e.g. $\frac{1}{4}$ is shown in figure 2.6.

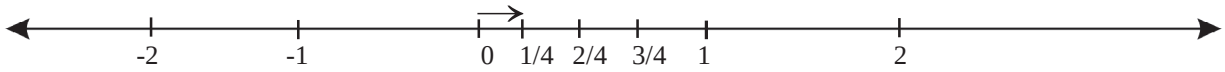

Figure 2.6

Similarly $\frac{-5}{7}$ will be shown on number line like this.

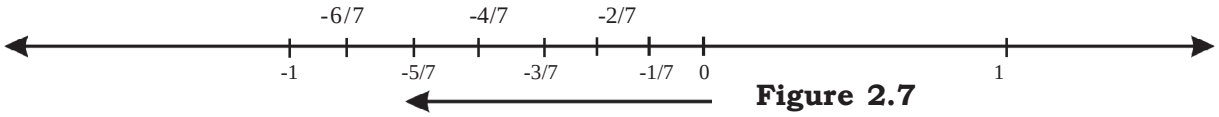

Figure 2.7

$\frac{-12}{7}$ is a mixed fraction. We can write this in the forms like

$\frac{-12}{7} = -(1+\frac{5}{7})$ or $-1\frac{5}{7}$ or $-1-\frac{5}{7}$ it will be shown on number line as:-

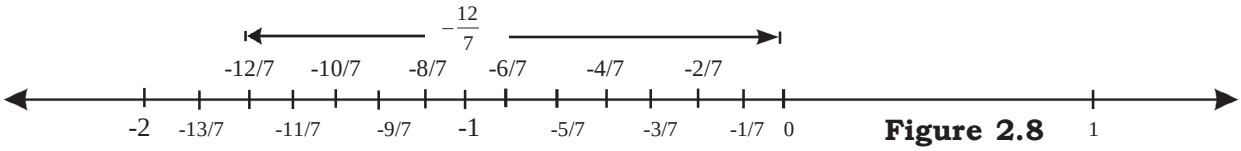

Figure 2.8

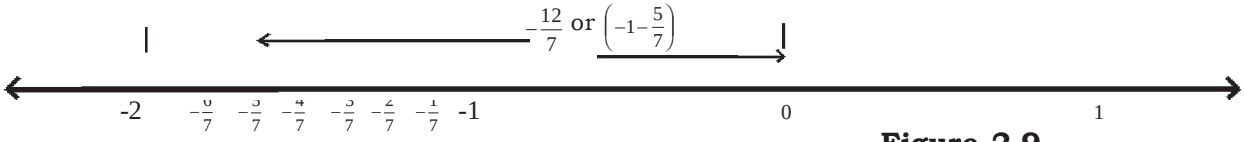

Figure 2.9

Mark the following numbers on number line.

(i) $\frac{3}{5}$ (ii) $\frac{-5}{8}$ (iii) $\frac{17}{9}$ (iv) $\frac{-15}{11}$

Write any three rational numbers and show them on number line.

Note: If the denominator of any rational number is negative, for example $\frac{p}{-q}$ then that number is shown on the number line in the same ways $\frac{-p}{q}$.

You have marked $\frac{-5}{7}$ on number line. Now mark $-\frac{5}{16}, -\frac{10}{8}, -1\frac{5}{9}, 1\frac{2}{8}$ on number line.

Activity 3

(i) Mark 4 points between -2 and -3 on number line.

(ii) Find 6 negative numbers between -5 and 3.

Which number is bigger?

We have already compared Natural numbers, integers and fractional numbers.

For comparing fractions, we have to convert them in such a manner that their values are unchanged and the denominators of all fractions are equal. After this we compare their numerators and then find out the bigger and smaller fraction.

Similarly for comparing rational numbers we convert them in such a manner that their values are unchanged but all of them have the same denominator. Then we compare their numerators and determine the bigger or smaller rational number.

Example 1: Compare $\frac{-5}{8}$ and $\frac{-3}{4}$

Solution: L.C.M. (8, 4) = 8

For converting $\frac{-5}{8}$ and $\frac{-3}{4}$ into same denominator, we multiply the numerator and denominator of $\frac{-5}{8}$ with 1 and multiply the numerator and denominator $\frac{-3}{8}$ with 2

$$\Rightarrow \frac{-5}{8}=\frac{-5\times1}{8\times1}=\frac{-5}{8}$$

$$\Rightarrow \frac{-3}{4}=\frac{-3\times2}{4\times2}=\frac{-6}{8}$$

Since - 5 > -6

$$\therefore \frac{-5}{8}>\frac{-6}{8}$$

or $$\frac{-5}{8}>\frac{-3}{4}$$

Example2: Which rational number is smaller, $\frac{-4}{7}$ or $\frac{-5}{-3}$.

Solution: Firstly, we change the negative fraction into positive.

$$\frac{-5}{-3}=\frac{-5\times(-1)}{-3\times(-1)}=\frac{5}{3}$$

LCM (7,3) = 21

$$\therefore \frac{5}{3}=\frac{5\times7}{3\times7}=\frac{35}{21} \text{ and } \frac{-4}{7}=\frac{-4\times3}{7\times3}=\frac{-12}{21}$$

(The reason for this can be that $\frac{-4}{7}$ is a negative number but $\frac{-5}{-3}=\frac{5}{3}$, a positive number. A negative number would always be smaller than a positive number.

Since -12 < 35

$$\therefore \frac{-12}{21}<\frac{35}{21}$$

$$\Rightarrow \frac{-4}{7}<\frac{5}{3}$$

$$\Rightarrow \frac{-4}{7}<\frac{-5}{-3}$$

(The reason for this can be that $\frac{-4}{7}$ is a negative number but $\frac{-5}{-3}=\frac{5}{3}$, a positive number. A negative number would always be smaller than a positive number.

Thus $\frac{-4}{7}$ is smaller than. $\frac{-5}{-3}$

Example 3. Write the following rational number is descending order.

$$\frac{3}{4}, \frac{-7}{8}, \frac{13}{-24}, \frac{-5}{-12}$$

Solution: Firstly, we convert the negative denominators into positive.

$$\Rightarrow \frac{13}{-24}=\frac{13\times(-1)}{-24\times(-1)}=\frac{-13}{24}$$

and $\frac{-5}{-12} = \frac{-5\times(-1)}{-12\times(-1)} = \frac{5}{12}$

Thus, the given rational numbers are

$$\frac{3}{4}, \frac{-7}{8}, \frac{-13}{24}, \frac{5}{12}$$

The denominators of the given numbers are 4, 8, 24, 12 and there LCM is = 24.

If we make all the denominators equal then

$$\frac{3}{4} = \frac{3\times6}{4\times6} = \frac{18}{24}$$

$$\frac{-7}{8} = \frac{-7\times3}{8\times3} = \frac{-21}{24}$$

$$\frac{-13}{24} = \frac{-13\times1}{24\times1} = \frac{-13}{24}$$

$$\frac{5}{12} = \frac{5\times2}{12\times2} = \frac{10}{24}$$

Since 18 > 10 > -13 > -21

$$\therefore \frac{3}{4} > \frac{-5}{-12} > \frac{13}{-24} > \frac{-7}{8}$$

Activity 4

1. Write any 5 rational numbers and arrange them in order.
2. Write the smallest and the largest rational number without finding out the LCM.

$$\frac{-1}{2}, \frac{-5}{1}, \frac{3}{2}, \frac{2}{7}, \frac{17}{12}, \frac{2}{1}, \frac{-2}{9}, \frac{-12}{6}$$ Give reasons for your answer.

3. Make 5 such exercises and ask your friends to solve them.

EXERCISE 2.2

1. Mark the following rational numbers on the number line.

$$\frac{2}{3}, \frac{-5}{9}, \frac{-3}{13}, \frac{-16}{-5}$$

2. Which rational number is smaller? Explain by showing on number line.

(i) $\frac{3}{5}, \frac{-7}{8}$ (ii) $\frac{-8}{7}, \frac{7}{5}$

3. Fill up the box with appropriate symbols (<, =, >)

(i) $\frac{3}{2} \square \frac{5}{4}$ (ii) $\frac{-6}{8} \square \frac{-2}{5}$ (iii) $\frac{1}{-2} \square \frac{-9}{18}$

(iv) $\frac{-15}{-7} \square \frac{3}{7}$ (v) $\frac{-10}{3} \square -9$

4. Between the two rational numbers, which is bigger?

(i) $\frac{-3}{13}, \frac{7}{13}$ (ii) $\frac{-4}{3}, \frac{-2}{-5}$

(iii) $\frac{-21}{20}, -6$ (iv) $\frac{7}{9}, \frac{3}{7}$ (v) $0, -\frac{1}{2}$

5. Between the two rational numbers, which is smaller?

(i) $5, \frac{13}{3}$, (ii) $\frac{4}{-6}, \frac{-7}{3}$, (iii) $\frac{-17}{11}, \frac{9}{7}$, (iv) $\frac{17}{19}, \frac{-3}{19}$

6. Write the given rational numbers in ascending order ?

$\frac{2}{6}, \frac{-4}{12}, \frac{-9}{-27}, \frac{-5}{18}$

7. Write the given rational numbers in descending order ?

$\frac{-8}{7}, \frac{2}{21}, \frac{-5}{14}, \frac{1}{28}$

8. Julie wrote some of the following statements and asked her friends to find out whether these are true or false statements ?

(i) The rational number $\frac{57}{23}$ is placed at the left side of 0 on number line.

(ii) $\frac{-8}{-3}$ is placed at the right side of 0 on number line.

(iii) $\frac{19}{-5}$ is placed at right side of 0 on number line.

(iv) $\frac{3}{4}$ and $\frac{-2}{7}$ are placed at right and left side of 0 respectively.

Write 4 such statements and ask your friends to check whether they are true.

Exercise 2.3

1. Satish takes $1\frac{1}{2}$ houre to reach his school from home, and his sister takes 90 minutes to reach school from home. Now tell who took more time to reach school.
2. Radhika eats $2\frac{1}{2}$ Chapatis in dinner and his sister Gitika eats $\frac{10}{4}$ Chapatis. Tell us whether they both eat equal Chapatis?
3. Ritesh go to the market by walk. After going $\frac{9}{2}$ kilometer in east direction he reminds crossed his destination then he turns back and walks $\frac{1}{2}$ kilometer in west direction by representing on number line tell the distance between his current position and home.
4. Saurbha travel $\frac{9}{3}$ kilometer by bus. After that $\frac{2}{3}$ kilometer by walk to reach his home from school. What is the total distance of school and home. Show it on number line.

We have learnt

1. The numbers which are expressed or can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq o$, are called rational numbers.
2. A rational number p/q is in the simplest form if there is no common factor between p and q except 1.
3. On comparing two or more than two rational numbers, we convert the given rational numbers into another rational number so that their values are unchanged but their denominators are equal then we compare their numerators.