RATIO - APPLICATIONS 6

6.0 Introduction

In your previous class, you have learnt how to use ratio and proportion to compare quantities. In this class, we will first review our understanding of the same and then learn about ratios expressed in the form of percentages.

6.1 Ratio

- Madhuri's weight is 50 kg and her daughter's weight is 10 kg. We say that Madhuri's weight is 5 times her daughter's weight. We can also say that the daughter's weight is one-fifth of her mother's weight. Thus, the ratio of Madhuri's weight and her daughter's weight is 50 : 10 or
  5 : 1. Inversely, the ratio of the daughter's weight and the mother's weight is 1 : 5.
- In a class there are 60 boys and 40 girls. The number of boys is $\frac{3}{2}$ times the number of girls. we can also say that the number of girls is two-thirds of the boys. Thus, the ratio of the number of boys and the number of girls is 60 : 40 or 3 : 2. Inversely, the ratio of number of girls and number of boys is 2 : 3.

Anand has a wire of length 100 cm and Rashmi has a wire of length 5 m. Anand said to Rashmi, "the wire with me is 20 times longer than yours." You know that this is not true as 5 m is much longer than 100 cm. The length of Rashmi's wire has been expressed in meters and that of Anand has been expressed in centimeters. Both have to be expressed in the same units before they are compared.

We know that 1 m = 100 cm. So the length of the wire with Rashmi is 5 m = 5 × 100 = 500 cm. Thus, the ratio of Rashmi and Anand's wire is 500 : 100 or 5 : 1. We can also say that the length of Rashmi's wire is 5 times that of Anand.

In all the above examples quantities have been comapred in the form of ratios. Thus, a ratio is an ordered comparison of quantities of the same units. We use the symbol ':' to represent a ratio. The ratio of two quantities a and b is a: b and we read this as "a is to b". The two quantities 'a' and 'b' are called terms of the ratio. The first quantity 'a' is called first term or antecedent and the second quantity 'b' is called second term or consequent.

Try This

Think of some real life situations in which you have to compare quantities in the form of a ratio.

Exercise 1

1. What is the ratio of ` 100 and `10 ? Express your answer in the simplest form.
2. Sudha has ` 5. Money with Radha is 3 times the money with Sudha. How much money does Radha have?
   (i) What is the ratio of Radha's money and Sudha's money?
   (ii) What is the ratio of Sudha's money and Radha's money?
3. Divide 96 chocolates between Raju and Ravi in the ratio 5 : 7
4. The length of a line segment AB is 38 cm. A point X on it divides it in the ratio 9 : 10. Find the lengths of the line segments AX and XB.

5. A sum of ` 1,60,000 is divided in the ratio of 3 : 5. What is the smaller share?
6. To make green paint, a painter mixes yellow paint and blue paint in the ratio of 3 : 2. If he used twelve liters of yellow paint, how much blue paint did he use?
7. A rectangle measures 40 cm at its length and 20 cm at its width. Find the ratio of the length to the width.
8. The speed of a Garden-Snail is 50 meters per hour and that of the Cheetah is 120 kilometers per hour. Find the ratio of the speeds.
9. Find (i) The ratio of boys and girls in your class.
   (ii) The ratio of number of doors and number of windows of your classroom.
   (iii) The ratio of number of text books and number of note books with you

Classroom Project

1. Take a tape and with the help of your friend measure the length and breadth of your classroom Find the ratio of length and breadth.

2. Take a ` 10 note. Find its length and breadth. Roundoff the answers to the nearest whole number, with the help of your teacher, find the ratio of the length and breadth.

Repeat this activity with ` 20 and ` 50 notes and record the lengths in your note book.

6.2 Proportion

Srilekha's mother prepares tea by using 2 spoons of tea powder for 1 cup of tea. One day 3 guests visited their home. How many spoons of tea powder must she use to prepare 3 cups of tea? Yes, you are right. She uses 6 spoons of tea powder to prepare 3 cups of tea. Here, Srilekha's mother used the 'law of proportion' to solve the problem.

Let us see one more example:

Ravi took a photo. He got the picture developed in a photo lab
in a size 4 cm × 6 cm .

He wanted to get the photo enlarged so he went to the photo lab again.
The lab-man gave him this photo. In turn Ravi said, “there seems to be something wrong with this picture”.
Do you think, is Ravi right?

Can you say what is wrong with this picture?

Ravi decided to measure the length and breadth of the photo. He knew that the ratio of length and breadth of the original photo should be equal to the ratio of length and breadth of the enlarged photo.

Ratio of length and breadth of the original photo = 4 : 6 = 2 : 3

Ratio of length and breadth of the enlarged photo = 4 : 12 = 1 : 3

Are the two ratios equal? Ravi also realised that the ratio of length and breadth of the enlarged photo was not equal to that of the original photo. He understood that the second picture was not proportionate to the first.

He asked the lab-man to develop another enlarged photo.

This time the photo was good. He again measured the length and breadth and calculated the ratio.
Ratio of length and breadth = 8 : 12 = 2 : 3
Now, Ravi understood that the original photo and the new
enlarged photo looked fine to him because the ratios of their length and breadth were equal i.e., they were in proportion.

Thus, two ratios are said to be in proportion when they are equal. The symbol we use for propotion is **': :'** (is as). If two ratios **a : b** and **c : d** are equal, we write **a : b = c : d** or **a : b : : c : d**. We read this as '**a is to b is proportionate to c is to d**'. This can also be read as '**a is to b is as c is to d**'.

The four quantities a, b, c and d are called first, second, third and fourth terms respectively. The first and fourth terms are known as extreme terms or extremes. The second and third terms are known as middle terms or means.

In a proportion, a : b = c : d

i.e. $\frac{a}{b} = \frac{c}{d}$

Therefore, ad = bc

Thus, The product of the means is equal to the product of the extremes.

i.e., Means

a : b = c : d

Extremes

Here 'd' is called the fourth proportional and d = $\frac{b.c}{a}$

Let us consider some examples

Example 1 : Find □ to complete the proportion.

(i) 2 : 5 = 6 : □

Solution: The product of the means is equal to the product of the extremes,

i.e. 2 : 5 = 6 :

Therefore, 2 × = 5 × 6

□ = $\frac{30}{2}$ = 15

(ii) 16 : 20 = □ × 35

The product of the means is equal to the product of the extremes,

i.e. 16 : 20 = : 35

Therefore, 20 × □ = 16 × 35

□ = $\frac{560}{20}$ = 28

∴ 6 : 20 = 28 × 35

Exercise 2

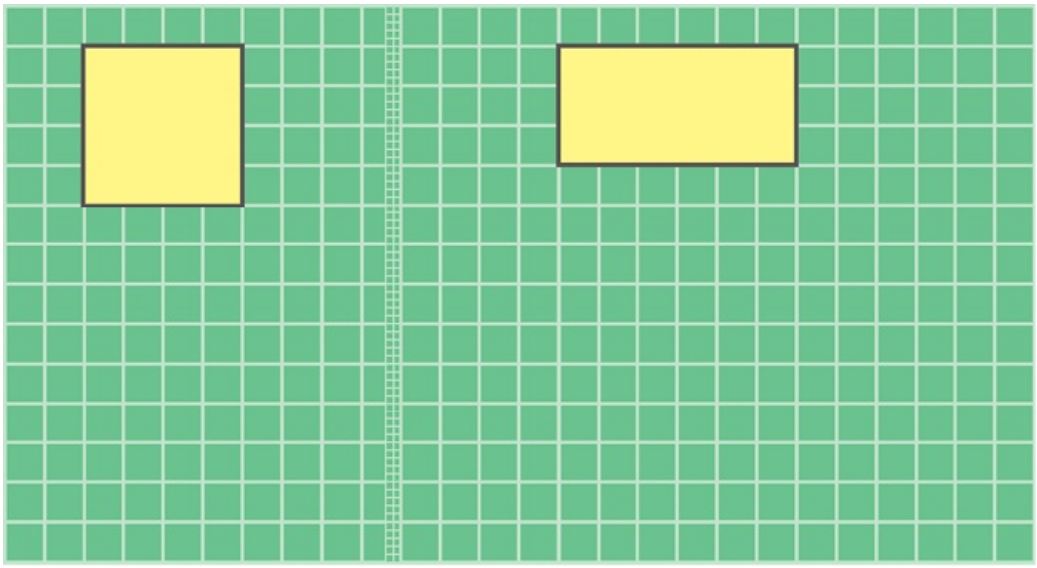
1. Find the missing numbers in the following proportions in the table given below .

S.No.	Proportion	Product of extremes	Product of means
(i)	1 : 2 : : 4 : 8		
(ii)	5 : 6 : : 75 : 90		
(iii)	3 : 4 : : 24 : 32		
(iv)	2 : 5 : : : 15	30	
(v)	3 : 6 : : 12 :		72

2. Write true or false.
   (i) 15 : 30 : : 30 : 40
   (ii) 22 : 11 : : 12 : 6
   (iii) 90 : 30 : : 36 : 12
   (iv) 32 : 64 : : 6 : 12
   (v) 25 : 1 : : 40 : 160
3. Madhu buys 5 kg of potatoes at the market. If the cost of 2 kg is ₹. 36, how much will Madhu pay?
4. Physics tells us that weights of an object on the moon is proportional to its weight on Earth. Suppose a 90 kg man weighs 15 kg on the moon what will a 60 kg woman weigh on the moon?
5. A disaster relief team consists of engineers and doctors in the ratio of 2 : 5.
   (i) If there are 18 engineers, find the number of doctors.
   (ii) If there are 65 doctors, find the number of engineers.
6. The ratio of two angles is 3 : 1. Find the
   (i) larger angle if the smaller is 180º (ii) smaller angle if the larger is 63º.

Do This

Enlarge the square and rectangle such that the enlarged square and rectangle remain proportional to the original square and rectangle.



6.3 Rate

Sometimes ratios appear as rates. Some examples are given below **:**

i) My father drives the vehicle with a speed of 60 km per hour.

ii) I bought apples at the rate of ₹ 120 per kg.

iii) My heart rate is 72 beats per minute.

iv) The cost of eggs is ₹ 60 per dozen.

v) The birth rate of India is 21 (approximately). (Birth rate is the number of live births per thousand people in a given time - Refer: *http://www.indexmundi.com/g/g.aspx?c=in&v=25*)

In the first example the distance travelled by the vehicle is compared with the time taken. In the second example cost of apples is compared to the quantity of apples. In the third example the number of heart beats is compared to the time taken. In the fourth example, the cost of eggs is compared to the quantity of eggs. In the fifth example, the number of live births is compared to 1000 poeple.

The word per can be replaced with the symbol '/' and the above examples can be written as 60km/ hour, ₹ 120/kg, 72 beats/ minute, ₹ 60/dozen and 21 births per1000 people.

6.4 Unitary Method

The method in which we first find the value of one unit and then the value of the required number of units is known as unitary method.

Example 2 : A shopkeeper sells 5 tumblers for ₹ 30. What would be the cost of 10 such tumblers?

Solution Cost of 5 tumblers = ₹ 30

Therefore, Cost of 1 tumbler = $\frac{30}{5}$ = ₹ 6

Thus, cost of 10 tumblers = 6 × 10 = ₹60.

Example 3 : What is the cost of 9 bananas, if the cost of a dozen bananas is ` 20?

Solution 1 dozen = 12 units.

Cost of 12 bananas = ₹ 20

Therefore, cost of 1 banana = ₹ $\frac{20}{12}$

Thus, cost of 9 bananas = $\frac{20}{12} \times 9$ = ₹ 15

Do This

1. 40 benches are required to seat 160 students. How many benches will be required to seat 240 students at the same rate?
2. When a Robin bird flies, it flaps wings 23 times in ten seconds. How many times will it flap its wings in two minutes?"
3. The average human heart beats at 72 times per minute. How many times does it beat in 15 seconds? How many in an hour? How many in a day?

6.5 Direct Proportion

There are various situations in day-to-day life, when a change in one quantity leads to a change in the other quantity.

For example**:**

• If the number of things purchased increases, the cost incurred also increases. Alternately, if the number of things purchased decreases, the cost incurred also decreases.

• If the money deposited with a bank increases, the interest earned on that sum also increases. Alternately, if the money in the bank decreases, the interest also decreases.

• At a constant speed, if the distance travelled increases, the time taken for it also increases. Alternately, if the distance travelled decreases, time also decreases.

In the above examples, when one quantity increases the other also increases and vice-versa. Let us understand such situations with the help of an example.

A tap takes 1 hour to fill 300 litres of a tank. How many litres will be filled up in 2 hours?

The tank will filled up by 600 litres. How many in 4 hours, how many in 8 hours? How do you make this calculation?

Look at the table given below **:**

Time taken to fill tank (hours)	1	2	4	8
Capacity filled (lts)	300	600	1200	2400

You will find that in each case above, if the time taken increases the quantity of water filled also increases such that the ratio of the time taken and the ratio of the quantity filled is same. Thus, when the time taken doubles, the quantity filled will also doubled; when the time taken is 4 times, the quantity filled is also four times the original. And when the time taken is 8 times, the quantity filled is also 8 times. The ratio of the time taken is 1 : 2 and the ratio of quantity filled is also 1 : 2. Thus, we can say that time taken to fill the tank and quantity filled are in direct proportion.

Example 4 : A shopkeeper sells 6 eggs for ₹ 30. What would be the cost of 10 eggs?

Solution : Let the cost of 10 eggs be ₹ x.

We know that as the number of eggs increases, the cost will also increases such that the ratio of the number of eggs and the ratio of their costs will remain the same. In other words, the ratio of the number of eggs and the ratio of the cost of eggs is in proportion.

Thus, $6 : 10 = 30 : x$

Since the product of the means is equal to the product of the extremes **:**

$6 \times x = 10 \times 30$

$6x = 10 \times 30$

$x = \frac{10 \times 30}{6} = 50$

$x =$ ₹ 50

Thus, the cost of 10 eggs is equal to ₹50.

This problem can be solved by using unitary method too i.e. finding the cost of one egg and then multiplying the unit cost with the number of eggs required.

Cost of 6 eggs is ₹ 30

Therefore, cost of 1 egg = $\frac{30}{6}$ = ₹ 5

Cost of 10 eggs = 5 × 10 = ₹ 50

Example 5: 20 kgs of rice is needed for a family of 4 members. How many kgs of rice will be required if the number of members in the house increases to 10?

Method 1 : Girija said as the number of members increases, the amount of rice required will also increase such that the ratio of number of members and the ratio of the amount of rice is the same. Thus, the number of members and amount of rice are in direct proportion.

Solution : Let x be the amount of rice required for 10 members

Then $x : 20 = 10 : 4$

Since the product of the means is equal to the product of the extremes:

$4x = 20 \times 10$

$x = \frac{20 \times 10}{4} = 50$

$x = 50$ kgs

$\therefore$ Amount of Rice required for 10 members = 50 Kgs.

Method 2 : Sarala used the unitary method to solve this problem **:**

Amount of Rice required for 4 members = 20 kgs.

Thus, amount of Rice required for one member = $\frac{20}{4}$ = 5 kgs.

$\therefore$ Amount of Rice required for 10 members = 10 × 5 = 50 kgs.

Example 6 : A jeep travels 90 km in 3 hours at a constant speed. In how many hours will the jeep covers 150 kms?

We know that as the distance travelled increases the time taken will also increases such that the ratio of the number of km and the ratio of the times taken is the same. Thus, the number of kms and the time taken is directly proportional.

Solution : Let x be the number of hours when the jeep covers 150 kms.

Thus, $x : 3 = 150 : 90$

Since the product of the means is equal to the product of the extremes

$90x = 150 \times 3$

$x = \frac{150 \times 3}{90} = 5$

$x = 5$

$\therefore$ Time taken to cover 150 Km = 5 hours.

Example 7 : The scale of a map is given as 1:30000. Two cities are 4 cm apart on the map. Find the actual distance between them.

Solution : Let the actual distance be x cm. Since the distance on the map is directly proportional to the actual distance,

1:30000 = 4 : x

Since the product of the means is equal to the product of the extremes

$x = 4 \times 30,000$

=1,20,000 cm

=1.2 kms (1 km = 1,00,000 cm)

Thus, two cities, which are 4 cm apart on the map, are actually 1.2 kms away from each other.

Try This

1. Place a 1litre empty bottle under a tap from which water is falling drop by drop due to leakage. How much time did it take to fill the bottle? Calculate how much water would be wasted in a year?
2. Take a clock and fix its minutes hand at 12.

Note the angles made by minutes hand in the given intervals of time **:**

Time Passed	(T_1)	(T_2)	(T_3)	(T_4)
(in minutes)	15	30	45	60
Angle turned	(A_1)	(A_2)	(A_3)	(A_4)
(in degree)	90	....	....	....

Is the angle turned through by the minute hand directly proportional to the time that has passed? Yes!

From the above table, you can also see

$T_1 : T_2 = A_1 : A_2$, because

$T_1 : T_2 = 15 : 30 = 1 : 2$

$A_1 : A_2 = 90 : 180 = 1 : 2$

Check if $T_2 : T_3 = A_2 : A_3$ and $T_3 : T_4 = A_3 : A_4$

You can repeat this activity by choosing your own time interval.

Exercise - 3

1. A length of a bacteria enlarged 50,000 times attains a length of 5 cm. What is the actual length of the bacteria? If the length is enlarged 20,000 times only, what would be its enlarged length?

2. Observe the following tables and find if x is directly proportional .

(i)	x	20	17	14	11	8	5	2
	y	40	34	28	22	16	10	4
(ii)	x	6	10	14	18	22	26	30
	y	4	8	12	16	20	24	28
(iii)	x	5	8	12	15	18	20	25
	y	15	24	36	60	72	100	125

3. Sushma has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?
4. On a Grid paper, draw five squares of different sizes. Write the following information in a tabular form.

	Square 1	Square 2	Square 3	Square 4	Square 5
Length of a side (L)					
Perimeter (P)					
Area (A)					

Find whether the length of a side is in direct proportion to:

(i) the perimeter of the square.

(ii) the area of the square.

Ratios also appear in the form of percentages. We will learn about percentages and the various ways in which we use them in day-to-day life.

6.6 Percentages

• Soumya got 65% marks in Mathematics and Ranjeet got 59% marks.

• A cloth seller in whole-sale market makes a profit of 25% on silk sarees in the retail-market makes a profit of 10%.

• Anita borrowed a loan of ` 10000 from the bank for one year. She has to pay a 10% interest at the end of the year.

• During festival season a T.V. seller was offering a discount of 10% and another was offening a discount of 15%.

The word '**percent**' means '**per every hundred**' or 'for a hundred'. The symbol '%' is used to represent percentage. Thus, 1% (one percent) means 1 out of a 100; 27% (27 percent) means 27 out of 100 and 93% (ninty three percent) means 93 out of a 100.

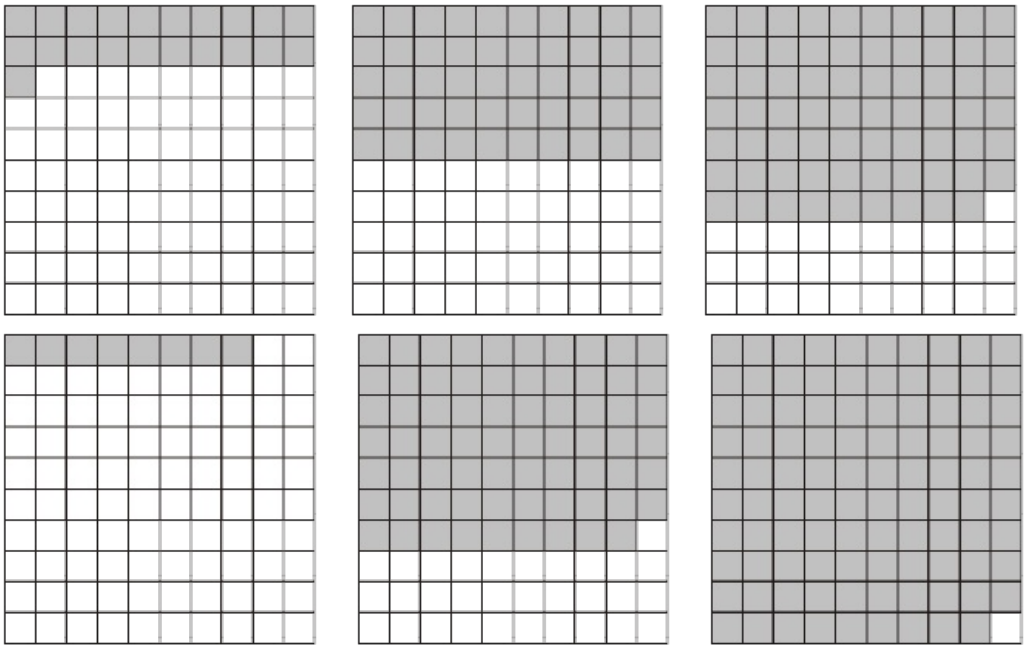
1% can also be written as $\frac{1}{100}$ or 0.01

27% can also be written as $\frac{27}{100}$ or 0.27

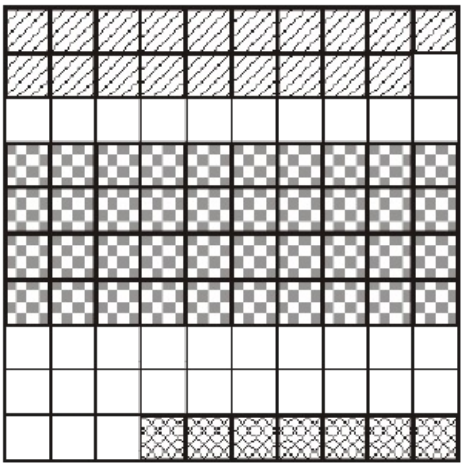
93% can also be written as $\frac{93}{100}$ or 0.93

Do This

1. Given below are various grids of 100 squares.
Each has a different number of squares coloured.
In each case, write the coloured and white part in the
form of a (1) Percentage, (2) Fraction and (3) Decimal.



2. Look at the grid paper given below. It is shaded in various designs.



Find the percentage of each design

What percent represents  *?*

What percent represents  *?*

What percent represents  *?*

What percent represents  *?*

3. The strength particular of a school are given below. Express the strength of each class as a fraction, percentage of total strength of the school.

Class	No. of children	As a fraction	As a percentage
VI	17		
VII	15		
VIII	20		
IX	30		
X	18		
Total	100		

In all the above examples the total number is 100. How do we find percentages when the total is not hundred?

Example. 8 : In a class there are 35 girls and 15 boys. What is the percentages of boys and what is the percentage of girls?

Sudhir solved it like this;

Table - 1

Solution:

Student	Number	Fraction	Converting denominator into hundred	As a percentage
Girls	35	$\frac{35}{50}$	$\frac{35}{50}\times\frac{100}{100}=\frac{70}{100}$	70%
Boys	15	$\frac{15}{50}$	$\frac{15}{50}\times\frac{100}{100}=\frac{30}{100}$	30%
Total	50			

Table - 2

Anwar found the percentage of girls and boys like this.

Total students 35 + 15 = 50

Out of 50 students there are 35 girls

Thus, out of 100 students there will be $\frac{35}{50}\times 100$ = 70 girls

Table - 3

Reena solved it like this

$\frac{35}{50}\times\frac{2}{2}=\frac{70}{100}=70\%$

We see that there are three methods that can be used to find percentage when the total does not add up to 100. In the first table, we multiply the fraction by $\frac{100}{100}$. This does not change the value of the fraction. Subsequently, only 100 remains in the denominator. Reena has multiplied by it $\frac{2}{2}$ to get 100 in the denominator. Anwar has used the unitary method. You can choose any of the methods or you can also find your own method.

Does Anwar's method work for all ratios? Does the method be used by Reena also work for all ratios?

Anwar says Reena's method can be used only if you can find a natural number which on multiplication with the denominator gives 100. Since denominator was 50, she could multiply it by 2 to get 100. If the denominator was 60, she would not have been able to use this method. Do you agree?

Example 9 : Shirt "A" has $\frac{3}{5}$ cotton where as shirt "B" has $\frac{3}{4}$ cotton.

(i) What is the percentage of cotton in each shirt?

(ii) Which shirt has more percentage of cotton?

Solution : The percentage of cotton in shirt "A" = $\frac{3}{5}\times 100 = 60\%$

The percentage of cotton in shirt "B" = $\frac{3}{4}\times 100 = 75\%$

shirt "B "has more percentage of cotton.

Example 10 : Ganga went to a tailor with 1 mt. cloth. She asked him to make a blouse to her. The tailor used 0.75 mts of cloth to make

the blouse and returned the remaining cloth to Ganga.

What percentage of the cloth (i) is used in making the blouse (ii) is given back to Ganga?

Solution : The tailor used 0.75 mts of cloth.

The percentage of cloth used = 0.75 × 100%

$$= \frac{75}{100} \times 100\ \%$$

$$= 75\%$$

The tailor returned 1 – 0.75 = 0.25 mts of cloth.

The percentage of cloth returned = 0.25 × 100%

$$= \frac{25}{100} \times 100\ \%$$

$$= 25\ \%$$

Example 11 : Last year the cost of a commodity was ` 40. This year, the cost of the comodity increased to ` 50. What is the percentage change in its price?

Solution : Percentage increase in price $= \frac{\text{amount changed}}{\text{original amount}} \times 100\ \%$

$$= \frac{50-40}{40} \times 100\ \%$$

$$= \frac{10}{40} \times 100\% = \frac{1000}{40}\ \% = 25\ \%$$

Example 12 : Shyam's monthly income is ` 10,000. He spends 60% of it on family expenses, 10% on medical expenses, 5% on donations and saves by 25%. Find the amount he spends on each item?

Solution : Amount spent on family expenses = 60% of total income

= 60% of ₹.10000

$$= \frac{60}{100} \times 10000 = ₹.6000$$

Similarly, amount spent on medical expenses $= \frac{10}{100} \times 10000 = ₹.1000$

Amount spent on donations $= \frac{5}{100} \times 10000 = ₹.500$

Amount saved $= \frac{25}{100} \times 10000 = ₹.\ 2500$

Exercise 4

1. In a school X, 48 students appeared for 10th class exam out of which 36 students passed. In another school Y, 30 students appeared and 24 students passed. If the District Educational Officer wants to give an award on the basis of pass percentage. To which school will he give the award?
2. Last year the cost of 1000 articles was ₹ 5000. This year it goes down to ₹ 4000. What is the percentage of decrease in price?
3. Sri Jyothi has a basket full of bananas, oranges and mangoes. If 50% are bananas, 15% are oranges, then what percent are mangoes?
4. 64% + 20% + ...?..... = 100%
5. On a rainy day, out of 150 students in a school 25 were absent. Find the percentage of students absent from the school? What percentage of students are present?
6. Out of 12000 voters in a constituency, 60% voted. Find the number of people voted in the constituency?
7. A local cricket team played 20 matches in one season. If it won 25% of them and lost rest. How many matches did it loose?
8. In every gram of gold, a goldsmith mixes 0.25 grams of silver and 0.05 grams of copper. What is the percentage of gold, silver and copper in every gram of gold?
9. 40% of a number is 800 then find the number?

Try This

1. Population of our country as per 2011 census is about 12×10^8 (120,00,00,000)

If the population of our country increases by 3% every year what will be the population by 2012?

2. (i) Can you eat 75% of a dosa?

(ii) Can the price of an item go up by 90%?

(iii) Can the price of an item go up by 100%?

Project Work

Fill up the following table showing the amount of time you spend on various activities in a day and calculate the percentage of time on each activity.

Activity	No. of hours	% of a day
For brushing bathing and getting ready for school		
In school		

For reading and doing home work

For playing / watching TV/helping parents

For sleeping

6.7 Some situations in which we use percentages

We use percentages to express profit and loss, discount and interest. Expressing these in percentages makes comparisons easy.

6.7.1 Profit and Loss

- A potter makes pots on the wheel, then bakes them in a kiln and decorates them with paint. He spends ₹ 3 on material, ₹ 2 on baking and ₹ 1 on painting the each pot. He sells each pot for ` 10. Does the potter make profit or loss?
- A toy maker makes a toy for ₹ 50 and sells it for ₹ 75. Does he make profit or loss?
- A trader buys shirts at ₹ 540 each. The shirts remain unsold till the end of the year. The trader sells them at ₹ 500 each at year end. Did the trader make a profit or a loss?
- Amar is a gold merchant. He bought 10 gms of gold worth ₹ 15000 in the last year. Now its rate has gone up to ₹ 20000. Will Amar make a profit or a loss on selling the gold?

For each of the above situations you can calculate the amount of profit or loss. However, many a times percentages are used in expressing the profit or loss made in a transaction.

Example 13 : Ramayya bought pens for ₹200 and he sold them for ₹ 240 whereas Somayya bought pens for ₹ 500 and he sold them for ₹ 575. Who made more profit?

Solution : Ramayya's Profit = ₹ 240 – ₹ 200 = ₹ 40

Somayya's Profit = ₹ 575 – ₹ 500 = ₹ 75

It appears like Somayya made more profit as he made a profit of ₹ 75 where as Ramayya made a profit of ₹ 40 only. Is this correct?

Ramayya made a profit of ₹ 40 when he invested an amount of ₹200 where as Somayya made a profit of ₹ 75 when he invested an amount of ₹ 500.

Thus, Ramayya's ratio of profit and cost = $\frac{40}{200}$ and

Somayya's ratio of profit and cost is = $\frac{75}{500}$

To compare profit, cost ratios we convert them in to percentages.

Thus,

Ramayya's profit percentage

$= \frac{40}{200} \times 100\%$

$= 20\ \%$

Somayya's profit percentage

$= \frac{70}{500} \times 100\%$

$= 15\%$

Ramayya earn a profit of 20% or ₹ 20 on investment of ₹ 100 and Somayya earns a profit of 15% or ₹ 15 on investment of ₹100. Thus, Ramaya earns more profit percent than Somayya.

Example 14 : A shop keeper bought a TV for ` 9000 and he sold it for ` 10,000. Find the profit or loss? calculate percentage.

Solution : Gopal solved the problem in the following way:

Cost price (CP) of the TV = ` 9000

Selling price (SP) of the TV = ` 10,000

As SP is greater than CP, the shopkeeper makes a profit:

Profit (P) = ₹ 10000 – ₹ 9000 = ₹ 1000

Thus, when the CP is ₹ 9000, the shopkeeper makes a profit of ₹ 1000

The ratio of profit and cost is $\frac{1000}{9000}$

To find the profit percentage we multiply this ratio with 100%

i.e. $\frac{1000}{9000} \times 100\% = \frac{100}{9}\% = 11\frac{1}{9}\%$

Madhu solved this problem using proportion.

When the CP is ₹ 9000, the profit is ₹ 1000.

Now, when CP is ₹ 100, let the profit be ₹ *x*.

We know that the CP and profit are directly proportional thus, ratio of profit and the ratio of cost price (CP) will be same in both cases.

Therefore, $x : 1000 = 100 : 9000$

$\frac{x}{1000} = \frac{100}{9000}$

$9000 \times x = 1000 \times 100$

$x = \frac{1000 \times 100}{9000} = 11-$.

Thus, the profit % = $11\frac{1}{9}$ %

Try This

The cost price of 12 mangoes is equal to the selling price of 15 mangoes. Find the loss percent?

Example 15 : Suppose a person buys an article for ₹ 650/- and gains 6% on selling it. Find the selling price?

Solution : Ravi solved it like this:

CP = ₹ 650

Gain % = 6%

So, if the CP is ₹ 100 then gain is ₹ 6 and SP is 100 + 6 = ₹ 106

Now, when the CP is ₹ 650 let the SP be ₹ x.

The CP and SP are directly proportional

Therefore, The ratio of CP = ratio of SP

100 : 650 = 106 : x

$\frac{100}{650}=\frac{106}{x}$

Therefore, 100 x = 106 × 650

Therefore, $x=\frac{106\times650}{100}=689$

Thus, the SP = ₹ 689

Arun solved it like this:

CP = ₹ 650

Profit % = 6%

Thus, profit = 6% of 650

$\frac{6}{100}\times650=39$

We know that SP = CP + Profit

= 650 + 39 = 689

Thus, the SP = ₹ 689

Example 16 : Ramesh sold a D.V.D player for ` 2800 at a gain of 12%. For how much did he buy it?

Solution : Naik uses proportion:

Gain % = 12%

SP = ₹ 2800

So, If CP is ₹ 100, then SP is ₹ 112

When SP = ₹ 2800, let its CP be ₹ x.

CP and SP are directly proportional

Thus, ratio of CP = ratio of SP

x : 100 = 2800 : 112

$\frac{x}{100}=\frac{2800}{112}$

Therefore, 112 × x = 100 × 2800

Therefore, x = $\frac{100\times2800}{112}=$ ₹ 2500

Thus, CP =₹ 2500

Meena uses the unitary method:

S.P = 2800

Gain = 12%

If CP is 100, then profit is 12

SP= 100+12=112

So, when SP in ₹ 112 then CP is ₹ 100

Therefore, when SP is 1 then CP is $\frac{100}{112}$

Thus, when SP is ₹2800 then CP is $\frac{100}{112}\times2800$ = ₹ 2500

CP = ₹ 2500

Example 17 : A man sold two cycles for ₹ 3000 each, gaining 20% on one and losing 20% on the other. Find his gain or loss percentage

on the whole transaction?

Solution: SP = ₹ 3000

Gain% on first cycle = 20%

Loss% on second cycle = 20%

Method (i) : Using the unitary method:

For first cycle

If CP is ₹ 100, then the profit is ₹ 20 and SP = 100 + 20 = ₹120

Thus, if SP is ₹ 120 then CP is ₹100

Now, if SP is 1 then CP is = $\frac{100}{120}$

Now, if SP is ₹ 3000 then CP = $\frac{100}{120} \times 3000$ = ₹ 2500

For second cycle

If CP is ₹ 100 then the loss is 20 and the SP is 100 – 20 = ₹ 80

Thus, if SP is ₹ 80 then CP is = ₹ 100

Now, if SP is Rs. 1 then CP is = $\frac{100}{80}$

Now, if SP is ₹ 3000 then CP is = $\frac{100}{80} \times 3000$ = ₹ 3750

Total CP = ₹ 2500 + ₹ 3750 = ₹ 6250

Total SP = ₹ 3000 + ₹ 3000 = ₹ 6000

Since SP is less than CP, loss = 6250 – 6000 = ₹ 250

Loss % = $\frac{loss}{CP} \times 100 = \frac{250}{6250} \times 100 = 4\%$

Method (ii): Using proportion:

When CP increases SP will increase, thus CP and SP are in direct proportion.

On the first cycle:

CP SP

100 120

x 3000

Thus, the ratio of CP = ratio of SP

100 : x = 120 : 3000

$\frac{100}{x} = \frac{120}{3000}$

100 × 3000 = 120 x

$\frac{100 \times 3000}{120} = x$

x = 2500

Thus, CP of first cycle = ₹ 2500

On the second cycle:

CP SP

100 80

x 3000

100 : x = 80 : 3000

$\frac{100}{x} = \frac{80}{3000}$

$x = \frac{100 \times 3000}{80}$ = ₹ 3750

Therefore, total CP of two cycles = ₹ 2500 + ₹ 3750

= ₹ 6250

Total SP of cycles = ₹ 6000

Since SP is less than CP, he has a loss

Loss = ₹ 6250 – ₹ 6000 = ₹ 250

Therefore, loss percentage = $\frac{Loss}{C.P} \times 100 = \frac{250}{6250} \times 100$ = 4%

Method (iii): SP of first cycle = ₹ 3000

Gain% = 20%

Let the CP be ₹ x

Then, the profit = $\frac{20}{100} \times x = \frac{20}{100} x$

We know that SP = CP + profit

Thus, $x+\frac{20}{100}x=3000$

$\frac{100x+20x}{100}=3000$

$\frac{120x}{100}=3000$

$x=\frac{3000\times100}{120}$ = ` 2500

Thus, CP of the first cycle = ` 2500

SP of second cycle = ` 3000

Loss % = 20%

Let the CP be ` x

Then, the loss $\frac{20}{100}\times x=\frac{20}{100}x$

We know that SP = CP – loss

Thus, $x-\frac{20}{100}x=3000$

$\frac{80}{100}x=3000$

80 x = 3000 ×100

$x=\frac{3000\times100}{80}$ = ` 3750

Thus, CP of the second cycle = ` 3750

Therefore, total CP of two cycles = ` 2500 + ` 3750 = ` 6250

Total SP of cycles = ` 6000

Since SP is less than CP, he has a loss

Loss = ` 6250 – ` 6000 = ` 250

Therefore, loss = $\frac{Loss}{C.P}\times100=\frac{250}{6250}\times100$ = 4%

Example 18 : The cost of an article goes down every year by 20% of its previous value. Find its original cost if the cost of it after 2 years is ₹19,200?

Solution : Cost of the article at the end of 2nd year = ₹ 19,200

The cost decreases every year by 20%

Let cost at the beginning of 1st year be 100. At the begining of 2nd year it will be ` 80 (i.e. 100–20% of 100)

At the begning of the 3rd year = ₹ 64 (80 – 20% of 80)

Thus, an article that costs ₹ 100 will cost ₹ 64 at the begining of third year.

The cost of article is ₹ 19200 after 2 years

Let the original cost be ₹ x.

Thus, ratio of the original cost = ratio of cost after 2 years

x : 100 = 19200 : 64

$\frac{x}{100}=\frac{19200}{64}$

64 x = 19200 × 100

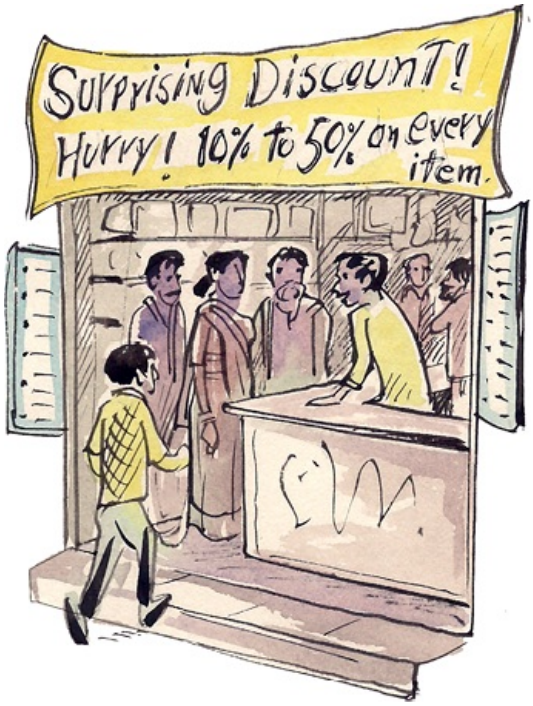
$x=\frac{19200\times100}{64}$

= 30000

Thus, the original cost of the article was ` 30000.

6.7.2 Discount

Situation 1 : Vijay opened a new cloth shop. To attract people, he advertised in the following way.



Situation 2 : On some special occassions such as dussera, deepawali, sankranthi business men offer discounts on marked price.

Situation 3 : Some times to clear their old stock or out dated stock, businessmen offer clearance sales in the form of discounts in the following way.

Example 19 : A shopkeeper marks his goods 25% above the cost price and allows a discount of 12% on them. What percent does he gain?

Solution: Let the cost price be ₹ 100.

Then marked price (MP) = ₹ 100 + ₹ 25 = ₹ 125.

Discount percent on marked price = 12%

Discount = $\frac{12}{100} \times 125$ = ₹ 15

SP = MP – Discount

= 125 – 15 = 110

Gain = SP – CP

=110 – 100

= ₹ 10

Gain% = $\frac{10}{100} \times 100 = 10\%$

Thus, the shopkeeper gains 10% after discount.

Exercise - 5

1. A shopkeeper bought a suit case for ₹ 480 and sold it for ₹ 540. Find his gain percent?
2. Ajay bought a TV for ₹ 15000 and sold it for ₹14100. Find the loss percent?
3. Ramu sold a plot of land for ₹ 2,40,000 gaining 20%. For how much did he purchase the plot?
4. On selling a mobile for ₹ 750, a shop keeper looses 10%. For what amount should he sell it to gain 5%?
5. A farmer sold 2 bullocks for ₹24000 each. On one bullock he gained 25% and on the other he lost 20%. Find his total profit or loss percent?
6. Sravya bought a watch for ₹ 480. She sold it to Ridhi at a gain of $6\frac{1}{4}$ %. Ridhi sold it to Divya at a gain of 10%. How much did Divya pay for it?
7. The marked price of a book is ₹ 225.The publisher allows a discount of ₹10% on it. Find the selling price of it?
8. A carpenter allows 15% discount on his goods. Find the marked price of a chair which is sold by him for ₹ 680?
9. A dealer allows a discount of ₹ 10% and still gains by 10%. What should be the marked price if the cost price is ₹ 900?

6.7.3 Simple Interest

Ramayya has ₹ 10,000. He requires ₹ 15,000 for agriculture. He approaches an agricultural bank manager. The conversation with the bank manager is as follows:

Ramayya: Sir, I need some money for agricultural purposes.

Bank manager :How much money do you require?

Ramayya : ₹ 5000

Bank manager :How long will you take to repay?

Ramayya : One year.

Bank manager :You have to pay an interest of 6% on the loan along with the lent amount after one year.

Ramayya : Yes sir, I will repay after one year the whole amount.

Bank manager :Do you know how much you have to pay after one year.

Ramayya : Yes, On ₹ 100 I have to pay ₹ 6.

So, on ₹ 1, I have to pay ₹ $\frac{6}{100}$ and on ₹ 5000, I have to pay ₹ $\frac{6}{100} \times 5000$ that is ₹ 300.Thus, I have to pay a total amount of ₹ 5300.

The money borrowed or lent out for a certain period is called the **Principle**. This money would be used by the borrower for some time before it is returned. For keeping this money for some time the borrower has to pay some extra money to the bank. This is known as **Interest**.

The amount that is to be repayed back is equal to the sum of the borrowed principle and the interest. That is, **Amount = Principle + Interest.**

Interest is generally expressed as percent of the principle for a period of one year. It is written as say 10% per year or per anmum or in short as 10% p.a.

10% p.a. means on every ₹100 borrowed, ₹ 10 is the interest you have to pay for one year. Let us take an example and see how this works.

Example 20 : Sunita takes a loan of ₹ 5000 at 12% rate of interest. Find the interest she has to pay at the end of one year.

Solution : Principle = ₹5000, Rate of interest = 12 % per year

If ₹ 100 is borrowed, sunita has to pay ₹ 12 interest for one year. Since the amount borrowed is ₹5000 the interest she has to pay for one year

$= \frac{12}{100} \times 5000$ = ₹ 600

So, at the end of the year she has to pay an amount of ₹ 5000 + ₹ 600 = ₹ 5600

In general, when P is principle, R% is rate of interest per annum and I is the interest, the amout be received at the end of the year is:

$$A = P + \frac{P \times R}{100}$$

If Ramayya, due to unavoidable circumstances, can not pay the total amount as requested by the manager in one year then the loan can be extended for one more year, The interest for next year will also be ₹ 300. Thus, Ramayya will pay ₹600 interest for 2 years.

For ₹100 borrowed for 3 years at 18%, the interest be paid at the end of 3 years will be 18 + 18 + 18 = 3 ×18 = ₹ 54

As the number of year increase the interest also increases. This interest being charged uniformly for each year is called simple interest.

In general, for Principle = P, Rate of Interest = R and Time = T years.

Interest to be paid (I) = P×R%×T or $P \times \frac{R}{100} \times T = \frac{PRT}{100} = \frac{PTR}{100}$

Do This

1. Find the interest on a sum of ₹ 8250 for 3 years at the rate of 8% per annum.
2. ₹ 3000 is lent out at 9% rate of interest. Find the interest which will be recieved at the end of 2½ years.

Example 21 : In what time will ₹ 6880 amount to ₹ 7224, if simple interest is calculated at 10% per annum?

Solution : Amount = ₹ 7224

Principle = ₹ 6880

S.I = Amount - Principle = ₹ 7224 – ₹ 6880 = ₹ 344

R% = 10%

Now $I = P \times \frac{R}{100} \times T$

$344 = 6880 \times \frac{10}{100} \times T$

344 × 100 = 6880 × 10 × T

Therefore, $T = \frac{344 \times 100}{6880 \times 10} = \frac{1}{2}$ year = 6 months

Example 22 : What sum will yield an interest of ₹ 3927 in 2 years and 4 months at 8% per annum?

Solution : S.I = ₹ 3927,

R% = 8 %

T = 2 year + 4 months

$\left(2 + \frac{4}{12}\right)$ Yrs.

$\left(2 + \frac{1}{3}\right)$ Yrs. $= \frac{7}{3}$ Yrs.

Substituting in $I = P \times \frac{R}{100} \times T$

$3927 = P \times \frac{8}{100} \times \frac{7}{3}$

3927 × 100 × 3 = P × 8 × 7

Therefore, $P = \frac{3927 \times 100 \times 3}{8 \times 7}$

Thus, P = ₹ 21037.50

Therefore, Principle = ₹ 21037.50

Example 23 : At what rate per annum will ₹ 6360 yield an interest of ₹1378 in $2\frac{1}{2}$ years?

Solution : Principle (P) = ₹ 6360

Time (T) = $2\frac{1}{2}$ years = $\frac{5}{2}$ years

Simple interest(S.I) = ₹ 1378

Substituting in $I = P \times \frac{R}{100} \times T$

$1378 = 6360 \times \frac{R}{100} \times \frac{5}{2}$

$1378 \times 100 \times 2 = 6360 \times 5 \times R$

Therefore, $R = \frac{1378 \times 100 \times 2}{6360 \times 5} = \frac{26}{3} = 8\frac{2}{3}\%$

Example 24 : At what rate per annum will the principle triples in 16 years?

Solution : Let the principle be ₹ x

Amount after 16 years = ₹ $3x$

Amount – Principle = Interest

Therefore, $3x - x = 2x$

For P = x, T = 16, I = $2x$

$I = P \times \frac{R}{100} \times T$

$2x = x \times \frac{R}{100} \times 16$

$2x \times 100 = x \times 16 \times R$

Therefore, $R = \frac{2x \times 100}{x \times 16} = \frac{25}{2} = 12\frac{1}{2}\ \%$

Exercise - 6

1. How long will it take for a sum of ₹ 12600 invested at 9% per annum become to ₹ 15624?
2. At what rate a sum doubles itself in 8 year 4 months?
3. A child friendly bank announces a savings scheme for school children. They will give kiddy banks to children. Children have to keep their savings in it and the bank collects all the money once in a year. To encourage children savings, they give 6% interest if the amount exceeds by ₹ 10000, and other wise 5%. Find the interest received by a school if they deposit is ₹ 9000 for one year.
4. A sum of money invested at 8% per annum for simple interest amounts to ₹ 12122 in 2 years. What will it amounts to in 2 year 8 months at 9% rate of interest?
5. In 4 years, ₹ 6500 amounts to ₹ 8840 at a certain rate of interest. In what time will ₹1600 amounts to ₹ 1816 at the same rate?

Lets earn Interest

Children! Let us play a game on simple interest.

5 members can play this game.

1. Take 3 bowls each labelled as P, R and T. Drop 5 pieces of paper in each bowl such that every paper is marked with a number.

(Hint: All the numbers in bowl P must be multiples of 100 or 1000.

2. Pick out 3 pieces of papers, one from each of the bowls, one after another.
3. The number on the paper picked from bowl P relates to principle, number on the paper picked from bowl T relates to time, number on the paper picked from bowl R relates to rate of interest.
4. Now calculate interest and tell I, P, T and R to every one.
5. If you say the right answer enter the interest amount in your account other wise put a 0 in your account.

Note : Repeat two or three rounds as per your wish and note down the values in the table given below:

	Interest amount			
Name	1st round	2nd round	3rd round	Total

Looking Back

- Many times in day-to-day life we compare quantities using ratios. For e.g., my income is ₹ 10000 and my friend's is ₹ 20000. Thus, my income is half of my friend's income or we can say that my friend's income is twice my income. The ratio of my income and my friends income is 1 : 2. and the ratio of my friend's income and my income is 2 : 1.
- When two ratio's are equal they are said to be in a proportion. The idea of proportion helps us solve various problems in our daily life.
- If some increase (decrease) in one quantity leads increase (decrease) in other quantity, the quantities are said to be in direct proportion.
- Ratio's can be expressed in the form of percentages. The word 'percent' means per hundred or out of every hundred. The symbol for percentage is '%'. 13% means 13 out of 100.

$13\% = \frac{13}{100} = 0.13$

- Percentages are used in various situations like profit and loss, discount and simple interest etc.,

RATIO - APPLICATIONS

6

12 cm

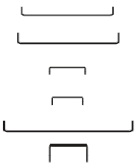
4 cm

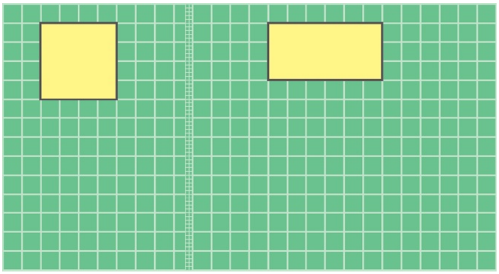
8 cm

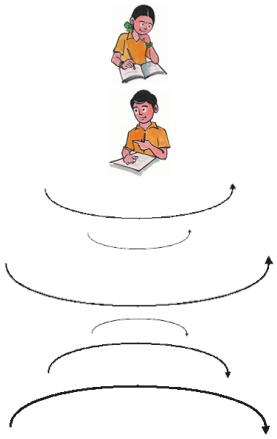
4 cm

12 cm

6 cm







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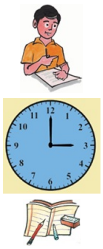
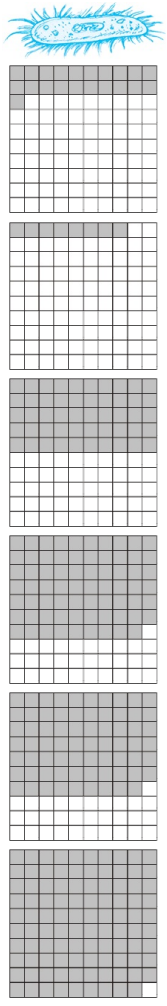
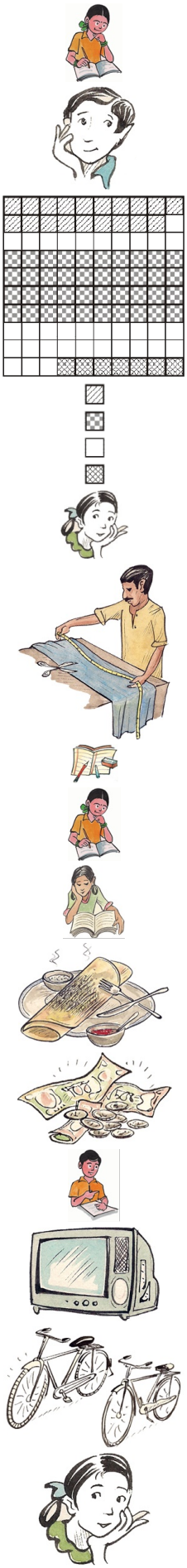


25
125





Fun with Fascinating Ratios

The digits 1,2,3,...9 can be arranged to form two numbers whose ratio is 1:2, as $\frac{7329}{14658} = \frac{1}{2} = 1:2$. This is interesting itself. But even more fascinating is the fact that the nine digits can also be arranged to form numbers whose ratio is 1:3, 1:4, 1:5, 1:6, 1:7, 1:8 and 1:9. Enjoy by finding them.