3 Fourier Transform

Multiple Choice Questions

Q.1 Match **List-I (Fourier transform)** with **List-II (Functions of time)** and select the correct answer using the codes given below the lists:

List-I

A. $\frac{\sin k\omega}{\omega}$

B. $e^{-j\omega d}$

C. $\frac{1}{(j\omega + 2)^2}$

D. $k\delta(\omega)$

List-II

1. A constant
2. Exponential function
3. *t*-multiplied exponential function
4. Rectangular pulse
5. Impulse function

Codes:

	A	B	C	D
(a)	4	5	3	1
(b)	4	5	3	2
(d)	3	4	2	1
(d)	3	4	2	5

[ESE-1999]

Q.2 Match **List-I (Type of signal)** with **List-II (Property of Fourier transform)** and select the correct answer using the codes given below the lists:

List-I

A. Real and even symmetric
B. Real and odd symmetric
C. Imaginary and even symmetric
D. Imaginary and odd symmetric

List-II

1. Imaginary and even symmetric
2. Real and even symmetric
3. Real odd even symmetric
4. Imaginary and odd symmetric

Codes:

	A	B	C	D
(a)	1	4	2	3
(b)	2	4	1	3
(c)	1	3	2	4
(d)	2	3	1	4

[ESE-2002]

Q.3 Match **List-I (Time domain property)** with **List-II (Frequency domain property pertaining to Fourier representation periodicity properties)** and select the correct answer using the codes given below the lists:

List-I	List-II
A. Continuous	1. Periodic
B. Discrete	2. Continuous
C. Periodic	3. Non-periodic
D. Non-periodic	4. Discrete

Codes:

	A	B	C	D
(a)	3	4	1	2
(b)	2	4	1	3
(c)	2	1	4	3
(d)	3	1	4	2

[ESE-2004]

Q.4 Match **List-I (Time Function)** with **List-II (Fourier Spectrum/Fourier Transform)** and select the correct answer using the codes given below the lists:

List-I

A. Periodic Function
B. Aperiodic Function
C. Unit Impulse $d(t)$
D. $\sin \omega_0 t$

List-II

1. Continuous spectrum at all frequencies
2. $\pi j(\delta(\omega+\omega_0)-\delta(\omega-\omega_0))$
3. Line discrete spectrum
4. 1

Codes:

	A	B	C	D
(a)	4	2	3	1
(b)	3	1	4	2
(c)	4	1	3	2
(d)	3	2	4	1

[ESE-2005]

Q.5 Match **List-I (CT Function)** with **List-II (CT Fourier Transform)** and select the correct answer using the code given below the lists:

List-I	List-II
A. $e^{-t}u(t)$	**1.** $2\pi e^{-\lvert\omega\rvert}$
B. $x(t)=\begin{cases}1, & \lvert t\rvert \le 1\\ 0, & \lvert t\rvert > 1\end{cases}$	**2.** $j\omega X(j\omega)$
C. $\dfrac{dx(t)}{dt}$	**3.** $\dfrac{1}{1+j\omega}$
D. $\dfrac{2}{1+t^2}$	**4.** $\dfrac{2\sin\omega}{\omega}$

Codes:

	A	B	C	D
(a)	1	4	2	3
(b)	3	2	4	1
(c)	1	2	4	3
(d)	3	4	2	1

[ESE-2006]

Q.6 If the Fourier transform of $x(t)$ is $\dfrac{2}{\omega}\sin(\pi\omega)$, then what is the Fourier transform of $e^{5t}x(t)$?

(a) $\dfrac{2}{\omega-5}\sin(\pi\omega)$

(b) $\dfrac{2}{\omega}\sin\{\pi(\omega-5)\}$

(c) $\dfrac{2}{\omega+5}\sin\{\pi(\omega+5)\}$

(d) $\dfrac{2}{\omega-5}\sin\{\pi(\omega-5)\}$

[ESE-2006]

Q.7 A real signal $x(t)$ has Fourier transform $X(f)$. Which one of the following is correct?

(a) Magnitude of $X(f)$ has even symmetry while phase of $X(f)$ has odd symmetry.
(b) Magnitude of $X(f)$ has odd symmetry while phase of $X(f)$ has even symmetry.
(c) Both magnitude and phase of $X(f)$ have even symmetry.
(d) Both magnitude and phase of $X(f)$ have odd symmetry.

[ESE-2007]

Q.8 Let $x(t) \leftrightarrow X(j\omega)$ be Fourier Transform pair. The Fourier Transform of the signal $x(5t-3)$ in terms of $X(j\omega)$ is given as

(a) $\dfrac{1}{5}e^{-\frac{j3\omega}{5}}X\left(\dfrac{j\omega}{5}\right)$ (b) $\dfrac{1}{5}e^{\frac{j3\omega}{5}}X\left(\dfrac{j\omega}{5}\right)$

(c) $\dfrac{1}{5}e^{-j3\omega}X\left(\dfrac{j\omega}{5}\right)$ (d) $\dfrac{1}{5}e^{j3\omega}X\left(\dfrac{j\omega}{5}\right)$

[GATE-2006]

Q.9 The signal $x(t)$ is described by

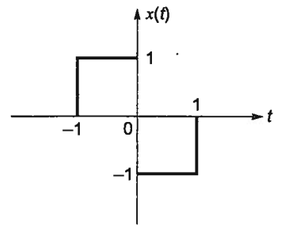
$$x(t)=\begin{cases}1 \text{ for } & -1\le t\le +1\\ 0 & \text{otherwise}\end{cases}$$

Two of the angular frequencies at which its Fourier transform becomes zero are

(a) $\pi, 2\pi$ (b) $0.5\pi, 1.5\pi$
(c) $0, \pi$ (d) $2\pi, 2.5\pi$

[GATE-2008]

Q.10 The fourier transform of given signal $x(t)$



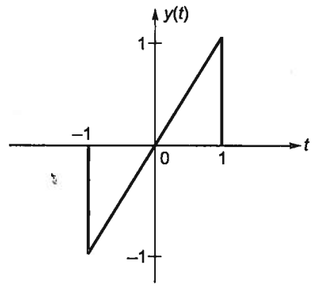
(a) $\dfrac{2\sin\omega-2}{\omega}$ (b) $\dfrac{2\cos\omega-2}{j\omega}$

(c) $2j\omega\cos\omega$ (d) $2j\omega\sin\omega$

Q.11 A Fourier transform pair is as follows:

$$x(t) \xleftrightarrow{F.T} X(j\omega)=\frac{2\sin\omega}{\omega}$$

where, $x(t)=\begin{cases}1, & \lvert t\rvert <\\ 0, & \text{otherwise}\end{cases}$



The fourier transform of given signal $y(t)$ is

(a) $4\pi j\left(\dfrac{\cos\omega}{\omega}-\dfrac{\sin\omega}{\omega}\right)$

(b) $2j\left(\dfrac{\cos\omega}{\omega}-\dfrac{\sin\omega}{\omega^2}\right)$

(c) $4\pi j\left(\dfrac{\cos\omega}{\omega^2}-\dfrac{\sin\omega}{\omega}\right)$

(d) $2j\left(\dfrac{\cos\omega}{\omega^2}-\dfrac{\sin\omega}{\omega}\right)$

Q.12 Suppose $y(t)=x(t)\cos t$ and

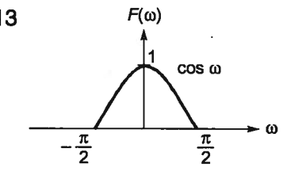
$$y(j\omega)=\begin{cases}2, & \lvert\omega\rvert\le 2\\ 0, & \text{otherwise}\end{cases}$$

then $x(t)$ will be

(a) $\dfrac{4\sin t}{\pi t}$ (b) $\dfrac{2\sin t}{t}$

(c) $\dfrac{4\sin t}{t}$ (d) $\dfrac{2\pi\sin t}{t}$

Q.13



Find $f(t)$

(a) $\left\{\dfrac{1}{4\pi}\left[\dfrac{2\sin((t-1)\pi/2)}{(t-1)}+\dfrac{2\sin((t+1)\pi/2)}{(t+1)}\right]\right\}$

(b) $\dfrac{(\omega_0^2t^2-2)\cos\omega_0t+2\omega_0t.\sin\omega_0t}{\pi t^3}$

(c) $\left\{\dfrac{1}{4\pi}\left[\dfrac{2\sin((t-1)\pi/2)}{(t+1)}+\dfrac{2\sin((t+1)\pi/2)}{(t-1)}\right]\right\}$

(d) $\dfrac{(\omega^2t^2-2)\cos\omega_0t+2\omega_0.\sin\omega_0t}{\pi t^3}$

Q.14 A signal $x(t)=\text{sinc}(\alpha t)$ where α is a real constant $\left(\text{sinc}(x)=\dfrac{\sin(\pi x)}{\pi x}\right)$ is the input to a Linear time invariant system whose impulse response $h(t)=\text{sinc}(\beta t)$, where β is a real constant. If min (α, β) denotes the minimum of α and β and similarly max (α, β) denotes the maximum of α and β, and K is a constant, which one of the following statements is true about the output of the system?

(a) It will be of the form K sinc(γt) where $\gamma=\min(\alpha, \beta)$.
(b) It will be of the $k$sinc(γt) where $\gamma=\max(\alpha, \beta)$.
(c) It will be of the form k sinc(αt).
(d) It cannot be a sinc type of signal.

[GATE-2008]

Q.15 Let $x(t)=\text{rect}\left(t-\dfrac{1}{2}\right)$, (where rect $(t)=1$ for $-\dfrac{1}{2}\le x\le\dfrac{1}{2}$ and zero otherwise).

Then $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, the Fourier Transformer of $x(t) + x(-t)$ will be given by

(a) $\text{sinc}\left(\frac{\omega}{2\pi}\right)$

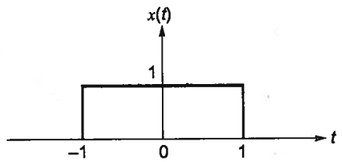
(b) $2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)$

(c) $2\,\text{sinc}\left(\frac{\omega}{2\pi}\right)\cos\left(\frac{\omega}{2}\right)$

(d) $\text{sinc}\left(\frac{\omega}{2\pi}\right)\sin\left(\frac{\omega}{2}\right)$

[GATE-2008]

Q.16 $x(t)$ is a positive rectangular pulse from $t = -1$ to $t = +1$ with unit height as shown in the figure. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2 \, d\omega$ {where $X(\omega)$ is the Fourier transform of $x(t)$} is



(a) 2 (b) 2π

(c) 4 (d) 4π

[GATE-2010]

Q.17 Let $f(t)$ be a continuous time signal and let $F(\omega)$ be its Fourier Transform defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}\,dt$$

Define $g(t)$ by

$$g(t) = \int_{-\infty}^{\infty} F(u)e^{-jut}\,du$$

What is the relationship between (t) and $g(t)$?

(a) $g(t)$ would always be proportional to $f(t)$.

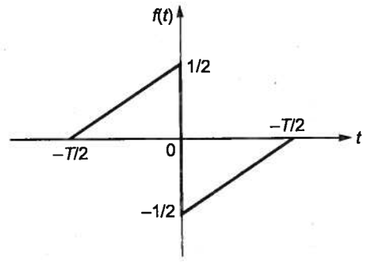
(b) $g(t)$ would be proportional to $f(t)$ if $f(t)$ is an even function.

(c) $g(t)$ would be proportional to $f(t)$ only if $f(t)$ is a sinusoidal function.

(d) $g(t)$ would never be proportional to $f(t)$.

[GATE-2014]

Q.18 A function $f(t)$ is shown in the figure.



The Fourier transform $F(\omega)$ of $f(t)$ is

(a) real and even function of ω

(b) real and odd function of ω

(c) imaginary and odd function of ω

(d) imaginary and even function of ω

[GATE-2014]

Q.19 A signal is represented by

$$x(t) = \begin{cases} 1 & |t| < 1 \\ 0 & |t| > 1 \end{cases}$$

The Fourier transform of the convolved signal

$y(t) = x(2t) * x\left(\frac{t}{2}\right)$ is

(a) $\frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)\sin(2\omega)$

(b) $\frac{4}{\omega^2}\sin\left(\frac{\omega}{2}\right)$

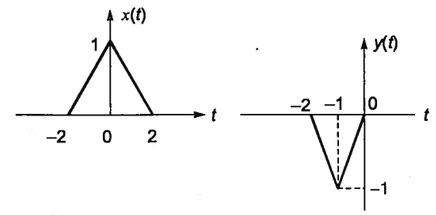
(c) $\frac{4}{\omega^2}\sin(2\omega)$

(d) $\frac{4}{\omega^2}\sin^2\omega$

[GATE-2014]

Q.20 Let $x(t)$ and $y(t)$ (with Fourier transforms $X(f)$ and $Y(f)$ respectively) be related as shown in the figure.

Then $Y(f)$ is



(a) $-\frac{1}{2}X(f/2)e^{-j2\pi f}$ (b) $-\frac{1}{2}X(f/2)e^{j2\pi f}$

(c) $-X(f/2)e^{j2\pi f}$ (d) $-X(f/2)e^{-j2\pi f}$

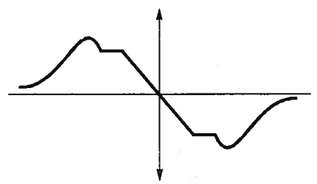
[GATE-2004]

Q.21 The Fourier transform of $x(t) \rightarrow \frac{\sin\omega}{a^2 + \cos^2\omega}$ and of $y(t) \rightarrow \frac{(a^2 + \cos^2\omega)\,5}{\sin\omega}$. The time domain signal obtain by the convolution of $x(t)$ and $y(t)$ is

(a) $5\sin t$ (b) $5\cos t$

(c) $5\,\delta(t)$ (d) $5\,u(t)$

Q.22 Following is the fourier transform plot of a signal. What is the time domain signal for this plot?



(a) $\frac{x(t)+x(-t)}{2}$ (b) $\frac{x(t)-x(-t)}{2}$

(c) $\frac{x(t)+x(-t)}{2j}$ (d) None

Statement for Linked Answer Questions (23 and 24):
The impulse response $h(t)$ of a linear time-invariant continuous time system is given by $h(t) = \exp(-2t)\,u(t)$, where $u(t)$ denotes the unit step function.

Q.23 The frequency response $H(\omega)$ of this system in terms of angular frequency ω, is given by, $H(\omega) =$

(a) $\frac{1}{1+j2\omega}$ (b) $\frac{\sin(\omega)}{\omega}$

(c) $\frac{1}{2+j\omega}$ (d) $\frac{j\omega}{2+j\omega}$

[GATE-2008]

Q.24 The output of this system, to the sinusoidal input $x(t) = 2\cos(2t)$ for all time t, is

(a) 0

(b) $2^{-0.25}\cos(2t-0.125\pi)$

(c) $2^{-0.5}\cos(2t-0.125\pi)$

(d) $2^{-0.5}\cos(2t-0.25\pi)$

[GATE-2008]

Q.25 Let a signal $a_1\sin(\omega_1 t + \phi_1)$ be applied to a stable linear time invariant system. Let the corresponding steady state output be represented as $a_2F(\omega_2 t + \phi_2)$. Then which of the following statements is true?

(a) F is not necessarily a "sine" or "cosine" function but must be periodic with $\omega_1 = \omega_2$

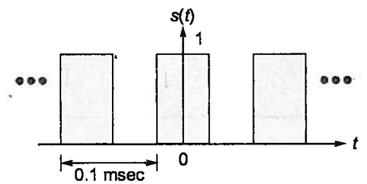
(b) F must be a "sine" or "cosine" function with $a_1 = a_2$

(c) F must be a "sine" function with $\omega_1 = \omega_2$ and $\phi_1 = \phi_2$

(d) F must be a "sine" or "cosine" function with $\omega_1 = \omega_2$

[GATE-2007]

Q.26 A rectangular pulse train $s(t)$ as shown in the figure is convolved with the signal $\cos^2(4\pi \times 10^3 t)$. The convolved signal will be a



(a) DC (b) 12 kHz sinusoid

(c) 8 kHz sinusoid (d) 14 kHz sinusoid

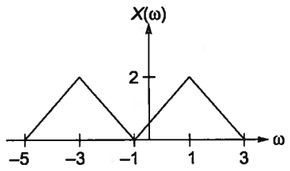
[GATE-2004]

Numerical Data Type Questions

Q.27 The Fourier transform of a signal $h(t)$ is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of $h(0)$ is _____.

[GATE-2012]

Q.28 $x(t) =$



(i) $\int_{-\infty}^{\infty} |x(t)|^2\, dt$ = _____

(ii) $\int_{-\infty}^{\infty} x(t)e^{j3t}\, dt$ is = _____

(iii) $\angle x(t)$ = _____

Q.29 A continuous, linear time-invariant filter has an impulse response $h(t)$ described by

$$h(t) = \begin{cases} 3 & \text{for } 0 \le t \le 3 \\ 0 & \text{otherwise} \end{cases}$$

When a constant input of value 5 is applied to this filter, the steady state output is ____.

[GATE-2014]

Q.30 For a function $g(t)$, it is given that,

$\int_{-\infty}^{+\infty} g(t)e^{-j\omega t}dt = \omega e^{-2\omega^2}$ for any real value ω. If

$y(t) = \int_{-\infty}^{t} g(\tau)d\tau$, then $\int_{-\infty}^{+\infty} y(t)dt$ is ____.

[GATE-2014]

Try Yourself

T1. Consider a continuous time signal $x(t)$ whose Fourier transform is

$X(j\omega)$ and $X(j\omega) = u(\omega) - u(\omega - 2)$

[Where $u(\omega)$ is unit step signal]

The signal $x(t)$ is
(a) real signal
(b) complex signal
(c) purely imaginary signal
(d) none of these

[Ans: (b)]

T2. Let $x(t)$ be a signal with its Fourier transform $X(j\omega)$. Suppose we are given the following facts.
1. $x(t)$ is real
2. $x(t) = 0$ for $t \le 0$
3. $\frac{1}{2\pi}\int_{-\infty}^{\infty} \text{Re}\{X(j\omega)\}e^{j\omega}d\omega = |t|e^{-|t|}$

then a closed form expression for $x(t)$ is

(a) $2e^{-t}u(t)$ (b) $e^{-|t|}$
(c) $te^{-2t}u(t)$ (d) $2te^{-t}u(t)$

[Ans: (d)]

T3. The Fourier transform of a signal $x(t) = e^{2t}u(-t)$ is given by

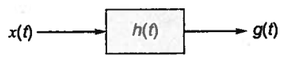
(a) $\frac{1}{2-j\omega}$ (b) $\frac{2}{1-j\omega}$

(c) $\frac{1}{j\omega-2}$ (d) $\frac{2}{2+j\omega}$

[Ans: (a)]

T4. Let $g(t) = x(t)\cos^2 t * \frac{\sin t}{\pi t}$

Assume $x(t)$ be real and $X(j\omega) = 0$, for $|\omega| \ge 1$. The system function $h(t)$ that will satisfy the condition given below is



(a) $\frac{1}{2}\delta(t)$ (b) $\frac{1}{2}\delta(t-1)+\frac{1}{2}\delta(t+1)$

(c) $[\cos^{-1}(t)]^2$ (d) $\frac{\sin ct}{t} * \cos^{-1}(t)$

[Ans: (a)]

T5. The Fourier transform $F\{e^{-t}u(t)\}$ is equal to $\frac{1}{1+j2\pi f}$. Therefore, $F\left\{\frac{1}{1+j2\pi t}\right\}$ is

(a) $e^{f}u(f)$ (b) $e^{-f}u(f)$
(c) $e^{f}u(-f)$ (d) $e^{-f}u(-f)$

[Ans: (c)]

T6. The fourier transform of a function $g(t)$ is given by

$G(\omega) = \frac{\omega^2+21}{\omega^2+9}$, the function $g(t)$ is

(a) $\delta(t) + 2\exp(-3|t|)$
(b) $\cos 3\omega t + 21\exp(-3t)$
(c) $\sin 3\omega t + 7\cos\omega t$
(d) $\sin 3\omega t + 21\exp(3t)$

[Ans: (a)]

T7. The Fourier transform of a conjugate symmetric function is always
(a) imaginary
(b) conjugate anti-symmetric
(c) conjugate symmetric
(d) real

[Ans: (d)]

T8. Fourier tranfform of $x(t) = \frac{2a}{a^2+t^2}$ is

(a) $2\pi e^{-a|\omega|}$ (b) $\pi e^{-2a|\omega|}$
(c) $2\pi e^{-a\omega}$ (d) $\pi e^{-2a\omega}$

[Ans: (a)]

T9. A signal is given as $x(t) = 5\,Sa(2t)$. The area of its Fourier transform plot in frequency domain is _____.

[Ans: (31.4)]

T10. Which of the following transfer functions gives a distortionless system

(a) $3\omega e^{-j5\omega}$ (b) $3e^{-j5\omega^2}$
(c) $3e^{-j8\omega}$ (d) $9\omega^2 e^{-5\omega j}$

[Ans:(c)]

T11. The time domain response of a LTI system is given as $h(t) = \cos t\, u(t)$. It is
(a) Static (b) Casual
(c) Stable (d) Both *b* and *c*

[Ans:(b)]

T12. The final value of $x(t) = [2 + e^{-3t}]u(t)$ is
(a) 2 (b) 3
(c) e^{-2t} (d) 0

[Ans: (a)]

T13. The transfer of differentiator has_____ magnitude
(a) constant (b) linear w.r.t. time
(c) non linear (d) linear w.r.t. frequency

[Ans:(d)]