Circle

Exercise – 4.1

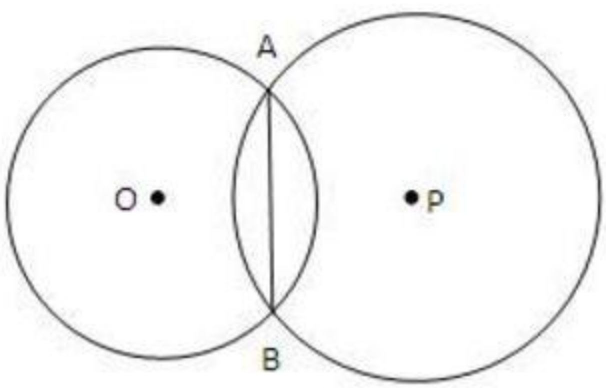
Solution 1:

Points in the interior: O, A, B, C.
Points in the exterior: E, D, G, F.
Points on the circle: P, Q, T, R.

Solution 2:

The radius of the circle is 6.7 cm.
d(P, R) = 5.7 cm.
∴5.7 cm < 6.7 cm.
∴ The distance between P and R is less than the radius of the circle.
∴Point R lies in the interior of the circle.
d(P, Q) = 7.6 cm.
∴7.6 cm> 6.7 cm.
∴ The distance between P and Q is greater than the radius of the circle.
∴Point Q lies in the exterior of the circle.

Solution 3:



Only one common chord (joining the points of intersection) can be drawn between two common points of two intersecting circles.

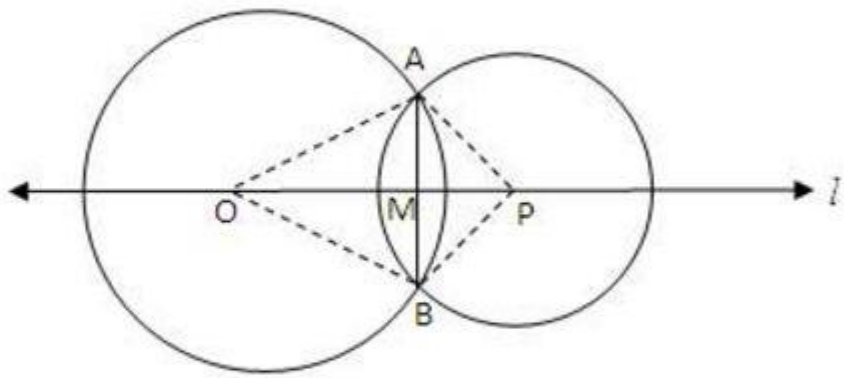
Solution 4:

Radius (r) = 7cm …(Given)
OP=4cm
∴4 < 7

∴OP < r
∴ Point P is in the interior of the circle.
OQ=9cm
∴ 9 > 7
∴OQ > r
∴ Point Q is on the exterior of the circle.

Solution 5:

Let the two circles with centre O and P intersect each other at the points A and B.
AB is the common chord.
Line l passes through the centre O and P.



Draw seg OA, seg OB, seg PA, seg PB.
Let line l intersect chord AB in point M.
In ΔAOP and ΔBOP,
seg OA ≅ seg OB … (Radii of the same circle)
seg PA ≅ seg PB … (Radii of the same circle)
seg OP ≅ seg OP … (Common side)
∴ Δ AOP ≅ ΔBOP … (sss test)
∴∠AOP ≅ ∠BOP … (c.a.c.t.)
i.e. ∠AOM ≅ ∠BOM
In ΔAOM and ΔBOM
seg OA ≅ seg OB … (Radii of the same circle)
∠AOM ≅ ∠BOM … (Proved)
seg OM ≅ seg OM … (Common side)
∴ ΔAOM ≅ ΔBOM… (SAS test)
∴seg AM ≅ seg BM …(c.s.c.t.)
and ∠AMO ≅ ∠BMO …(c.a.c.t.)
But m∠AMO + m∠BMO = 180° … (Angles in a linear pair)
∴m∠AMO = m∠BMO=90°
∴seg OP is the perpendicular bisector of seg AB.
i.e. line l is the perpendicular bisector of the common chord AB.

Solution 6:

BD = 3DC ... (Given)

Let DC be x. Then BD = 3x ... (1)

BC = BD + DC = 3x + x

$\therefore$ BC = 4x $\therefore$ $BC^2 = 16x^2$ ... (2)

In right angled ΔADB, by Pythagoras' Theorem,

$AB^2 = AD^2 + BD^2$ ... (3)

Similarly, in right angled ΔADC,

$AC^2 = AD^2 + DC^2$

$\therefore AD^2 = AC^2 - DC^2$ ... (4)

From (3) and (4),

$AB^2 = AC^2 - DC^2 + BD^2$

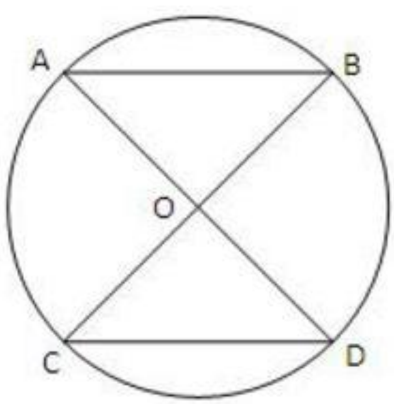
$AB^2 = AC^2 - x^2 + (3x)^2$ ... [From (1)]

$= AC^2 - x^2 + 9x^2$

$= AC^2 + 8x^2 = AC^2 + \frac{1}{2} \times 16x^2$ ...(5)

$\therefore AB^2 = AC^2 + \frac{1}{2}BC^2$ ... [From (5) and (2)]

Solution 7:



In ΔAOB and ΔCOD,

seg OA ≅ seg OD; seg OB ≅ seg OC … (Radii of the same circle)

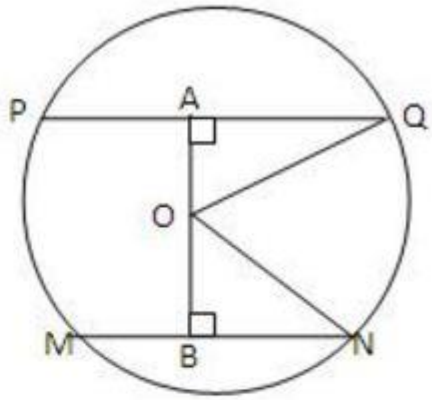
∠AOB ≅ ∠COD … (Vertically opposite angles)

∴ ΔAOB ≅ ΔCOD … (SAS test)

∴ seg AB ≅ seg CD … (c.s.c.t.)

i.e. chord AB ≅ chord CD.

Solution 8:



Let O be the centre of the circle. Seg OA $\perp$ chord PQ and seg OB $\perp$ chord MN.

The perpendicular from the centre of the circle to a chord bisects the chord.

$\therefore$ Point A is the midpoint of chord PQ and point B is the midpoint of chord MN.

$\therefore AQ = \frac{1}{2}PQ = \frac{1}{2} \times 11$ cm $= 5.5$ cm and

Chord PQ $\parallel$ chord MN ... (Given)

$m\angle OAQ = m\angle OBN = 90^\circ$ ... (Construction)

$\therefore$ A, O and B are collinear points.

$\therefore AB = OA + OB = 6$ cm ... (Given)

Suppose $OA = x$ cm. Then $OB = (6 - x)$cm.

Let the radius of the circle be r.

In right angled ΔOAQ,

by Pythagoras' Theorem,

$OQ^2 = OA^2 + AQ^2$

$\therefore r^2 = x^2 + (5.5)^2$ ...(1)

Similarly, in right angled ΔOBN,

$ON^2 = OB^2 + BN^2$

$\therefore r^2 = (6 - x)^2 + (2.5)^2$ ...(2)

From (1) and (2),

$x^2 + (5.5)^2 = (6 - x)^2 + (2.5)^2$

$\therefore x^2 + 30.25 = 36 - 12x + x^2 + 6.25$

$\therefore 12x = 42.25 - 30.25$

$\therefore 12x = 12 \qquad \therefore x = 1.$

Substituting $x = 1$ in equation (1),

$r^2 = (1)^2 + (5.5)^2 = 1 + 30.25$

$\therefore r^2 = 31.25$

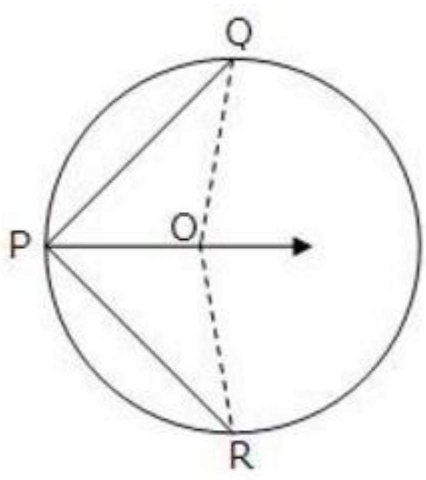
$\therefore r^2 = 31\frac{1}{4}$

$\therefore r^2 = \frac{125}{4}$

$\therefore r = \frac{5\sqrt{5}}{2}$

The radius of the circle is $\frac{5\sqrt{5}}{2}$ cm.

Solution 9:



Join OQ and OR.

In ∆OPQ and ∆OPR

seg OQ ≅ seg OR … (Radii of the same circle)

seg PQ ≅ seg PR … (Given)

seg PO ≅ seg PO … (Common side)

∴ ΔOPQ ≅ ΔOPR … (SSS test)
∴∠OPQ ≅ ∠OPR … (c.a.c.t.)
∴seg PO is the bisector of ∠QPR.
i.e. the bisector of ∠RPQ passes through the centre of the circle.

Solution 10:

Let r be the radius of the circle.

The length of chord PQ = r and OP = OQ = r ... (1)

∴ ΔOPQ is an equilateral triangle.

Draw seg OM ⊥ seg PQ.

In ΔOPM, $m\angle P = 60°$ ... (Angle of an equilateral triangle)

$m\angle OMP = 90°$ ... (Construction)

∴ $m\angle POM = 30°$

∴ ΔOPM is a 30°-60°-90° triangle.

∴ $PM = \frac{1}{2}OP$ ... (Side opposite to 30°)

∴ $PM = \frac{1}{2}r$ ...[From (1)] ... (2)

In right angled ΔOPM, by Pythagoras theorem,

$OP^2 = OM^2 + PM^2$

$r^2 = OM^2 + \frac{1}{4}r^2$

∴ $OM^2 = r^2 - \frac{1}{4}r^2$

∴ $OM^2 = \frac{3}{4}r^2$

∴ $OM = \frac{\sqrt{3}}{2}r$ ... (Taking square root)

The distance of chord PQ from the centre O in terms of radius is $\frac{\sqrt{3}}{2}r$.

Solution 11:

PA and PB are the radii of the circle .

Suppose PA = PB = x cm.

Then from the given condition,

AB = PA + PB - 4

=(2x - 4) cm.

Now, the perimeter of ΔPAB is given to be 144 cm.

$\therefore$ P A + PB + AB = 144 cm

$\therefore$ (x + x + 2x - 4) cm = 144 cm

$\therefore$ 4x - 4 = 144 $\therefore$ 4x = 148 $\therefore$ x = 37

$\therefore$ PA = PB = 37 cm ... (1)

and AB = 2 x 37 - 4 = 74 - 4 = 70 cm.

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore AM = \frac{1}{2}AB = \frac{1}{2} \times 70 \text{ cm} = 35 \text{ cm}$ ... (2)

In right angled ΔPAM, by Pythagoras' Theorem,

$PA^2 = PM^2 + AM^2$

$\therefore (37)^2 = PM^2 + (35)^2$ ... [From (1) and (2)]

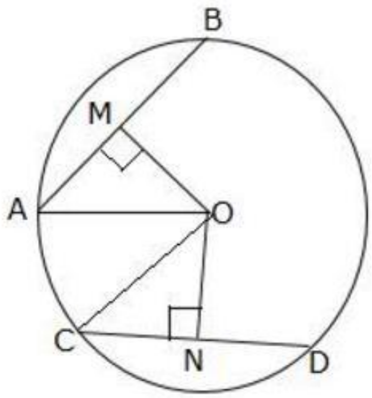
$\therefore PM^2 = (37)^2 - (35)^2$

$\therefore PM^2 = 144$

$\therefore PM = 12$

The length of side PM is 12 cm.

Solution 12:



Consider a circle with centre O having chord AB ≅ chord CD.
Draw seg OM ⊥ chord AB and seg ON ⊥ chord CD.
Join radius OA and radius OC.

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

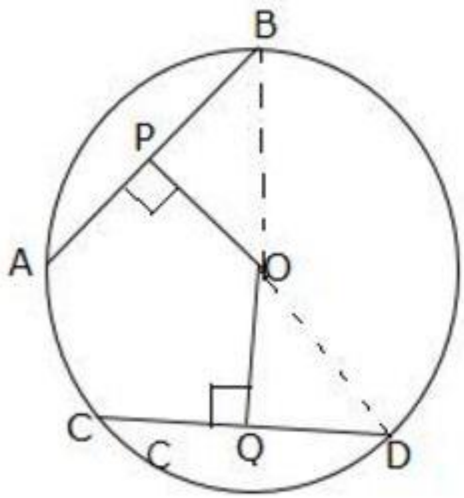
∴ $AM = \frac{1}{2}AB$ and $CN = \frac{1}{2}CD$ ....(1)

AB = CD ... (Given) ....(2)

From (1) and (2) AM = CN

In right angled triangles,
ΔOMA and ΔONC,
$m\angle ONC = m\angle OMC = 90°$
hypotenuse OA ≅ hypotenuse OC
... (Radii of the same circle)
side AM ≅ side CN ... [From (3)]
ΔOMA ≅ ΔONC ... (Hypotenuse-side theorem)
seg OM ≅ seg ON ... (c.s.c.t.)
i.e. OM = ON
∴ Congruent chords of a circle are equidistant from the centre of the circle.

Solution 13:



Draw radius OB and radius OD.

In right angled triangles, ΔOPB and ΔOQD,

$m\angle OPB = m\angle OQD = 90^\circ$

hypotenuse OB $\cong$ hypotenuse OD

... (Radii of the same circle)

seg OP $\cong$ seg OQ ... (Given)

$\therefore \Delta OPB \cong \Delta OQD$ ... (Hypotenuse-side theorem)

$\therefore$ seg PB $\cong$ seg QD ... (c.s.c.t.)

i.e. PB = QD ... (1)

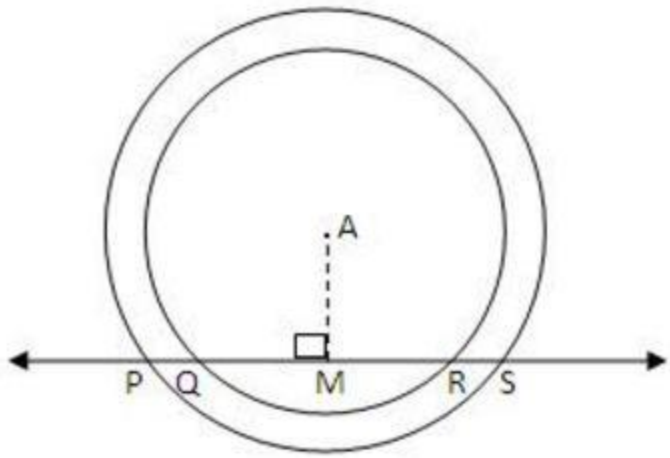
The perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore PB = \frac{1}{2}AB$ and $QD = \frac{1}{2}CD$ ... (2)

From (1) and (2), AB = CD

i.e. chord AB $\cong$ chord CD.

Solution 14:



Draw seg AM ⊥ chord PS.
The perpendicular drawn from the centre of a circle to a chord bisects the chord.
∴In the larger circle, PM = MS …(1)
and in the smaller circle, QM = MR …(2)
Now, PM – QM=PQ … (P-Q-M) …(3)
AndMS – MR=RS … (M-R-S) …(4)
From (1), (2), (3) and (4), PQ = RS.

Solution 15:

Circles with centres O and P intersect each other in points Q and R
Seg QR is the common chord.
Seg OP intersects the common chord QR in point S.
OQ = OR … (Radii of the same circle)
∴Point O is equidistant from points Q and R. …(1)
Similarly, PQ=PR
∴Point P is equidistant from points Q and R … (2)
From (1) and (2) and by Perpendicular Bisector Theorem, OP is the perpendicular bisector of QR.
∴point S is the midpoint of QR and
m∠QSO = m∠QSP = 90° …(3)
QR=12cm …(Given)…(4)
From (3) and (4), QS = 6cm …(5)
In right angled ∆PQS,
By Pythagoras' Theorem
PQ2 = QS2 + PS2
∴(8)2 = (6)2 + PS2…[Given and from (5)]
∴PS2 = 82 – 62 = 64 -36
∴PS2 = 28
∴PS= √28

∴PS= 2√7 cm …(6)
Similarly, in right angled ∆OQS,
QO2 = QS2 +OS2
∴ (10) 2=(6) 2+OS2 …[Given and from (5)]
∴OS2 = 10 2 – 6 2 = 100 – 36
∴OS2 =64
∴OS= 8cm …(7)
OP = OS +SP …[O – S – P]
= (8+2√7) cm
The distance between their centres is (8+2√7) cm

Solution 16:

The perpendicular drawn from the centre of a circle to a chord bisects the chord.
∴Point E is the midpoint of chord AB.
∴ seg AE ≅ seg BE … (1)
Diameter CD ⊥ chord AB … (Given)
In ∆CAE and ∆CBE,
seg AE ≅ seg BE … [From (1)]
∠CEA ≅ ∠CEB
seg CE ≅ seg CE … (Common side)
∴ ∆CAE≅ ∆CBE … (SAS test)
∴seg CA ≅ seg CB … (c.s.c.t.)
i.e. CA= CB
∴ ∆ABC is an isosceles triangle.