**CBSE Test Paper 01**

**Chapter 9 Differential Equations**

1. In a bank, principal increases continuously at the rate of r% per year. Find the value of r if Rs 100 double itself in 10 years (loge2 = 0.6931).
   a. 9.93%
   b. 7.93%
   c. 6.93%
   d. 8.93%
2. General solution of $cos^2 x \frac{dy}{dx} + y = \tan x \ \left(0 \leqslant x < \frac{\pi}{2}\right)$ is
   a. $y = (\tan x - 1) + Ce^{-\tan x}$
   b. $y = (\tan x + 1) + Ce^{-\tan x}$
   c. $y = (\tan x + 1) - Ce^{-\tan x}$
   d. $y = (\tan x - 1) - Ce^{-\tan x}$
3. The number of arbitrary constants in the general solution of a differential equation of fourth order are:
   a. 3
   b. 2
   c. 1
   d. 4
4. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs1000 is deposited with this bank, how much will it worth after 10 years $\left(e^{0.5} = 1.648\right)$.
   a. Rs 1848
   b. Rs 1648
   c. Rs 1748
   d. Rs 1948
5. What is the order of differential equation : $\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = e^x$.
   a. 2
   b. 3
   c. 1
   d. 0

6. F(x, y) = $\frac{\sqrt{x^2+y^2}+y}{x}$ is a homogeneous function of degree ________.
7. The degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = x$ is ________.
8. The order of the differential equation of all circles of given radius a is ________.
9. Verify that the function is a solution of the corresponding differential equation
   $y = x\sin x;\ xy' = y + x\sqrt{x^2-y^2}$.
10. Find order and degree. $\frac{d^4y}{dx^2} + \sin(y''') = 0$.
11. Write the solution of the differential equation $\frac{dy}{dx} = 2^{-y}$.
12. Verify that the given function (explicit) is a solution of the corresponding differential equation: y = $x^2$ + 2x + C : y' - 2x - 2 = 0.
13. Find the differential equation of all non-horizontal lines in a plane.
14. Verify that the function is a solution of the corresponding differential equation
   $y = \sqrt{1+x^2}; y' = \frac{xy}{1+x^2}$ .
15. Solve the following differential equation.
   $\left(y+3x^2\right)\frac{dx}{dy} = x$
16. Solve the differential equation (1 + $y^2$) $\tan^{-1}$x dx + 2y (1 + $x^2$) dy = 0.
17. Find the particular solution of the differential equation (1 + $e^{2x}$)dy + (1 + $y^2$)$e^x$ dx = 0, given that y = 1, when x = 0.
18. Solve $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$.

## Solution

1. c. 6.93%

   **Explanation:** Let P be the principal at any time t. then,
   $\frac{dP}{dt} = \frac{rP}{100} \Rightarrow \frac{dP}{dt} = \frac{P}{100}$
   $\Rightarrow \int \frac{1}{P} dP = \int \frac{r}{100} dt$
   $\Rightarrow \log P = \frac{r}{100} t + \log c$
   $\Rightarrow \log \frac{P}{c} = \frac{r}{100} t$
   $\Rightarrow P = ce^{\frac{r}{100}}$

   When P = 100 and t = 0., then, c = 100, therefore, we have:

   $\Rightarrow P = 100\ e^{r/100}$

   Now, let t = T, when P = 100., then;

   $\Rightarrow 200 = 100e^{\frac{T}{100}}$

   $\Rightarrow e^{\frac{T}{100}} = 2$

   $\Rightarrow T = 100 \log 2$ = 100(0.6931) = 6.93%

2. a. $y = (\tan x - 1) + Ce^{-\tan x}$

   **Explanation:** $\frac{dy}{dx} + \sec^2 x . y = \tan x . \sec^2 x \Rightarrow P = \sec^2 x, Q = \tan x . \sec^2 x$
   $\Rightarrow I.F. = e^{\int \sec^2 x dx} = e^{\tan x}$
   $\Rightarrow y. e^{\tan x} = \int \tan x \sec^2 x e^{\tan x} dx \Rightarrow y. e^{\tan x} = (\tan x - 1)e^{\tan x} + C$
   $\Rightarrow y = (\tan x - 1) + Ce^{-\tan x}$

3. d. 4

   **Explanation:** 4, because the no. of arbitrary constants is equal to order of the differential equation.

4. b. Rs 1648

   **Explanation:** Here P is the principal at time t
   $\frac{dP}{dt} = \frac{5P}{100} \Rightarrow \frac{dP}{dt} = \frac{P}{20}$
   $\Rightarrow \int \frac{1}{P} dP = \int \frac{1}{20} dt$
   $\Rightarrow \log P = \frac{1}{20} t + \log c$
   $\Rightarrow \log \frac{P}{c} = \frac{1}{20} t$

$\Rightarrow P = ce^{\frac{1}{100}}$

When P = 1000 and t = 0 ., then ,

c = 1000, therefore, we have :

$\Rightarrow P = 1000e^{\frac{T}{100}}$

$\Rightarrow A = 1000e^{\frac{5}{10}}$

$\Rightarrow e^{\frac{5}{10}} = A$

$\Rightarrow A = 1000 \log 0.5$

= 1000(1.648)

= 1648

5. a. 3

   **Explanation:** Order = 3. Since the third derivative is the highest derivative present in the equation. i.e. $\frac{d^3y}{dx^3}$

6. Zero
7. not defined
8. 2
9. $y = x.\sin x$...(1)

   $y' = x.\cos x + \sin x.1$

   $\Rightarrow xy' = x^2 \cos x + x.\sin x$

   $xy' = x^2\sqrt{1 - \sin^2 x} + x.\sin x$

   $xy' = x^2\sqrt{1 - \left(\frac{y}{x}\right)^2} + x.\sin x \left[\because \frac{y}{x} = \sin x\right]$

   $xy' = x^2\frac{\sqrt{x^2 - y^2}}{x} + x.\sin x$

   $xy' = x\sqrt{x^2 - y^2} + y$

   Hence proved.

10. order = 4 ,degree = not defined
11. Given differential equation is

    $\frac{dy}{dx} = 2^{-y}$

    on separating the variables, we get

    $2^y dy = dx$

    On integrating both sides, we get

    $\int 2^y dy = \int dx$

$\Rightarrow \quad \frac{2^y}{\log 2} = x + C_1$

$\Rightarrow 2^y = x \log 2 + C_1 \log 2$

$\therefore 2^y = x \log 2 + C$, where $C = C_1 \log 2$

12. Given: $y = x^2 + 2x + C$ ...(i)
    To prove: y is a solution of the differential equation y' - 2x - 2 = 0 ...(ii)
    Proof:From, eq. (i),
    y' = 2x + 2
    L.H.S. of eq. (ii),
    = y' - 2x - 2
    = (2x + 2) - 2x - 2
    = 2x + 2 - 2x - 2 = 0 = R.H.S.
    Hence, y given by eq. (i) is a solution of y' - 2x - 2 = 0.
13. The general equation of all non-horizontal lines in a plane is ax + by = c, where $a \neq 0$.
    differentiating both sides w.r.t. y on both sides,we get
    $a\frac{dy}{dx} + b = 0$
    Again, differentiating both sides w.r.t. y, we get
    $a\frac{d^2x}{dy^2} = 0 \Rightarrow \frac{d^2x}{dy^2} = 0.$
14. $y = \sqrt{1+x^2}$ ......(i)
    $y' = \frac{1}{2\sqrt{1+x^2}}.2x$ ......(ii)
    $(ii) \div (i)$,we get,
    $\Rightarrow \frac{y'}{y} = \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+x^2}}$
    $\Rightarrow \frac{y'}{y} = \frac{x}{1+x^2}$
    $y' = \frac{xy}{1+x^2}$
    Hence given value of y is the solution of given differential equation.
15. According to the question,we have to solve the differential equation ,
    $\left(y + 3x^2\right)\frac{dx}{dy} = x \Rightarrow \frac{dy}{dx} = \frac{y}{x} + 3x$
    $\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = 3x$
    which is a linear differential equation of the form
    $\frac{dy}{dx} + Py = Q.$
    Here, $P = \frac{-1}{x}$ and Q = 3x

$\therefore \quad \text{IF} = e^{\int P dx} = e^{\int -\frac{1}{x} dx} = e^{-\log|x|} = e^{\log x^{-1}} = x^{-1}$

$\Rightarrow \quad \text{IF} = x^{-1} = \frac{1}{x}$

The solution of linear differential equation is given by

$y \times IF = \int (Q \times IF) dx + C$

$\Rightarrow \quad y \times \frac{1}{x} = \int \left(3x \times \frac{1}{x}\right) dx + C$

$\Rightarrow \quad \frac{y}{x} = \int 3dx + C \Rightarrow \frac{y}{x} = 3x + C$

$\therefore \quad y = 3x^2 + Cx$

which is the required solution.

16. Given differential equation is

$(1 + y^2) \tan^{-1}x \, dx + 2y (1 + x^2) \, dy = 0$

$\Rightarrow \left(1 + y^2\right) \tan^{-1} x dx = -2y \left(1 + x^2\right) dy$

$\Rightarrow \frac{\tan^{-1} x dx}{1+x^2} = -\frac{2y}{1+y^2} dy$

On integrating both sides, we get

$\int \frac{\tan^{-1} x}{1+x^2} dx = -\int \frac{2y}{1+y^2} dy$

Put $\tan^{-1} x = t \ in \ LHS,$ we get

$\frac{1}{1+x^2} dx = dt$

and put $1 + y^2 = u$ in RHS, we get

$2ydy = du$

$\Rightarrow \int t dt = -\int \frac{1}{u} \Rightarrow \frac{t^2}{2} = -\log u + C$

$\Rightarrow \frac{1}{2} \left(\tan^{-1} x\right)^2 = -\log\left(1 + y^2\right) + C$

$\Rightarrow \frac{1}{2} \left(\tan^{-1} x\right)^2 + \log\left(1 + y^2\right) = C$

17. Given differential equation is,

$(1 + e^{2x})dy + (1 + y^2)e^x \, dx = 0$

Above equation may be written as

$\frac{dy}{1+y^2} = \frac{-e^x}{1+e^{2x}} dx$

On integrating both sides, we get

$\int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}} dx$

On putting $e^x = t \Rightarrow e^x \, dx = dt$ in RHS, we get

$\tan^{-1} y = -\int \frac{1}{1+t^2} dt$

$\Rightarrow \quad \tan^{-1} y = -\tan^{-1} t + C$

$\Rightarrow \quad \tan^{-1} y = -\tan^{-1}(e^x) + C$ ...(i) [put $t = e^x$]

Also, given that y = 1, when x = 0.

On putting above values in Eq. (i), we get

$\tan^{-1}1 = -\tan^{-1}(e^0) + C$

$\Rightarrow \quad \tan^{-1}1 = -\tan^{-1}1 + C \quad [\because e^0 = 1]$

$\Rightarrow \quad 2\tan^{-1}1 = C$

$\Rightarrow \quad 2\tan^{-1}\left(\tan\frac{\pi}{4}\right) = C$

$\Rightarrow \quad C = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

On putting $C = \frac{\pi}{2}$ in Eq. (i), we get

$\tan^{-1}y = -\tan^{-1}e^x + \frac{\pi}{2}$

$\Rightarrow \quad y = \tan\left[\frac{\pi}{2} - \tan^{-1}(e^x)\right] = \cot\left[\tan^{-1}(e^x)\right]$

$= \cot\left[\cot^{-1}\left(\frac{1}{e^x}\right)\right] \left[\because \tan^{-1}x = \cot^{-1}\frac{1}{x}\right]$

$\therefore \quad y = \frac{1}{e^x}$

which is the required solution.

18. $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$

$\Rightarrow \frac{dx}{dy} = -\frac{e^{x/y}\left(1-\frac{x}{y}\right)}{1+e^{x/y}}$

$\Rightarrow \frac{dx}{dy} = \frac{e^{x/y}\left(\frac{x}{y}-1\right)}{1+e^{x/y}}$ ........(1)

Let x = vy, then,

$\frac{dx}{dy} = v + y\frac{dv}{dy}$

Put $\frac{dx}{dy}$ in eq (1),we get,

$v + y\frac{dv}{dy} = \frac{e^v(v-1)}{e^v+1}$

$\Rightarrow y\frac{dv}{dy} = \frac{ve^v-e^v}{e^v+1} - v$

$\Rightarrow y\frac{dv}{dy} = \frac{ve^v-e^v-ve^v-v}{e^v+1}$

$\Rightarrow -\int\frac{dy}{y} = \int\frac{e^v+1}{v+e^v}dv$

$\Rightarrow \log(e^v + v) = -\log(y) + c$

$\Rightarrow \log((e^v + v).y) = c$

$\Rightarrow (e^v + v)y = e^c$

$\Rightarrow (e^v + v)y = A$ [Putting $e^c$ = A]

$\Rightarrow \left(e^{x/y} + \frac{x}{y}\right)y = A$

$\Rightarrow ye^{x/y} + x = A$