Chapter 5 – Coordinate Geometry

Practice Set 5.1

1. Find the distance between each of the following pairs of points.
(1)A(2, 3), B(4, 1)
(2) P(-5, 7), Q(-1, 3)
(3) R(0, -3), S(0, 5/2)
Solution:
(1) Let $A(x_1, y_1)$ and $B(x_2 , y_2)$ be the given points
By distance formula $d(A,B) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
Here $x_1 = 2$, $y_1 = 3$, $x_2 = 4$, $y_2 = 1$
$\therefore$ $d(A,B) = \sqrt{[(4-2)^2+(1-3)^2]}$
$= \sqrt{[2^2+(-2)^2]}$
$= \sqrt{8}$
$= 2\sqrt{2}$
Hence, the distance between A and B is $2\sqrt{2}$ units.

(2) Let $P(x_1, y_1)$ and $Q(x_2 , y_2)$ be the given points
By distance formula $d(P,Q) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
Here $x_1 = -5$, $y_1 = 7$, $x_2 = -1$, $y_2 = 3$
$\therefore$ $d(A,B) = \sqrt{[(-1-(-5))^2+(3-7)^2]}$
$= \sqrt{[4^2+(-4)^2]}$
$= \sqrt{32}$
$= 4\sqrt{2}$
Hence, the distance between P and Q is $4\sqrt{2}$ units.

(3) Let $R(x_1, y_1)$ and $S(x_2 , y_2)$ be the given points
By distance formula $d(R,S) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
Here $x_1 = 0$, $y_1 = -3$, $x_2 = 0$, $y_2 = 5/2$
$d(A,B) = \sqrt{[(0-0)^2+((5/2) -(-3))^2]}$
$= \sqrt{[0^2+(11/2)^2]}$
$= \sqrt{(121/4)}$
$= 11/2$
Hence, the distance between P and Q is 11/2 units.

2. Determine whether the points are collinear.
(1) A(1, -3), B(2, -5), C(-4, 7)
(2) L(-2, 3), M(1, -3), N(5, 4)

Solution:

(1) If the sum of any two distances out of d(A, B), d(B, C) and d(A, C) is equal to the third , then the three points A, B and C are collinear.

$\therefore$ we will find d(A, B), d(B, C) and d(A, C).

Co-ordinates of A = (1,-3)

Co-ordinates of B = (2,-5)

Co-ordinates of C = (-4,7)

By distance formula $d(A,B) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$d(A,B) = \sqrt{[(2-1)^2+(-5-(-3))^2]}$

$d(A,B) = \sqrt{[(1)^2+(-2)^2]}$

$d(A,B) = \sqrt{(1+4)}$

$d(A,B) = \sqrt{5}$ ……………(i)

By distance formula $d(B,C) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$d(B,C) = \sqrt{[(-4-2)^2+(7-(-5))^2]}$

$d(B,C) = \sqrt{[(-6)^2+(12)^2]}$

$d(B,C) = \sqrt{(36+144)}$

$d(B,C) = \sqrt{180}$

$d(B,C) = 6\sqrt{5}$ …………(ii)

By distance formula $d(A,C) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$d(A,C) = \sqrt{[(-4-1)^2+(7-(-3))^2]}$

$d(A,C) = \sqrt{[(-5)^2+(10)^2]}$

$d(A,C) = \sqrt{(25+100)}$

$d(A,C) = \sqrt{125}$

$d(A,C) = 5\sqrt{5}$…………(iii)

From (i) , (ii) and (iii)

$\sqrt{5} + 5\sqrt{5} = 6\sqrt{5}$

d(A,C)+d(A,B) = d(B,C)

Hence, Points A, B, C are collinear.

(2) If the sum of any two distances out of d(L, M), d(M, N) and d(L, N) is equal to the third , then the three points L, M and N are collinear.

we will find d(L, M), d(M, N) and d(L, N).

Co-ordinates of L = (-2, 3)

Co-ordinates of M = (1, -3)

Co-ordinates of N = (5, 4)

By distance formula $d(L,M) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$d(L,M) = \sqrt{[(1-(-2))^2+(-3-3)^2]}$

$d(L,M) = \sqrt{[3^2+(-6)^2]}$

$d(L,M) = \sqrt{(9+36)}$

$d(L,M) = \sqrt{45}$

$d(L,M) = 3\sqrt{5}$……..(i)

By distance formula $d(M,N) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$d(M,N) = \sqrt{[(5-1)^2+(4-(-3))^2]}$
$d(M,N) = \sqrt{[4^2+7^2]}$
$d(M,N) = \sqrt{(16+49)}$
$d(M,N)= \sqrt{65}$……..(ii)
By distance formula $d(L,N) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$d(L,N) = \sqrt{[(5-(-2))^2+(4-3)^2]}$
$d(L,N) = \sqrt{[7^2+1^2]}$
$d(L,N) = \sqrt{(49+1)}$
$d(L,M) = \sqrt{50}$
$d(L,M) =5 \sqrt{2}$……..(iii)
Adding (i) and (iii)
$d(L,M)+d(L,N) = 3\sqrt{5}+\sqrt{50} \neq \sqrt{65}$
$d(L,M)+d(L,N) \neq d(M,N)$
Points L,M,N are not collinear.

3. Find the point on the X-axis which is equidistant from A(-3, 4) and B(1, -4).
Solution:
Let C be the point on X axis equidistant from A(-3,4) and B(1,-4).
Since C lies on X axis, the Y co-ordinate of C is 0.
Let C = (x,0)
Co-ordinates of A = (-3, 4)
Co-ordinates of B = (1, -4)
Since C is equidistant from A and B,
AC = BC
By distance formula, $d(A,C) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$d(A,C) = \sqrt{[(x-(-3))^2+(0-4)^2]}$
$d(A,C) = \sqrt{[(x+3)^2+(4)^2]}$
$d(A,C) = \sqrt{[(x+3)^2+16]}$ ……(i)

By distance formula, $d(B,C) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$d(B,C) = \sqrt{[(x-1)^2+(0-(-4))^2]}$
$d(B,C) = \sqrt{[(x-1)^2+(4)^2]}$
$d(B,C) = \sqrt{[(x-1)^2+16]}$ ……(ii)
Equating (i) and (ii) [$\because$AC = BC]
$\sqrt{[(x+3)^2+16]} = \sqrt{[(x-1)^2+16]}$
Squaring both sides
$(x+3)^2+16 = (x-1)^2+16$
$x^2+6x+9+16 = x^2-2x+1+16$
$x^2+6x+25 = x^2-2x+17$
$8x = -8$
$x = -8/8 = -1$
Hence, the point on X axis which is equidistant from A and B is C(-1,0)

4. Verify that points P(-2, 2), Q(2, 2) and R(2, 7) are vertices of a right angled triangle.

Solution:

Given P(-2, 2), Q(2, 2) and R(2, 7).

If the square of the length of the longest side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

By distance formula, distance between two points = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

PQ = $\sqrt{[(2-(-2))^2+(2-2)^2]}$

PQ = $\sqrt{[(4^2+0^2]}$

PQ = $\sqrt{16}$

PQ = 4 …..(i)

QR = $\sqrt{[(2-2)^2+(7-2)^2]}$

QR = $\sqrt{[(0)^2+(5)^2]}$

QR = $\sqrt{25}$

QR = 5 …….(ii)

PR = $\sqrt{[(2-(-2))^2+(7-2)^2]}$

PR = $\sqrt{[4^2+5^2]}$

PR = $\sqrt{16+25}$

PR = $\sqrt{41}$…….(iii)

$PQ^2+QR^2 = 4^2+5^2$

= 16+25

=41

$PR^2 = 41$

$PQ^2+QR^2 = PR^2$

PQR is a right triangle.

P,Q,R are the vertices of a right angled triangle.

Practice Set 5.2

1. Find the coordinates of point P if P divides the line segment joining the points A(-1,7) and B(4,-3) in the ratio 2:3.
Solution:
Let the co-ordinates of P be (x, y).
A(-1,7) and B(4,-3) are the given points.
$x_1 = -1$, $y_1 = 7$, $x_2 = 4$, $y_2 = -3$, m = 2 and n = 3
By Section formula $x = (mx_2+nx_1)/(m+n)$
$x = (2\times4+3\times-1)/(2+3)$
$x = (8-3)/5$
$x = 5/5$
$x= 1$
By Section formula $y = (my_2+ny_1)/(m+n)$
$y = (2\times-3+3\times7)/5$
$y = (-6+21)/5$
$y = 15/5$
$y = 3$
Hence, the co-ordinate of point P is (1,3).

2. In each of the following examples find the co-ordinates of point A which divides segment PQ in the ratio a:b.
(1) P(-3, 7), Q(1, -4), a:b = 2:1
(2) P(-2, -5), Q(4, 3), a:b = 3:4
(3) P(2, 6), Q(-4, 1), a:b = 1:2

Solution:
(1) Let the co-ordinates of A be (x, y).
P(-3, 7) and Q(1, -4) are the given points.
$x_1 = -3$, $y_1 = 7$, $x_2 = 1$, $y_2 = -4$, m = 2 and n = 1
By Section formula $x = (mx_2+nx_1)/(m+n)$
$x = (2\times1+1\times-3)/(2+1)$
$x = -1/3$
By Section formula $y = (my_2+ny_1)/(m+n)$
$y = (2\times-4+1\times7)/3$
$y = (-8+7)/3$
$y = -1/3$
Hence the co-ordinates of A is (-1/3 , -1/3)

(2) Let the co-ordinates of A be (x, y).
P(-2, -5), and Q(4, 3), are the given points.
$x_1 = -2$, $y_1 = -5$, $x_2 = 4$, $y_2 = 3$, m = 3 and n = 4

By Section formula $x = (mx_2+nx_1)/(m+n)$
$x = (3×4+4×-2)/(3+4)$
$x = (12+-8)/(3+4)$
$x = 4/7$
By Section formula $y = (my_2+ny_1)/(m+n)$
$y = (3×3+4×-5)/(3+4)$
$y = (9-20)/7$
$y = -11/7$
Hence the co-ordinates of A is (4/7 , -11/7)

(3) Let the co-ordinates of A be (x, y).
P(2, 6), and Q(-4, 1), are the given points.
$x_1 = 2$, $y_1 = 6$, $x_2 = -4$, $y_2 = 1$, m =1 and n = 2
By section formula $x = (mx_2+nx_1)/(m+n)$
$x = (1×-4+2×2)/(1+2)$
$x = (-4+4)/(1+2)$
$x = 0$
By Section formula $y = (my_2+ny_1)/(m+n)$
$y = (1×1+2×6)/(1+2)$
$y = (1+12)/3$
$y = 13/3$
Hence, the co-ordinates of A is (0 , 13/3)

3. Find the ratio in which point T(-1, 6)divides the line segment joining the points P(-3, 10) and Q(6, -8).
Solution:
Let, point T(-1,6) divides segment PQ in the ratio m:n
Given P(-3, 10) and Q(6, -8).
x = -1 , y = 6
$x_1 = -3$, $y_1 = 10$
$x_2 = 6$, $y_2 = -8$
By Section formula, $x = (mx_2+nx_1)/(m+n)$
$-1 = m×6+n×-3/(m+n)$
$-1 = 6m-3n/(m+n)$
$-m+-n = 6m-3n$
$2n = 7m$
$m/n = 2/7$
m:n = 2:7
Point T divides PQ is the ratio 2:7.

4. Point P is the center of the circle and AB is a diameter. Find the coordinates of point B if coordinates of point A and P are (2, -3) and (-2, 0) respectively.
Solution:

Given co-ordinates of P = (-2,0) = (x,y)
Co-ordinates of A = (2,-3) = (x_1, y_1)
Let co-ordinates of B = (x_2,y_2)
Since P is the midpoint of diameter AB,
By midpoint formula $x = (x_1+x_2)/2$
$-2 = 2+x_2/2$
$x_2 = -4-2$
$x_2 = -6$
$y = (y_1+y_2)/2$
$0 = (-3+y_2)/2$
$-3+y_2 = 0$
$y_2 = 3$
Hence the co-ordinates of point B is (-6,3).

5. Find the ratio in which point P(k, 7) divides the segment joining A(8, 9) and B(1, 2). Also find k .
Solution:
Let P (x, y) , A (x_1, y_1) and B (x_2, y_2) and be the given points.
Here, x = k, y = 7, $x_1 = 8$, $y_1 = 9$, $x_2 = 1$, $y_2 = 2$,
By Section formula $y = (my_2+ny_1)/(m+n)$
$7 = m\times2+n\times9/(m+n)$
$7 = 2m+9n/(m+n)$
$7m+7n = 2m+9n$
$7m-2n = 9n-7n$
$5m = 2n$
$m/n = 2/5$
Hence m:n = 2:5.

By Section formula, $x = (mx_2+nx_1)/(m+n)$
$k = (2\times1+5\times8)/2+5$
$k = (2+40)/7$
$k = 42/7 = 6$
Hence, value of k is 6.
Hence P(6,7) divides the segment in the ration 2:5.

6. Find the coordinates of midpoint of the segment joining the points (22, 20) and (0, 16).
Solution:
Let $(x_1,y_1) = (22,20)$
$(x_2,y_2) = (0,16)$
Let co-ordinate of midpoint be A(x,y)
By Midpoint formula $x = (x_1+x_2)/2$ and $y = (y_1+y_2)/2$
$x = (22+0)/2 = 11$
$y = (20+16)/2 = 36/2 = 18$
Hence co-ordinates of midpoint are (11,18).

Practice Set 5.3

1. Angles made by the line with the positive direction of X-axis are given. Find the slope of these lines.
(1) 45°
(2) 60°
(3) 90°
Solution:
(1) Given angle made by line with positive direction of X axisθ, = 45˚.
Slope of the line, m = tanθ
m = tan 45˚ = 1
Hence, slope of the line is 1.

(2) Given angle made by line with positive direction of X axisθ, = 60˚.
Slope of the line ,m = tanθ
m = tan 60˚ = √3
Hence, slope of the line is √3.

(3) Given angle made by line with positive direction of X axisθ, = 90˚.
Slope of the line ,m = tanθ
m = tan 90˚ = not defined
Hence, slope of the line cannot be determined.

2. Find the slopes of the lines passing through the given points.
(1) A (2, 3) , B (4, 7)
(2) P (-3, 1) , Q (5, -2)
(3) C (5, -2) , D (7, 3)
(4) L (-2, -3) , M (-6, -8)
(5) E(-4, -2) , F (6, 3)
(6) T (0, -3) , S (0, 4)

Solution:
(1) Given A(2,3) and B(4,7)
x_1 = 2
y_1 = 3
x_2 = 4
y_2 = 7
Slope of the line AB = $(y_2-y_1)/ (x_2-x_1)$
= (7-3)/(4-2)
= 4/2 = 2
Hence, Slope of line AB is 2.

(2) Given P(-3,1) and Q(5,-2)
$x_1 = -3$
$y_1 = 1$
$x_2 = 5$
$y_2 = -2$
Slope of the line PQ = $(y_2-y_1)/ (x_2-x_1)$
= (-2-1)/(5-(-3))
= -3/8
Hence, Slope of line PQ is -3/8.

(3) Given C(5,-2) and D(7,3)
$x_1 = 5$
$y_1 = -2$
$x_2 = 7$
$y_2 = 3$
Slope of the line CD = $(y_2-y_1)/ (x_2-x_1)$
= (3-(-2))/(7-5)
= 5/2
Hence, Slope of line CD is 5/2.

(4) Given L(-2,-3) and M(-6,-8)
$x_1 = -2$
$y_1 = -3$
$x_2 = -6$
$y_2 = -8$
Slope of the line LM = $(y_2-y_1)/ (x_2-x_1)$
= (-8-(-3))/(-6-(-2))
= -5/-4 = 5/4
Hence, Slope of line LM is 5/4.

(5) Given E(-4,-2) and F(6,3)
$x_1 = -4$
$y_1 = -2$
$x_2 = 6$
$y_2 = 3$
Slope of the line EF = $(y_2-y_1)/ (x_2-x_1)$
= (3-(-2))/(6-(-4))
= 5/10 = 1/5
Hence, Slope of line EF is 1/2.

(6) Given T(0,-3) and S(0,4)
$x_1 = 0$
$y_1 = -3$
$x_2 = 0$
$y_2 = 4$
Slope of the line TS = $(y_2-y_1)/ (x_2-x_1)$
= (4-(-3))/(0-0)
= 7/0 = not defined
Hence, Slope of line TS cannot be determined.

3. Determine whether the following points are collinear.
(1) A(-1, -1), B(0, 1), C(1, 3)
(2) D(-2, -3), E(1, 0), F(2, 1)
(3) L(2, 5), M(3, 3), N(5, 1)
Solution:
(1) A(-1, -1), B(0, 1), C(1, 3) are the given points.
Slope of line AB = $(y_2-y_1)/ (x_2-x_1)$
= (1-(-1))/(0-(-1))
= 2/1 = 2
Slope of line BC = $(y_2-y_1)/ (x_2-x_1)$
= (3-1)/(1-0)
= 2
Slope of line AB and BC are equal.
Point B lies on both lines.
Point A,B,C are collinear.

(2) D(-2, -3), E(1, 0), F(2, 1) are the given points.
Slope of line DE = $(y_2-y_1)/ (x_2-x_1)$
= (0-(-3))/(1-(-2))
= 3/3 = 1
Slope of line EF = $(y_2-y_1)/ (x_2-x_1)$
= (1-0)/(2-1)
= 1/1 = 1
Slope of line DE and EF are equal.
Point E lies on both lines.
Point D,E,F are collinear.

(3) L(2, 5), M(3, 3), N(5, 1)are the given points.
Slope of line LM = $(y_2-y_1)/ (x_2-x_1)$
= (3-5)/(3-2)
= -2/1 = -2

Slope of line MN = $(y_2-y_1)/(x_2-x_1)$
= (1-3)/(5-3)
= -2/2 = -1
Slope of line LM and MN are not equal.
Point L,M,N are not collinear.

4. If A (1, -1),B (0, 4),C (-5, 3) are vertices of a triangle then find the slope of each side.
Solution:
A (1, -1),B (0, 4),C (-5, 3) are the given points.
Slope of line AB = $(y_2-y_1)/(x_2-x_1)$
= (4-(-1))/(0-1)
= 5/-1 = -5
Slope of line BC = $(y_2-y_1)/(x_2-x_1)$
= (3-4)/(-5-0)
= -1/-5 = 1/5
Slope of line AC = $(y_2-y_1)/(x_2-x_1)$
= (3-(-1))/(-5-1)
= 4/-6 = -2/3
Hence the slopes of the sides AB, BC and AC are -5, 1/5 , -2/3 respectively.

Problem Set 5

1. Fill in the blanks using correct alternatives.

(1)Seg AB is parallel to Y-axis and coordinates of point A are (1,3) then co-ordinates of point B can be …….. .

(A) (3,1) (B) (5,3) (C) (3,0) (D) (1,-3)

Solution:

Given AB parallel to Y axis. So x- coordinate of all points on A will be same.

Co-ordinates of A = (1,3)

Co-ordinates of B can be (1,-3).

Hence, Option D is the answer.

(2) Out of the following, point …….. lies to the right of the origin on X- axis.

(A) (-2,0) (B) (0,2) (C) (2,3) (D) (2,0)

Solution:

If a point is on X axis, y co-ordinate will be zero.

Since the point lies to right of origin, x co-ordinate will be positive.

So (2,0) lies to the right of the origin on X- axis.

Hence, option D is the answer

(3) Distance of point (-3,4) from the origin is …… .

(A) 7 (B) 1 (C) 5 (D) -5

Solution:

Co-ordinates of origin are (0, 0).

Hence if co-ordinates of point P are (x, y) then d(O, P) = $\sqrt{(x^2+y^2)}$

Distance of (-3,4) from origin = $\sqrt{(-3^2+4^2)} = \sqrt{(9+16)} = \sqrt{25} = 5$

Hence, option C is the answer.

(4) A line makes an angle of 30° with the positive direction of X- axis. So the slope of the line is ………. .

(A) 1/2 (B) √3/2 (C) 1/√3 (D) √3

Solution:

Given angle made by line with positive direction of X axis, = 30˚.

Slope of the line ,m = tan

m = tan 30˚ = $1/\sqrt{3}$

Hence, option C is the answer.

2. Determine whether the given points are collinear.
(1) A(0,2) , B(1,-0.5), C(2,-3)
(2) P(1, 2) , Q(2, 8/5) , R(3, 6/5)
(3) L(1,2) , M(5,3) , N(8,6)
Solution:
(1) A(0,2) , B(1,-0.5), C(2,-3) are the given points.
Slope of line AB = $(y_2-y_1)/(x_2-x_1)$
= (-0.5-2)/(1-0))
= -2.5/1 = -2.5
Slope of line BC = $(y_2-y_1)/(x_2-x_1)$
= (-3-(-0.5))/(2-1)
= -2.5/1 = -2.5
Slope of line AB and BC are equal.
Point B lies on both lines.
Point A,B,C are collinear.

(2) P(1, 2) , Q(2, 8/5) , R(3, 6/5) are the given points.
Slope of line PQ = $(y_2-y_1)/(x_2-x_1)$
= ((8/5)-2)/(2-1))
= (-2/5)/1 = -2/5
Slope of line QR = $(y_2-y_1)/(x_2-x_1)$
= (6/5-(8/5))/(3-2)
=(-2/5)/1 = -2/5
Slope of line PQ and QR are equal.
Point Q lies on both lines.
Point P,Q,R are collinear.

(3) L(1,2) , M(5,3) , N(8,6) are the given points.
Slope of line LM = $(y_2-y_1)/(x_2-x_1)$
= (3-2)/(5-1))
= 1/4
Slope of line MN = $(y_2-y_1)/(x_2-x_1)$
= (6-3)/(8-5)
=3/3 = 1
Slope of line LM $\neq$ Slope of MN
Point L,M,N are not collinear.

3. Find the coordinates of the midpoint of the line segment joining P(0,6) and Q(12,20).

Solution:

Given $P(x_1,y_1) = (0,6)$

$Q(x_2,y_2) = (12,20)$

Let co-ordinate of midpoint be M(x,y)

By Midpoint formula $x = (x_1+x_2)/2$ and $y = (y_1+y_2)/2$

$x = (0+12)/2 = 6$

$y = (6+20)/2 = 26/2 = 13$

Hence co-ordinates of midpoint of PQ are (6,13).

4. Find the ratio in which the line segment joining the points A(3,8) and B(-9, 3) is divided by the Y- axis.

Solution:

Suppose, P be the point on Y axis divides segment AB in the ratio m:n.

Since P lies on Y axis, its x co-ordinate is zero.

Let P = (0,y)

By Section formula, $x = (mx_2+nx_1)/(m+n)$

Given $A(x_1,y_1) = (3,8)$

$B(x_2,y_2) = (-9,3)$

$0 = m\times-9+n\times3/(m+n)$

$0 = (-9m+3n)/(m+n)$

$-9m+3n = 0$

$-9m = -3n$

$m/n = 3/9 = 1/3$

The required ratio m:n = 1:3.

5. Find the point on X-axis which is equidistant from P(2,-5) and Q(-2,9).

Solution:

Let M be the point on X-axis which is equidistant from P(2,-5) and Q(-2,9).

Since the point M is on X-axis, its y co-ordinate is zero.

M = (x,0)

Since M is equidistant from P and Q, PM = QM ………..(i)

by Distance formula, $PM = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$= \sqrt{[(x-2)^2+(0-(-5))^2]}$

$= \sqrt{[(x-2)^2+(5)^2]}$

$= \sqrt{(x^2-4x+4+25)}$

$= \sqrt{(x^2-4x+29)}$

by Distance formula, $QM = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$= \sqrt{[(x-(-2))^2+(0-9)^2]}$

$= \sqrt{[(x+2)^2+(9)^2]}$

$= \sqrt{(x^2+4x+85)}$

From (i)

$\sqrt{(x^2-4x+29)} = \sqrt{(x^2+4x+85)}$
Squaring both sides
$x^2-4x+29 = x^2+4x+85$
$-8x = 85-29$
$-8x = 56$
$x = 56/-8$
$x = -7$
Hence the point on X axis equidistant from P(2,-5) and Q(-2,9) is (-7,0).

6. Find the distances between the following points.
(i) A(a, 0), B(0, a)
(ii) P(-6, -3), Q(-1, 9)
(iii) R(-3a, a), S(a, -2a)

Solution:
(i) Given points are A(a,0) and B(0,a)
$x_1 = a$, $y_1= 0$, $x_2 = 0$, $y_2 = a$
By Distance formula,
$d(A,B) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$d(A,B) = \sqrt{[(0-a)^2+(a-0)^2]}$
$= \sqrt{(a^2+a^2)} = \sqrt{(2a^2)}$
$= a\sqrt{2}$ units.
Hence, the distance between the points A and B is $a\sqrt{2}$ units.

(ii) Given points are P(-6,-3) and Q(-1,9)
$x_1 = -6$, $y_1= -3$, $x_2 = -1$, $y_2 = 9$
By Distance formula,
$d(P,Q) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$d(P,Q) = \sqrt{[(-1-(-6))^2+(9-(-3))^2]}$
$= \sqrt{(5^2+12^2)}$
$= \sqrt{(25+144)}$
$= \sqrt{169} = 13$
Hence, the distance between the points P and Q is 13 units.

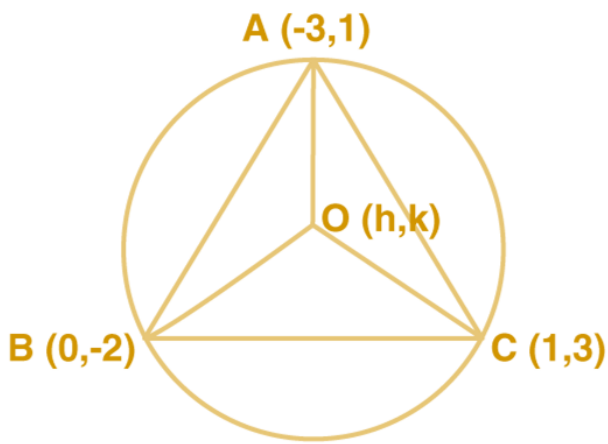
(iii) Given points are R(-3a, a) and S(a, -2a)
$x_1 = -3a$, $y_1= a$, $x_2 = a$, $y_2 = -2a$
By Distance formula,
$d(R,S) = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
$d(R,S) = \sqrt{[(a-(-3a))^2+(-2a-a)^2]}$
$= \sqrt{(4a)^2+(-3a)^2)}$
$= \sqrt{(16a^2+9a^2)}$
$= \sqrt{(25a^2)} = 5a$
Hence, the distance between the points R and S is 5a units.

7. Find the coordinates of the circumcenter of a triangle whose vertices are (-3,1), (0,-2) and (1,3).

Solution:

Let A(-3,1), B(0,-2) and C(1,3) be the vertices of the triangle.

Let O(h,k) be the circumcenter of the triangle.



OA , OB and OC are the radii of the circle.

OA = OB………………….(i)

By distance formula, OA = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$\sqrt{[(h-3)^2+(k-1)^2]} = \sqrt{[(h+3)^2+(k-1)^2]}$ ……..(ii)

OB = $\sqrt{[(h-0)^2+(k-(-2))^2]}$

= $\sqrt{[(h^2+(k+2)^2]}$ …………………………..(iii)

Equating (ii) and (iii)

$\sqrt{[(h+3)^2+(k-1)^2]} = \sqrt{[(h^2+(k+2)^2]}$

Squaring both sides

$[(h+3)^2+(k-1)^2] = [(h^2+(k+2)^2]$

$h^2+6h+9+k^2-2k+1 = h^2+k^2+4k+4$

6h-2k+10 = 4k+4

6h-6k = -6h-k = -1 ……….(a) [Dividing both sides by 6]

OB = OC [radii of same circle]

OC = $\sqrt{[(h-1)^2+(k-3)^2]}$

= $\sqrt{[h^2-2h+1+k^2-6k+9]}$

=$\sqrt{[h^2-2h+k^2-6k+10]}$ ……..(iv)

Equating (iii) and (iv)

$\sqrt{[(h^2+(k+2)^2]} = \sqrt{[h^2-2h+k^2-6k+10]}$

Squaring both sides

$(h^2+(k+2)^2 = h^2-2h+k^2-6k+10$

$(h^2+k^2+4k+4 = h^2-2h+k^2-6k+10$

4k+4 = -2h-6k +10

4k+6k+2h = 10-4

10k+2h = 6

Divide by 2 on both sides, we get
5k+h = 3 ……… (b)
Solving (a) and (b)
h-k = -1…….(a)
h+5k = 3 ……..(b)
(a)-(b)
h-k = -1
-h-5k = -3
-6k = -4
k = -4/-6 = 2/3
Substitute the value of k in (a)
h-k = -1
h-(2/3) = -1
h = -1+(2/3)
h= (-3/3)+(2/3)
h = -1/3
Hence the co-ordinates of circumcenter of triangle are (-1/3 , 2/3)

8. In the following examples, can the segment joining the given points form a triangle? If triangle is formed, state the type of the triangle considering sides of the triangle.
(1) L(6,4) , M(-5,-3) , N(-6,8)
(2) P(-2,-6) , Q(-4,-2), R(-5,0)
(3) A(√2 , √2), B(– √2 , – √2), C(– √6 , √6)

Solution:
(1) Given points are L(6,4) , M(-5,-3) , N(-6,8).
By distance formula LM = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
= $\sqrt{[(-5-6)^2+(-3-4)^2]}$
= $\sqrt{[(-11)^2+(-7)^2]}$
= √[121+49]
= √170
LM = √170 …………(i)
By distance formula MN = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
= $\sqrt{[(-6-(-5))^2+(8-(-3))^2]}$
= $\sqrt{[(-1)^2+(11)^2]}$
= √(1+121)
= √(122)
MN = √122 ……………(ii)
By distance formula LN = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
= $\sqrt{[(-6-6)^2+(8-4)^2]}$
= $\sqrt{[(-12^2+(4)^2]}$
= √(144+16)

$= \sqrt{(160)}$

$LN = \sqrt{160}$ …………..(iii)

(MN +LN) LM

These points are not collinear.

We can construct a triangle through 3 non collinear points.

$LM \neq MN \neq LN$

Triangle formed is a scalene triangle.

(2) Given points are P(-2,-6) , Q(-4,-2), R(-5,0)

By distance formula $PQ = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$= \sqrt{[(-4-(-2)^2+(-2-(-6))^2]}$

$= \sqrt{[(-2)^2+(4)^2]}$

$= \sqrt{[4+16]}$

$= \sqrt{20}$

$= 2\sqrt{5}$

$PQ = 2\sqrt{5}$ …………(i)

By distance formula $QR = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$= \sqrt{[(-5-(-4))^2+(0-(-2))^2]}$

$= \sqrt{[(-1)^2+(2)^2]}$

$= \sqrt{(1+4)}$

$= \sqrt{(5)}$

$QR = \sqrt{5}$ ……………(ii)

By distance formula $PR = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

$= \sqrt{[(-5-(-2))^2+(0-(-6))^2]}$

$= \sqrt{[(-3^2+(6)^2]}$

$= \sqrt{(9+36)}$

$= \sqrt{(45)}$

$= 3\sqrt{5}$

$PR = 3\sqrt{5}$ …………..(iii)

Add (i) and (ii)

$2\sqrt{5}+\sqrt{5} = 3\sqrt{5}$

d(P,Q)+d(Q,R) = d(P,R)

P,Q,R are collinear points.

So we cannot construct a triangle with these collinear points.

9. Find k if the line passing through points P(-12,-3) and Q(4, k)has slope 1/2 .

Solution:

Given points are P(-12,-3) and Q(4, k).

$x_1 = -12$, $y_1 = -3$, $x_2 = 4$, $y_2 = k$

Slope of line $PQ = (y_2-y_1)/ (x_2-x_1)$

$= (k-(-3))/(4-(-12)$

$= (k+3)/16$

Given slope of line passing through P and Q is 1/2.
½ = (k+3)/16
2(k+3) = 16
k+3 = 8
k = 8-3 = 5 Hence the value of k is 5.

10. Show that the line joining the points A(4, 8) and B(5, 5) is parallel to the line joining the points C(2,4) and D(1,7).

Solution:
Proof
Given co-ordinates of A = (4,8)
Co-ordinates of B = (5,5)
Slope of line AB = $(y_2-y_1)/ (x_2-x_1)$
= (5-8)/(5-4)
= -3/1 = -3
Given co-ordinates of C = (2,4)
Co-ordinates of D = (1,7)
Slope of line CD = $(y_2-y_1)/ (x_2-x_1)$
= (7-4)/(1-2)
= 3/-1 = -3
Slope of line AB = Slope of line CD
Line AB is parallel to line CD.
Hence proved.

11. Show that points P(1,-2), Q(5,2), R(3,-1), S(-1,-5) are the vertices of a parallelogram.

Solution:
Proof
Given points are P(1,-2), Q(5,2), R(3,-1), S(-1,-5).
By distance formula PQ = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
PQ = $\sqrt{[(5-1)^2+(2-(-2))^2]}$
= $\sqrt{[(4)^2+(4)^2]}$
= $\sqrt{[(16+16)]}$
= $\sqrt{32}$
PQ = $\sqrt{32}$ ……………… (i)

By distance formula QR = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$
QR = $\sqrt{[(3-5)^2+(-1-2)^2]}$
= $\sqrt{[(-2)^2+(-3)^2]}$
= $\sqrt{[(4+9)]}$ = $\sqrt{13}$
QR = $\sqrt{13}$ …………… (ii)

By distance formula RS = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

RS = $\sqrt{[(-1-3)^2+(-5-(-1))^2]}$

= $\sqrt{[(-4)^2+(-4)^2]}$

= $\sqrt{[(16+16)]}$

= $\sqrt{32}$

RS = $\sqrt{32}$ ……………… (iii)

By distance formula PS = $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$

PS = $\sqrt{[(-1-1)^2+(-5-(-2))^2]}$

= $\sqrt{[(-2)^2+(-3)^2]}$

= $\sqrt{[(4+9)]}$

= $\sqrt{13}$

PS = $\sqrt{13}$ ……………… (iv)

Here PQ = RS [From (i) and (iii)]

And QR = PS [From (ii) and (iv)]

Hence, PQRS is a parallelogram. [For a parallelogram, opposite sides are equal]

Points P, Q, R, and S are the vertices of a parallelogram.

Hence proved.