LIMIT
DPP - 1

1. $\lim\limits_{x\to 3}\left([x-3]+[3-x]-x\right)$, where [.] denotes the greatest integer function, is equal to

(A) 4 (B) – 4 (C) 0 (D) does not exist

2. Let f(x) = $\begin{cases} x+1, & x>0 \\ 2-x, & x\le 0 \end{cases}$

and g(x) = $\begin{cases} x+3, & x<1 \\ x^2-2x-2, & 1\le x<2 \\ x-5, & x\ge 2 \end{cases}$ then $\lim\limits_{x\to 0} g(f(x))$ is

(A) 2 (B) 1 (C) −3 (D) does not exist

3. $\lim\limits_{x\to 0}\dfrac{\sin[\cos x]}{1+[\cos x]}$ ([.] denotes the greatest integer function)

(A) equal to 1 (B) equal to 0 (C) does not exist (D) None of these

4. If f: R → R is defined by f(x) = [x – 3] + |x – 4| for $x\in R$, then $\lim\limits_{x\to 3^-} f(x)$ is equal to (where [.] represents the greatest integer function)

(A) – 2 (B) – 1 (C) 0 (D) 1

5. $\lim\limits_{x\to -7}\dfrac{[x]^2+15[x]+56}{\sin(x+7)\sin(x+8)}$ = (where [.] denotes the greatest integer function)

(A) is 0 (B) is 1 (C) is – 1 (D) does not exist

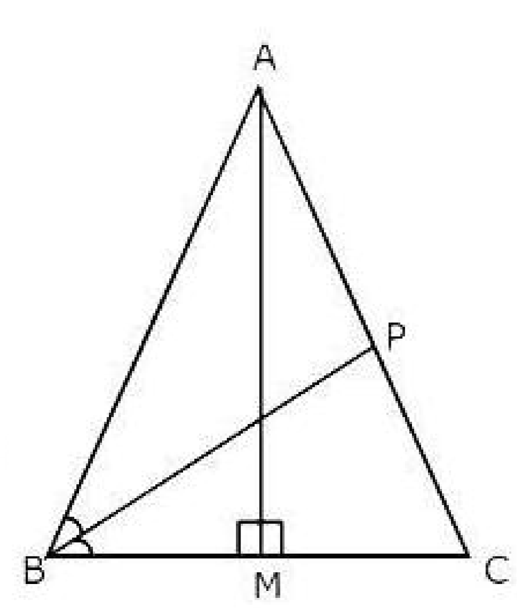
6. The value of $\lim\limits_{x\to 0^+}(1+[x])^{1/x}$, where [x] is the greatest integer $\le$ x, is

(A) e (B) 1 (C) does not exist (D) 0

7. $\lim\limits_{x\to\infty}\left(\dfrac{1}{e}-\dfrac{x}{1+x}\right)^x$ is equal to

(A) $\dfrac{e}{1-e}$ (B) 0 (C) $e^{\frac{e}{1-e}}$ (D) does not exist

8. The figure shows an isosceles triangle ABC with $\angle B=\angle C$. The bisector of angle B intersects the side AC at the point P. Suppose that BC remains fixed but the altitude AM approaches 0, so that A → M (mid-point of BC). Limiting value of BP, is (where a is fixed side BC)

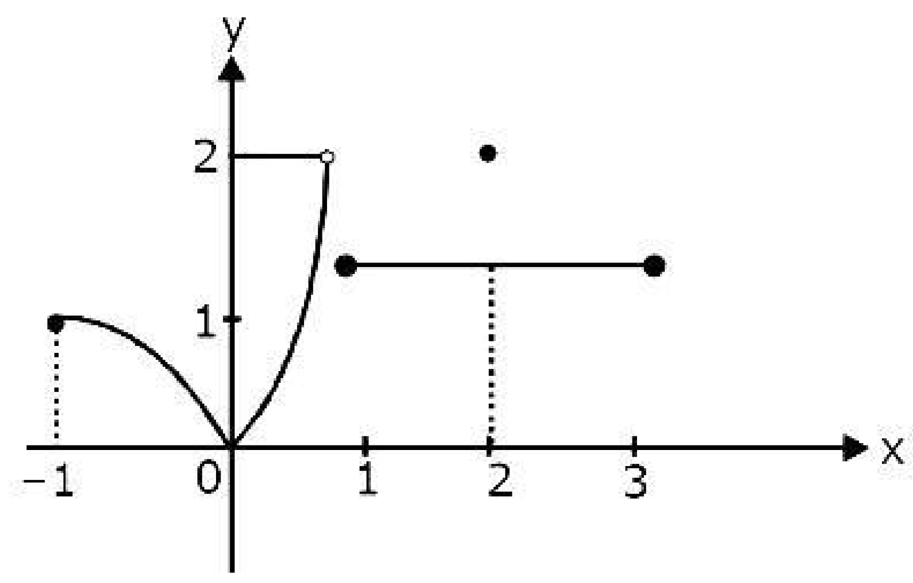


(A) $\dfrac{a}{3}$ (B) $\dfrac{a}{2}$ (C) $\dfrac{2a}{3}$ (D) $\dfrac{3a}{4}$

9. If $\lim\limits_{x\to 0}\left(\ln\left(x^2+5x\right)-2\ln\left(cx+1\right)\right)$ = − 2, then which is not correct ?

(A) $c = e^{-1}$ (B) $c = e^{-2}$ (C) $c = e$ (D) $c = e^2$

10. Which of the following statements are true for the function f defined for $-1 \le x \le 3$ in the figure shown.



(A) $\lim\limits_{x\to -1^+} f(x) = 1$

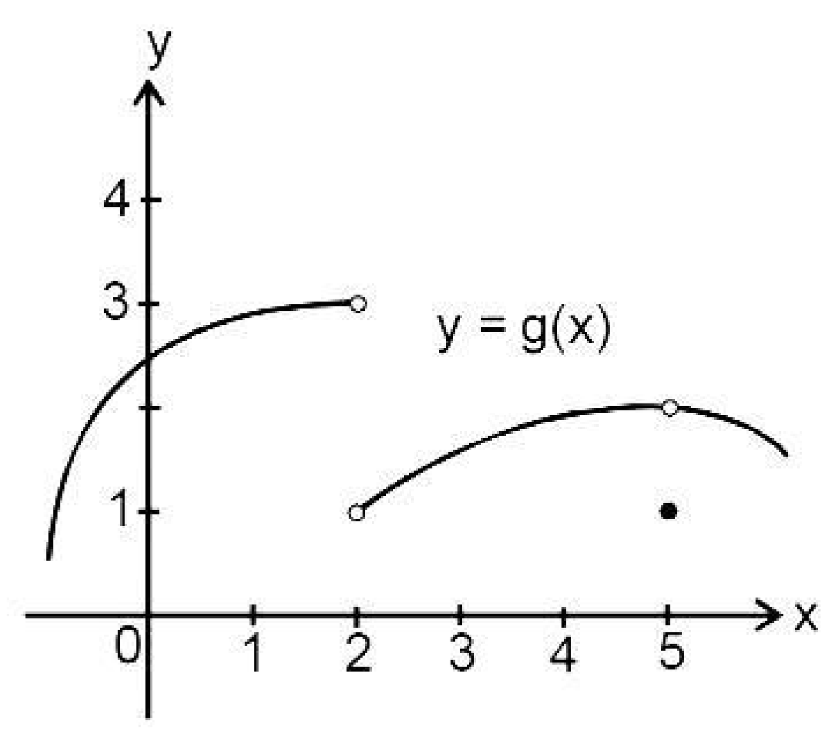
(B) $\lim\limits_{x\to 2} f(x)$ does not exist

(C) $\lim\limits_{x\to 1^-} f(x) = 2$

(D) $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^-} f(x)$

LIMIT
DPP - 2

1. The graph of a function g is shown in the figure. Use it to



state the values (if they exist) of the following

(a) $\lim\limits_{x\to 2^-} g(x)$ **(b)** $\lim\limits_{x\to 2^-} g(x)$ **(c)** $\lim\limits_{x\to 2} g(x)$ **(d)** $\lim\limits_{x\to 5^-} g(x)$

(e) $\lim\limits_{x\to 5^-} g(x)$ **(f)** $\lim\limits_{x\to 5} g(x)$

2. If f(x) = $\begin{cases} \sqrt{x-4} & \text{if} \quad x > 4 \\ 8-2x & \text{if} \quad x < 4 \end{cases}$ determine whether $\lim\limits_{x\to 4} f(x)$ exists.

(A) 1 (B) 0 (C) -1 (D) Does not exist

3. Evaluate $\lim\limits_{x\to 0}\left[\dfrac{\sin^{-1}x}{x}\right]$ (where [$*$] denotes the greatest integer function)

(A) – 1 (B) 1 (C) 0 (D) Does not exist

4. If f(x) = $\begin{cases} x^2+1 & x \ge 1 \\ 3x-1 & x < 1 \end{cases}$ then the value of $\lim\limits_{x\to 1} f(x)$ is -

(A) 1 (B) 2 (C) 3 (D) Does not exist

5. The value of $\lim\limits_{x\to 1} [x]$ is-

(A) 1 (B) 2 (C) 4 (D) Does not exist

6. Evaluate $\lim\limits_{x\to 2}\dfrac{x-2}{\sqrt{x^2-4}-\sqrt{x-2}}$

(A) 1 (B) – 1 (C) 0 (D) Does not exist

7. Evaluate : $\lim\limits_{x\to 0}\dfrac{\sqrt{(1+\sin 3x)}-1}{\ln(1+\tan 2x)}$

(A) $\dfrac{3}{2}$ (B) $\dfrac{3}{4}$ (C) 1 (D) $\dfrac{1}{3}$

8. $\lim\limits_{x \to a} \dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{x+3a}-2\sqrt{x}}$ equals-

(A) $\dfrac{1}{\sqrt{3}}$ (B) $\dfrac{2}{\sqrt{3}}$ (C) $\dfrac{2}{3}\sqrt{3}$ (D) 2/3

9. Let $f(x) = \begin{cases} 1+\dfrac{2x}{a}, & 0 \le x < 1 \\ ax, & 1 \le x < 2 \end{cases}$. If $\lim\limits_{x \to 1} f(x)$ exists, then a is

(A) 1 (B) − 1 (C) 2 (D) − 2

10. If $\lim\limits_{x \to 1}$ (2 − x + a [x − 1] + b [1 + x]) exists, then a and b can take the values (where [·] denotes the greatest integer function)

(A) a = $\dfrac{1}{3}$, b = 1 (B) a = 1, b = −1 (C) a = 9, b = − 9 (D) a = 2, b = $\dfrac{2}{3}$

LIMIT
DPP - 3

1. If $\lim\limits_{x\to\infty} \dfrac{x^2+3x+5}{4x+1+x^k}$ exists then

(A) $k = 2$ (B) $k < 2$ (C) $k > 2$ (D) $k \ge 2$

2. $\lim\limits_{n\to\infty} \dfrac{1.n+2(n-1)+3(n-2)+......+n.1}{1^2+2^2+3^2+......+n^2}$ has the value

(A) $\dfrac{1}{2}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{4}$ (D) 1

3. If $\alpha = \lim\limits_{n\to\infty}\left(\dfrac{1}{n^3+1}+\dfrac{4}{n^3+1}+\dfrac{9}{n^3+1}+......+\dfrac{n^2}{n^3+1}\right)$ and $\beta = \lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 8x}$, then the quadratic equation whose roots are α, β is

(A) $12x^2 - 7x + 1 = 0$ (B) $x^2 + 19x - 120 = 0$
(C) $x^2 - 17x + 66 = 0$ (D) $x^2 - 7x + 12 = 0$

4. The limit of $x^3\left\{\sqrt{x^2+\sqrt{x^4+1}}-x\sqrt{2}\right\}$ as $x\to\infty$ equals

(A) $\dfrac{1}{2\sqrt{2}}$ (B) $\dfrac{1}{4\sqrt{2}}$ (C) zero (D) $\dfrac{1}{\sqrt{2}}$

5. $\lim\limits_{x\to 0^+}\left(\dfrac{3\ln^2 x+5\ln x+6}{1+\ln^2 x}\right)$ is equal to

(A) 0 (B) 3 (C) 1 (D) does not exist

6. If $\lim\limits_{n\to\infty}\left(\sqrt{2n^2+n}-\lambda\sqrt{2n^2-n}\right)=\dfrac{1}{\sqrt{2}}$ where $\lambda>0$, then λ is equal to

(A) -1 (B) 1 (C) $\dfrac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

7. If $\lim\limits_{x\to 0}\dfrac{p\sin 2x+(1-\cos 2x)}{x+\tan x} = 1$, then p is equal to

(A) 0 (B) 1 (C) 2 (D) -2

8. Let $L = \lim\limits_{x\to -2}\dfrac{3x^2+ax+a+1}{x^2+x-2}$. If L is finite, then

(A) $L = \dfrac{4}{3}$ (B) $L = 13$ (C) $L = -2$ (D) $L = \dfrac{-1}{3}$

9. If $\lim\limits_{x\to 3}\left(\dfrac{\sqrt{2x+3}-x}{\sqrt{x+1}-x+1}\right)^{\frac{x-1-\sqrt{x^2-5}}{x^2-5x+6}}$ can be expressed in the form $\dfrac{a\sqrt{b}}{c}$ where a, b, c $\in$ N, then find the least value of $(a^2 + b^2 + c^2)$.

10. Let $f(x) = \dfrac{ax^2+bx+c}{x+1}$ such that $\lim\limits_{x\to 0} f(x) = 2$ and $\lim\limits_{x\to \infty} f(x) = 1$, then find the value of $(a + b + c)$.

LIMIT
DPP - 4

1. If $\lim\limits_{x\to 0}$ $(x^{-3}\sin 3x + ax^{-2} + b)$ exists and is equal to zero then

(A) $a = -3$ & $b = 9/2$
(B) $a = 3$ & $b = 9/2$
(C) $a = -3$ & $b = -9/2$
(D) $a = 3$ & $b = -9/2$

2. $\lim\limits_{x\to\infty} x\left(\frac{1}{e}-\left(\frac{x}{x+1}\right)^x\right)$ equals

(A) $\frac{1}{e}$ (B) $\frac{-1}{2e}$ (C) $\frac{1}{2e}$ (D) $\frac{-1}{e}$

3. If a is non-zero real number and $\lim\limits_{x\to 0}\frac{1-\cos ax}{x^2} = \lim\limits_{x\to\pi}\frac{\sin x}{\pi - x}$, then sum of the squares of all possible values of a, is

(A) 2 (B) 4 (C) 8 (D) 9

4. Which one of the following limit does not have value unity ?

(A) $\lim\limits_{x\to 0}\frac{\sin(\tan x)}{\sin x}$
(B) $\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin(\cos x)}{\cos x}$
(C) $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$
(D) $\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$

5. $\lim\limits_{x\to\infty}(2^x+3^x+5^x+7^x)^{\frac{1}{x}}$ is equal to

(A) 2 (B) 3 (C) 5 (D) 7

6. $\text{Limit}_{x\to 0}\frac{\cos^{-1}(1-x)}{\sqrt{x}} =$

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 1 (D) 0

7. $\lim\limits_{n\to\infty}\left(\frac{\sqrt[n]{p}+\sqrt[n]{q}}{2}\right)^n$, p, q > 0 equals

(A) 1 (B) $\sqrt{pq}$ (C) pq (D) $\frac{pq}{2}$

8. If $\lim\limits_{x \to a}\left(2-\frac{a}{x}\right)^{a\tan\left(\frac{\pi x}{2a}\right)} = \frac{1}{e}$, then a is equal to

(A) $-\pi$ (B) π (C) $\frac{\pi}{2}$ (D) $\frac{-2}{\pi}$

9. The value of the $\lim\limits_{x\to 0}\frac{30^x - 15^x - 10^x - 6^x + 5^x + 3^x + 2^x - 1}{x \sin x \tan x} = \lambda \ln 2 \ln k \ln 5$, where $\lambda, k \in \mathbb{N}$, then find the value of $\lambda + k$.

10. If the value of $\lim\limits_{x\to\infty} x^4\left(\left(\frac{x^2+1}{x^2-1}\right)^{x^2} - e^2\right)$ is equal to $\frac{ae^2}{b}$ (where a and b are coprime) then find the value of (a + b).

LIMIT
DPP - 5

1. Let $\lim\limits_{x\to 1}\dfrac{x^a - ax + a - 1}{(x-1)^2} = f(a)$. The value of $f(101)$ is

(A) 5050 (B) 5151 (C) 4950 (D) 101

2. $\lim\limits_{n\to\infty}\left(\dfrac{n}{n^2+1}+\dfrac{n}{n^2+2}+......+\dfrac{n}{n^2+n}\right)$

(A) is equal to zero (B) is equal to 1/2
(C) is equal to 1 (D) does not exist .

3. $\lim\limits_{x\to\infty}\sqrt[x]{2\sum\limits_{n=0}^{x}\dfrac{x^n}{n!}}$ is

(A) 1 (B) e (C) $2e^{-1}$ (D) 0

4. Let $P(n) = \prod\limits_{n=2}^{n}\left(1-\dfrac{4}{(2n-1)^2}\right)$ then $\lim\limits_{n\to\infty}P(n)$ equals

(A) $\dfrac{1}{4}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{5}$ (D) $\dfrac{1}{6}$

5. The value of $\lim\limits_{n\to\infty}\left(\sum\limits_{r=1}^{n}\left(\dfrac{\sum\limits_{k=1}^{r}k}{\sum\limits_{k=1}^{r}k^3}\right)\right)$ is equal to

(A) 2 (B) 4 (C) 9 (D) 10

6. The value of $\lim\limits_{x\to 0^-}\left(\left[\dfrac{e^x-1}{x}\right]+\left[\dfrac{\sin x}{x}\right]+\left[\dfrac{\ln(1+x)}{x}\right]\right)$ equals

[**Note:** [x] denotes the greatest integer function.]
(A) 0 (B) 1 (C) 2 (D) 3

7. If a, b, c are real numbers such that $\lim\limits_{x\to 0}\dfrac{ax^3-\cos bx-\cos cx+2}{x^3+2x^4+3x^5}=4$, then the value of $(a^2+b^2+c^2)$ is
(A) 8 (B) 16 (C) 32 (D) 64

8. Let $n \in N$ be such that $L = \lim\limits_{x\to 0}\dfrac{(1-\tan x-\cos x+x)(\ln(1+x)-x)}{x^n(x-\sin x)}$
where L is finite and non zero, then the value of $(n + 2\,|L|)$ is equal to
(A) 3 (B) 4 (C) 5 (D) 6

9. $\lim\limits_{n\to\infty}(a^n+b^n)^{\frac{1}{n}}$, $0 < a < b$, is equal to

10. Use Sandwich theorem to evaluate: $\lim\limits_{n\to\infty}\sum\limits_{k=n^2}^{(n+1)^2}\dfrac{1}{\sqrt{k}}$.