Geometric Progression 10

STUDY NOTES

- A sequence $a_1, a_2, a_3, \ldots a_n$ is called a geometric progression (GP), if each term is non-zero and

  $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_4}{a_3} \ldots\ldots\ldots \frac{a_n}{a_{n-1}} = \text{constant.}$

  The constant is called the common ratio.

- For a GP with first term a, common ratio r, the nth term is given by $T_n = ar^{n-1}$
- For a GP with first term a, common ratio r and last term l, the nth term from the end is given by $\frac{l}{r^{n-1}}$.
- It is always convenient to take (i) three numbers in GP as $\frac{a}{r}$, a, ar (ii) the numbers in GP as a, ar, ar^2, ar^3 …, when their product is not given.
- Sum of n terms of a GP with first term a and common difference r is given by

$$S_n = \begin{cases} na, \quad when\ r = 1 \\ \dfrac{a\left(1-r^n\right)}{1-r}, \quad when\ r < 1 \\ \dfrac{a\left(r^n-1\right)}{r-1}, \quad when\ r > 1. \end{cases}$$

- If a GP contains n terms with first term a, common ratio r and last term l, then

$$S_n = \begin{cases} \dfrac{a-lr}{1-r}, \quad when\ r < 1 \\ \dfrac{lr-a}{r-1}, \quad when\ r > 1. \end{cases}$$

QUESTION BANK

A. Multiple Choice Questions [1 Mark]

Choose the correct option:

1. A list of numbers in which each term is obtained by ___________ its preceding term by a fixed (non-zero) number, except the first term, is called a geometric progression.

(a) adding (b) multiplying (c) subtracting (d) dividing

2. The list of numbers $a_1, a_2, a_3, \ldots$ forms a G.P. if and only if _________ $= r$, a fixed number.

(a) $\frac{a_{n+1}}{a_n}$ (b) $\frac{a_n}{a_{n+1}}$ (c) $\frac{a_{n-1}}{a_n}$ (d) All of these

3. If $a, ar, ar^2, \ldots$ is a G.P., then its general term is denoted by $T_n =$

(a) ar^n (b) ar^{n+1} (c) ar^{n-1} (d) ar

4. If a finite G.P. a, ar, ar^2, ... contains n terms and its last term is l, then l =

(a) $\frac{a}{r^{n-1}}$ (b) ar^{n-1} (c) ar^{n+1} (d) ar^n

5. If a, ar, ar^2, ... is a finite G.P. consisting of m terms, then nth term from end is :

(a) ar^{m-n} (b) ar^m (c) ar^{m+n} (d) ar^{m+n-2}

6. The numbers a, b, c are in G.P. if :

(a) $a^2 = bc$ (b) $b^2 = ac$ (c) $c^2 = ab$ (d) $a - b = b - c$

7. If a, ar, ar^2, ... is a finite G.P. with last term l, then nth term from end =

(a) $\left(\frac{1}{r}\right)^{n-1}$ (b) $l\left(\frac{1}{r}\right)^{n}$ (c) $l\left(\frac{1}{r}\right)^{n-1}$ (d) lr^{n-1}

8. If the product of numbers in G.P. is given, then four numbers are taken as __________ respectively.

(a) $ar^2, ar^3, ar, \frac{a}{r}$ (b) $\frac{a}{r^2}, \frac{a}{r}, ar, ar^3$ (c) $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$ (d) $\frac{a}{r^2}, \frac{a}{r}, ar, ar^2$

9. The list of numbers $\frac{1}{9}, \frac{-1}{3}, 1, -3, \ldots$ is a G.P. with r =

(a) –3 (b) $\frac{-1}{3}$ (c) 3 (d) $\frac{1}{3}$

10. The 11th term of the G.P. $\frac{1}{8}, \frac{-1}{4}, 2, -1, \ldots$ is :

(a) 64 (b) –64 (c) 128 (d) 1024

11. The 5th term from the end of the G.P. 2, 6, 18, ..., 13122 is :

(a) 162 (b) 486 (c) 54 (d) 1458

12. If k, $2(k + 1)$, $3(k + 1)$ are three consecutive terms of a G.P., then the value of k is :

(a) –1 (b) – 4 (c) 1 (d) 4

13. ___________ term of the G.P. 18, 12, 8, ... is $\frac{512}{729}$.

(a) 12th (b) 10th (c) 9th (d) 11th

14. If x, $2y$, $3z$ are in A.P. where the distinct numbers x, y, z are in G.P., then the common ratio of the G.P. is:

(a) 3 (b) $\frac{1}{3}$ (c) –3 (d) None of these

15. If the first and the nth terms of a G.P. are a and b respectively, and if p is the product of first n terms, then p^2 =

(a) ab (b) ab^n (c) $(ab)^n$ (d) None of these

16. If a is the first term and r is the common ratio of a G.P., then S_n = __________, where $r \neq 1$.

(a) $\frac{a(1 - r^{n-1})}{1 - r}$ (b) $\frac{a(1 - r^{n})}{1 - r}$ (c) $\frac{a(1 - r^{n+1})}{1 - r}$ (d) ar^{n-1}

17. If l is the last term of a G.P with a as the first term and r as the common ratio, then S_n = ___________, $r \neq 1$.

(a) $\frac{a + lr}{1 + r}$ (b) $\frac{a - lr}{1 + r}$ (c) $\frac{a - lr}{1 - r}$ (d) $\frac{a + lr}{1 - r^2}$

18. The sum of the first 5 terms of the list of numbers 3, 6, 12, ... is :

(a) 93 (b) 31 (c) 96 (d) 33

19. The sum of the first 8 terms of the series $1 + \sqrt{3} + 3 + \ldots$ is :

(a) $40(\sqrt{3} - 1)$ (b) $40(\sqrt{3} + 1)$ (c) $80(\sqrt{3} - 1)$ (d) $\frac{40}{(\sqrt{3} + 1)}$

20. The sum of first 6 terms of the G.P. $1, -\frac{2}{3}, -\frac{4}{9}, \ldots$ is :

(a) $\frac{-133}{243}$ (b) $\frac{133}{243}$ (c) $\frac{793}{1215}$ (d) $\frac{667}{1215}$

21. If the sum of the G.P. 1, 4, 16 .... is 341, then the number of terms in the G.P. is :

(a) 8 (b) 6 (c) 5 (d) 10

22. ____________ terms of the G.P. $3, \frac{3}{2}, \frac{3}{4}, ...$ are needed to give the sum $\frac{3069}{512}$.

(a) 9 (b) 10 (c) 11 (d) 12

23. Given a G.P. with $a = 729$ and 7th term as 64, then $S_7 =$

(a) 2059 (b) 462 (c) –2061 (d) 2060

24. $\sum_{k=1}^{11}\left(2+3^k\right) =$

(a) $\frac{41 + 3^{10}}{2}$ (b) $\frac{41 + 3^{12}}{2}$ (c) $41 + 3^{12}$ (d) $\frac{44 + 3^{12}}{2}$

Answers :

1. (b)	**2.** (d)	**3.** (c)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (c)
11. (a)	**12.** (b)	**13.** (c)	**14.** (b)	**15.** (c)	**16.** (b)	**17.** (c)	**18.** (a)	**19.** (b)	**20.** (b)
21. (c)	**22.** (b)	**23.** (a)	**24.** (b)						

B. Short Answer Type Questions [3 Marks]

1. Which term of the GP 5, 10, 20, … is 20480?

Sol. $T_n = ar^{n-1}$.

$\Rightarrow 20480 = 5 \times 2^{n-1}$

$\Rightarrow 2^{n-1} = \frac{20480}{5} = 4096 = 2^{12}$

$\Rightarrow n - 1 = 12 \Rightarrow n = 13.$

2. Find the 6th term from the end of the GP $8, 4, 2, 1, \frac{1}{2} \dots \frac{1}{1024}$.

Sol. Required term $= \dfrac{\frac{1}{1024}}{\left(\frac{1}{2}\right)^{6-1}} = \frac{1}{1024} \times 2^5 = \frac{32}{1024} = \frac{1}{32}.$

3. If the 4th and 7th terms of a GP are $\frac{1}{18}$ and $\frac{-1}{486}$ respectively, then find the GP.

Sol. $ar^3 = \frac{1}{18}$ and $ar^6 = \frac{-1}{486}$

$\Rightarrow \frac{ar^6}{ar^3} = \frac{-1}{486} \times \frac{18}{1} = \frac{1}{-27} \Rightarrow r^3 = \frac{1}{(-3)^3} \Rightarrow r = \frac{-1}{3}$

$ar^3 = \frac{1}{18} \Rightarrow a = \frac{1}{18} \times (-3^3) = \frac{-3}{2}$

Thus, the GP is $\frac{-3}{2}, \frac{1}{2}, \frac{-1}{6}, \dots$

4. The 4th, 9th and last terms of a GP are 10, 320 and 2560 respectively, find the first term and the number of terms in the GP.

Sol. $ar^3 = 10$, $ar^8 = 320$ and $ar^{n-1} = 2560$

$\Rightarrow \frac{ar^8}{ar^3} = \frac{320}{10} \Rightarrow r^5 = 32 \Rightarrow r^5 = 2^5 \Rightarrow r = 2$

$ar^3 = 10 \Rightarrow a = \frac{10}{2^3} = \frac{5}{4}$

Also, $ar^{n-1} = 2560$

$\Rightarrow r^{n-1} = \dfrac{2560}{\frac{5}{4}} = 2048 = 2^{11}$

$\Rightarrow n - 1 = 11 \Rightarrow n = 12.$

5. The third term of a GP is 4. Find the product of its first five terms.

Sol. $ar^2 = 4$

Now, $a \times ar \times ar^2 \times ar^3 \times ar^4 = a^5 \times r^{10} = (ar^2)^5 = 4^5 = 1024.$

6. The first term of a GP is 1. The sum of the third and fifth terms is 90. Find the common ratio of the GP.

Sol. $a = 1$

$ar^2 + ar^4 = 90$

$\Rightarrow r^4 + r^2 - 90 = 0$

$\Rightarrow r^4 + 10r^2 - 9r^2 - 90 = 0$

$\Rightarrow r^2(r^2 + 10) - 9\,(r^2 + 10) = 0$

$\Rightarrow (r^2 + 10)(r^2 - 9) = 0$

$\Rightarrow r^2 = -10$ or $r^2 = 9$

$\Rightarrow r = \pm\, 3$ [Rejecting $r^2 = -10$]

7. Find the sum: 243 + 324 + 432 + … up to n terms.

Sol. Here, $a = 243,\ r = \dfrac{324}{243} = \dfrac{4}{3}$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{243\left[\left(\frac{4}{3}\right)^n - 1\right]}{\frac{4}{3} - 1}$$

$$= 243 \times 3\ \frac{[4^n - 3^n]}{3^n} = \frac{3^6}{3^n}\,[4^n - 3^n].$$

8. Determine the number of terms n in the GP $T_1, T_2 \ldots\ldots T_n$, if $T_1 = 3$, $T_n = 96$ and $S_n = 189$.

Sol. $a = 3$

$T_n = ar^{n-1} \Rightarrow \dfrac{96}{3} = r^{n-1}$

$\Rightarrow r^{n-1} = 32 = 2^{6-1} \Rightarrow n = 6.$

9. How many terms of the GP 1, 4, 16, 64, … will make their sum 5461?

Sol. Here, $a = 1$, $r = 4$

$S_n = \dfrac{a(r^n - 1)}{r - 1} \Rightarrow 5461 = \dfrac{1(4^n - 1)}{4 - 1}$

$\Rightarrow 4^n - 1 = 5461 \times 3 = 16383$

$\Rightarrow 4^n = 16384 = 4^7 \Rightarrow n = 7.$

10. The sum of n terms of a progression is $3^n - 1$. Show that it is a GP. Find its common ratio.

Sol. $S_1 = 3^1 - 1 = 2$

$S_2 = 3^2 - 1 = 8$

$S_3 = 3^3 - 1 = 26$

$T_1 = S_1 = 2$, $T_2 = S_2 - S_1 = 8 - 2 = 6$, $T_3 = S_3 - S_2 = 26 - 8 = 18.$

Here, $\dfrac{T_2}{T_1} = \dfrac{T_3}{T_2} = 3$

So, 2, 6, 18, … form a GP, whose common ratio is 3.

11. The first term of a GP is 27 and its 8th term is $\dfrac{1}{81}$. Find the sum of its first 10 terms.

Sol. $T_1 = a = 27, \qquad T_8 = ar^7 = \dfrac{1}{81}$

$$\Rightarrow r^7 = \frac{1}{81 \times 27} = \frac{1}{3^4 \times 3^3} = \frac{1}{3^7} = \left(\frac{1}{3}\right)^7$$

$$\Rightarrow r = \frac{1}{3}.$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \Rightarrow S_{10} = \frac{27\left[1 - \left(\frac{1}{3}\right)^{10}\right]}{1 - \frac{1}{3}}$$

$$= 27 \times \frac{3}{2} \times \left(1 - \frac{1}{3^{10}}\right) = \frac{81}{2}\left(1 - \frac{1}{3^{10}}\right).$$

C. Long Answer Type Questions [4 Marks]

1. The sum of three numbers which are consecutive terms of an AP is 21. If the second number is reduced by 1, and the third is increased by 1, we obtain three consecutive terms of a GP. Find the numbers.

Sol. Let the numbers be $a - d, a, a + d$.

Then, $a - d + a + a + d = 21 \Rightarrow a = 7$

So, the numbers are $7 - d$, 7 and $7 + d$

But $7 - d, 7 - 1, 7 + d + 1$ are in GP

i.e., $7 - d$, 6 and $8 + d$ are in G.P.

$\Rightarrow (7 - d) \times (8 + d) = 6^2$

$\Rightarrow d^2 + d - 20 = 0 \Rightarrow (d + 5)(d - 4) = 0$

$\Rightarrow d = -5$ or 4.

So, the numbers are 12, 7, 2 or 3, 7, 11.

2. The sum of three numbers in GP is 38 and their product is 1728. Find the numbers.

Sol. Let the numbers be $\frac{a}{r}, a, ar$.

$\Rightarrow \frac{a}{r} \times a \times ar = 1728$

$\Rightarrow a^3 = 12^3 \Rightarrow a = 12$

Also, $\frac{12}{r} + 12 + 12r = 38$

$\Rightarrow 12 + 12r + 12r^2 - 38r = 0$

$\Rightarrow 12r^2 - 26r + 12 = 0$

$\Rightarrow 6r^2 - 13r + 6 = 0$

$\Rightarrow 6r^2 - 9r - 4r + 6 = 0$

$\Rightarrow 3r(2r - 3) - 2(2r - 3) = 0$

$\Rightarrow (2r - 3)(3r - 2) = 0$

$\Rightarrow r = \frac{3}{2}$ or $\frac{2}{3}$

So, the numbers are 8, 12, 18, or 18, 12, 8

3. Find the least value of n for which the sum $1 + 3 + 3^2 + \ldots$ to n terms is greater than 7000.

Sol. Here, $a = 1, r = 3$ $\quad S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1.(3^n - 1)}{3 - 1} = \frac{3^n - 1}{2}$

$S_n > 7000 \Rightarrow \frac{3^n - 1}{2} > 7000 \Rightarrow 3^n = 14001$

But, $3^8 = 6561$ and $3^9 = 19683$.

So, the least value of n for which $3^n > 14001$ is 9.

So, the required value is 9.

4. Find the sum to n terms of the series whose nth term is $2^n + 3n$.

Sol. $T_n = 2^n + 3n$

$\Rightarrow \quad T_1 = 2^1 + 3 \times 1$

$T_2 = 2^2 + 3 \times 2$

$T_3 = 2^3 + 3 \times 3$

$T_n = 2^n + 3 \times n$

$S_n = T_1 + T_2 + \ldots\ldots T_n$

$= (2 + 2^2 + 2^3 + \ldots 2^n) + 3 \times 1 + 3 \times 2 + 3 \times 3 + \ldots 3 \times n.$

$$= \frac{2\times(2^n - 1)}{2-1} + \frac{3n(n+1)}{2} = \frac{4(2^n - 1) + 3n^2 + 3n}{2}$$

5. Find the sum of the following to n terms 7 + 77 + 777 + … .

Sol. $S_n = 7 + 77 + 777 + \ldots$ to n terms

$= \frac{7}{9}$ [9 + 99 + 999 + … to n terms]

$= \frac{7}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) + … n terms]

$= \frac{7}{9}$ [($10 + 10^2 + 10^3 + \ldots\ldots$ to n terms) – (1 + 1 + 1 + …to n terms)]

$$= \frac{7}{9}\left(\frac{10\times(10^n - 1)}{10-1} - 1\times n\right) = \frac{7}{81}(10^{n+1} - 9n - 10).$$

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