Sets

Exercise 2.1

Q. 1. Which of the following are sets? Justify your answer.

(i) The collection of all the months of a year beginning with the letter "J"
(ii) The collection of ten most talented writers of India.
(iii) A team of eleven best cricket batsmen of the world.
(iv) The collection of all boys in your class.
(v) The collection of all even integers.

Answer : We know that a set is a collection of things that have something in common or follow a rule. Set notation uses brace {} with elements separated by commas.

(i) Collection of all months of a year beginning with the letter "J" = {January, June, July}

∴ This is a set.

(ii) Collection of ten most talented writers in India cannot be accurately defined.

∴ This is not a set.

(iii) Team of eleven best cricket batsmen of the world cannot be accurately defined.

∴ This is not a set.

(iv) Collection of boys in class = {Rohan, Sushant, Mohan, Rehan}

∴ This is a set.

(v) Collection of all even integers = {… -4, -2, 0, 2, 4, …}

∴ This is a set.

Q. 2. If A = {0, 2, 4, 6}, B = {3, 5, 7} and C = {p, q, r} then fill the appropriate symbol, $\in$ or $\notin$ in the blanks.

(i) 0 ….. A (ii) 3 ……… C
(iii) 4 …… B (iv) 8 ……… A
(v) p ……. C (vi) 7 ……… B

Answer : We know that ∈ is read as "belongs to" and is read as "does not belong to".

(i) 0 ∈ A

(ii) 3 C

(iii) 4 B

(iv) 8 A

(v) p ∈ C

(vi) 7 ∈ B

Q. 3. Express the following statements using symbols.

(i) The elements 'x' does not belong to 'A'.
(ii) 'd' is an element of the set 'B'.
(iii) '1' belongs to the set of Natural numbers N.
(iv) '8' does not belong to the set of prime numbers P.

Answer : We know that ∈ is read as "belongs to" and is read as "does not belong to".

(i) x A

(ii) d ∈ B

(iii) 1 ∈ N

(iv) 8 P

Q. 4. State whether the following statements are true or false. Justify your answer

(i) 5 $\not\in$ set of prime numbers
(ii) S = {5, 6, 7} implies 8 ∈S.
(iii) -5 $\not\in$ W where 'W' is the set of whole numbers
(iv) $\frac{8}{11} \in$ Z where 'Z' is the set of integers.

Answer : We know that ∈ is read as "belongs to" and is read as "does not belong to".

(i) False

Prime numbers = {2, 3, 5, 7, 11, …}

So, 5 ∈ set of prime numbers.

(ii) False

Given S = {5, 6, 7}

S does not include 8.

∴ 8 S

(iii) True

We know that W = {0, 1, 2, 3, 4 …}.

There are no negative integers.

∴ -5 W

(iv) False

We know that Z = {… -2, -1, 0, 1, 2 …}.

Integers do not include fractions or decimals.

∴ $\frac{8}{11}$ Z

Q. 5. Write the following sets in roster form.

(i) B = {x : x is a natural number smaller than 6}
(ii) C = {x : x is a two-digit natural number such that the sum of its digits is 8}.
(iii) D = {x : x is a prime number which is a divisor of 60}.
(iv) E = {x : x is an alphabet in BETTER}.

Answer : We know that in roster method, we write all the elements of a set in curly brackets. Each of the element is written only once and separated by commas. The order of the element is not important but it is necessary to write all the elements of the set.

(i) B = {x: x is a natural number smaller than 6}

Natural numbers are 1, 2, 3 …

∴ B = (1, 2, 3, 4, 5}

(ii) C = {x: x is a two-digit natural number such that the sum of its digits is 8}.

8 can be the sum of digits when the two digits are 1, 7 (or) 2, 6 (or) 3, 5 (or) 4, 4.

∴ C = {17, 26, 35, 44, 53, 62, 71}

(iii) D = {x: x is a prime number which is a divisor of 60}.

The prime factorization of 60 = $2^2 \times 3 \times 5$

∴ D = {3, 5}

(iv) E = {x: x is an alphabet in BETTER}.

∴ E = {B, E, T, R}

Q. 6. Write the following sets in the set-builder form.

(i) {3, 6, 9, 12}
(ii) {2, 4, 8, 16, 32}
(iii) {5, 25, 125, 625}
(iv) {1, 4, 9, 16, 25 … 100}

Answer : We know that when we write a set by defining its elements with a "common property", we can say that the set is in set builder form.

(i) {3, 6, 9, 12}

⇒ 3, 6, 9, 12 are the multiples of 3 and they are less than 13.

∴ A={x: x is multiple of 3 & less than 13}

(ii) {2, 4, 8, 16, 32}

⇒ Here 2, 4, 8, 16, 32 are in the form of 2^1, 2^2, 2^3, 2^4, 2^5

Where 1, 2, 3, 4, 5 are the powers of 2 and power is less than 6.

∴ B = {x: x is a power of 2^x & x is less than 6}

(iii) {5, 25, 125, 625}

⇒ Here 5, 25, 125, 625 are in the form of 5^1, 5^2, 5^3, 5^4.

Where 1, 2, 3, 4 are the powers of 5 and power is less than 5.

∴ C = {x: x is a power of 5 & x is less than 5}

(iv) {1, 4, 9, 16, 25 … 100}

⇒ Here 1, 4, 9 16, 25 … 100 are in the form of $1^2, 2^2, 3^2, 4^2, 5^2$ … 10^2 where 1, 2, 3, 4, 5 … 10 are natural numbers and the given numbers are squares and not greater than 10.

∴ D = {x : x in square of natural number and not greater than 10}

Q. 7. Write the following sets in roster form.

(i) A = {x : is a natural number greater than 50 but smaller than 100}
(ii) B = {x : x is an integer, x^2 = 4}
(iii) D = {x : x is a letter in the word "LOYAL"}

Answer : We know that in roster method, we write all the elements of a set in curly brackets. Each of the element is written only once and separated by commas. The order of the element is not important but it is necessary to write all the elements of the set.

(i) A = {x: x is a natural number greater than 50 but smaller than 100}

⇒ Natural numbers greater than 50 are 51, 52, 53… and smaller than 100 are … 97, 98, 99.

∴ A = {51, 52, 53 … 97, 98, 99}

(ii) B = {x: x is an integer, x^2 = 4}

⇒ Given $x^2 = 4$

⇒ $x = \sqrt{4}$

∴ $x = \pm 2$

∴ B = {+2, -2}

(iii) D = {x : x is a letter in the word "LOYAL"}

∴ D = {L, O, Y, A}

Q. 8. Match the roster form with set builder form.

(i) {1, 2, 3, 6}
(ii) {2, 3}
(iii) {M, A, T, H, E, I, C, S}
(iv) {1, 3, 5, 7, 9}
(a) {x : x is prime number and a divisor of 6}
(b) {x : x is an odd natural number smaller than 10}

(c) {x : x is a natural number and divisor of 6}
(d) {x : x is a letter of the word MATHEMATICS}

Answer : Let us consider the set builder forms.

(a) {x : x is prime number and a divisor of 6}

⇒ Prime factorization of 6 = 2 × 3

where 2 and 3 are prime numbers.

∴ Roster form = {2, 3} … (ii)

(b) {x : x is an odd natural number smaller than 10}

Odd natural numbers smaller than 10 = 1, 3, 5, 7, 9

∴ Roster form = {1, 3, 5, 7, 9} … (iv)

(c) {x : x is a natural number and divisor of 6}

⇒ Factors of 6 are 1, 2, 3 and 6.

∴ Roster form = {1, 2, 3, 6} … (i)

(d) {x : x is a letter of the word MATHEMATICS}

∴ Roster form = {M, A, T, H, E, I, C, S} … (iii)

Exercise 2.2

Q. 1. If A = {1, 2, 3, 4}; B = {1, 2, 3, 5, 6} then find $A \cap B$ and $B \cap A$. Are they equal?

Answer : We know that the intersection of sets A and B is the set of all elements which are common to A and B.

The common elements in both A and B are 1, 2, 3.

⇒ A ∩ B = {1, 2, 3, 4} ∩ {1, 2, 3, 5, 6} = {1, 2, 3}

⇒ B ∩ A = {1, 2, 3, 5, 6} ∩ {1, 2, 3, 4} = {1, 2, 3}

∴ A ∩ B = B ∩ A

Ans. Yes, $A \cap B$ & $B \cap A$ are same.

Q. 2. A = {0, 2, 4}, find $A \cap \phi$ and $A \cap A$. Comment.

Answer : We know that the intersection of sets A and B is the set of all elements which are common to A and B.

We know that ϕ is a Null set which has no elements.

Given A = {0, 2, 4}

$\Rightarrow A \cap \phi = \{0, 2, 4\} \cap \phi = \phi$

$\Rightarrow A \cap A = \{0, 2, 4\} \cap \{0, 2, 4\} = \{0, 2, 4\} = A$

Ans. $A \cap \phi = \phi$

$A \cap A = A$

Q. 3. If A = {2, 4, 6, 8, 10} and B = {3, 6, 9, 12, 15}, find A – B and B – A.

Answer : We know that the **(A-B)** is the set of elements which belongs to A but does not belong to B and **(B-A)** is the set of elements which belongs to B but does not belong to A

$\Rightarrow A - B = \{2, 4, 6, 8, 10\} - \{3, 6, 9, 12, 15\} = \{2, 4, 8, 10\}$

$\Rightarrow B - A = \{3, 6, 9, 12, 15\} - \{2, 4, 6, 8, 10\} = \{3, 9, 12, 15\}$

Ans. **A – B = {2, 4, 8, 10}**

B – A = {3, 9, 12, 15}

Q. 4. If A and B are two sets such that $A \subset B$ then what is $A \cup B$?

Answer : We know that if all elements of set A are present in B, then A is said to be subset of B denoted by $A \subset B$.

We know that the union of A and B is the set which consists of all the elements of A and B.

$\Rightarrow A \cup B = B \cup B = B$

Ans. $A \cup B = B$

Q. 5. If A = {x : x is a natural number}, B = {x : x is an even natural number} C = {x : x is an odd natural number} and D = {x : x is a prime number} Find A ∩ B, A ∩ C, A ∩ D, B ∩ C, B ∩ D, C ∩ D.

Answer : A = {1, 2, 3, 4, 5 …}

B = {2, 4, 6, 8 …}

C = {1, 3, 5, 7 …}

D = {3, 5, 7, 11 …}

We know that the intersection of sets A and B is the set of all elements which are common to A and B.

A ∩ B = {1, 2, 3, 4, 5 …} ∩ {2, 4, 6, 8 …} = {2, 4, 6 …} = Even natural number

A ∩ C = {1, 2, 3, 4, 5 …} ∩ {1, 3, 5, 7 …} = {1, 3, 5 …} = Odd natural number

A ∩ D = {1, 2, 3, 4, 5 …} ∩ {3, 5, 7, 11 …} = {3, 5, 7 …} = Prime number

B ∩ C = {2, 4, 6, 8 …} ∩ {1, 3, 5, 7 …} = ϕ

B ∩ D = {2, 4, 6, 8 …} ∩ {3, 5, 7, 11 …} = ϕ

C ∩ D = {1, 3, 5, 7 …} ∩ {3, 5, 7, 11 …} = {3, 5, 7 …} = Prime number

Q. 6. If A = {3, 6, 9, 12, 15, 18, 21}; B = {4, 8, 12, 16, 20} C = {2, 4, 6,8, 10, 12, 14, 16}; D = {5, 10, 15, 20} find

(i) A – B (ii) A – C
(iii) A – D (iv) B – A
(v) C – A (vi) D – A
(vii) B – C (viii) B – D
(ix) C – B (x) D – B

Answer : We know that the difference set of sets A and B is the set of elements which belongs to A but does not belong to B.

(i) A – B = {3, 6, 9, 12, 15, 18, 21} – {4, 8, 12, 16, 20}

= {3, 6, 9, 15, 18, 21}

(ii) A – C = {3, 6, 9, 12, 15, 18, 21} – {2, 4, 6, 8, 10, 12, 14, 16}

= {3, 9, 15, 18, 21}

(iii) A – D = {3, 6, 9, 12, 15, 18, 21} – {5, 10, 15, 20}

= {3, 6, 9, 12, 18, 21}

(iv) B – A = {4, 8, 12, 16, 20} - {3, 6, 9, 12, 15, 18, 21}

= {4, 8, 16, 20}

(v) C – A = {2, 4, 6, 8, 10, 12, 14, 16} - {5, 10, 15, 20}

= {2, 4, 8, 10, 14, 16}

(vi) D – A = {5, 10, 15, 20} - {3, 6, 9, 12, 15, 18, 21}

= {5, 10, 20}

(vii) B – C = {4, 8, 12, 16, 20} - {2, 4, 6, 8, 10, 12, 14, 16}

= {20}

(viii) B – D = {4, 8, 12, 16, 20} - {5, 10, 15, 20}

= {4, 8, 12, 16}

(ix) C – B = {2, 4, 6, 8, 10, 12, 14, 16} - {4, 8, 12, 16, 20}

= {2, 6, 10, 14}

(x) D – B = {5, 10, 15, 20} - {4, 8, 12, 16, 20}

= {5, 10, 15}

Q. 7. State whether each of the following statement is true or false. Justify you answers.

(i) {2, 3, 4, 5} and {3, 6} are disjoint sets.
(ii) {a, e, i, o, u} and {a, b, c, d} are disjoint sets.
(iii) {2, 6, 10, 14} and {3, 7, 11, 15} are disjoint sets.
(iv) {2, 6, 10} and {3, 7, 11} are disjoint sets.

Answer : We know that when there are no common elements in A and B, the sets are known as disjoint sets.

(i) False

Explanation: In {2, 3, 4, 5} and {3, 6}, 3 is a common element.

∴ These sets are not disjoint sets.

(ii) False

Explanation: In {a, e, i, o, u} and {a, b, c, d}, a is a common element.

∴ These sets are not disjoint sets.

(iii) True

Explanation: In {2, 6, 10, 14} and {3, 7, 11, 15}, there is no common element.

∴ These sets are disjoint sets.

(iv) True

Explanation: In {2, 6, 10} and {3, 7, 11}, there is no common element.

∴ These sets are disjoint sets.

Exercise 2.3

Q. 1. Which of the following sets are equal?

(i) A = {x: x is a letter in the word FOLLOW}
(ii) B = {x: x is a letter in the word FLOW}
(iii) C = {x: x is a letter in the word WOLF}

Answer : We know that two sets A and B are equal if every element in A belongs to B and every element in B belongs to A.

⇒ Roster form of A = {F, O, L, W}

⇒ Roster form of B = {F, L, O, W}

⇒ Roster form of C = {W, O, L, F}

So, the elements of A, B and C are same.

∴ A, B and C are equal sets.

Q. 2. **Consider the following the following sets and fill up the blank in the statement given below with = or ≠ so as to make the statement true.**

A = {1, 2, 3};
B = {The first three natural numbers};

C = {a, b, c, d};
D = {d, c, a, b}
E = {a, e, i, o, u};
F = {set of vowels in English Alphabet}

(i) A ….. B **(ii) A ….. E**
(iii) C ….. D **(iv) D ….. F**
(v) F ….. A **(vi) D ….. E**
(vii) F ….. B

Answer : We know that two sets A and B are equal if every element in A belongs to B and every element in B belongs to A.

(i) First three natural numbers are 1, 2, 3.

∴ A = B

(ii) First three natural numbers are 1, 2, 3.

∴ A ≠ B

(iii) C = {a, b, c, d}

D = {d, c, a, b}

∴ C = D

(iv) D = {d, c, a, b}

F = set of vowels = {a, e, i, o, u}

∴ D ≠ F

(v) F = set of vowels = {a, e, i, o, u}

A = {1, 2, 3}

∴ F ≠ A

(vi) D = {d, c, a, b}

E = (a, e, i, o, u}

∴ D ≠ E

(vii) F = set of vowels = {a, e, i, o, u}

B = first three natural numbers = {1, 2, 3}

∴ F ≠ B

Q. 3. In each of the following, state whether A = B or not.

(i) A = {a, b, c, d}
B = {d, c, a, b}

(ii) A = {4, 8, 12, 16}
B = {8, 4, 16, 18}

(iii) A = {2, 4, 6, 8, 10}
B = {x: x is a positive even integer and x < 10}

(iv)A = {x: x is a multiple of 10}
B = {10, 15, 20, 25, 30, ….}

Answer : We know that two sets A and B are equal if every element in A belongs to B and every element in B belongs to A.

(i) A = {a, b, c, d}

B = {d, c, a, b}

∴ A = B

(ii) A = {4, 8, 12, 16}

B = {8, 4, 16, 18}

∴ A ≠ B

(iii) A = {2, 4, 6, 8, 10}

B = {x: x is a positive even integer and x < 10}

B = {2, 4, 6, 8}

∴ A ≠ B

(iv) A = {x: x is a multiple of 10}

B = {10, 15, 20, 25, 30, ….}

A = {10, 20, 30 …}

∴ A ≠ B

Q. 4. State the reasons for the following:

(i) {1, 2, 3, …., 10} ≠ {x: x ∈ N and 1 < x < 10}
(ii) {2, 4, 6, 8, 10} ≠ {x: x = 2n + 1 and x ∈ N}
(iii) {5, 15, 30, 45} ≠ {x: x is a multiple of 15}
(iv) {2, 3, 5, 7, 9} ≠ {x: x is a prime number}

Answer : We know that two sets A and B are equal if every element in A belongs to B and every element in B belongs to A.

(i) {x: x ∈ N and 1 < x < 10}

= {2, 3, 4, 5, 6, 7, 8, 9}

But the first set is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

The second set does not have 1, 10.

∴ {1, 2, 3 … 10} ≠ {x: x ∈ N and 1 < x < 10}

(ii) {x: x = 2n + 1 and x ∈ N}

We know that N = 1, 2, 3, 4 …

Let n = 0, 1, 2, 3 …

∴ When n = 0; x = 2(0) + 1 = 2

When n = 1; x = 2(1) + 1 = 3

When n = 2; x = 2(2) + 2 = 6

When n = 3; x = 2(3) + 2 = 8

When n = 4; x = 2(4) + 2 = 10

∴ Set = {2, 3, 6, 8, 10}

The first given set = {2, 4, 6, 8, 10}

The first set does not have 3.

∴ {2, 4, 6, 8, 10} ≠ {x: x = 2n + 1 and x ∈ N}

(iii) {x: x is a multiple of 15}

∴ Second set = {15, 30, 45, 60 …}

First set = {5, 15, 30, 45}

The second set does not contain 5.

∴ {5, 15, 30, 45} ≠ {x: x is a multiple of 15}

(iv) {x: x is a prime number}

Second set = {2, 3, 5, 7, 11 …}

First set = {2, 3, 5, 7, 9}

The second set i.e. prime number does not contain 9.

∴ {2, 3, 5, 7, 9} ≠ {x: x is a prime number}

Q. 5. List all the subsets of the following sets.

(i) B = {p, q}
(ii) C = {x, y, z}
(iii) D = {a, b, c, d}
(iv) E = {1, 4, 9, 16}
(v) F = {10, 100, 1000}

Answer : We know that every set is subset of itself.

(i) B = {p, q}

There are $2^2 = 4$ subsets.

Subsets: {p}, {q}, {p, q}, ϕ

(ii) C = {x, y, z}

There are $2^3 = 8$ subsets.

Subsets: {x}, {y}, {z}, {x, y}, {y, z}, {x, z}, {x, y, z}, ϕ

(iii) D = {a, b, c, d}

There are $2^4 = 16$ subsets.

Subsets: {a}, {b}, {c}, {d}, {a, b}, {b, c}, {c, d}, {d, a}, {a, c}, {b, d}, {a, b, c}, {b, c, d}, {c, d, a}, {a, b, d}, {a, b, c, d}, ϕ

(iv) E = {1, 4, 9, 16}

There are $2^4 = 16$ subsets.

Subsets: {1}, {4}, {9}, {16}, {1, 4}, {1, 9}, {1, 16}, {4, 9}, {4, 16}, {9, 16}, {1, 4, 9}, {1, 9, 16}, {4, 9, 16}, {1, 4, 16}, {1, 4, 9, 16}, ϕ

(v) F = {10, 100, 1000}

There are $2^3 = 8$ subsets.

Subsets: {10}, {100}, {1000}, {10, 100}, {10, 1000}, {100, 1000}, {10, 100, 1000}, ϕ

Exercise 2.4

Q. 1. State which of the following sets are empty and which are not?

(i) The set of lines passing through a point.
(ii) Set of odd natural numbers divisible by 2.
(iii) {x : x is a natural number x < 5 and x > 7}
(iv) {x : x is a common point to any two parallel lines}
(v) Set of even prime numbers.

Answer : We know that an empty set is a finite set. A set which does not contain any element is called an empty set.

(i) Let 'L' be the set of lines passing through a point. Then L is infinite. So, it is not an empty set.

(ii) Let 'O' be the set of odd natural numbers divisible by 2.

But only even numbers are divisible by 2.

∴ O = ϕ = Empty set

(iii) {x: x is a natural number x < 5 and x > 7}

⇒ If x < 5 and x > 7, then

⇒ x = ϕ = Empty Set

(iv) {x : x is a common point to any two parallel lines}

We know that two parallel lines do not intersect. So, there is no common point.

∴ x = ϕ = Empty set

(v) Set of even prime numbers

Let ‘E’ be the set of even prime numbers.

⇒ E = {2} ≠ ϕ

∴ It is not an empty set.

Q. 2. Which of the following sets are finite or infinite.

(i) The set of months in a year.
(ii) {1, 2, 3, …, 99, 100}
(iii) The set of prime numbers smaller than 99.

Answer : We know that a finite set is the set in which the number of elements of the set are finite and an infinite set is the set in which the number of elements of the set are not finite.

(i) Let ‘M’ be the set of months in a year.

∴ M = {January, February, March, April, May, June, July, August, September, October, November, December}

There are 12 elements in ‘M’.

∴ ‘M’ is finite.

(ii) {1, 2, 3 … 99, 100}

There are 100 elements in the above set.

∴ The given set is finite.

(iii) Let ‘P’ be the set of prime numbers less than 99.

⇒ P = {2, 3, 5, 7, 11 … 89, 97}

There are 25 elements i.e. prime numbers less than 99.

∴ P is finite.

Q. 3. State whether each of the following sets is finite or infinite.

(i) The set of letters in the English alphabet.
(ii) The set of lines which are parallel to the X-Axis.
(iii) The set of numbers which are multiplies of 5.
(iv) The set of circles passing through the origin (0, 0).

Answer : We know that a finite set is the set in which the number of elements of the set are finite and an infinite set is the set in which the number of elements of the set are not finite.

(i) Let 'E' be the set of letters in the English alphabet.

$\Rightarrow$ E = {A, B, C, D … X, Y, Z}

There are 26 alphabets.

$\therefore$ E is a finite set.

(ii) Let 'L' be the sets of lines which are parallel to X-axis.

Then L is an infinite set.

(iii) Let 'M' be the set of multiples of 5.

$\Rightarrow$ M = {5, 10, 15, 20 …}

$\therefore$ M is an infinite set.

(iv) Let 'S' be the set of circles passing through the origin (0, 0).

Then S is an infinite set.