GEOMETRIC PROGRESSION - I

1. If the p^{th}, q^{th}, r^{th} terms of a G.P. be a, b, c respectively, then prove that $a^{q-r}b^{r-p}c^{p-q}=1$.

2. The fifth term of a G.P. is 81, and the second term is 24; find the series.

3. Find the sum of the series : $3, -4, \frac{16}{3}, \ldots$ to 2n terms.

4. The sum of the first 6 terms of a G.P. is 9 times the sum of the first 3 terms; find the common ratio.

5. The sum of a G.P. whose common ratio is 3 is 728, and the last term is 486; find the first term.

6. In a G.P. the first term is 7, the last term 448, and the sum 889; find the common ratio.

7. The sum of infinite number of terms of a G.P. is 4 and the sum of their cubes is 192; find the series.

8. The sum of three numbers in G.P. is 38, and their product is 1728; find them.

9. The continued product of three nubmers in G.P. is 216, and the sum of the products of them in pairs is 156; find the numbers.

10. The sum of three numbers in G.P. is 70; if the two extremes be multipled each by 4, and the mean by 5, the products are in A.P., find the numbers.

11. If the p^{th}, q^{th}, r^{th}, s^{th} terms of an A.P. are in G.P., show that p – q, q – r, r – s are in G.P.

12. The sum of first three terms of a G.P. is to the sum of the first six terms as 125 : 152. Find the common ratio of the G.P.

13. Sum the series : (a) $\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+\ldots\left(x^n+\frac{1}{x^n}\right)^2$

(b) $1+(1+x)+(1+x+x^2)+(1+x+x^2+x^3)+\ldots$ to n terms.

14. Find the sum of n terms of the following series

(a) $\cdot 7+\cdot 77+\cdot 777+\ldots$

(b) 6 + 66 + 666 + ...

15. (a) Find the value of $\cdot 1\dot{2}\dot{3}$ regarding it as geometric series.

(b) Find the value of $\cdot 4\dot{2}\dot{3}$.

GEOMETRIC PROGRESSION - II

1. If p^{th}, q^{th} and r^{th} terms of an A.P. are in G.P., then the common ratio of G.P. is

(A) $\frac{q-r}{p-q}$ (B) $\frac{r-q}{p-q}$ (C) $\frac{q-r}{q-p}$ (D) $\frac{q-p}{q-r}$

2. If the roots of cubic equation $ax^3 + bx^2 + cx + d = 0$ are in G.P., then

(A) $c^3a = b^3d$ (B) $ca^3 = bd^3$ (C) $a^3b = c^3d$ (D) $ab^3 = cd^3$

3. If $\frac{p+q.5^x}{p-q.5^x}=\frac{q+r.5^x}{q-r.5^x}=\frac{r+s.5^x}{r-s.5^x}$ then p, q, r, s are in

(A) A.P. (B) G.P. (C) H.P. (D) none of these

4. If the sum of the series $\sum_{n=0}^{\infty} r^n$, $|r| < 1$, is S, then sum of the series $\sum_{n=0}^{\infty} r^{2n}$ is

(A) S^2 (B) $\frac{2S}{S^2-1}$ (C) $\frac{S^2}{2S+1}$ (D) $\frac{S^2}{2S-1}$

5. If S denotes the sum of infinity and S_n the sum of n terms of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+$ ...such that $S - S_n < \frac{1}{1000}$, then the least value of n is

(A) 11 (B) 9 (C) 10 (D) 8

6. If a, b, c are in G.P. then the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root if $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in

(A) A.P. (B) G.P. (C) H.P (D) None of these

7. A certain number is inserted between the number 3 and the unknown number so that the three numbers form an A.P. If the middle term is diminished by 6 then the number are in G.P. The unknown number can be

(A) 3 (B) 15 (C) 18 (D) 27

8. Let the numbers a_1, a_2, a_3 ..... a_n constitute a geometric progression.

If $S = a_1 + a_2 + ... + a_n$, $T = \frac{1}{a_1} + \frac{1}{a_2} + ... + \frac{1}{a_n}$ and $P = a_1 a_2 a_3 ..... a_n$ then P^2 is equal to

(A) $\left(\frac{S}{T}\right)^n$ (B) $\left(\frac{T}{S}\right)^n$ (C) $\left(\frac{2S}{T}\right)^n$ (D) $\left(\frac{2T}{S}\right)^n$

9. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If α, β, γ, δ are in G.P., then the integral values of p and q respectively, are

(A) – 2, – 32 (B) – 2, 3 (C) – 6, 3 (D) – 6, – 32

10. a, b, c, d are in increasing G.P. If the AM between a and b is 6 and the AM between c and d is 54., then the AM of a and d is

(A) 15 (B) 48 (C) 44 (D) 42

11. Insert 3 geometric means between $\frac{9}{4}$ and $\frac{4}{9}$.

12. If the arithmetic mean between a and b is twice as great as the geometric mean, show that $a : b = 2 + \sqrt{3} : 2 - \sqrt{3}$.

13. If a, b, c, d be in G.P. Prove that

(a) $(a^2 + ac + c^2)(b^2 + bd + d^2) = (ab + bc + cd)^2$.

(b) $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$.

14. If a, b, c, d be in G.P. $(a \neq b \neq c \neq d)$. Prove that

(a) $(a - d)^2 = (b - c)^2 + (c - a)^2 + (d - b)^2$

(b) $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in G.P.

15. (a) If one geometric mean G and two arithmetic means p and q be inserted between any two given numbers, then show that $G^2 = (2p - q)(2q - p)$.

(b) If one arithmetic mean A and two geometric means p and q be inserted between any two given numbers, then show that $p^3 + q^3 = 2\ Apq$.

16. Find the $\prod_{i=1}^{3} Gi$ (Geometric means) inserted between 'a' and 'b' which satisfy the equation $(G_1+2)^4 + (G_2-4)^2 + |G_3 + 8| = 0$. Also find ab =

Answers

RACE # 12

GEOMETRIC PROGRESSION - I

2. 16, 24, 36.... **3.** $\frac{9}{7}\left(1-\left(\frac{4}{3}\right)^{2n}\right)$ **4.** 2 **5.** 2 **6.** 2 **7.** 6, –3, $1\frac{1}{2}$,.....

8. 8, 12, 18 **9.** 2, 6, 18 **10.** 40, 20, 10 **12.** 3/5

13. (a) $\frac{x^{2n}-1}{x^2-1}\left[\frac{x^{2n+2}+1}{x^{2n}}\right]+2n$ (b) $\frac{1}{(1-x)^2}\left[n(1-x)-x(1-x^n)\right]$

15. (a) $\frac{7n}{9}-\frac{7}{81}\left(1-\frac{1}{10^n}\right)$ (b) $\frac{2}{27}\left[10^{n+1}-9n-10\right]$ **15.** (a) $\frac{122}{990}$ (b) $\frac{419}{990}$

GEOMETRIC PROGRESSION - II

1. (A) **2.** (A) **3.** (B) **4.** (D) **5.** (A) **6.** (A) **7.** (D) **8.** (A) **9.** (A) **10.** (D)

11. $\frac{3}{2}, 1, \frac{2}{3}$ **16.** 64,16