Chapter 2

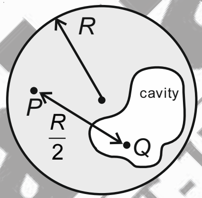
Electrostatic Potential and Capacitance

Solutions

SECTION - A

Objective Type Questions (One option is correct)

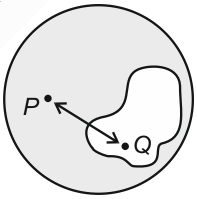
1. There is a non-spherical cavity inside a solid conducting neutral sphere of radius R and a charge $+Q$ is placed inside the cavity as shown in figure. Consider a point P which is inside the sphere and at a distance of $\frac{R}{2}$ from point charge Q as shown in figure. Potential at P due to charge on the inner surface of the cavity will be $\left(k = \frac{1}{4\pi\varepsilon_0}\right)$



(1) $-\frac{kQ}{2R}$ (2) $-\frac{2kQ}{R}$ (3) $-\frac{2kQ}{3R}$ (4) $-\frac{kQ}{R}$

Sol. Answer (2)

Potential at point P due to point charge Q and induced charge on inner surface is zero.

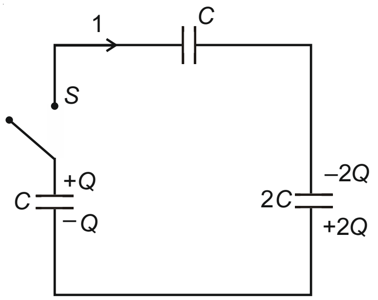


$\therefore \quad V_P' = V_Q + V_{\text{induced}} = 0$

$\Rightarrow \quad V_{\text{induced}} = -\frac{kQ}{(R/2)}$

$= \frac{-2kQ}{R}$

2. Initially switch S was open and capacitors C, $2C$ were having charges Q and $2Q$ respectively as shown in figure. Find charge flown in the direction 1 shown in figure after switch S is closed.

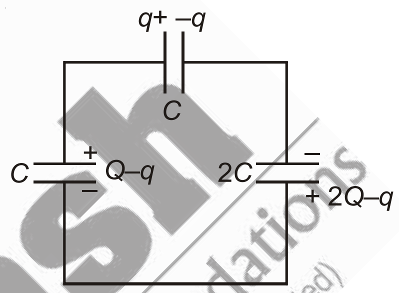


(1) $\left(\frac{2Q}{5}\right)$ (2) $\left(-\frac{4Q}{5}\right)$ (3) $\left(\frac{4Q}{5}\right)$ (4) $\left(-\frac{2Q}{5}\right)$

Sol. Answer (3)

Let the charge flown in direction 1 is q.

then by KVL,



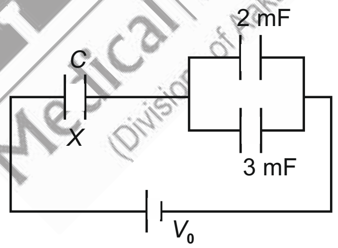
$$\frac{Q-q}{C}-\frac{q}{C}+\frac{2Q-q}{2C}=0$$

$$\Rightarrow\ Q-q-q+Q-\frac{q}{2}=0$$

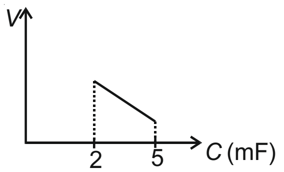
$$\Rightarrow\ 2Q=\frac{5q}{2}$$

$$\Rightarrow\ q=\frac{4Q}{5}$$

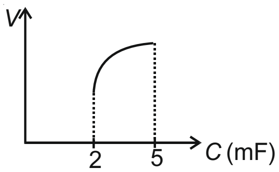
3. In the circuit shown, potential difference V across capacitor X changes its capacitance C is varied from 2 μF to 5 μF. V as a function of C is given by



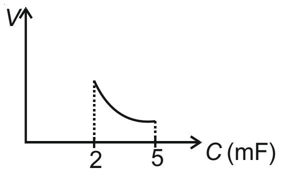
(1)



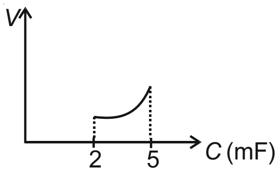
(2)



(3)



(4)



Sol. Answer (3)

$$\frac{Q}{C}+\frac{Q}{5}=V_0$$

$$Q=\frac{5CV_0}{C+5} = \frac{5V_0}{C+5}$$

4. The region between two concentric spheres of radii a and $2a$ has volume charge density, $\rho = \frac{A}{r}$, where A is constant and r is distance from the centre. A point charge Q is placed at the centre. If the potential in the region $a \le r < 2a$ varies linearly then value of A is

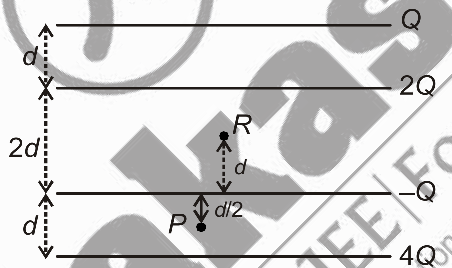
(1) $\frac{Q}{6\pi a^2}$ (2) $\frac{2Q}{3\pi a^2}$ (3) $\frac{2Q}{\pi a^2}$ (4) $\frac{Q}{2\pi a^2}$

Sol. Answer (4)

$$E=\frac{1}{\varepsilon_0}\left(Q+2\pi A(r^2-a^2)\right)$$

$Q = 2\pi Aa^2$

5. Four non-conducting plates each of area A having charges Q, $2Q$, $-Q$ and $4Q$ distributed uniformly are placed parallel to each other as shown in figure. Find potential difference across P and R.

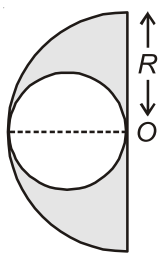


(1) $\frac{3Qd}{2A\varepsilon_0}$ (2) $\frac{Qd}{2A\varepsilon_0}$ (3) $\frac{3Qd}{A\varepsilon_0}$ (4) $\frac{Qd}{A\varepsilon_0}$

Sol. Answer (2)

$$(V_P - V_R) = \left(\frac{2Q}{2A\varepsilon_0}\right)\left(\frac{d}{2}\right) + 0 = \left(\frac{Qd}{2A\varepsilon_0}\right)$$

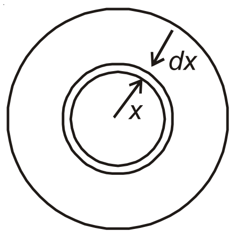
6. Consider a half circular disc of radius R having uniform surface charge density σ. A disc of radius $\frac{R}{2}$ is removed from the half disc as shown in figure. Find potential at O (in V)(centre of complete disc) $\left(\frac{\sigma R}{2\varepsilon_0} = \frac{54\pi}{\pi-2}\text{V}\right)$



(1) 9 (2) 27 (3) 18 (4) 36

Sol. Answer (2)

$$dV = \frac{(k)(\sigma)(2\pi x)dx}{x}$$



$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)(\sigma)(2\pi)\int_0^R dx = \left(\frac{\sigma R}{2\varepsilon_0}\right)$$

Also, potential at edge of disc is given by $\frac{\sigma R}{\pi\varepsilon_0}$.

$$\therefore\ V_0 = \left(\frac{1}{2}\right)\left[\frac{\sigma R}{2\varepsilon_0} - \frac{\sigma R}{\pi\varepsilon_0}\right]$$

$$V_0 = \frac{\sigma R}{2\varepsilon_0}\left[\frac{1}{2} - \frac{1}{\pi}\right]$$

7. A cylindrical air capacitor has outer radius b. What should be the inner radius of capacitor so that for a applied potential difference, the electric field strength very close to the inner surface is minimum?

(1) $\frac{b}{e}$ (2) $\frac{b}{\sqrt{e^2+1}}$ (3) $\frac{b}{2e}$ (4) $\frac{eb}{e+1}$

Sol. Answer (1)

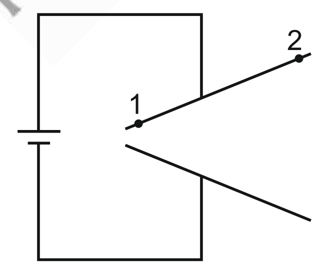
$$Q = CV = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{x}\right)} \times V$$

$$\therefore\quad E \times (2\pi x l) = \frac{Q}{\varepsilon_0} = \frac{2\pi l V}{\ln\left(\frac{b}{x}\right)}$$

$\therefore$ For E to be minimum,

$$x = \frac{b}{e}$$

8. Two metallic plates are arranged as shown in figure forming a non-parallel capacitor. Then



(1) Surface charge density on plates is uniform

(2) Electric field between the plates is uniform

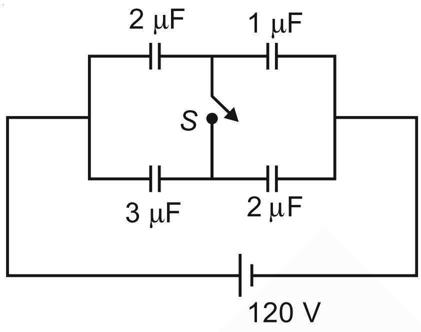
(3) Surface charge density at point 1 is greater than that at point 2

(4) Surface charge density at point 2 is greater than that at point 1

Sol. Answer (3)

Field strength is stronger near point 1, so surface charge density is greater near this point.

9. In the given circuit, the switch S was initially open. If the switch is closed now, then amount of charge flown through the switch S is



(1) 15 μC (2) 30 μC (3) 40 μC (4) 90 μC

Sol. Answer (1)

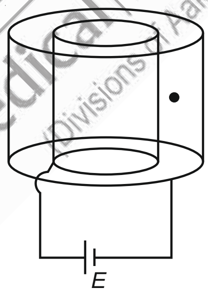
When switch is closed,

then, $q_2 = \dfrac{2}{(2+3)} \times \left[120 \times \dfrac{5\times 3}{(5+3)}\right] = 90\ \mu\text{C}$

$q_1 = \dfrac{1}{(1+2)} \times \left[120 \times \dfrac{5\times 3}{(5+3)}\right] = 75\ \mu\text{C}$

$\therefore\ \Delta q = 90 - 75 = 15\ \mu\text{C}$

10. A cylindrical air capacitor of length ℓ, inner radius R and outer radius $2R$ is connected to a battery of emf E as shown in figure. A particle of charge $+q$ and mass m is released from rest from inner surface of capacitor. The speed of particle when it reaches to the mid-point of plates, is

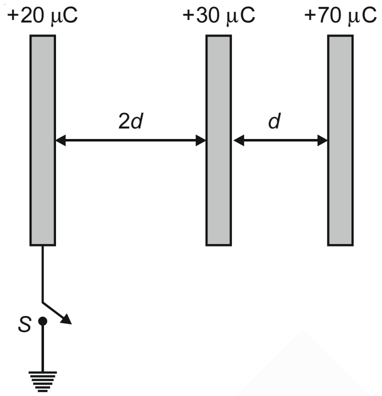


(1) $\sqrt{\dfrac{qE}{m}}$ (2) $\sqrt{\dfrac{qE}{m}\left(\dfrac{\ln 3}{\ln 2}-1\right)}$ (3) $\sqrt{\dfrac{2qE}{m}\left(\dfrac{\ln 3}{\ln 2}-1\right)}$ (4) $\sqrt{\dfrac{qE}{m}(\ln 2+\ln 3)}$

Sol. Answer (3)

$$\frac{1}{2}mv^2 = \frac{Eq}{\ln 2}\int_a^{\frac{3a}{2}} \frac{1}{r}\,dr$$

11. Three identical conducting plates are arranged as shown in figure and they are given charges as indicated. When switch S is closed, the amount of charge flown through the switch is



(1) 20 μC (2) 60 μC (3) 120 μC (4) Zero

Sol. Answer (3)

E inside conductor should be zero.

$\therefore$ Charge on first plate will be –100 μC.

$\therefore$ Δq = 120 μC.

12. A charged particle having charge q and mass m accelerated by a potential difference V_0 enters inside a parallel plate capacitor (parallel to length ℓ of plates) at midpoint of its separation between plates at t = 0. Potential difference across capacitor varies with time t as, $V = a\,d\,t$, where a is constant and d is separation between plates. Angle of deviation as it comes out of plates, is $\frac{\pi}{4}$. Value of $\frac{ma^2l^4}{2qV_0^3}$, will be

(1) 16 (2) 32 (3) 8 (4) 4

Sol. Answer (1)

$$l = V_x t,\ V_x = \sqrt{\frac{2qV_0}{m}}$$

$$V_y = \int \frac{qE}{m}dt$$

$$= \frac{q}{m}\int at\ dt$$

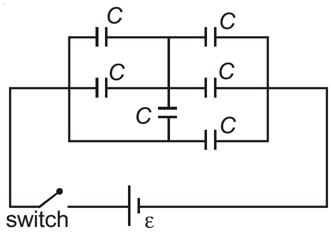
$$V_y = \frac{q}{m}a\frac{t^2}{2}$$

$$V_y = \frac{q}{m}a\frac{l^2}{2V_x^2} = \frac{qal^2}{2m}\left(\frac{m}{2qV_0}\right)$$

$$V_y = \frac{al^2}{4V_0}$$

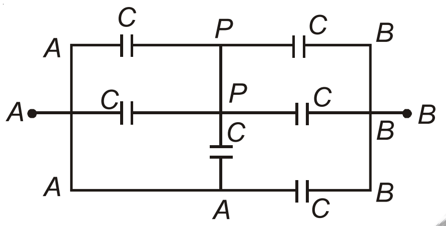
$$\tan\theta = \frac{V_y}{V_x} = \frac{al^2}{4}\sqrt{\frac{m}{2qV_0^3}}$$

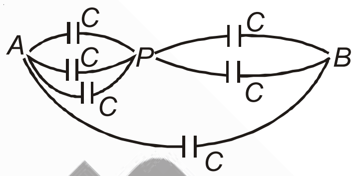
13. Six capacitors each of capacitance 'C' is connected as shown in the figure and initially all the capacitors are uncharged. Now a battery of emf = ε is connected. How much charge will flow through the battery if the switch is closed?



(1) $\dfrac{9\,C\varepsilon}{5}$ (2) $\dfrac{11\,C\varepsilon}{5}$ (3) $\dfrac{13\,C\varepsilon}{5}$ (4) $\dfrac{7\,C\varepsilon}{5}$

Sol. Answer (2)

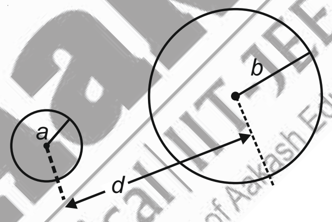




Get $C_{eq} = \dfrac{11C}{5}$

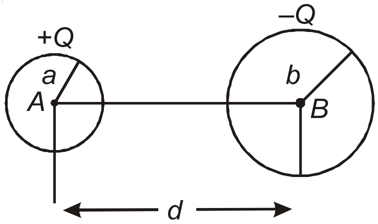
Charge flow = $C_{eq}\,\varepsilon = \dfrac{11C\varepsilon}{5}$.

14. There are two conducting spheres of radius a and b ($b > a$) carrying equal and opposite charges. They are placed at a separation d (>>> a and b). The capacitance of system is



(1) $\dfrac{4\pi\varepsilon_0}{a-b-d}$ (2) $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}-\frac{1}{b}-\frac{1}{d}}$ (3) $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{1}{d}}$ (4) $\dfrac{4\pi\varepsilon_0}{\frac{1}{a}+\frac{1}{b}-\frac{2}{d}}$

Sol. Answer (4)



$$V_B = \frac{KQ}{d} - \frac{KQ}{b}$$

$$V_A = \frac{KQ}{a} - \frac{KQ}{d}$$

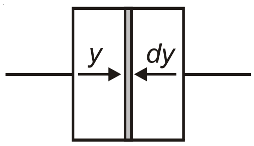
$$V_A - V_B = KQ\left[\frac{1}{a}+\frac{1}{b}-\frac{2}{d}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{b}-\frac{2}{d}\right]$$

or $$\frac{Q}{V_A - V_B} = C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a}+\frac{1}{b}-\frac{2}{d}\right)}$$

15. The area of the plates of a parallel plate capacitor is A and the gap between them is d. The gap is filled with a non–homogeneous dielectric whose dielectric constant varies with the distance 'y' from one plate as : $K = \lambda \sec\left(\dfrac{\pi y}{2d}\right)$, where λ is a dimensionless constant. The capacitance of this capacitor is

(1) $\dfrac{\pi\varepsilon_0\lambda A}{2d}$ (2) $\dfrac{\pi\varepsilon_0\lambda A}{d}$ (3) $\dfrac{2\pi\varepsilon_0\lambda A}{d}$ (4) None of these

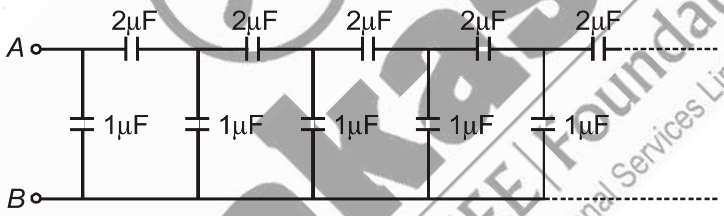
Sol. Answer (1)



$$\int \frac{1}{dC} = \int \frac{dy}{K\varepsilon_0 A} = \int_0^d \frac{dy}{\lambda\varepsilon_0 A \sec\left(\frac{\pi y}{2d}\right)}$$

$$\Rightarrow C = \frac{\lambda\varepsilon_0 A\pi}{2d}$$

16. Find the equivalent capacitance of the infinite ladder shown in figure between the points A & B.

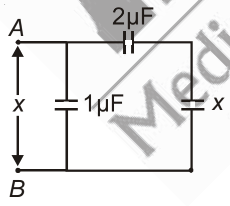


(1) $\left(\sqrt{3}+1\right)\,\mu F$ (2) 2 μF (3) 4 μF (4) 8 μF

Sol. Answer (2)

$$x = \frac{2x}{2+x} + 1$$

(Let $C_{eq} = x$)



$$x = \frac{2x+2+x}{2+x}$$

$\Rightarrow$ $x(2 + x) = 3x + 2$

$\Rightarrow$ $2x + x^2 = 3x + 2 \Rightarrow x^2 - x - 2 = 0$

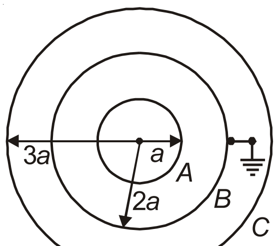
Use

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2} = 2$$

and –1

$x = 2$, $C_{eq} = 2\mu F$

17. Figure shows a system of three concentric metal shells *A*, *B* and *C* with radii *a*, 2*a* and 3*a* respectively. Shell *B* is earthed and shell *C* is given a charge *Q*. Now if shell *C* is connected to shell *A*, then the final charge on the shell *B* is equal to



(1) $-\frac{4Q}{13}$ (2) $-\frac{8Q}{11}$ (3) $-\frac{5Q}{3}$ (4) $-\frac{3Q}{7}$

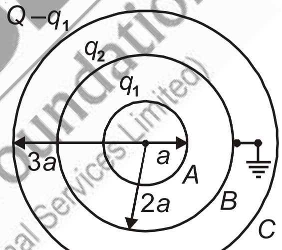
Sol. Answer (2)

Let q_1 and q_2 are charges on *A* and *B* respectively.

From given conditions: Charge on *A* & *C* after connection with wire are q_1 and $Q - q_1$, on *B* charge is q_2

$V_A = V_C$ and $V_B = 0$

$$\Rightarrow \quad V_B = \frac{K(Q-q_1)}{3a} + \frac{Kq_2}{2a} + \frac{Kq_1}{2a} = 0$$



$\Rightarrow \quad 2Q + q_1 + 3q_2 = 0$ ...(i)

Using $V_C = V_A$

$$\frac{K(Q-q_1)}{3a} + \frac{Kq_2}{3a} + \frac{Kq_1}{3a} = \frac{Kq_1}{a} + \frac{K(Q-q_1)}{3a} + \frac{Kq_2}{2a}$$

$\Rightarrow \quad q_1 = -\frac{q_2}{4}$ ...(ii)

Using it in (i), $q_2 = -\frac{8}{11}Q$

SECTION - B

Objective Type Questions (More than one options are correct)

1. A non-conducting sphere has a total charge *Q*, uniformly distributed throughout its volume. The centre of the sphere is at origin and its radius is *R*. Let U_1 be the electrostatic potential energy in the region inside the sphere and U_2 be the electrostatic potential energy in another imaginary spherical shell, having inner radius *R* and outer radius infinity, centred at origin. Select the correct alternative(s).

(1) $U_1 = \frac{Q^2}{8\pi\varepsilon_0 R}$ (2) $U_2 = \frac{Q^2}{8\pi\varepsilon_0 R}$

(3) $U_1 + U_2 = \frac{3}{5} \times \frac{Q^2}{4\pi\varepsilon_0 R}$ (4) $U_1 = \frac{3Q^2}{20\pi\varepsilon_0 R}$

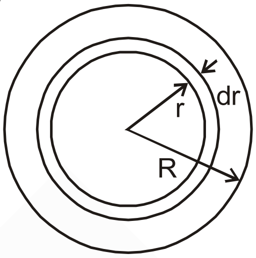
Sol. Answer (2, 3)

Energy density in electric field,

$$U = \frac{1}{2}\varepsilon_0 E^2$$

If r < R,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$$



Energy in the element shown in figure,

$$dU_1 = \frac{1}{2}\varepsilon_0 \cdot \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{R^3}\right)^2 .(4\pi r^2 dr) = \frac{1}{8\pi\varepsilon_0}\cdot\frac{Q^2}{R^6}.r^4.dr$$

$$\therefore\ U_1 = \frac{1}{8\pi\varepsilon_0}\cdot\frac{Q^2}{R^6}\int_{r=0}^{R} r^4.dr = \frac{Q^2}{40\pi\varepsilon_0 R}$$

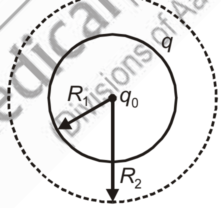
If $r \geq R$, $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}$

Energy in the element, $dU_2 = \frac{1}{2}\varepsilon_0 \cdot \left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\right)^2 4\pi r^2.dr$

$$\therefore\ U_2 = \frac{Q^2}{8\pi\varepsilon_0}\int_R^{\infty}\frac{1}{r^2}.dr = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{R}-\frac{1}{\infty}\right] = \frac{Q^2}{8\pi\varepsilon_0 R}$$

$$\therefore\ U_1 + U_2 = \frac{Q^2}{8\pi\varepsilon_0 R}\left(1+\frac{1}{5}\right) = \frac{3}{5}\times\frac{Q^2}{4\pi\varepsilon_0 R}$$

2. A spherical shell is uniformly charged by a charge q. A point charge q_0 is at its centre, then



(1) The work done by electric forces upon the expansion of shell from radius R_1 to R_2 is $\frac{q\left(q_0 + \frac{q}{2}\right)}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(2) Electric energy of system will decreased upon expansion

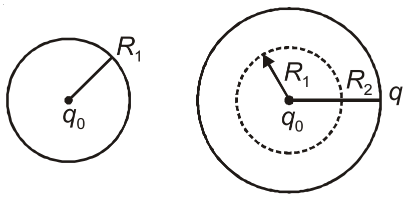
(3) Electric energy of system will ramain same upon expansion of shell

(4) The electric field and energy changes is localised only in spherical region of inner radius R_1 and outer radius R_2

Sol. Answer (1, 2, 4)

Field and energy will change only in spherical layer of inner radius r_1 and outer radius R_2

Let E_1 and E_2 are field intensities in the hatched region at a distance r from the centre of system before and after expansion.



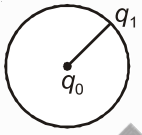
$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{q+q_0}{r^2}, \quad E_2 = \frac{q_0}{4\pi\varepsilon_0 r^2}$$

$$U_i - U_f = \int_{R_1}^{R_2} \frac{\varepsilon_0}{2}(E_1^2 - E_2^2)4\pi r^2 dr$$

$$U_i - U_f = \frac{\varepsilon_0}{2}\int_{R_1}^{R_2} \left\{\left(\frac{q+q_0}{4\pi\varepsilon_0 r^2}\right)^2 - \left(\frac{q_0}{4\pi\varepsilon_0 r^2}\right)^2\right\} 4\pi r^2 dr$$

$$U_i - U_f = \frac{q\left(q_0 + q/2\right)}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

2nd method



$$U_i = \frac{q^2}{2c} + \frac{qq_0}{4\pi\varepsilon_0 R_1}$$

$$U_i = \frac{q^2}{8\pi\varepsilon_0 R_1} + \frac{qq_0}{4\pi\varepsilon_0 R_1}$$

$$U_f = \frac{q^2}{8\pi\varepsilon_0 R_2} + \frac{qq_0}{8\pi\varepsilon_0 R_2}$$

$$W_{\text{electric}} = U_i - U_f = \frac{q(q_0 + q/2)}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

3. The field potential inside a charged ball of radius R and centre at O depends only on distance from its centre as $V(r) = \alpha r^2 + \beta$ when α, β are +ve constant. Now choose correct options

(1) Electric field inside the ball $E_r = -2\alpha r$

(2) Electric flux passing through an imaginary sphere of radius r centre at O will be $-2\pi\alpha r^4$

(3) Volume charge density $\rho(r)$ inside ball is $-6\varepsilon_0\alpha$

(4) Electric energy of charged ball will be $\frac{48}{5}\pi\varepsilon_0\alpha^2 R^5$

Sol. Answer (1, 3, 4)

$V(r) = \alpha r^2 + \beta$

$$E_r = -\frac{\partial v}{\partial r} = -2\alpha r$$

$\phi = E_r . 4\pi r^2 = -8\pi\alpha r^3$

for volume charge density

$$4\pi r^3 E = \frac{q}{\varepsilon_0}$$

$$4\pi d(r^2 E) = \frac{dq}{\varepsilon_0}$$

$dq = \rho 4\pi r^2 dr$

$$4\pi\left(r^2 dE + 2rE_{dr}\right) = \frac{4\pi\rho r^2 dr}{\varepsilon_0}$$

$$r^2(-2\alpha)dr + 2rEdr = \frac{\rho r^2}{\varepsilon_0}dr$$

$$-2\alpha r^2 dr - 4\alpha r^2 dr = \frac{\rho r^2}{\varepsilon_0}dr$$

$$-6\alpha r^2 dr = \frac{\rho r^2}{\varepsilon_0}dr$$

$\rho = -\,6\alpha\varepsilon_0$

For electric energy

(i) Inside $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \times 4\alpha^2 r^2$

$dV = 4\pi r^2 dr$, $dU = 2\varepsilon_0\alpha^2 r^2 4\pi r^2 dr$

$= 8\pi\varepsilon_0\alpha^2 r^4 dr$

$$U = \frac{8}{5}\pi\varepsilon_0\alpha^2 R^5 = \frac{8\pi\varepsilon_0}{5}\alpha^2 R^5$$

(ii) Total charge $= -6\varepsilon_0\alpha \times \frac{4}{3}\pi R^3$

$= -8\varepsilon_0\alpha\pi R^3$

Electric field outside the ball

$$E = \frac{8\pi\varepsilon_0 dR^3}{4\pi\varepsilon_0 r^2} = \frac{2\alpha R^3}{r^2}$$

$$u = \frac{1}{2}\varepsilon_0\frac{4\alpha^2 R^6}{r^4}$$

$$= 2\varepsilon_0\alpha^2 R^6\frac{1}{r^4}$$

$dv = 4\pi r^2 dr$

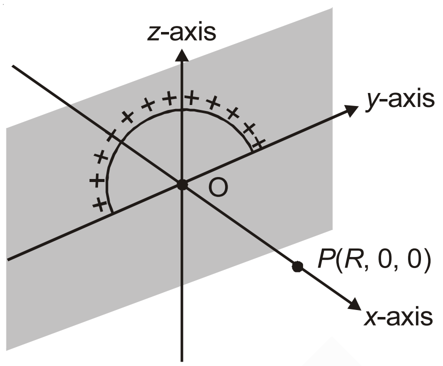
$$dU = 2\varepsilon_0\alpha^2 R^6 \times 4\pi\frac{r^2 dr}{r^4}$$

$$V = 8\pi\varepsilon_0\alpha^2 R^6 \times \left[\frac{1}{R}\right]$$

$V = 8\pi\varepsilon_0\alpha^2 R^5$

$$V_{\text{total}} = \frac{48}{5}\pi\varepsilon_0\alpha^2 R^5$$

4. A thread, bent in the form of a semi circle of radius R, is placed in y-z plane, with its centre at the origin. The charge per unit length of the thread is λ. Select the correct alternatives.



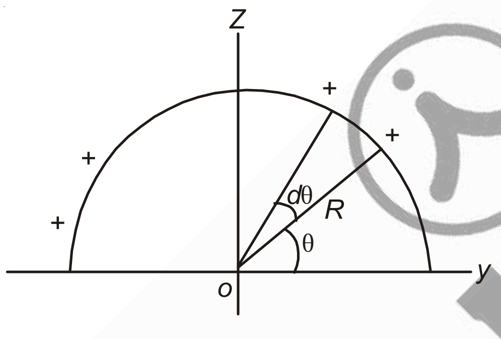
(1) Electric field at O is $\dfrac{-\lambda}{2\pi\varepsilon_0 R}\hat{k}$

(2) Electric potential at O is $\dfrac{\lambda}{4\varepsilon_0}$

(3) Electric field at P lies completely in x-z plane

(4) Electric field at P is along – z-axis only

Sol. Answer (1, 2, 3)



Let us consider an element of angular width $d\theta$ as shown in the figure.

$d\theta = \lambda R d\theta$

Electric field at O due to this element,

$$dE' = \frac{KdQ}{R^2}$$

Due to symmetry about z–axis, the net field will be along negative z–axis only

$$\therefore \quad dE = dE' \sin\theta = \frac{K\lambda R \sin\theta . d\theta}{R^2}$$

$$E = \frac{K\lambda R}{R^2}\int_0^{\pi} \sin\theta . d\theta = \frac{K\lambda}{R}.\left[-\cos\theta\right]_0^{\pi} = \frac{2K\lambda}{R} = \frac{\lambda}{2\pi\varepsilon_0 R}$$

$$\Rightarrow \vec{E} = -\frac{\lambda}{2\pi\varepsilon_0 R}.\hat{k}$$

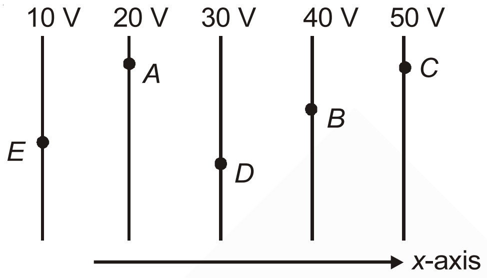
Potential at O due to the element is,

$$dV = \frac{KdQ}{R} = \frac{K\lambda R d\theta}{R} = K\lambda d\theta$$

$$V = K\lambda\int_0^{\pi} d\theta = \frac{1}{4\pi\varepsilon_0}.\lambda.\pi = \frac{\lambda}{4\varepsilon_0}$$

Due to symmetry, components of electric field along y–axis due to various elements at P will be mutually cancelled out. So, only components along x–axis and z–axis will remain. Hence, the electric field at P will lie completely in x–z plane.

5. Figure shows a set of equipotential surfaces. There are a few points marked on them. An electron is being moved from one point to other. Which of the following statements is/are correct?



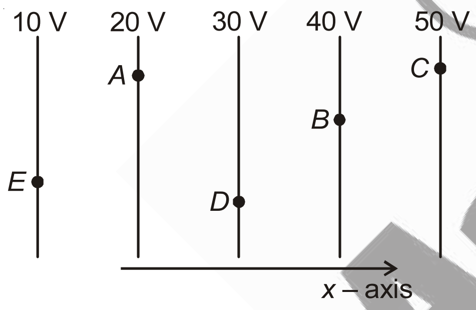
(1) Work done by the electric field, in moving the electron from A to D, is positive

(2) Work done by the electric field, in moving the electron from D to B is same as from B to C

(3) The electric field is directed along + x-axis

(4) As the electron moves from C to E, the potential energy increases

Sol. Answer (1, 2, 4)



Electric field is directed from higher potential to lower potential i.e., along negative x–direction.

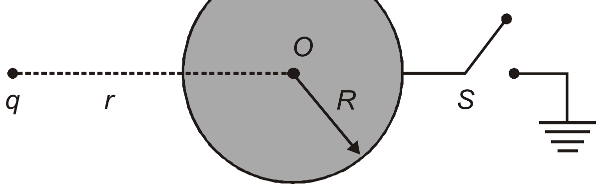
$$W = -e\vec{E}.\vec{S}$$

For displacement A to D, angle between $\vec{E}$ and $\vec{S}$ is obtuse. Hence W is positive.

Let $\vec{S}_{DB}$ and $\vec{S}_{BC}$ represent the displacements from D to B and B to C respectively. $\vec{S}_{DB}$ and $\vec{S}_{BC}$ have same components along $\vec{E}$ (uniform). Hence work done is same for both displacements.

If electrons moves from C to E work is done against electrostatic force. Hence the potential energy increases.

6. A point charge q, is placed at a distance r from the centre of an aluminium sphere, of radius R ($r > R$). Select the correct alternatives



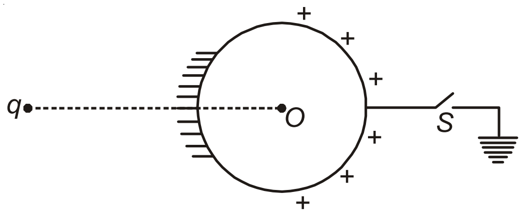
(1) When the switch is open, the potential at O is $\dfrac{q}{4\pi\varepsilon_0 r}$

(2) When the switch is closed, the potential at O is zero

(3) A charge $-q$ is induced on left half of the sphere, when switch is kept open

(4) When switch is closed, sphere acquires a net charge $\dfrac{-qR}{r}$

Sol. Answer (1, 2, 4)



When the switch S is open, the induced charge on sphere is as shown in the figure. Negative charge does not appear on entire left-half of the sphere.

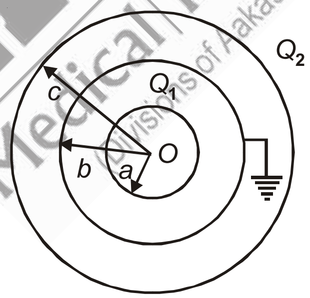
Potential at O due to induced charges is zero. Hence net potential at O is due to q which is given by.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = 0$$

When the switch is closed, the sphere comes to the potential of earth which is zero. Let Q charge appears on it in that case. Then,

$$\frac{1}{4\pi\varepsilon_0}.\frac{q}{r} + \frac{1}{4\pi\varepsilon_0}.\frac{Q}{R} = 0 \Rightarrow Q = -\frac{qR}{r}$$

7. Figure shows three concentric conducting spherical shells of radii a, b and c ($c > b > a$). The innermost and outermost shell are given charges Q_1 and Q_2 respectively, and the middle shell has been earthed. Select the correct alternative(s).



(1) The total charge on middle shell is zero

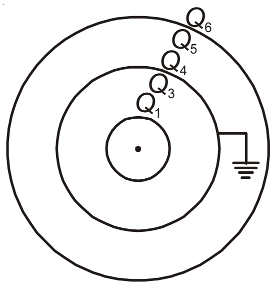
(2) If $|Q_1| > |Q_2|$, total charge on middle shell has opposite sign as that of Q_1

(3) The charge on the outer surface of outermost shell is $Q_2\left[1-\dfrac{b}{c}\right]$

(4) The potential at a point outside the outermost shell is $\dfrac{\left(1-\dfrac{b}{c}\right)Q_2}{4\pi\varepsilon_0 r}$, where r is distance from O

Sol. Answer (2, 3, 4)

Charge on different surfaces have been shown in the figure



From Gauss law,

$Q_1 + Q_3 = 0$

and $Q_1 + Q_3 + Q_4 + Q_5 = 0$

$\Rightarrow \; Q_3 = -Q_1$ ...(i)

$Q_5 = -Q_4$ ...(ii)

Conservation of charge on outer-most shell $\Rightarrow Q_5 + Q_6 = Q_2$ ...(iii)

Potential of middle shell is zero $\Rightarrow \dfrac{KQ_1}{b} + \dfrac{K(Q_3+Q_4)}{b} + \dfrac{K(Q_5+Q_6)}{c} = 0$ ...(iv)

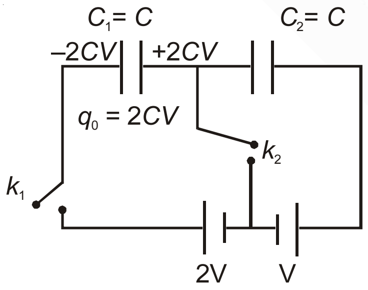
Solving we get, $\dfrac{KQ_1}{b} - \dfrac{KQ_1}{b} + \dfrac{(-KQ_5)}{b} + \dfrac{KQ_2}{c} = 0$

$\Rightarrow \; Q_5 = Q_2\left(\dfrac{b}{c}\right)$ $\qquad \therefore \; Q_6 = Q_2 - Q_5 = Q_2\left(1 - \dfrac{b}{c}\right)$

Potential at a point outside the outer-most shell is,

$$V = \frac{1}{4\pi\varepsilon_0 r}\,[Q_1 + Q_3 + Q_4 + Q_5 + Q_6] = \frac{Q_6}{4\pi\varepsilon_0 r} = \frac{\left(1-\dfrac{b}{c}\right)Q_2}{4\pi\varepsilon_0 r}$$

8.



Initially C_1 has charge $2CV$ and k_1 & k_2 are open.

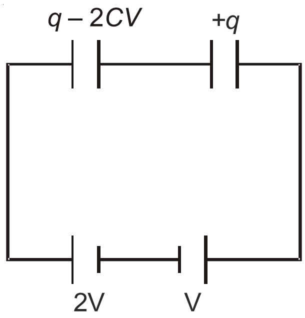
(1) If k_1 is closed and k_2 remains open, magnitude of charge flown through k_1 will be $\dfrac{3CV}{2}$

(2) For case (1), heat dissipated in circuit would be $\dfrac{9CV^2}{4}$

(3) Now k_2 is also closed (k_1 already closed), magnitude of charges flown through k_2 would be $5CV$

(4) Heat dissipated in circuit in case (3) would be $8.75CV^2$

Sol. Answer (1, 2, 3)



$$-\left(\frac{q-2CV}{C}\right)-\frac{q}{C}+V=0$$

$$-\frac{q}{C}+2V-\frac{q}{C}+V=0$$

$$-\frac{2q}{C}+3V=0$$

$$-2q = -3CV$$

$$q=+\frac{3CV}{2}$$

Work done by cells = $\frac{3CV}{2}\times V=+\frac{3CV^2}{2}$

Initial energy stored in capacitor $=\frac{1}{2}C.(2V)^2=2CV^2$

Finally stored energy :

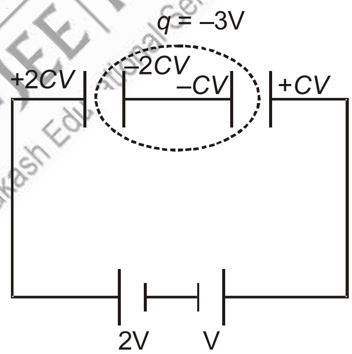
$$V_f=\frac{1}{2}C\left(\frac{V}{2}\right)^2+\frac{1}{2}C\times\left(\frac{3V}{2}\right)^2=\frac{1}{2}C.\frac{V^2}{4}+\frac{1}{2}C\times\frac{9}{4}V^2=\frac{5}{4}CV^2$$

Heat dissipated = $2CV^2+\frac{3}{2}CV^2-\frac{5}{4}CV^2=\frac{9}{4}CV^2$

Now k_2 is also closed :

Charge flown through k_2 = $-3CV-2CV=-5CV$

Magnitude = $5CV$



$$V_f=\frac{1}{2}C(2V)^2+\frac{1}{2}CV^2=\frac{5}{2}CV^2$$

Work done by cell

$$(2V)=(2V)\times\frac{5}{2}CV=5CV^2$$

Work done by cell

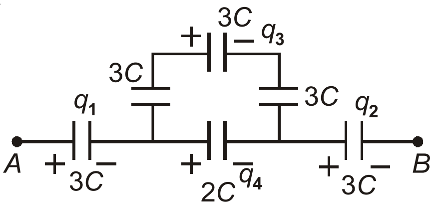
$$(V)=V\times\frac{5CV}{2}=\frac{5CV^2}{2}$$

Total work done $=\frac{15}{2}CV^2$

$$V_f=\frac{5}{2}CV^2$$

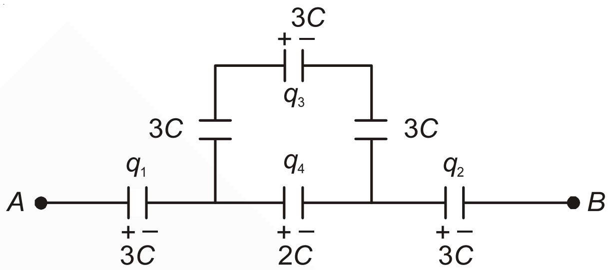
Heat dissipated $=\frac{25}{4}CV^2$

9. Consider the electrical circuit shown. A potential difference V exists between A and B. The charges on various capacitors are shown. Select the correct relationship(s).



(1) $q_1 = q_2$ (2) $q_3 = q_4$ (3) $q_1 = q_3 + q_4$ (4) $q_3 = \dfrac{q_4}{2}$

Sol. Answer (1, 3, 4)



Capacitors with charges q_1 and q_2 are in series.

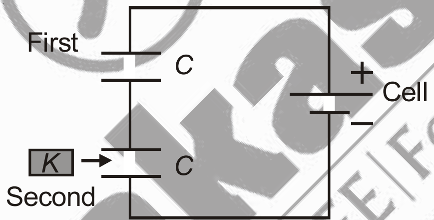
$\therefore\ q_1 = q_2$

Capacitors with charge q_1 are in series with a parallel combination of capacitors of charges q_3 and q_4

$\therefore\ q_1 = q_3 + q_4$

and $\dfrac{q_3}{3C/3} = \dfrac{q_4}{2C} = q_3 \Rightarrow \quad q_3 = \dfrac{q_4}{2}$

10. When a dielectric slab is inserted between the plates of one of the two identical capacitors in figure, which of the following properties increase?



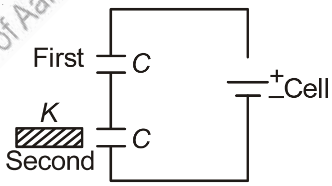
(1) Charge on the first capacitor

(2) Charge on the second capacitor

(3) Potential difference across the first capacitor

(4) Potential difference across the second capacitor

Sol. Answer (1, 2, 3)



Before dielectric is inserted : $C_{net} = \dfrac{C}{2}$

Charge on each capacitor is $\dfrac{C}{2}V$

After dielectric is inserted $C_{net} = \dfrac{KC}{1+K}$

We know $C_{net} > \dfrac{C}{2}$ if $K > 1$, which is true.

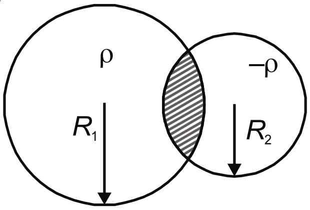
$\therefore$ Net capacitance increases and hence charge. Since capacitors are in series, therefore charge of both capacitors increases.

Potential difference across first capacitor, initially = $\dfrac{V}{2}$

Potential difference across first capacitor, finally = $\dfrac{Q}{C} = \dfrac{KV}{1+K}$

$\because\ K > 1$ therefore $\dfrac{KV}{1+K} > \dfrac{V}{2}$

11. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,



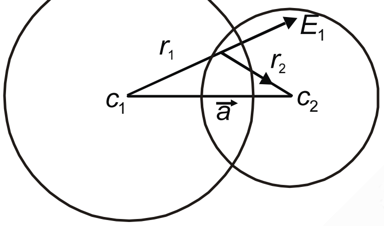
(1) The electrostatic field is zero

(2) The electrostatic potential is constant

(3) The electrostatic field is constant in magnitude

(4) The electrostatic field has same direction

Sol. Answer (3, 4)



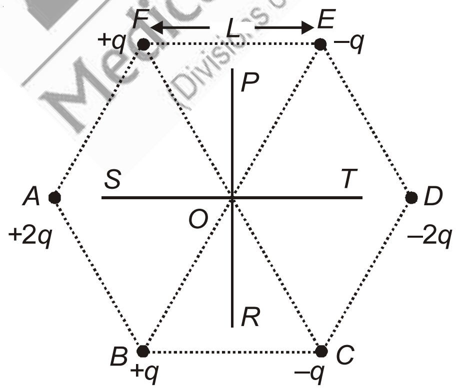
$\vec{E}_1 = \frac{\rho}{3\varepsilon_0}\vec{r}_1 \qquad \vec{E}_2 = -\frac{\rho}{3\varepsilon_0}\vec{r}_2$

$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{a}$

Electric field of constant magnitude and direction.

12. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in the figure.

Given that $K = \frac{1}{4\pi\varepsilon_0}\frac{q}{L^2}$, which of the following statement(s) is(are) correct?



(1) The electric field at O is $6K$ along OD

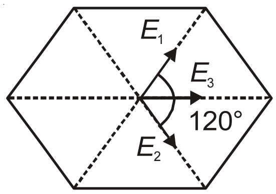
(2) The potential at O is zero

(3) The potential at all points on the line PR is same

(4) The potential at all points on the line ST is same

Sol. Answer (1, 2, 3)

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$



$E_1 = E_2 = 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{q}{L^2}$

$E_3 = 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{2q}{L^2}$

Net electric field at *O*.

$E_3 + 2E_1 \cos 60° = 6$ K

Potential at *O* is zero.

Any point on line *PR* is equidistant from a pair of equal and opposite charge.

13. A spherical metal shell *A* of radius R_A and a solid metal sphere *B* of radius $R_B(< R_A)$ are kept far apart and each is given charge '+*Q*'. Now they are connected by a thin metal wire. Then

(1) $E_A^{\text{inside}} = 0$ (2) $Q_A > Q_B$ (3) $\frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$ (4) $E_A^{\text{on surface}} < E_B^{\text{on surface}}$

Sol. Answer (1, 2, 3, 4)

After connection,

$V_A = V_B$

$\Rightarrow \frac{q_A}{R_A} = \frac{q_B}{R_B}$

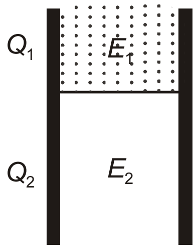
$E_{A(\text{inside})} = 0$

$\Rightarrow q_A > q_B$

$\frac{\sigma_A}{\sigma_B} = \frac{q_A}{q_B}\times\frac{R_B^{\ 2}}{R_A^{\ 2}} = \frac{R_B}{R_A}$

$\frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} = \frac{R_B}{R_A}$

14. A parallel plate capacitor has a dielectric slab of dielectric constant *K* between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is *C* while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects.



(1) $\frac{E_1}{E_2} = 1$ (2) $\frac{E_1}{E_2} = \frac{1}{K}$ (3) $\frac{Q_1}{Q_2} = \frac{3}{K}$ (4) $\frac{C}{C_1} = \frac{2+K}{K}$

Sol. Answer (1, 4)

$$C = C_1 + C_2$$

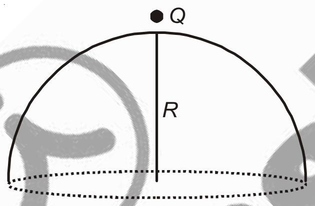
$$C_1 = \frac{K\varepsilon_0 A/3}{d},\ C_2 = \frac{\varepsilon_0 2A/3}{d}$$

$$\Rightarrow\ C = \frac{(K+2)\varepsilon_0 A}{3d}$$

$$\Rightarrow\ \frac{C}{C_1} = \frac{K+2}{K}$$

Also, $E_1 = E_2 = \frac{V}{d}$, where V is potential difference between the plates.

15. A point charge $+Q$ is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?



(1) The electric flux passing through the *curved* surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

(2) The component of the electric field normal to the flat surface is constant over the surface

(3) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$

(4) The circumference of the flat surface is an equipotential

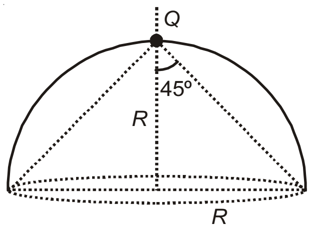
Answer (1, 4)

Sol. Net flux through curved surface and flat surface = 0

$$\Rightarrow\ \phi_{Curved} = -\phi_{Plane}$$

$$= -\left[\frac{Q}{2\varepsilon_0}(1-\cos\theta)\right]$$

$$= -\left[\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)\right]$$



The circumference points are equidistant from Q.

$\therefore$ All points will be at the same potential.

$\therefore$ Options (1) and (4) are correct.

SECTION - C

Linked Comprehension Type Questions

Comprehension-I

A particle *P*, of charge *q* and mass *m*, is placed at a point in gravity free space and it is free to move. Another particle *Q*, of same charge and mass, is projected from a distance *r* from *P* with an initial speed v_0 towards *P*. Initially the distance between *P* and *Q* decreases and then increases.

1. What are the speeds of particles *P* and *Q*, when their separation is minimum?

(1) $\frac{v_0}{2}, \frac{v_0}{2}$ (2) 0, v (3) $\frac{v_0}{3}, \frac{2v_0}{3}$ (4) 0, 0

Sol. Answer (1)

When the separation is minimum, *P* and *Q* both move with same velocities. Applying conservation of linear momentum principle,

$mv_0 + 0 = (m + m)\,v \qquad v = \frac{v_0}{2}$

2. The potential energy of the system of particles *P* and *Q*, at closest separation is

(1) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}+\frac{1}{2}mv_0^2$ (2) $\frac{-1}{4\pi\varepsilon_0}\frac{q^2}{r}+\frac{1}{2}mv_0^2$ (3) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r}+\frac{mv_0^2}{4}$ (4) $\frac{-1}{4\pi\varepsilon_0}\frac{q^2}{r}+\frac{mv_0^2}{4}$

Sol. Answer (3)

Kinetic energy of system at minimum separation = $2\times\frac{1}{2}m.\frac{v_0^2}{4}=\frac{mv_0^2}{4}$

From conservation of mechanical energy principle, the potential energy at minimum separation,

$$U=\left(\frac{1}{4\pi\varepsilon_0}.\frac{q^2}{r}+\frac{1}{2}mv_0^2\right)-\frac{mv_0^2}{4} = \frac{1}{4\pi\varepsilon_0}.\frac{q^2}{r}+\frac{mv_0^2}{4}$$

3. If *U* is the potential energy of the system of *P* and *Q* at any instant, then

(1) *U* first increases, becomes maximum and then decreases

(2) *U* first decreases, becomes minimum and then decreases

(3) *U* increases continuously

(4) *U* decreases continuously

Sol. Answer (1)

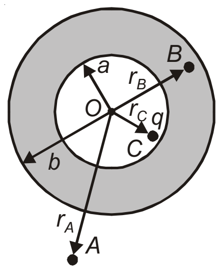
First two particles come closer. So work is done against electrostatic force. This increases potential energy.

When separation is minimum, potential energy is maximum.

When the two particles start receding away, work is done by electrostatic force. So, potential energy decreases.

Comprehension-II

A point charge q is placed off centre at point C inside a thick spherical shell of inner radius a and outer radius b. Shell is neutral and conducting.



$OA = r_A$

$OB = r_B$

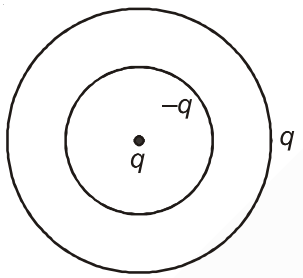
$OC = r_C$

1. Potential at centre of shell is

(1) $\dfrac{Kq}{r_C}$ (2) $\dfrac{Kq}{r_C} - \dfrac{Kq}{a}$

(3) $\dfrac{Kq}{r_C} - \dfrac{Kq}{a} + \dfrac{Kq}{b}$ (4) Cannot be calculated

Sol. Answer (3)



Total charge induced on the inner surface equal $-q$ and on outer surface = q

2. Electric field at B due to induced charges on the inner surface is

(1) Zero (2) $\dfrac{Kq}{(CB)^2}$, towards C (3) $\dfrac{Kq}{CB^2}$, towards B (4) Cannot be determined

Sol. Answer (2)

Net field at 'B' equal zero, field due to charges induced on outer surface of shell at 'B' is zero. Thus, field at 'B' due to induced charges equals to negative of field at 'B' due to point charge q.

3. Choose the incorrect option

(1) Field at A is independent of the position of point charge q inside the cavity

(2) Field at B is independent of the position of point charge q inside the cavity

(3) Field at A is independent of the magnitude of point charge q

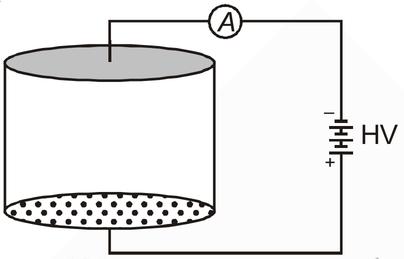
(4) Field at B is independent of the magnitude of point charge q

Sol. Answer (3)

$$E_A = \frac{1}{4\pi\varepsilon_0}\frac{r}{r_A^2}$$

Comprehension-III

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius $r << h$. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. (Ignore gravity)



1. Which one of the following statements is correct?

   (1) The balls will execute simple harmonic motion between the two plates

   (2) The balls will bounce back to the bottom plate carrying the same charge they went up with

   (3) The balls will stick to the top plate and remain there

   (4) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

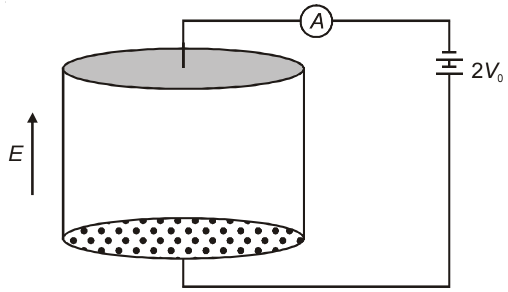
2. The average current in the steady state registered by the ammeter in the circuit will be

   (1) Proportional to V_0^2 (2) Proportional to the potential V_0

   (3) Zero (4) Proportional to $V_0^{1/2}$

Solution of Comprehension-III

$$E = \frac{2V_0}{h}$$



When sphere will be at bottom, charge is $q = KV_0$

When it will touch the top plate, charge would be $q' = -KV_0$

Current $i = \frac{q}{T}$

Where T is time taken by spheres to move from bottom to top.

$$h = \frac{1}{2}\left(\frac{2V_0 q}{hm}\right)T^2$$

$$\Rightarrow \quad h = \left(\frac{KV_0^2}{mh}\right)T^2$$

$$T^2 = \frac{mh^2}{KV_0^2} \Rightarrow T = \left(\sqrt{\frac{m}{K}}\right)\frac{h}{V_0}$$

$$i = \frac{KV_0}{\left(\sqrt{\frac{m}{K}}\frac{h}{V_0}\right)} \Rightarrow \boxed{i \propto V_0^2}$$

1. Answer (4)
2. Answer (1)

SECTION - D

Matrix-Match Type Questions

1. Match the following :

Column I	**Column II**
(A) $E \propto \frac{1}{r^2}$	(p) Point charge
(B) $E \propto r$	(q) Spherically symmetric charge distribution
(C) $V \propto \frac{1}{r}$	(r) Long line charge
(D) $V_2 - V_1 = f\left(\frac{r_2}{r_1}\right)$	(s) Plane sheet of charge
(E) $V_2 - V_1 \propto r_2 - r_1$	(t) Electric dipole

Sol. Answer A(p, q), B(q), C(p, q), D(r), E(s)

For a point charge,

$$E = \frac{KQ}{r^2} \quad \text{and} \quad V = \frac{KQ}{r}$$

A spherically symmetric charge distribution behaves like a point charge for outside points.

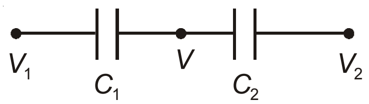
For a long line of charge , $V_2 - V_1 = \frac{-1}{2\pi\varepsilon_0}\ln\frac{r_2}{r_1}$

For a plane sheet of charge, $V_2 - V_1 = -\frac{\sigma}{2\varepsilon_0}(r_2 - r_1)$

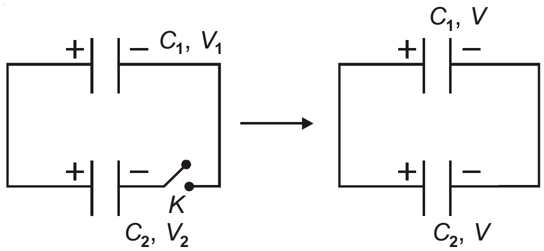
2. Match the following:

Column I — **Column II**

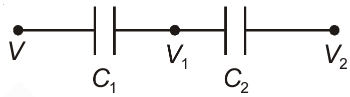
(A) $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

(p) 

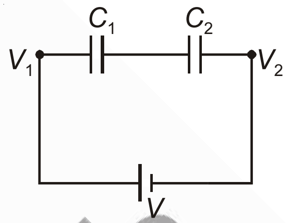
(B) $\Delta U = \dfrac{1}{2}\dfrac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$

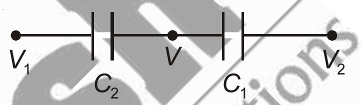
(q) 

(C) $\dfrac{(V-V_1)}{C_2} = \dfrac{V_2-V}{C_1}$

(r) 

(D) Total energy = $\dfrac{1}{2}\dfrac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$

(s) 

(t) 

Sol. Answer A(p, q), B(q), C(p, q), D(p)

In figure represented by (*p*),

$q = C_1(V_1 - V) = C_2(V - V_2)$

$\Rightarrow \dfrac{V-V_1}{C_2} = \dfrac{V_2-V}{C_1}$

Also, $V = \dfrac{C_1V_1+C_2V_2}{C_1+C_2}$

$C_{eq.} = \dfrac{C_1C_2}{C_1+C_2}$

$\therefore\ U = \dfrac{1}{2}\ C_{eq.}\ (V_1 - V_2)^2 = \dfrac{1}{2}\dfrac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$

In figure represented by (*q*)

$V = \dfrac{C_1V_1+C_2V_2}{C_1+C_2} \Rightarrow \dfrac{V-V_1}{C_2} = \dfrac{V_2-V}{C_1}$

$\Delta U = \dfrac{1}{2}\dfrac{C_1C_2}{C_1+C_2}\cdot(V_1-V_2)^2$

3. If some charge is given to a conducting object of any shape, then match the following colums, under electrostatic condition

Column I	Column II
(A) $\vec{E}$ at a point within conducting material	(p) Must be zero
(B) $\vec{E}$ at a point outside the material	(q) May be Zero
(C) $\vec{E}$ at a point in the cavity having no charge	(r) May be uniform but not zero
(D) $\vec{E}$ at a point in the cavity having some charge	(s) May be non-uniform
	(t) Cannot predict

Sol. Answer A(p), B(r, s), C(p), D(s)

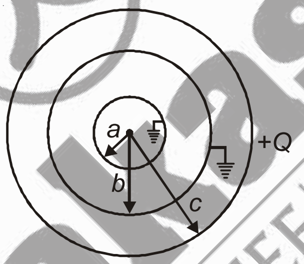
Inside the conducting region, $E = 0$

In a cavity without charge $E = 0$, but in a cavity containing charge, $E \neq 0$.

SECTION - E

Assertion-Reason Type Questions

1. The following figure shows three concentric conducting shells. The two inner shells are grounded and a charge Q is given to the outermost shell.



STATEMENT-1 : The net charge appearing on the innermost shell is $\frac{-Qa}{c}$.

and

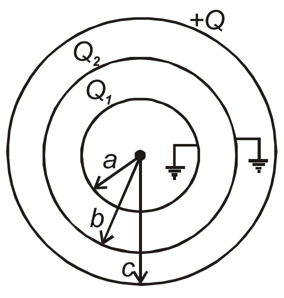
STATEMENT-2 : The net charge appearing on the central shell is $\frac{-Qb}{c}$.

Sol. Answer (4)

Let Q_1 and Q_2 be the charges on inner-most and middle – shells respectively. Then

$$\frac{KQ}{c} + \frac{KQ_2}{b} + \frac{KQ_1}{a} = 0 \quad \text{.... (i)}$$

$$\frac{KQ_1}{b} + \frac{KQ_2}{b} + \frac{KQ}{c} = 0 \quad \text{.... (ii)}$$



(i) – (ii) $\Rightarrow \frac{KQ_1}{a} - \frac{KQ_1}{b} = 0$

$\Rightarrow Q_1 = 0$

$\Rightarrow Q_2 = -\frac{Qb}{c}$

2. STATEMENT-1 : When some charge is given to an irregular shaped conductor, it distributes itself so that charge density is same everywhere.

and

STATEMENT-2 : A conductor has to be equipotential under static condition.

Sol. Answer (4)

Statement-1 is wrong. Because charge density for a conductor, $\sigma \propto \frac{1}{r}$, where r is radius of curvature.

3. STATEMENT-1 : Inside an isolated uniformly charged hollow spherical shell, electrostatic potential is constant everywhere.

and

STATEMENT-2 : Electric field inside a isolated uniformly charged hollow spherical shell is always zero.

Sol. Answer (1)

In an ISOLATED hollow sphere of charge, electric field is zero.

4. STATEMENT-1 : Potential difference between two concentric charged metal spherical shells is directly proportional to the charge of inner sphere.

and

STATEMENT-2 : The field inside the space between the two spheres described above will be only due to charge of inner sphere.

Sol. Answer (1)

In the region between the two shells, $E \propto q_{inner}$.

5. STATEMENT-1 : At a point electrostatic field is zero, then potential at that point may be zero.

and

STATEMENT-2 : Negative of the potential gradient is equal to electrostatic field.

Sol. Answer (2)

$E = 0 \Rightarrow dV = 0$ this does not imply $V = 0$.

6. STATEMENT-1 : Total charge of a conductor can be transferred to the other conductor by connecting them with a conducting wire.

and

STATEMENT-2 : On connecting the two conductors, their potentials become equal.

Sol. Answer (2)

When one conductor is kept inside other and connected, complete charge transfer takes place.

7. STATEMENT-1 : If distance between plates of charged and isolated parallel plate capacitor increases then force between plates decreases.

and

STATEMENT-2 : Force between two point charges is inversely proportional to the square of distance of separation.

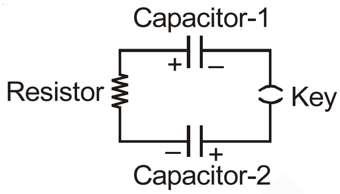
Sol. Answer (4)

Force is independent of separation, in a parallel plate capacitor.

SECTION - F

Integer Answer Type Questions

1. Consider the circuit shown with key opened. Capacitor-1 (3 μF) is charged to potential difference of 9 V. Capacitor - 2 (6 μF) is charged to potential difference of 3 *V*. The key is now closed. What will be the potential difference (in V) across capacitor-1 in steady state?



Sol. Answer (1)

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$

$$= \frac{3\times 9 - 6\times 3}{3+6}$$

$$= 1$$

2. Suppose that electric potential due to a small charge configuration varies inversely with square of distance from the charge distribution. Electric field intensity will vary inversely with what power of distance from the charge configuration in this case?

Sol. Answer (3)

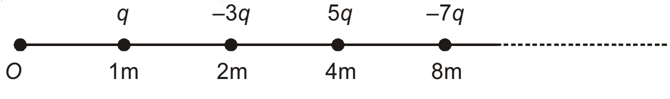
$$E = -\frac{dv}{dr}$$

3. Consider an electric dipole located in uniform electric field in stable equilibrium position. It is now slowly rotated to the position of unstable equilibrium. Work done by the external agent in the process is numerically how many times the maximum electric torque experienced by the dipole during the process?

Sol. Answer (2)

$$W = pE\left[\cos\theta_1 - \cos\theta_2\right], \tau = pE\sin\theta$$

4. Find the magnitude of electric potential at the origin due to following charge distribution (If q = 1 nC).



Sol. Answer (2)

$$E = \frac{1}{4\pi\varepsilon_0}q\left[\frac{1}{1}+\frac{-3}{2}+\frac{5}{2^2}+\frac{-7}{2^3}+\cdots\cdots\infty\right] = \frac{q}{4\pi\varepsilon_0}S_\infty \qquad \because\left[S_\infty = \frac{a}{1-r}+\frac{dr}{(1-r)^2} = \frac{2}{9}\right] \text{where } a = 1,\ r = \frac{-1}{2},\ d = 2$$

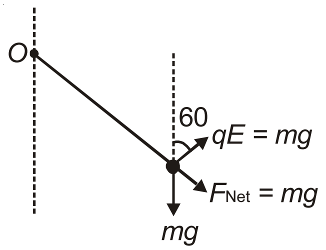
$$= 9 \times 10^9 \times 10^{-9} \times \frac{2}{9} \text{ V/m}$$

$$= 2 \text{ V/m}$$

5. The bob of mass m, charge q is circulating in a vertical circle of radius R. With the help of a string. If the maximum speed of the bob is V then the period of revolution is T_1. If an electric field of magnitude (mg/q) is setup which makes an angle 60° with upward vertical. Again the bob is circulating in same circle and its maximum speed is also V, then the period of revolution is T_2. Find then T_1/T_2.

Sol. Answer (1)

$\therefore$ Period of revolution is same only position of bob when speed is maximum will change.



6. A uniform charged shell is reassembled in the form of a sphere of same radius but charge uniformly distributed through out of its volume. Find the ratio of initial potential energy to work required for it.

Sol. Answer (5)

$$\text{Required ratio} = \frac{\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{R}}{\frac{3}{5}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{R} - \frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{R}} = \frac{\frac{1}{2}}{\frac{3}{5}-\frac{1}{2}} = 5$$

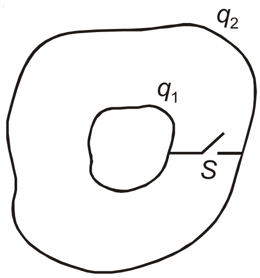
7. Three point charges q, $q/3$ and $16q$ have to be arranged on positive x-axis within 20 cm, so that system's potential energy is minimum. Find the distance of charge $q/3$ from charge q.

Sol. Answer (4)

For minimum potential energy the position of q and $16q$ are 0 and 20 cm and position of $\left(\frac{q}{3}\right)$ is that place where field due to q and $16q$ is zero.

$$\therefore \quad x = \frac{20\sqrt{16q} + \sqrt{q} \times 0}{\sqrt{16q} + \sqrt{q}} = \frac{20}{5} = 4 \text{ cm.}$$

8. Two conducting closed surfaced shells are shown in figure. They are connected by wire through switch S. If q_1 and q_2 charges are given to inner and outer shells respectively. The switch is closed find the ratio of final charge to initial charge of inner shell.



Sol. Answer (0)

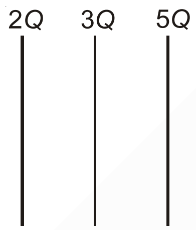
After switch is closed then all charge of inner shell will flow to the other shell.

9. There are three conducting and concentric spherical shells of radii R, $2R$ and $3R$. The charge on inner and outer most shells are q qnd $3q$ while middle shell is earthed. Find the ratio of number of field lines emitted by outermost shell and inner shell.

Sol. Answer (3)

Since middle shere is earthed so there is no any change of charge on other shell. Required ratio = charge ratio.

10. Three charged conducting plates are separated by small distances as shown in figure. The charges on the plates are shown. Find the ratio of charge on right surface and left surface of the middle plate.



Sol. Answer (0)

The charge distribution is shown here