Waves

15
Chapter

WAVE

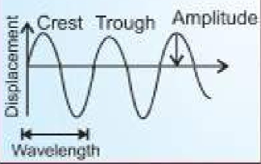
It is a disturbance produced, which transfer energy and momentum without transfer of matter.

(1) TYPES OF WAVES

- **Electromagnetic Wave** – wave propagates in the form of time varying electric and magnetic fields. It require no medium.
- **Matter waves** – wave associated with the particles having momentum.
- **Mechanical waves** – The waves which require a material medium for their propagation.

MECHANICAL WAVES

Transverse waves	Longitudinal waves
The individual particle of medium vibrate perpendicular to direction of propagation.	The individual particle of medium vibrate parallel to direction of propagation



- Trasverse waves are possible in solids like strings (under tension), due to shear modulus.
- Longitudinal waves, involve compressive stress, i.e. (Bulk modulus), so is possible in both solids and fluids
- Waves on the surface of water are of two kinds capillary waves and gravity waves

(2) DISPLACEMENT RELATION IN A PROGRESSIVE WAVES

- $y(x, t) = a \sin(kx - \omega t + \phi)$

  a = amplitude of wave

  is linear combination of sine and cosine function

  $y(x, t) = A \sin(kx - \omega t) + B \cos(kx - \omega t)$

  Amplitude of resultant wave, $a = \sqrt{A^2 + B^2}$

  $\phi = \tan^{-1}\left(\frac{B}{A}\right)$

  Speed of wave $v = \frac{\omega}{k} = v\lambda$

  $k = \frac{2\pi}{\lambda}$ called angular wave number)

  $(kx - \omega t + \phi)$ = Phase of wave

- **Speed of a Transverse Wave on a Stretched String**

  $v = \sqrt{\frac{T}{\mu}}$

  Here

  T = tension in string (in newton)

  $\mu = \frac{m}{l}$ (mass per unit length of string)

Speed of a Longitudinal wave

- Speed of longitudinal wave in a solid bar $v = \sqrt{\frac{Y}{\rho}}$

  where Y = Young's modulus of material of bar

  ρ = Density of material of bar

- Speed of longitudinal wave in gases

  According to Newton, $v = \sqrt{\frac{P}{\rho}}$ (Isothermal)

  According to Laplace, $v = \sqrt{\frac{\gamma P}{\rho}}$ (Adiabatic)

(3) PRINCIPLE OF SUPERPOSITION OF WAVES

- If $y_1(x, t)$ and $y_2(x, t)$ be the displacement due to two wave disturbances in the medium and the waves arrive in a region simultaneously and overlap, the net displacement $y(x, t)$ is given by

  $y(x, t) = y_1(x, t) + y_2(x, t)$

  Similarly, resultant waveform

  $y = \sum_{i=1}^{n} f_i (x - vt)$

  In the phenomenon of **interference** of two waves

  $y_1(x, t) = a \sin(kx - \omega t)$

  and $y_2(x, t) = a \sin(kx - \omega t - \phi)$

  The net displacement

  $y(x, t) = 2a \cos\frac{\phi}{2} \sin\left(kx - \omega t + \frac{\phi}{2}\right)$

  So, amplitude is a function of phase difference

  $A(\phi) = 2a \cos\left(\frac{\phi}{2}\right)$

  For $\phi_i = 0$, $A = 2a$ (Constructive interference)

  For constructive interference, path difference between two waves, $\Delta x = 0, \lambda, 2\lambda, \ldots n\lambda$.

  For $\phi = \pi$, $A = 0$ (Destructive interference)

  For destructive interference, path difference between two waves,

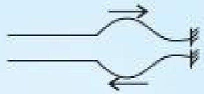
  $\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2} \ldots (2n-1)\frac{\lambda}{2}$

(4) REFLECTION OF WAVES

- Rigid Boundary – At rigid boundary wave suffer a phase change of π.

$y_i = a\sin(\omega t - kx)$

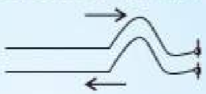
$y_r = -a\sin(\omega t + kx)$



- Open Boundary or Free boundary : At open boundary phase change is 0.

$y_i = a\sin(\omega t - kx)$

$y_r = a\sin(\omega t + kx)$



Standing Waves and Normal Modes

When two waves of same amplitude and of same frequency travel in opposite direction then resultant wave pattern from their superposition is called standing waves.

From open boundary.

$y_i(x, t) = a\sin(\omega t - kx)$,

$y_r(x, t) = a\sin(\omega t + kx)$

$y = y_i + y_r$

$y(x, t) = 2a\sin\omega t\cos kx$

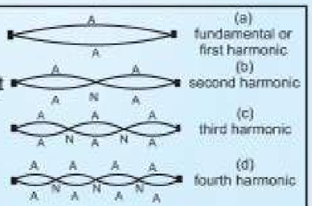
- The amplitude varies from point to point, but each element of string oscillate with same angular frequency (ω)
- **Nodes** – The point at which amplitude is zero or there is no motion called nodes. Distance between two consecutive nodes is $\lambda/2$.
- **Antinodes** – The points at which amplitude is maximum called antinodes. Distance between two consecutive antinodes is $\lambda/2$

Normal modes of stretched string Fixed At Both Ends

$L = \frac{n\lambda}{2}$, $n = 1, 2, 3$

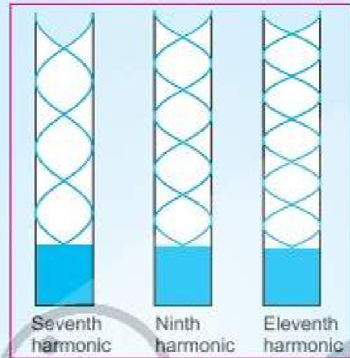
Frequencies of different modes

$\nu = \frac{nv}{2L}$, $n = 1, 2, 3$--



(5) NORMAL MODES OF ORGAN PIPES

Closed organ pipe



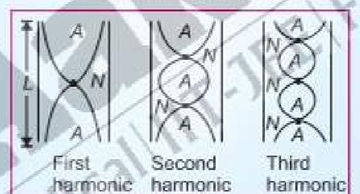
$L = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}$; for $n = 0, 1, 2, 3$,.....

Possible wavelengths $\lambda = \frac{2L}{\left(n + \frac{1}{2}\right)}$

for $n = 0, 1, 2, 3$

Natural frequencies $\nu = \left(n + \frac{1}{2}\right)\frac{v}{2L}$, for n or $= 0, 1, 2, 3$

Open Organ Pipe



$L = n\frac{\lambda}{2}$, for $n = 1, 2, 3$...

Wavelength of stationary wave, $\lambda = \frac{2L}{n}$, for $n = 1, 2, 3$...

Frequencies of different modes, $\nu = \frac{nv}{2L}$, for $n = 1, 2, 3$...

- A compression is reflected as compression from the closed end of the organ pipe and as rarefaction from the open end.
- A rarefaction is reflected as rarefaction from the closed end of the organ pipe and as compression from the open end.

(6) BEATS

- When two harmonic sound waves of nearly same frequencies travel in the same direction then the intensity of resultant wave produced from their superposition increase and decrease continuously at same point with time. It is called beat formation.
- Two waves of angular frequencies ω_1 and ω_2 superimpose at, $x = 0$ at time t

$s_1 = a_1\cos\omega_1 t$, $s_2 = a_2\cos\omega_2 t$

from superposition, $s = s_1 + s_2$

$$s = 2a\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t.\cos\left(\frac{\omega_1 + \omega_2}{2}\right)t$$

$$\omega_b = \frac{(\omega_1 - \omega_2)}{2} \text{ and } \omega_a = \frac{(\omega_1 + \omega_2)}{2}$$

Beat frequency, $\nu_{beat} = |\nu_1 - \nu_2|$

- We hear a waxing and warning of sound with frequency equal to difference between the frequencies of superposing waves.

(7) DOPPLER EFFECT

Generally, if there is relative motion between a source(s) and observer then observed frequency will be other than real frequency. This apparent change in frequency is called Doppler effect.

- Both source and observer moving

$$\nu = \nu_0\left(\frac{v + v_0}{v + v_s}\right)$$

here v is the speed of sound through the medium, v_0 is the velocity of observer relative to the medium, and v_s is the source velocity relative to the medium. In using this formula, velocities in the direction O to S should be treated as positive and those opposite to it should be taken to be negative.

- When source and observer stationary and wind is blowing towards stationary observer with speed v_w, apparent wavelength

$$\lambda_a = \frac{(V_0 - V_w)}{\nu}$$

- When source is moving towards the stationary observer with medium at rest, apparent wavelength

$$\lambda_a = \frac{(v - v_s)}{\nu}$$

Sharpen Your Understanding

NCERT Based MCQs

1. Some examples of wave motion are given in the following options. In which case wave motion is a combination of both transverse and longitudinal waves?

[NCERT XI Pg. 370]

(1) Motion of a kink in a longitudinal spring produced by displacing one end of the spring side ways

(2) Waves produced in a cylinder containing a liquid by moving its piston back and forth

(3) Waves produced by a motorboat sailing in water

(4) Both (1) and (3)

2. Longitudinal waves in a medium propagate due to [NCERT XI Pg. 390]

(1) Shear modulus

(2) Bulk modulus

(3) Both Shear and Bulk modulus

(4) Young's modulus

3. Modification in Newton's formula for speed of sound in air was made by

[NCERT XI Pg. 376]

(1) Stefan

(2) Boltzman

(3) Laplace

(4) Edison

4. At what temperature will the speed of sound in air becomes 3 times of its value at 0°C?

[NCERT XI Pg. 391]

(1) 1184°C (2) 1148°C

(3) 2184°C (4) 2148°C

5. A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, the wavelength of the reflected and transmitted sound are (speed of sound in air = 340 m/s and in water 1500 m/s)

[NCERT XI Pg. 391]

(1) 3.4 mm, 30 mm (2) 6.8 mm, 15 mm

(3) 0.34 mm, 1.5 mm (4) 6.8 mm, 30 mm

6. A pipe 30 cm long, is open at both the ends. Which harmonic mode of the pipe resonates with 1.1 kHz source?

(v = 330 m s^{-1}) [NCERT XI Pg. 382]

(1) First

(2) Second

(3) Third

(4) Forth

7. A progressive wave is represented by $y = 2\sin(100\pi t - 2\pi x)$, where x and y are in cm and t is in second. The maximum particle velocity and wave velocity respectively are

[NCERT XI Pg. 373]

(1) 628 cm/s, 628 cm/s

(2) 50 cm/s, 50 cm/s

(3) 628 cm/s, 50 cm/s

(4) 50 cm/s, 628 cm/s

8. Equation of a plane progressive wave is given by $y = 0.6\sin 2\pi\left(t - \frac{x}{2}\right)$. On reflection from a denser medium its amplitude becomes $\left(\frac{2}{3}\right)^{rd}$ of the amplitude of incident wave. The equation of reflected wave is

[NCERT XI Pg. 379]

(1) $y = 0.6\sin 2\pi\left(t + \frac{x}{2}\right)$

(2) $y = 0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

(3) $y = -0.4\sin 2\pi\left(t - \frac{x}{2}\right)$

(4) $y = -0.4\sin 2\pi\left(t + \frac{x}{2}\right)$

9. A sound is produced by plucking a string in a musical instrument, then

[NCERT XI Pg. 381]

(1) The velocity of wave in string is equal to the sound velocity in string

(2) The frequency of wave in string is equal to the frequency of sound produced

(3) The wave in string is progressive

(4) The frequency of the wave in string is double the frequency of sound

10. A glass tube of 100 cm length is filled with water. The water can be drained out slowly at the bottom of the tube. If a vibrating tuning fork of frequency 500 Hz is brought at the upper end of the tube and the velocity of sound in air is 330 m/s, then the total number of resonances obtained will be [NCERT XI Pg. 382]

(1) 4

(2) 3

(3) 2

(4) 1

11. A tuning fork A of frequency 512 Hz produces 5 beats per second when sounded with another tuning fork B of unknown frequency. If B is loaded with wax the number of beats is again 5 per second. The frequency of fork B before it was loaded is [NCERT XI Pg. 384]

(1) 507 Hz (2) 502 Hz

(3) 517 Hz (4) 522 Hz

12. The equation of a stationary wave along a stretched string is given by $y = 5\sin\frac{2\pi x}{3}\cos 40\pi t$ in, where x and y are cm and t is in second. The separation between two adjacent nodes is

[NCERT XI Pg. 379]

(1) 1.5 cm (2) 3 cm

(3) 6 cm (4) 4 cm

13. A second harmonic has to be generated in a string of length L stretched between two rigid support. The point where the string has to be plucked and touched are

[NCERT XI Pg. 381]

(1) Plucked at $\frac{L}{4}$ and touch at $\frac{L}{2}$

(2) Plucked at $\frac{L}{4}$ and touch at $\frac{3L}{8}$

(3) Plucked at $\frac{L}{2}$ and touch at $\frac{L}{4}$

(4) Plucked at $\frac{L}{2}$ and touch at $\frac{3L}{4}$

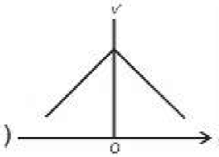
14. An observer moves towards a stationary source of sound with a velocity one fifth of the velocity of sound. The percentage change in apparent frequency is [NCERT XI Pg. 386]

(1) 0% (2) 5%

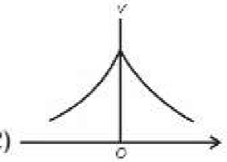
(3) 10% (4) 20%

15. A railway engine whistling at a constant frequency moves with a constant speed.It goes past a stationary observer standing beside the railway track. The frequency (ν') of the sound heard by observer is plotted against time (t). Which of the following graph best represent the variation in apparent frequency with time? [NCERT XI Pg. 385]

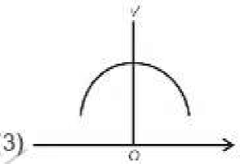
(1)



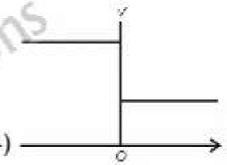
(2)



(3)



(4)



16. If a wave is incident on a surface and a part of the incident wave is reflected back and a part is transmitted into the second medium, then [NCERT XI Pg. 378]

(1) Incident and refracted waves obey Snell's law of refraction

(2) Incident and refracted waves doesn't obey laws of refraction

(3) Incident and reflected waves obey the usual laws of reflection

(4) Both (1) and (3)

17. Two sitar strings *A* and *B* playing a note are slightly out of tune and produce beats of frequency 5 Hz. When the tension in the string *B* is slightly increased, the beat frequency is found to reduce to 3 Hz. If the frequency of string *A* is 427 Hz. The original frequency of string *B* is [NCERT XI Pg. 392]

(1) 422 Hz

(2) 424 Hz

(3) 430 Hz

(4) 432 Hz

18. The transverse displacement of a string clamped at its both ends is given by $y = 0.06\sin\left(\frac{2\pi x}{3}\right)\cos(120\pi t)$, where *x* and *y* are in metre and *t* is in second. The length of the string is 1.5 m and its mass is 3×10^{-2} kg. The tension in string is

[NCERT XI Pg. 392]

(1) 324 N

(2) 648 N

(3) 832 N

(4) 972 N

19. In longitudinal stationary waves, displacement nodes are the points where there is [NCERT XI Pg. 379]

(1) Maximum displacement and maximum pressure

(2) Minimum displacement and minimum pressure change

(3) Minimum displacement and maximum pressure change

(4) Maximum displacement and maximum pressure change

20. Newton assumed that sound propagation in a gas takes under [NCERT XI Pg. 376]

(1) Isothermal condition

(2) Adiabatic condition

(3) Isotropic condition

(4) Isochoric condition

Thinking in Context

1. In any mechanical wave, _______ and not _______ is transferred from one point to the other. [NCERT XI Pg. 367]

2. Transverse wave can propagate only in a medium which can sustain _______ stress. [NCERT XI Pg. 370]

3. The lowest possible natural frequency of a system is called its _______ mode.

[NCERT XI Pg. 380]

4. Relative to an observer at rest in a medium the speed of a mechanical wave in that medium depends only on _______ and Inertial properties of medium.

[NCERT XI Pg. 374]

5. At rigid boundary there is a phase difference of _______ between incident and reflected wave. [NCERT XI Pg. 379]

6. When two or more waves traverse in opposite direction in the same medium, the net displacement of any element of the medium is the _______ of displacement due to each wave. [NCERT XI Pg. 377]

7. In stationary waves, wavelength is equal to _______ the distance between two consecutive nodes or antinodes.

[NCERT XI Pg. 379]

8. The speed of sound in air at constant temperature is independent of _______.

[NCERT XI Pg. 391]

9. The propagation constant represents 2π times the _______ that can be accommodated per unit length.

[NCERT XI Pg. 372]

10. For a travelling wave, minimum distance between two point having the _______ is called the wavelength of wave. [NCERT XI Pg. 367]

11. The argument of trigonometric function representing a travelling wave is called the _______ of the wave. [NCERT XI Pg. 370]

12. The phase determine the _______ of the wave at any position and at any instant. [NCERT XI Pg. 371]

13. Beats arise when two waves having _______ frequencies and comparable amplitudes are superposed. [NCERT XI Pg. 389]

14. The waves in an ocean are the combination of both _______ and _______ waves. [NCERT XI Pg. 370]

15. In a harmonic progressive wave of a given frequency, all the particles have the same amplitude but different _______ at a given instant of time. [NCERT XI Pg. 371]

16. In a stationary wave, all particles between two nodes have the same _______ at a given instant but have different _______. [NCERT XI Pg. 379]

17. In a stationary wave, all the particles on the two sides of a node oscillates in _______ phase. [NCERT XI Pg. 379]

18. If a wave is incident obliquely on the boundary between two different media, the transmitted wave is called the _______ wave [NCERT XI Pg. 378]

19. If there is no medium present Doppler shifts are _______ irrespective of whether the source moves or the observer moves. [NCERT XI Pg. 385]

20. In stationary waves, the points at which the amplitude is largest are called _______. [NCERT XI Pg. 379]

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