1. Real number system

Exercise 1.1

1 A. Question

Identify which of the following are surds and which are not with reasons

$\sqrt{8} \times \sqrt{6}$

Answer

Given, $\sqrt{8} \times \sqrt{6}$

Need to find $\sqrt{8} \times \sqrt{6}$ is surd or not

⇒ we know $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

⇒ $\sqrt{8} \times \sqrt{6}$ can be written as $\sqrt{8 \times 6}$

⇒ $\sqrt{2 \times 2 \times 2 \times 2 \times 3}$

$= 4\sqrt{3}$, which is irrational number

⇒ since, $4\sqrt{3}$ cannot be expressed as squares or cubes of any rational numbers

⇒ Hence, $\sqrt{8} \times \sqrt{6}$ is surd

1 B. Question

Identify which of the following are surds and which are not with reasons

$\sqrt{90}$

Answer

Given, $\sqrt{90}$

Need to find $\sqrt{90}$ is surd or not

⇒ we know $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

⇒ $\sqrt{90}$ can be written as $\sqrt{9 \times 10}$

⇒ $\sqrt{3 \times 3 \times 2 \times 5}$

⇒ $3\sqrt{10}$, which is irrational numbers

since, $3\sqrt{10}$ cannot be expressed as squares or cubes of any rational numbers

⇒ Hence, it is surd.

1 C. Question

Identify which of the following are surds and which are not with reasons

$\sqrt{180} \times \sqrt{5}$

Answer

Given, $\sqrt{180}$ × √5

Need to find $\sqrt{180}$ × √5 is surd or not

⇒ we know √a × √b = $\sqrt{ab}$

⇒ $\sqrt{180}$ × √5 can be written as $\sqrt{180 \times 5}$

⇒ $\sqrt{2 \times 90 \times 5}$

⇒ $\sqrt{2 \times 3 \times 3 \times 2 \times 5 \times 5}$

⇒ $\sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5}$

= 2 × 3 × 5 = 30 which is not a irrational number as it can be expressed in squares form

Hence, it is not a surd

1 D. Question

Identify which of the following are surds and which are not with reasons

$4\sqrt{5} \div \sqrt{8}$

Answer

Given, 4√5 ÷ √8

Need to find 4√5 ÷ √8 is surd or not

⇒ we know √a ÷ √b = $\frac{\sqrt{a}}{\sqrt{b}}$

⇒ 4√5 ÷ √8 can be written as $\frac{4\sqrt{5}}{\sqrt{8}}$

⇒ $\frac{\sqrt{2\times2\times2\times2\times5}}{\sqrt{8}}$

$= \frac{\sqrt{80}}{\sqrt{8}}$

$= \frac{\sqrt{8} \times \sqrt{10}}{\sqrt{8}}$

$= \sqrt{10}$ is irrational number

since, $\sqrt{10}$ cannot be expressed as squares or cubes of any rational numbers

⇒ Hence, it is surd.

1 E. Question

Identify which of the following are surds and which are not with reasons

$\sqrt[3]{4} \times \sqrt[3]{16}$

Answer

Given, $\sqrt[3]{4} \times \sqrt[3]{16}$

Need to find $\sqrt[3]{4} \times \sqrt[3]{16}$ is surd or not

⇒ we know $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

⇒ $\sqrt[3]{4} \times \sqrt[3]{16}$ can be written as $\sqrt[3]{4 \times 16}$

⇒ $\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2}$

= 2× 2 × 2 = 8 is not irrational number as it can be expressed in cubes form

⇒ Hence, it is not a surd

2 A. Question

Simplify$(10+ \sqrt{3})(2 + \sqrt{5})$

Answer

Given, $(10+ \sqrt{3})(2 + \sqrt{5})$

Need to simplify it

⇒ the given expression can be written in expanded form

⇒ $20+10\sqrt{5} + 2\sqrt{3} +(\sqrt{3} \times \sqrt{5})$

⇒ We know $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

= $20+10\sqrt{5} + 2\sqrt{3} + \sqrt{15}$

Hence, $(10+ \sqrt{3})(2 + \sqrt{5})$ is simplified into $20+10\sqrt{5} + 2\sqrt{3} + \sqrt{15}$

2 B. Question

Simplify

$$\left(\sqrt{5}+\sqrt{3}\right)^2$$

Answer

Given, $(\sqrt{5}+\sqrt{3})^2$

Need to simplify it

⇒ we know that $(a+b)^2 = a^2+2ab+b^2$

⇒ simplifying the given expression we get

⇒ $(\sqrt{5})^2+2(\sqrt{5})(\sqrt{3})+ (\sqrt{3})^2$

$= 5+2\sqrt{15}+3$

$= 8+2\sqrt{15}$

Hence, $(\sqrt{5}+\sqrt{3})^2$ is simplified into $8+2\sqrt{15}$

2 C. Question

Simplify

$$\left(\sqrt{13}-\sqrt{2}\right)\left(\sqrt{13}+\sqrt{2}\right)$$

Answer

Given, $(\sqrt{13} - \sqrt{2})\ (\sqrt{13} + \sqrt{2})$

Need to simplify it

⇒ we know that $(a–b)(a+b) = a^2 – b^2$

⇒ the given expression can be written in this form

⇒ $(\sqrt{13})^2 –(\sqrt{2})^2$

$= 13 –2$

$= 11$

Hence, $(\sqrt{13} - \sqrt{2})\ (\sqrt{13} + \sqrt{2})$ is simplified into 11

2 D. Question

Simplify

$\left(8+\sqrt{3}\right)\left(8-\sqrt{3}\right)$

Answer

Given, $(8+\sqrt{3})\ (8-\sqrt{3})$

Need to simplify it

⇒ we know that $(a-b)(a+b) = a^2 - b^2$

⇒ the given expression can be written in this form

⇒ $(8)^2 - (\sqrt{3})^2$

$= 64-3$

$= 61$

Hence, $(8+\sqrt{3})\ (8-\sqrt{3})$ is simplified into 61

3 A. Question

Simplify the following.

$5\sqrt{75}+8\sqrt{108}-\frac{1}{2}\sqrt{48}$

Answer

Given, $5\sqrt{75} + 8\sqrt{108} - \frac{1}{2}\sqrt{48}$

Need to simplify it

⇒ the given expression is written as follows

⇒ $5\sqrt{5\times5\times3} + 8\sqrt{2\times2\times3\times3\times3} - \frac{1}{2}\sqrt{2\times2\times2\times2\times3}$

$= 25\sqrt{3} + 48\sqrt{3} - 2\sqrt{3}$

$= (25+48-2)\sqrt{3}$

$= 71\sqrt{3}$

Hence, $5\sqrt{75} + 8\sqrt{108} - \frac{1}{2}\sqrt{48}$ is simplified into $71\sqrt{3}$

3 B. Question

Simplify the following.

$7\sqrt{2}+6\sqrt[3]{16}-\sqrt[3]{54}$

Answer

Given, $7\sqrt[3]{2} + 6\sqrt[3]{16} - \sqrt[3]{54}$

Need to simplify it

⇒ the given expression is written as follows

⇒ $7\sqrt[3]{2} + 6\sqrt[3]{2 \times 2 \times 2 \times 2} - \sqrt[3]{2 \times 3 \times 3 \times 3}$

$= 7\sqrt[3]{2} + 12\sqrt[3]{2} - 3\sqrt[3]{2}$

$= 16\sqrt[3]{2}$

Hence, $7\sqrt[3]{2} + 6\sqrt[3]{16} - \sqrt[3]{54}$ is simplified into $16\sqrt[3]{2}$

3 C. Question

Simplify the following.

$$4\sqrt{72} + 8\sqrt{50} - 7\sqrt{128}$$

Answer

Given, $4\sqrt{72} - \sqrt{50} - 7\sqrt{128}$

Need to simplify it

⇒ the given expression is written as follows

⇒ $4\sqrt{2 \times 2 \times 2 \times 3 \times 3} - \sqrt{5 \times 2 \times 5} - 7\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= 24\sqrt{2} - 5\sqrt{2} - 56\sqrt{2}$

$= (24 - 5 - 56)\sqrt{2}$

$= -37\sqrt{2}$

Hence, $4\sqrt{72} - \sqrt{50} - 7\sqrt{128}$ is simplified into $-37\sqrt{2}$

3 D. Question

Simplify the following.

$$2\sqrt[3]{40} + 3\sqrt[3]{625} - 4\sqrt[3]{320}$$

Answer

Given, $2\sqrt[3]{40} + 3\sqrt[3]{625} - 4\sqrt[3]{320}$

Need to simplify it

⇒ the given expression is written as follows

$\Rightarrow 2\sqrt[3]{2 \times 2 \times 2 \times 5} + 3\sqrt[3]{5 \times 5 \times 5 \times 5} - 4\sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5}$

$\Rightarrow 4\sqrt[3]{5} + 15\sqrt[3]{5} - 16\sqrt[3]{5}$

$= (4 + 15 - 16)\sqrt[3]{5}$

$= 3\sqrt[3]{5}$

Hence, $2\sqrt[3]{40} + 3\sqrt[3]{625} - 4\sqrt[3]{320}$ is simplified into $3\sqrt[3]{5}$

4 A. Question

Express the following surds in its simplest form.

$\sqrt[3]{108}$

Answer

Given, $\sqrt[3]{108}$

Need to simplify it

$\Rightarrow$ the given number can be written as follows

$\Rightarrow \sqrt[3]{2 \times 2 \times 3 \times 3 \times 3}$

$= 3\sqrt[3]{4}$

Hence, $\sqrt[3]{108}$ is simplified into $3\sqrt[3]{4}$

4 B. Question

Express the following surds in its simplest form.

$\sqrt{98}$

Answer

Given, $\sqrt{98}$

Need to simplify it

$\Rightarrow$ the given number can be written as follows

$\Rightarrow \sqrt{2 \times 7 \times 7}$

$= 7\sqrt{2}$

Hence, $\sqrt{98}$ is simplified into $7\sqrt{2}$

4 C. Question

Express the following surds in its simplest form.

$\sqrt{192}$

Answer

Given, $\sqrt{192}$

Need to simplify it

⇒ the given number can be written as follows

⇒ $\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3}$

= $8\sqrt{3}$

Hence, $\sqrt{192}$ is simplified into $8\sqrt{3}$

4 D. Question

Express the following surds in its simplest form.

$\sqrt[3]{625}$

Answer

Given, $\sqrt[3]{625}$

Need to simplify it

⇒ the given number can be written as follows

⇒ $\sqrt[3]{5 \times 5 \times 5 \times 5}$

= $5\sqrt[3]{5}$

Hence, $\sqrt[3]{625}$ is simplified into $5\sqrt[3]{5}$

5 A. Question

Express the following as pure surds.

$6\sqrt{5}$

Answer

Given, $6\sqrt{5}$

Need to express it as pure surd

⇒ $6\sqrt{5}$ can be expressed as $(\sqrt{6})^2 .\sqrt{5}$

⇒ $\sqrt{6^2.5}$

$= \sqrt{36.5}$

$= \sqrt{180}$

$\therefore \sqrt{180}$ is pure surd

$\because$ a surd with rational coefficient as unity is pure surd

Hence, $6\sqrt{5}$ is expressed as pure surd

5 B. Question

Express the following as pure surds.

$5\sqrt[3]{4}$

Answer

Given, $5\sqrt[3]{4}$

Need to express it as pure surd

$\Rightarrow 5\sqrt[3]{4}$ can be expressed as $(\sqrt[3]{5})^3.\sqrt[3]{4}$

$\Rightarrow \sqrt[3]{(5)^3 .4}$

$= \sqrt[3]{125 .4}$

$= \sqrt[3]{500}$

$\because$ a surd with rational coefficient as unity is pure surd

$\therefore \sqrt[3]{500}$ is a pure surd

Hence, $5\sqrt[3]{4}$ is expressed as pure surd

5 C. Question

Express the following as pure surds.

$3\sqrt[4]{5}$

Answer

Given, $3\sqrt[4]{5}$

Need to express it as pure surd

$\Rightarrow 3\sqrt[4]{5}$ can be written as $\left(\sqrt[4]{3}\right)^4.\sqrt[4]{5}$

$\Rightarrow \sqrt[4]{(3)^4 \times 5}$

$\Rightarrow \sqrt[4]{81 \times 5}$

$= \sqrt[4]{405}$

$\because$ a surd with rational coefficient as unity is pure surd

$\therefore \sqrt[4]{405}$ is a pure surd

Hence, $3\sqrt[4]{5}$ is expressed as pure surd

5 D. Question

Express the following as pure surds.

$\frac{3}{4}\sqrt{8}$

Answer

Given, $\frac{3}{4}\sqrt{8}$

Need to express it as pure surd

$\Rightarrow \frac{3}{4}\sqrt{8}$ can be expressed as follows

$\Rightarrow \left(\sqrt{\frac{3}{4}}\right)^2 \sqrt{8}$

$\Rightarrow \sqrt{\left(\frac{3}{4}\right)^2 .8} = \sqrt{\left(\frac{9}{16}\right).8}$

$= \sqrt{\frac{9}{2}}$

$\because$ a surd with rational coefficient as unity is pure surd

$\therefore \sqrt{\frac{9}{2}}$ is pure surd

Hence, $\frac{3}{4}\sqrt{8}$ is expressed as pure surd

6 A. Question

Simplify the following.

$\sqrt{5} \times \sqrt{18}$

Answer

Given, $\sqrt{5} \times \sqrt{18}$

Need to simplify it

⇒ we know $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

⇒ $\sqrt{5} \times \sqrt{18}$ can be written as $\sqrt{5 \times 18}$

⇒ $\sqrt{90}$

$= \sqrt{9.10}$

$= 3\sqrt{10}$

Hence, $\sqrt{5} \times \sqrt{18}$ is simplified into $3\sqrt{10}$

6 B. Question

Simplify the following.

$\sqrt[3]{7} \times \sqrt[3]{8}$

Answer

Given, $\sqrt[3]{7} \times \sqrt[3]{8}$

Need to simplify it

⇒ we know $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

⇒ $\sqrt[3]{7} \times \sqrt[3]{8}$ can be expressed as $\sqrt[3]{7.8}$

⇒ $\sqrt[3]{7.(2)^3}$

$= 2\sqrt[3]{7}$

Hence, $\sqrt[3]{7} \times \sqrt[3]{8}$ is simplified into $2\sqrt[3]{7}$

6 C. Question

Simplify the following.

$\sqrt[4]{8} \times \sqrt[4]{12}$

Answer

Given,$\sqrt[4]{8} \times \sqrt[4]{12}$

Need to simplify it

⇒ we know $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\Rightarrow \sqrt[4]{8} \times \sqrt[4]{12}$ can be expressed as $\sqrt[4]{8.12}$

$\Rightarrow \sqrt[4]{(2 \times 2 \times 2).(2 \times 2 \times 3)}$

$\Rightarrow \sqrt[4]{2 \times 2 \times 2 \times 2 \times 2 \times 3}$

$= \sqrt[4]{(2)^4 \times 6}$

$= 2\sqrt[4]{6}$

Hence, $\sqrt[4]{8} \times \sqrt[4]{12}$ is simplified into $2\sqrt[4]{6}$

6 D. Question

Simplify the following.

$\sqrt[3]{3} \times \sqrt[6]{5}$

Answer

Given, $\sqrt[3]{3} \times \sqrt[6]{5}$

Need to simplify it

$\Rightarrow$ we know $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\Rightarrow \sqrt[3]{3} \times \sqrt[6]{5}$ can be expressed as $\sqrt[6]{(\sqrt[3]{3})^6 \times 5}$

$\Rightarrow \sqrt[6]{\left((3)^{\frac{1}{3}}\right)^6 \times 5}$

$= \sqrt[6]{(3)^{\frac{6}{3}} \times 5}$

$= \sqrt[6]{(3)^2 \times 5}$

$= \sqrt[6]{9 \times 5}$

$= \sqrt[6]{45}$

Hence, $\sqrt[3]{3} \times \sqrt[6]{5}$ is simplified into $\sqrt[6]{45}$

7 A. Question

Which is greater ?

$\sqrt{2}$ or $\sqrt[3]{3}$

Answer

Given, $\sqrt{2}$ or $\sqrt[3]{3}$

Need to find the greater number

⇒ The order of the given irrational number is 2 and 3

⇒ now, we have to convert each irrational number into irrational number with same order

⇒ First we need to do the LCM of 2 and 3 is 6

⇒ now, each irrational number is converted into order of 6

$\Rightarrow \sqrt{2} = \left(\sqrt[6]{(2)^{\frac{1}{2}}}\right)^6$

$= \sqrt[6]{\left(2^{\frac{6}{2}}\right)}$

$= \sqrt[6]{2^3}$

$= \sqrt[6]{8}$

and

$\Rightarrow \sqrt[3]{3} = \left(\sqrt[6]{(\sqrt[3]{3})}\right)^6 = \sqrt[6]{(\sqrt[3]{3})^6}$

$= \sqrt[6]{(3)^{\frac{6}{3}}}$

$= \sqrt[6]{3^2}$

$= \sqrt[6]{9}$

⇒ $\sqrt[6]{9}$ is greater than $\sqrt[6]{8}$

∴ $\sqrt[3]{3} > \sqrt{2}$

Hence, $\sqrt[3]{3}$ is greater than $\sqrt{2}$

7 B. Question

Which is greater ?

$\sqrt[3]{3}$ or $\sqrt[4]{4}$

Answer

Given, $\sqrt[3]{3}$ or $\sqrt[4]{4}$

Need to find the greater number

⇒ The order of the given irrational number is 3 and 4

⇒ now, we have to convert each irrational number into irrational number with same order

⇒ First we need to do the LCM of 3 and 4 is 12

⇒ now, each irrational number is converted into order of 12

$\Rightarrow \sqrt[3]{3} = \left(\sqrt[12]{(3)^{\frac{1}{3}}}\right)^{12}$

$= \sqrt[12]{\left(3^{\frac{12}{3}}\right)}$

$= \sqrt[12]{(3^4)}$

$= \sqrt[12]{81}$

And

$\Rightarrow \sqrt[4]{4} = \left(\sqrt[12]{(4)^{\frac{1}{4}}}\right)^{12}$

$= \sqrt[12]{\left(4^{\frac{1}{4}}\right)^{12}}$

$= \sqrt[12]{\left(4^{\frac{12}{4}}\right)}$

$= \sqrt[12]{(4^3)}$

$= \sqrt[12]{64}$

$\therefore \sqrt[12]{81}$ is greater than $\sqrt[12]{64}$

$\Rightarrow \sqrt[3]{3}$ is greater than $\sqrt[4]{4}$

$\therefore \sqrt[3]{3} > \sqrt[4]{4}$

Hence, $\sqrt[3]{3}$ is greater than $\sqrt[4]{4}$

7 C. Question

Which is greater ?

$\sqrt{3}$ or $\sqrt[4]{10}$

Answer

Given, $\sqrt{3}$ or $\sqrt[4]{10}$

Need to find the greater number

⇒ The order of the given irrational number is 2 and 4

⇒ now, we have to convert each irrational number into irrational number with same order

⇒ First we need to do the LCM of 2 and 4 is 4

⇒ now, each irrational number is converted into order of 4

$\Rightarrow \sqrt{3} = \left(\sqrt[4]{(3)^{\frac{1}{2}}}\right)^4$

$= \sqrt[4]{(3)^{\frac{4}{2}}}$

$= \sqrt[4]{3^2}$

$= \sqrt[4]{9}$

And

$\Rightarrow \sqrt[4]{10}$

∴ the greater number between $\sqrt[4]{9}$ and $\sqrt[4]{10}$ is $\sqrt[4]{10}$

$\Rightarrow \sqrt[4]{10} > \sqrt[4]{9}$

$\therefore \sqrt[4]{10} > \sqrt{3}$

Hence, $\sqrt[4]{10}$ is greater than $\sqrt{3}$

8 A. Question

Arrange in descending and ascending order.

$\sqrt[4]{5}, \sqrt{3}\ \sqrt[3]{4}$

Answer

Given, $\sqrt[4]{5}$, $\sqrt{3}$, $\sqrt[3]{4}$

Need to arrange the given numbers in ascending and descending order

⇒ The order of the given irrational number is 4, 2 and 3 respectively.

⇒ now, we have to convert each irrational number into irrational number with same order

⇒ First we need to do the LCM of 4,2 and 3 is 12

⇒ now, each irrational number is converted into order of 12

$\Rightarrow \sqrt[4]{5} = \left(\sqrt[12]{\sqrt[4]{5}}\right)^{12}$

$= \left(\sqrt[12]{\left(5^{\frac{1}{4}}\right)}\right)^{12}$

$= \sqrt[12]{5^{\frac{12}{4}}}$

$= \sqrt[12]{5^3}$

$= \sqrt[12]{125}$

And

$\Rightarrow \sqrt{3} = \left(\sqrt[12]{(3)^{\frac{1}{2}}}\right)^{12}$

$= \sqrt[12]{\left(3^{\frac{12}{2}}\right)}$

$= \sqrt[12]{3^6}$

$= \sqrt[12]{729}$

And

$\Rightarrow \sqrt[3]{4} = \left(\sqrt[12]{\left(4^{\frac{1}{3}}\right)}\right)^{12}$

$= \sqrt[12]{\left(4^{\frac{12}{3}}\right)}$

$= \sqrt[12]{4^4}$

$= \sqrt[12]{256}$

∴ Ascending order is $\sqrt[4]{5}, \sqrt[3]{4}, \sqrt{3}$

∴ Descending order is $\sqrt{3}, \sqrt[3]{4}, \sqrt[4]{5}$

8 B. Question

Arrange in descending and ascending order.

$\sqrt[3]{2}, \sqrt[3]{4}, \sqrt[4]{4}$

Answer

Given, $\sqrt[3]{2}, \sqrt[3]{4}, \sqrt[4]{4}$

Need to arrange the given numbers in ascending and descending order

⇒ The order of the given irrational number is 3, 3 and 4 respectively.

⇒ now, we have to convert each irrational number into irrational number with same order

⇒ First we need to do the LCM of 3,3 and 4 is 12

⇒ now, each irrational number is converted into order of 12

$\Rightarrow \sqrt[3]{2} = \left(\sqrt[12]{\left(\sqrt[3]{2}\right)}\right)^{12}$

$= \left(\sqrt[12]{\left(2^{\frac{1}{3}}\right)}\right)^{12}$

$= \sqrt[12]{\left(2^{\frac{12}{3}}\right)}$

$= \sqrt[12]{2^4}$

$= \sqrt[12]{16}$

And

$\Rightarrow \sqrt[3]{4} = \sqrt[12]{\left(4^{\frac{1}{3}}\right)^{12}}$

$= \sqrt[12]{\left(4^{\frac{12}{3}}\right)}$

$= \sqrt[12]{4^4}$

$= \sqrt[12]{256}$

And

$\Rightarrow \sqrt[4]{4} = \left(\sqrt[12]{\left(4^{\frac{1}{4}}\right)}\right)^{12}$

$= \sqrt[12]{\left(4^{\frac{12}{4}}\right)}$

$= \sqrt[12]{(4^3)}$

$= \sqrt[12]{64}$

∴ Ascending order is $\sqrt[3]{2}, \sqrt[4]{4}, \sqrt[3]{4}$

∴ Descending order is $\sqrt[3]{4}, \sqrt[4]{4}, \sqrt[3]{2}$

8 C. Question

Arrange in descending and ascending order.

$\sqrt[3]{2}, \sqrt[9]{4}, \sqrt[6]{3}$

Answer

Given, $\sqrt[3]{2}, \sqrt[9]{4}, \sqrt[6]{3}$

Need to arrange the given numbers in ascending and descending order

⇒ The order of the given irrational number is 3, 9 and 6 respectively.

⇒ now, we have to convert each irrational number into irrational number with same order

⇒ First we need to do the LCM of 3, 9 and 6 is 18

⇒ now, each irrational number is converted into order of 18

$\Rightarrow \sqrt[3]{2} = \left(\sqrt[18]{\left(2^{\frac{1}{3}}\right)}\right)^{18}$

$= \sqrt[18]{\left(2^{\frac{18}{3}}\right)}$

$= \sqrt[18]{(2^6)}$

$= \sqrt[18]{64}$

And

$\Rightarrow \sqrt[9]{4} = \left(\sqrt[18]{\left(4^{\frac{1}{9}}\right)}\right)^{18}$

$= \sqrt[18]{\left(4^{\frac{18}{9}}\right)}$

$= \sqrt[18]{4^2}$

$= \sqrt[18]{14}$

And

$\Rightarrow \sqrt[6]{3} = \left(\sqrt[18]{\left(3^{\frac{1}{6}}\right)}\right)^{18}$

$= \sqrt[18]{3^{\frac{18}{6}}}$

$= \sqrt[18]{3^3}$

$= \sqrt[18]{27}$

∴ Ascending order is $\sqrt[9]{4}$, $\sqrt[6]{3}$, $\sqrt[3]{2}$

∴ Descending order is $\sqrt[3]{2}$, $\sqrt[6]{3}$, $\sqrt[9]{4}$,

Exercise 1.2

1 A. Question

Write the rationalizing factor of the following.

$3\sqrt{2}$

Answer

Given, $3\sqrt{2}$

Need to find the rationalizing factor

⇒ We know that if the product of two surds is rational then each is called a rationalizing factor of each other

⇒ $3\sqrt{2} \times \sqrt{2} = (3)(\sqrt{2})^2 = 6$ is rational number

Hence, rationalizing factor of $3\sqrt{2}$ is $\sqrt{2}$

1 B. Question

Write the rationalizing factor of the following.

$\sqrt{7}$

Answer

Given, $\sqrt{7}$

Need to find the rationalizing factor

⇒ We know that if the product of two surds is rational then each is called a rationalizing factor of each other

⇒ $\sqrt{7} \times \sqrt{7} = 7$

∴ $\sqrt{7}$ is rationalizing factor

1 C. Question

Write the rationalizing factor of the following.

$\sqrt{75}$

Answer

Given, $\sqrt{75}$

Need to find the rationalizing factor

⇒ We know that if the product of two surds is rational then each is called a rationalizing factor of each other

⇒ $\sqrt{75}$ can be written as $\sqrt{3 \times 5 \times 5}$

⇒ $5\sqrt{3}$

∴ $5\sqrt{3} \times \sqrt{3}$

⇒ (5)(3) = 15

Hence, $\sqrt{3}$ is rationalizing factor

1 D. Question

Write the rationalizing factor of the following.

$2\sqrt[3]{5}$

Answer

Given, $2\sqrt[3]{5}$

Need to find the rationalizing factor

⇒ We know that if the product of two surds is rational then each is called a rationalizing factor of each other

⇒ $2\sqrt[3]{5} \times \sqrt[3]{25}$

= $2(\sqrt[3]{5 \times 25})$

= 2 ($\sqrt[3]{125}$)

= 2 (5) = 10 is rational number

Hence, $\sqrt[3]{25}$ is rationalizing factor

1 E. Question

Write the rationalizing factor of the following.

$5-4\sqrt{3}$

Answer

Given, $5-4\sqrt{3}$

Need to find the rationalizing factor

⇒ We know that if the product of two surds is rational then each is called a rationalizing factor of each other

⇒ $(5-4\sqrt{3})(5+4\sqrt{3})$

⇒ $25+20\sqrt{3}-20\sqrt{3}-(16)(3)$

= −23 is rational number

∴ Rationalizing factor of $(5-4\sqrt{3})$ is $(5 + 4\sqrt{3})$

1 F. Question

Write the rationalizing factor of the following.

$\sqrt{2}+\sqrt{3}$

Answer

Given, $\sqrt{2} +\sqrt{3}$

Need to find the rationalizing factor

⇒ We know that if the product of two surds is rational then each is called a rationalizing factor of each other

⇒ $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

= $4 -\sqrt{6} + \sqrt{6} -3$

= 1 is a rational number

∴ Rationalizing factor of $(\sqrt{2} + \sqrt{3})$ is $(\sqrt{2} - \sqrt{3})$

1 G. Question

Write the rationalizing factor of the following.

$\sqrt{5}-\sqrt{2}$

Answer

Given, $\sqrt{5}-\sqrt{2}$

Need to find the rationalizing factor

⇒ We know that if the product of two surds is rational then each is called a rationalizing factor of each other

⇒ $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$= 25 + \sqrt{10} - \sqrt{10} - 2$

= 23 is a rational number

∴ Rationalizing factor of $(\sqrt{5}-\sqrt{2})$ is $(\sqrt{5}+\sqrt{2})$

1 H. Question

Write the rationalizing factor of the following.

$2+\sqrt{3}$

Answer

Given, $2+\sqrt{3}$

Need to find the rationalizing factor

⇒ We know that if the product of two surds is rational then each is called a rationalizing factor of each other

⇒ $(2+\sqrt{3})(2-\sqrt{3})$

$= 4 + 2\sqrt{3} - 2\sqrt{3} - 3$

= 1 is a rational number

∴ Rationalizing factor of $(2+\sqrt{3})$ is $(2-\sqrt{3})$

2 A. Question

Rationalize the denominator of the following

$$\frac{3}{\sqrt{5}}$$

Answer

Given, $\frac{3}{\sqrt{5}}$

Need to rationalize the denominator

⇒ To rationalize the denominator of $\frac{3}{\sqrt{5}}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{3\sqrt{5}}{(\sqrt{5})^2}$

$= \frac{3\sqrt{5}}{5}$

Hence, rationalizing the denominator of $\frac{3}{\sqrt{5}}$ we get $\frac{3\sqrt{5}}{5}$

2 B. Question

Rationalize the denominator of the following

$$\frac{2}{3\sqrt{3}}$$

Answer

Given, $\frac{2}{3\sqrt{3}}$

Need to rationalize the denominator

⇒ To rationalize the denominator of $\frac{2}{3\sqrt{3}}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{2}{3\sqrt{3}} \times \frac{3\sqrt{3}}{3\sqrt{3}}$

$= \frac{2\times 3\sqrt{3}}{(3\sqrt{3})^2}$

$= \frac{6\sqrt{3}}{27}$

$= \frac{2\sqrt{3}}{9}$

Hence, rationalizing the denominator of $\frac{2}{3\sqrt{3}}$ we get $\frac{2\sqrt{3}}{9}$

2 C. Question

Rationalize the denominator of the following

$$\frac{1}{\sqrt{12}}$$

Answer

Given, $\frac{1}{\sqrt{12}}$

Need to rationalize the denominator

⇒ To rationalize the denominator of $\frac{1}{\sqrt{12}}$ we must multiply the number with its denominator as follows

⇒ here, $\sqrt{12}$ can be written as $2\sqrt{3}$

$\Rightarrow \frac{1}{2\sqrt{3}} \times \frac{2\sqrt{3}}{2\sqrt{3}}$

$= \frac{2\sqrt{3}}{(2\sqrt{3})^2}$

$= \frac{2\sqrt{3}}{12}$

$= \frac{\sqrt{3}}{6}$

Hence, rationalizing the denominator of $\frac{1}{\sqrt{12}}$ we get $\frac{\sqrt{3}}{6}$

2 D. Question

Rationalize the denominator of the following

$$\frac{2\sqrt{7}}{\sqrt{11}}$$

Answer

Given, $\frac{2\sqrt{7}}{\sqrt{11}}$

Need to rationalize the denominator

⇒ To rationalize the denominator of $\frac{2\sqrt{7}}{\sqrt{11}}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{2\sqrt{7}}{\sqrt{11}} \times \frac{\sqrt{11}}{\sqrt{11}}$

$= \frac{(2\sqrt{7})(\sqrt{11})}{(\sqrt{11})^2}$

$= \frac{2\sqrt{77}}{11}$

Hence, rationalizing the denominator of $\frac{2\sqrt{7}}{\sqrt{11}}$ we get $\frac{2\sqrt{77}}{11}$

2 E. Question

Rationalize the denominator of the following

$$\frac{3\sqrt[3]{5}}{\sqrt[3]{9}}$$

Answer

Given, $\frac{3\sqrt[3]{5}}{\sqrt[3]{9}}$

Need to rationalize the denominator

⇒ To rationalize the denominator of $\frac{3\sqrt[3]{5}}{\sqrt[3]{9}}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{3\sqrt[3]{5}}{\sqrt[3]{9}} \times \frac{\sqrt[3]{9}}{\sqrt[3]{9}}$

$= \frac{3\sqrt[3]{45}}{\left(\sqrt[3]{9}\right)^2}$

⇒ Now, we can write 3 as $\left(\sqrt[3]{3}\right)^3 = \sqrt[3]{27}$

$\therefore \frac{\sqrt[3]{27}\times\sqrt[3]{45}}{\sqrt[3]{9}\times\sqrt[3]{9}}$

$= \frac{\sqrt[3]{27\times45}}{\sqrt[3]{81}}$

$\because \frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\left(\frac{a}{b}\right)}$

$= \sqrt[3]{\frac{(27\times45)}{81}}$

$= \sqrt[3]{15}$

Hence, rationalizing the denominator of $\frac{3\sqrt[3]{5}}{\sqrt[3]{9}}$ we get $\sqrt[3]{15}$

3 A. Question

Simplify by rationalizing the denominator.

$$\frac{1}{11+\sqrt{3}}$$

Answer

Given, $\frac{1}{11+\sqrt{3}}$

Need to simplify by rationalizing denominator

⇒ To rationalize the denominator of $\frac{1}{11+\sqrt{3}}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{1}{11+\sqrt{3}} \times \frac{11-\sqrt{3}}{11-\sqrt{3}}$

$= \frac{11-\sqrt{3}}{(11)^2-(\sqrt{3})^2}$

⇒ we know the denominator is in the form of $a^2-b^2 = (a+b)(a-b)$

$= \frac{11-\sqrt{3}}{121-3}$

$= \frac{11-\sqrt{3}}{118}$

Hence, $\frac{1}{11+\sqrt{3}}$ is simplified by rationalizing denominator as $\frac{11-\sqrt{3}}{118}$

3 B. Question

Simplify by rationalizing the denominator.

$$\frac{1}{9+3\sqrt{3}}$$

Answer

Given, $\frac{1}{9+3\sqrt{5}}$

Need to simplify by rationalizing denominator

⇒ To rationalize the denominator of $\frac{1}{9+3\sqrt{5}}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{1}{9+3\sqrt{5}} \times \frac{9-3\sqrt{5}}{9-3\sqrt{5}}$

$= \frac{9-3\sqrt{5}}{(9)^2-(3\sqrt{5})^2}$

⇒ we know the denominator is in the form of $a^2-b^2 = (a+b)(a-b)$

$= \frac{3(3-\sqrt{5})}{81-45}$

$= \frac{3(3-\sqrt{5})}{36}$

$= \frac{(3-\sqrt{5})}{12}$

Hence, $\frac{1}{9+3\sqrt{5}}$ is simplified by rationalizing denominator as $\frac{(3-\sqrt{5})}{12}$

3 C. Question

Simplify by rationalizing the denominator.

$$\frac{1}{\sqrt{11}+\sqrt{13}}$$

Answer

Given, $\frac{1}{\sqrt{11}+\sqrt{13}}$

Need to simplify by rationalizing denominator

⇒ To rationalize the denominator of $\frac{1}{\sqrt{11}+\sqrt{13}}$ we must multiply the number with its denominator as follows

⇒ $\frac{1}{\sqrt{11}+\sqrt{13}}$ can be written as $\frac{1}{\sqrt{13}+\sqrt{11}}$

⇒ $\frac{1}{\sqrt{13}+\sqrt{11}} \times \frac{\sqrt{13}-\sqrt{11}}{\sqrt{13}-\sqrt{11}}$

$= \frac{\sqrt{13}-\sqrt{11}}{(\sqrt{13})^2-(\sqrt{11})^2}$

⇒ we know the denominator is in the form of $a^2-b^2 = (a+b)(a-b)$

$= \frac{\sqrt{13}-\sqrt{11}}{13-11}$

$= \frac{\sqrt{13}-\sqrt{11}}{2}$

Hence, $\frac{1}{\sqrt{11}+\sqrt{13}}$ is simplified by rationalizing denominator as $\frac{\sqrt{13}-\sqrt{11}}{2}$

3 D. Question

Simplify by rationalizing the denominator.

$$\frac{\sqrt{5}+1}{\sqrt{5}-1}$$

Answer

Given, $\frac{\sqrt{5}+1}{\sqrt{5}-1}$

Need to simplify by rationalizing denominator

⇒ To rationalize the denominator of $\frac{\sqrt{5}+1}{\sqrt{5}-1}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{\sqrt{5}+1}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$= \frac{(\sqrt{5}+1)^2}{(\sqrt{5})^2-1}$

⇒ we know that numerator is in the form of $(a+b)^2 = a^2+2ab+b^2$

⇒ we know the denominator is in the form of $a^2-b^2 = (a+b)(a-b)$

$= \frac{5+2\sqrt{5}+1}{4}$

$= \frac{6+2\sqrt{5}}{4}$

$= \frac{2(3+\sqrt{5})}{4}$

$= \frac{3+\sqrt{5}}{2}$

Hence, $\frac{\sqrt{5}+1}{\sqrt{5}-1}$ is simplified by rationalizing denominator as $\frac{3+\sqrt{5}}{2}$

3 E. Question

Simplify by rationalizing the denominator.

$$\frac{3-\sqrt{3}}{2+5\sqrt{3}}$$

Answer

Given, $\frac{3-\sqrt{3}}{2+5\sqrt{3}}$

Need to simplify by rationalizing denominator

⇒ To rationalize the denominator of $\frac{3-\sqrt{3}}{2+5\sqrt{3}}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{3-\sqrt{3}}{2+5\sqrt{3}} \times \frac{2-5\sqrt{3}}{2-5\sqrt{3}}$

⇒ we know the denominator is in the form of $a^2-b^2 = (a+b)(a-b)$

$= \frac{(3-\sqrt{3})(2-5\sqrt{3})}{2^2-(5\sqrt{3})^2}$

$= \frac{6-15\sqrt{3}-2\sqrt{3}+5(3)}{4-25(3)}$

$= \frac{21-17\sqrt{3}}{4-75}$

$= \frac{-(17\sqrt{3}-21)}{-(75-4)}$

$= \frac{17\sqrt{3}-21}{71}$

Hence, $\frac{3-\sqrt{3}}{2+5\sqrt{3}}$ is simplified by rationalizing denominator as $\frac{17\sqrt{3}-21}{71}$

4 A. Question

Find the values of the following upto 3 decimal places. Given that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$.

$$\frac{1}{\sqrt{2}}$$

Answer

Given, $\frac{1}{\sqrt{2}}$

Need to find the values upto 3 decimal places

⇒ substitute the value of $\sqrt{2} \approx 1.414$

$\Rightarrow \frac{1}{1.414}$

$= 0.707$

Hence, the value of $\frac{1}{\sqrt{2}}$ is 0.707

4 B. Question

Find the values of the following upto 3 decimal places. Given that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$.

$$\frac{6}{\sqrt{3}}$$

Answer

Given, $\frac{6}{\sqrt{3}}$

Need to find the values upto 3 decimal places

⇒ substitute the value of $\sqrt{3} \approx 1.732$

$\Rightarrow \frac{6}{1.732}$

= 3.46

Hence, the value of $\frac{6}{\sqrt{3}}$ is 3.46

4 C. Question

Find the values of the following upto 3 decimal places. Given that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$.

$$\frac{5-\sqrt{3}}{\sqrt{3}}$$

Answer

Given, $\frac{5-\sqrt{3}}{\sqrt{3}}$

Need to find the values upto 3 decimal places

⇒ substitute the value of $\sqrt{3} \approx 1.732$

$\Rightarrow \frac{5-1.732}{1.732}$

= 1.887

Hence, the value of $\frac{5-\sqrt{3}}{\sqrt{3}}$ is 1.887

4 D. Question

Find the values of the following upto 3 decimal places. Given that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$.

$$\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$$

Answer

Given, $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$

Need to find the values upto 3 decimal places

⇒ Substitute the value of $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$ and

$\sqrt{2} \approx 1.414$

$\Rightarrow \frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$

$= \frac{3.162-2.236}{1.414}$

$= \frac{0.926}{1.414}$

$= 0.655$

Hence, the value of $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$ is 0.655

4 E. Question

Find the values of the following upto 3 decimal places. Given that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$.

$$\frac{3-\sqrt{5}}{3+2\sqrt{5}}$$

Answer

Given, $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$

Need to find the values upto 3 decimal places

⇒ Substitute the value of $\sqrt{5} \approx 2.236$, $\sqrt{2} \approx 1.414$

$\Rightarrow \frac{3-\sqrt{5}}{3+2\sqrt{5}}$

$= \frac{3-2.236}{3+2(2.236)}$

$= \frac{0.764}{3+4.472}$

$= \frac{0.764}{7.472}$

$= 0.1022$

Hence, the value of $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ is 0.1022

4 F. Question

Find the values of the following upto 3 decimal places. Given that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$.

$$\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

Answer

Given, $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

Need to find the values upto 3 decimal places

$\Rightarrow$ Substitute the value of $\sqrt{5} \approx 2.236$, $\sqrt{2} \approx 1.414$

$\Rightarrow \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

$= \frac{2.236+1.414}{2.236-1.414}$

$= \frac{3.65}{0.822}$

$= 4.441$

Hence, the value of $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ is 4.441

4 G. Question

Find the values of the following upto 3 decimal places. Given that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$.

$$\frac{\sqrt{3}+1}{\sqrt{3}-1}$$

Answer

Given, $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Need to find the values upto 3 decimal places

$\Rightarrow$ Substitute the value of $\sqrt{3} \approx 1.732$

$\Rightarrow \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \frac{1.732+1}{1.732-1}$

$= \frac{2.732}{0.732}$

= 3.732

Hence, the value of $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ is 3.732

4 H. Question

Find the values of the following upto 3 decimal places. Given that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$.

$$\frac{1}{\sqrt{10}+\sqrt{5}}$$

Answer

Given, $\frac{1}{\sqrt{10}+\sqrt{5}}$

Need to find the values upto 3 decimal places

⇒ Substitute the value of $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$

$\Rightarrow \frac{1}{\sqrt{10}+\sqrt{5}}$

$= \frac{1}{3.162+2.236}$

$= \frac{1}{5.392}$

= 0.1854

Hence, the value of $\frac{1}{\sqrt{10}+\sqrt{5}}$ is 0.1854

5. Question

If $\frac{5+\sqrt{6}}{5-\sqrt{6}} = a + b\sqrt{6}$ find the values of a and b.

Answer

Given, $\frac{5+\sqrt{6}}{5-\sqrt{6}} = a+b\sqrt{6}$

Need to find the value of a and b

⇒ Now, we can find by rationalizing the denominator

⇒ Since, we know to rationalize the denominator of $\frac{5+\sqrt{6}}{5-\sqrt{6}}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{5+\sqrt{6}}{5-\sqrt{6}} \times \frac{5+\sqrt{6}}{5+\sqrt{6}}$

$= \frac{(5+\sqrt{6})^2}{5^2-\sqrt{6}^2}$

∵ we know that numerator is in the form of $(a+b)^2 = a^2+2ab+b^2$

And denominator in the form of $a^2-b^2 = (a+b)(a-b)$

$\Rightarrow \frac{25+2(5)(\sqrt{6})+6}{25-6}$

$= \frac{30+10\sqrt{6}}{19}$

$= \frac{30}{19}+\frac{10}{19}\sqrt{6}$

∴ $a = \frac{30}{19}$ and $b = \frac{10}{19}\sqrt{6}$

6. Question

If $\frac{\left(\sqrt{3}+1\right)^2}{4-2\sqrt{3}} = a + b\sqrt{3}$ find the values of a and b.

Answer

Given, $\frac{(\sqrt{3}+1)^2}{4-2\sqrt{3}} = a+b\sqrt{3}$

Need to find the value of a and b

⇒ Now, we can find by rationalizing the denominator

⇒ Since, we know to rationalize the denominator of $\frac{(\sqrt{3}+1)^2}{4-2\sqrt{3}}$ we must multiply the number with its denominator as follows

$\Rightarrow \frac{(\sqrt{3}+1)^2}{4-2\sqrt{3}} \times \frac{4+2\sqrt{3}}{4+2\sqrt{3}}$

$= \frac{(3+2\sqrt{3}+1)(4+2\sqrt{3})}{4^2-(2\sqrt{3})^2}$

$= \frac{(4+2\sqrt{3})(4+2\sqrt{3})}{16-12}$

$= \frac{(4+2\sqrt{3})^2}{4}$

$= \frac{16+2(4)(2\sqrt{3})+4(3)}{4}$

$= \frac{16+16\sqrt{3}+12}{4}$

$= \frac{28+16\sqrt{3}}{4}$

$= \frac{4(7+4\sqrt{3})}{4}$

$= 7+4\sqrt{3}$

$\therefore 7+4\sqrt{3} = a+b\sqrt{3}$

Hence, the value of a = 7 and b = 4

7. Question

If $\frac{\sqrt{5}+1}{\sqrt{3}-1}+\frac{\sqrt{5}-1}{\sqrt{5}+1} = a+b\sqrt{5}$, find the values of a and b.

Answer

Given, $\frac{\sqrt{5}+1}{\sqrt{5}-1}+\frac{\sqrt{5}-1}{\sqrt{5}+1} = a+b\sqrt{5}$

Need to find the value of a and b

⇒ ⇒ Now, we can find by rationalizing the denominator

⇒ Since, we know to rationalize the denominator of $\frac{\sqrt{5}+1}{\sqrt{5}-1}+\frac{\sqrt{5}-1}{\sqrt{5}+1}$ we must multiply the number with its denominator as follows

$\Rightarrow (\frac{\sqrt{5}+1}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1})+(\frac{\sqrt{5}-1}{\sqrt{5}+1}\times\frac{\sqrt{5}-1}{\sqrt{5}-1})$

$= \frac{(\sqrt{5}+1)^2}{5-1}+\frac{(\sqrt{5}-1)^2}{5-1}$

$= \frac{5+2\sqrt{5}+1}{4}+\frac{5-2(\sqrt{5})(1)+1}{4}$

$= \frac{6+2\sqrt{5}}{4}+\frac{6-2\sqrt{5}}{4}$

$= \frac{6+2\sqrt{5}+6-2\sqrt{5}}{4}$

$= \frac{12}{4}$

= 3

∴ a = 3 b = 0

8. Question

If $\frac{4+\sqrt{5}}{4-\sqrt{3}}-\frac{4-\sqrt{5}}{4+\sqrt{5}}=a+b\sqrt{5}$, find the values of a and b.

Answer

Given, $\frac{4+\sqrt{5}}{4-\sqrt{5}}-\frac{4-\sqrt{5}}{4+\sqrt{5}}=a+b\sqrt{5}$

Need to find the value of a and b

⇒ ⇒ Now, we can find by rationalizing the denominator

⇒ Since, we know to rationalize the denominator of $\frac{4+\sqrt{5}}{4-\sqrt{5}}-\frac{4-\sqrt{5}}{4+\sqrt{5}}$ we must multiply the number with its denominator as follows

$\Rightarrow(\frac{4+\sqrt{5}}{4-\sqrt{5}}\times\frac{4+\sqrt{5}}{4+\sqrt{5}})-(\frac{4-\sqrt{5}}{4+\sqrt{5}}\times\frac{4-\sqrt{5}}{4-\sqrt{5}})$

$=\frac{(4+\sqrt{5})^2}{4^2-\sqrt{5^2}}-\frac{(4-\sqrt{5})^2}{4^2-\sqrt{5^2}}$

$=\frac{16+2(4)(\sqrt{5})+25}{16-5}-\frac{16-2(4)(\sqrt{5})+25}{16-5}$

$=\frac{41+8\sqrt{5}}{11}-\frac{41-8\sqrt{5}}{11}$

$=\frac{41+8\sqrt{5}-41+8\sqrt{5}}{11}$

$=\frac{16\sqrt{5}}{11}$

$\Rightarrow\frac{16\sqrt{5}}{11}=a+b\sqrt{5}$

Hence, the value of a = 0 and b = $\frac{16}{11}$

9. Question

If $x=2+\sqrt{3}$, find the values of $x^2+\frac{1}{x^2}$

Answer

Given, x= 2 + √3

Need to find the values of $x^2+\frac{1}{x^2}$

⇒ By substituting the given values of x in the equation we get

⇒ $x^2 = (2+\sqrt{3})^2 = 4+2(2)(\sqrt{3})+(3) = 7+4\sqrt{3}$

⇒ $\frac{1}{x^2} = \frac{1}{(2+\sqrt{3})^2}$

$= \frac{1}{7+4\sqrt{3}}$

⇒ By rationalizing method we can write as

⇒ $\frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$= \frac{7-4\sqrt{3}}{49-16(3)}$

$= \frac{7-4\sqrt{3}}{49-48}$

$= \frac{7-4\sqrt{3}}{1}$

$= 7-4\sqrt{3}$

⇒ $x^2+\frac{1}{x^2} = 7+4\sqrt{3}+7-4\sqrt{3}$

$= 14$

Hence, the value of $x^2+\frac{1}{x^2}$ is 14

10. Question

$x = \sqrt{3}+1$, find the values of $\left(x-\frac{2}{x}\right)^2$

Answer

Given, $x = \sqrt{3}+1$

Need to find the value of $\left(x-\frac{2}{x}\right)^2$

⇒ $\frac{2}{x} = \frac{2}{\sqrt{3}+1}$

$= \frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{2(\sqrt{3}-1)}{3-1}$

$= \frac{2(\sqrt{3}-1)}{2}$

$= \sqrt{3} - 1$

$\Rightarrow \left(x - \frac{2}{x}\right)^2 = ((\sqrt{3}+1) - (\sqrt{3}-1))^2$

$= (\sqrt{3}+1-\sqrt{3}+1)^2$

$= 4$

Hence, the value of $\left(x - \frac{2}{x}\right)^2$ is 4

Exercise 1.3

1. Question

Using division algorithm, find the quotient and remainder of the following pairs.

(i) 10, 3 (ii) 5, 12 (iii) 27, 3

Answer

(i) 10,3

We write the given pair in the form a = bq + r, $0 \leq r < b$ as follows.

10 = 3(3) + 1[3 divides 10 three time and leaves the remainder 1]

quotient = 3; remainder = 1

(ii) 5,12

We write the given pair in the form a = bq + r, $0 \leq r < b$ as follows.

5 = 12(0) + 5 [12 divides 5 Zero time and leaves the remainder 5]

quotient = 0; remainder = 5

(iii) 27,3

We write the given pair in the form a = bq + r, $0 \leq r < b$ as follows.

27 = 3(9) + 0 [3 divides 27 Nine time and leaves the remainder 1]

quotient = 3; remainder = 0

Exercise 1.4

1. Question

Which one of the following is not a surd?

A. $\sqrt[3]{8}$

B. $\sqrt[3]{30}$

C. $\sqrt[5]{4}$

D. $\sqrt[8]{3}$

Answer

As we know

surd is a number in which we cant remove its square root(cube root ...etc)

A.$\sqrt[3]{8} = 8^{\frac{1}{3}} = 2$ is not surd

B. $\sqrt[3]{30}$ cant be simplified hence surd

C.$\sqrt[5]{4}$ cannot be simplified hence surd

D. $\sqrt[8]{3}$ cannot be simplified hence surd

Hence A is the answer.

2. Question

The simplest form of $\sqrt{50}$ is

A. $5\sqrt{10}$

B. $5\sqrt{2}$

C. $10\sqrt{5}$

D. $25\sqrt{2}$

Answer

On simplification

$\sqrt{50}$

$= \sqrt{5 \times 5 \times 2}$

$= 5\sqrt{2}$

Hence B is the answer

3. Question

$\sqrt[4]{11}$ is equal to

A. $\sqrt[8]{11^2}$

B. $\sqrt[8]{11^4}$

C. $\sqrt[8]{11^8}$

D. $\sqrt[8]{11^6}$

Answer

On Simplification

$\sqrt[4]{11}$

$= 11^{\frac{1}{4}}$

$= 11^{\frac{2}{8}}$

$= \sqrt[8]{11^2}$

Hence A is the answer

4. Question

$\frac{2}{\sqrt{2}}$ is equal to

A. $2\sqrt{2}$

B. $\sqrt{2}$

C. $\frac{\sqrt{2}}{2}$

D. 2

Answer

On rationalizing $\frac{2}{\sqrt{2}}$

$= \frac{2\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$ (rationalizing)

$= \frac{2\sqrt{2}}{2}$

$= \sqrt{2}$

Hence B is the answer

5. Question

The rationlizing factor of $\frac{5}{\sqrt[3]{3}}$ is

A. $\sqrt[3]{6}$

B. $\sqrt[3]{3}$

C. $\sqrt[3]{9}$

D. $\sqrt[3]{27}$

Answer

As we know

Prime factorization of 27 = $3 \times 3 \times 3$

$\frac{5}{\sqrt[3]{3}} = \frac{5}{\sqrt[3]{3}} \times \frac{\sqrt[3]{3} \times \sqrt[3]{3}}{\sqrt[3]{3} \times \sqrt[3]{3}}$

$= \frac{5}{\sqrt[3]{3}} \times \frac{\sqrt[3]{9}}{\sqrt[3]{9}}$

Hence C is the answer

6. Question

Which one of the following is not true?

A. $\sqrt{2}$ is an irrational number

B. $\sqrt{17}$ is an irrational number

C. 0.10110011100011110... is an irrational number

D. $\sqrt[4]{16}$ is an irrational number

Answer

As we know an irrational number are those number which cannot be represented in a simple fraction

A. $\sqrt{2}$ cannot be represent in simple fraction hence it is an irrational number

Hence A is true

B. $\sqrt{17}$ cannot be represent in simple fraction hence it is an irrational number

Hence B is true

C. 0.10110011100011110....cannot be represent in simple fraction hence it is a irrational number . Hence C is true

D. on simplification

$$\sqrt[4]{16} = \sqrt[4]{2 \times 2 \times 2 \times 2} = \sqrt[4]{2^4} = 2$$

Hence is rational number and not irrational.

Hence D is the right answer

7. Question

The order and radicand of the surd $\sqrt[8]{12}$ are respectively

A. 8,12

B. 12,8

C. 16,12

D. 12,16

Answer

As we know

$\sqrt[a]{n}$ here a is the order of surd and n is the radicand

$\sqrt[8]{12}$ order = 8

Radicand = 12

A is the answer

8. Question

The surd having radicand 9 and order 3 is

A. $\sqrt[9]{3}$

B. $\sqrt[3]{27}$

C. $\sqrt[3]{9}$

D. $\sqrt[3]{81}$

Answer

As we know

$\sqrt[a]{n}$ here a is the order of surd and n is the radicand

$\sqrt[3]{9}$ is the answer

Hence C is the option

9. Question

$5\sqrt[3]{3}$ represents the pure surd

A. $\sqrt[3]{15}$

B. $\sqrt[3]{375}$

C. $\sqrt[3]{75}$

D. $\sqrt[3]{45}$

Answer

$5\sqrt[3]{3} = \sqrt[3]{5 \times 5 \times 5 \times 3} = \sqrt[3]{125 \times 3} = \sqrt[3]{375}$

Which is B

Hence B is the answer

10. Question

Which one of the following is not true?

A. $\sqrt{2}$ is an irrational number

B. If a is a rational number and $\sqrt{b}$ is an irrational number

C. Every surd is an irrational number.

D. The square root of every positive integer is always irrational

Answer

Option A is incorrect because $\sqrt{2}$ cannot be written in simple fraction.

hence is irrational number, hence true.

option B is incorrect as it can be written in simple form hence it is a rational number

And also √b cannot be written in simple form hence it is a irrational number

Option C

As we know surd is a number in which we cant remove its square root(cube root …etc) hence cannot represent, in simple fraction hence it is irrational number

Hence C is true

Option D

D is not true as square root of every positive integer is not always irrational

For example, square root of 4 is 2 which is rational number hence D is not true.

Hence D is the answer

11. Question

Which one of the following is not true?

A. When x is not a perfect square, $\sqrt{x}$ is an irrational number

B. The index form of $\sqrt[m]{x^n}$ is $x^{\frac{n}{m}}$

C. The radical form of $\left(x^{\frac{1}{n}}\right)^{\frac{1}{m}}$ is $\sqrt[mn]{x}$

D. Every real number is an irrational number

Answer

As in

Option A

As we know an irrational number are those number which cannot be represented in simple fraction

Hence they are not perfect square

For example 3 is not a perfect square

also $\sqrt{3}$ is irrational number

hence A is true

option B

index form of number means power form

as $\sqrt[m]{x^n} = \sqrt[m]{x}^n = x^{\frac{m}{n}}$

hence B is true

Option C

Radical form means surd form

$x^{\frac{1}{n}^{\frac{1}{m}}} = x^{\frac{1}{m \times n}} = \sqrt[mn]{x}$

Hence C is also true

Option D

It is not true that every real number is a rational number

For example 2 is real number but not irrational number

Hence D is not true

Hence D is the right answer

12. Question

$\left(\sqrt{5}-2\right)\left(\sqrt{5}+2\right)$ is equal to

A. 1

B. 3

C. 23

D. 21

Answer

using identity

$(a + b)(a-b) = a^2 - b^2$

$\left(\sqrt{5}-2\right)\left(\sqrt{5}+2\right) = \left(\left(\sqrt{5}\right)^2 - 2^2\right)$

= (5-4)

= 1

Hence A is the answer.