SIMPLE HARMONIC MOTION

KEY CONCEPTS

PERIODIC MOTION

- Any motion which repeats itself after regular interval of time is called periodic motion.
- The constant interval of time after which the motion is repeated is called time period.

  Examples : (i) Motion of planets around the sun. (ii) Motion of the pendulum of wall clock.

OSCILLATORY MOTION

- The motion of body is said to be oscillatory if it moves back and forth (to and fro) about a fixed point. Oscillation of very high frequency & small amplitude is called vibration.
- The fixed point about which the body oscillates is called mean position or equilibrium position.

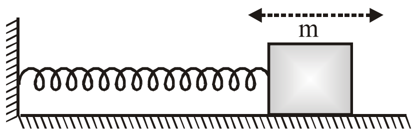
  Examples : (i) Vibration of the wire of 'Sitar'. (ii) Oscillation of the mass suspended from spring.

SIMPLE HARMONIC MOTION (S.H.M.)

Simple harmonic motion is the simplest form of oscillatory motion.

(i) S.H.M. are of two types

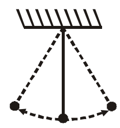
- **Linear S.H.M. :** When a particle moves to and fro about a fixed point (called equilibrium position) along a straight line then its motion is called linear simple harmonic motion.



  Example : Motion of a mass connected to spring.

- **Angular S.H.M.**

  When a system oscillates angularly with respect to a fixed axis then its motion is called angular simple harmonic motion.
  Example :- Motion of a bob of simple pendulum.



(ii) Necessary Condition to execute S.H.M.

- **In linear S.H.M. :** The restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position

  $\therefore \quad F \propto -x \quad \text{or} \quad a \propto -x$

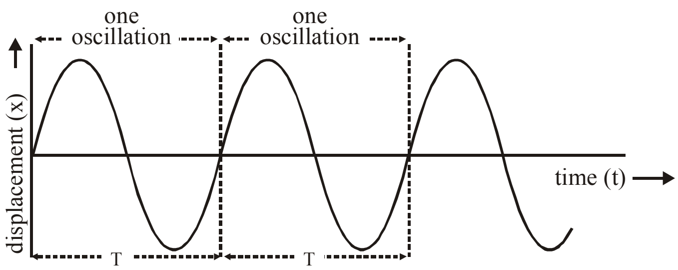
  Negative sign shows that direction of force and acceleration is towards equilibrium position and x is displacement of particle from equilibrium position.

- **In angular S.H.M. :** The restoring torque (or angular acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position

  $\therefore \quad \tau \propto -\theta \quad \text{or} \quad \alpha \propto -\theta$

SOME BASIC TERMS

- **Mean Position :** The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.
- **Restoring Force**
  - The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.
  - Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.
- **Amplitude :** The maximum value of displacement of particle from mean position is define as amplitude.
- **Time period (T)**
  - The minimum time after which the particle keeps on repeating its motion is known as time period.
  - The smallest time taken to complete one oscillation or vibration is also define as time period.
  - It is given by $T = \frac{2\pi}{\omega} = \frac{1}{n}$ where ω is angular frequency and n is frequency.
- **Oscillation :** When a particle goes on one side from mean position and returns back and then it goes to other side and again returns back to mean position, then this process is known as one oscillation.



- **Frequency (n or f)**
  - The number of oscillations per second is define as frequency.
  - It is given by $n = \frac{1}{T}$, $n = \frac{\omega}{2\pi}$
  - **SI UNIT :** hertz (Hz), 1 hertz = 1 cycle per second.
  - **Dimensions :** $M^0 L^0 T^{-1}$.
- **Phase** :
  - Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.
  - In the equation $x = A \sin(\omega t + \theta)$, $(\omega t + \theta)$ is the phase of the particle.
  - The phase angle at time t = 0 is known as initial phase.
  - The difference of total phase angles of two particles executing S.H.M. with respect to the mean position is known as phase difference.
  - Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π, i.e., $\Delta\phi = 2n\pi$ Where n = 0, 1, 2, 3,....
  - Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π i.e., $\Delta\phi = (2n + 1)\pi$ Where n = 0, 1, 2, 3,....

- **Angular frequency (ω)** :
  The rate of change of phase angle of a particle with respect to time is define as its angular frequency.
  SI unit **:** radian/second, *Dimensions* **:** $M^0 L^0 T^{-1}$,

DISPLACEMENT IN S.H.M.

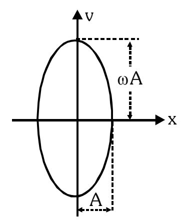
(i) The displacement of a particle executing linear S.H.M. at any instant is defined as the distance of the particle from the mean position at that instant.

(ii) It can be given by relation x = Asinωt or x = Acosωt.
The first relation is valid when the time is measured from the mean position and the second relation is valid when the time is measured from the extreme position of the particle executing S.H.M. along a straight line path.

VELOCITY IN SHM

(i) It is define as the time rate of change of the displacement of the particle at the given instant.

(ii) Velocity in S.H.M. is given by $v = \frac{dx}{dt} = \frac{d}{dt}(A\sin\omega t) \Rightarrow v = A\omega\cos\omega t$



$$v = \pm A\omega\sqrt{1-\sin^2\omega t} \Rightarrow v = \pm A\omega\sqrt{1-\frac{x^2}{A^2}} = \pm\omega\sqrt{(A^2-x^2)}\ [\because\ x = A\sin\omega t]$$

Squaring both the sides $v^2 = \omega^2(A^2 - x^2) \Rightarrow \frac{v^2}{\omega^2} = A^2 - x^2 \Rightarrow \frac{v^2}{\omega^2A^2} = 1 - \frac{x^2}{A^2} \Rightarrow \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$

This is equation of ellipse. So curve between displacement and velocity of particle executing S.H.M. is ellipse.

(iii) The graph between velocity and displacement is shown in figure. If particle oscillates with unit angular frequency (ω = 1) then curve between v and x will be circular.

Note :

(i) The direction of velocity of a particle in S.H.M. is either towards or away from the mean position.

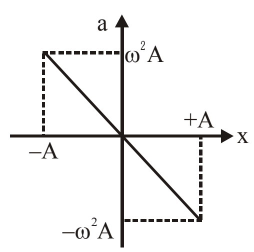
(ii) At mean position (x = 0), velocity is maximum (=Aω) and at extreme position (x = ±A), the velocity of particle executing S.H.M. is zero (minimum).

ACCELERATION IN SHM

(i) It is define as the time rate of change of the velocity of the particle at given instant.

(ii) Acceleration in S.H.M. is given by $a = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t)$

$a = -\omega^2 A\sin\omega t \Rightarrow \quad a = -\omega^2 x$



(iii) The graph between acceleration and displacement as shown in figure

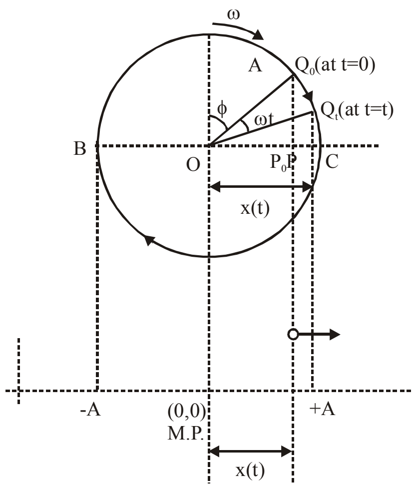
Note

(i) The acceleration of a particle executing S.H.M. is always directed towards the mean position.

(ii) The acceleration of the particle executing S.H.M. is maximum at extreme position (= $\omega^2 A$) and minimum at mean position (= zero)

SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION

Consider a particle Q, moving on a circle of radius A with constant angular velocity ω. The projection of Q on a diameter BC is P. It is clear from the figure that as Q moves around the circle the projection P executes a simple harmonic motion on the x-axis between B and C. The angle that the radius OQ makes with the +ve vertical in clockwise direction in at t = 0 is equal to phase constant (ϕ).



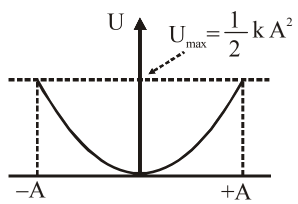
Let the radius OQ_0 makes an angle ωt with the OQ_t at time t. Then x(t)=Asin(ωt+ϕ)
In the above discussion the foot of projection is x-axis so it is called horizontal phasor. Similarly the foot of perpendiuclar on y-axis will also executes SHM of amplitude A and angular frequency ω[y(t)=Acosωt]. This is called vertical phasor. The phaser of the two SHM differ by π/2.

ENERGY OF PARTICLE IN S.H.M.

- **Potential Energy (U or P.E.)**

(i) In terms of displacement

The potential energy is related to force by the relation $F = -\frac{dU}{dx} \Rightarrow \int dU = -\int F dx$

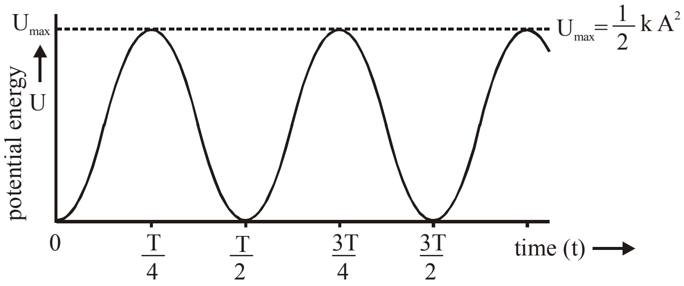


For S.H.M. F = – kx so $\int dU = -\int(-kx)dx = \int kx\, dx \Rightarrow U = \frac{1}{2}kx^2 + C$

At x = 0, U = U_0 $\Rightarrow$ C = U_0 So $U = \frac{1}{2}kx^2 + U_0$

Where the potenital energy at equilibrium position = U_0 when $U_0 = 0$ then $U = \frac{1}{2}kx^2$

(ii) In terms of time



Since $x = A\sin(\omega t + \phi)$, $U = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$

If initial phase (ϕ) is zero then $U = \frac{1}{2}kA^2\sin^2\omega t = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

Note :

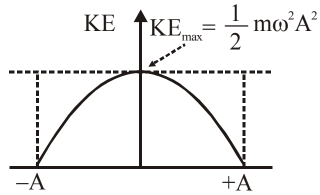
(i) In S.H.M. the potential energy is a parabolic function of displacement, the potential energy is minimum at the mean position ($x = 0$) and maximum at extreme position ($x = \pm A$)

(ii) The potential energy is the periodic function of time. For $x = A \sin(\omega t)$, it is minimum at $t = 0, \frac{T}{2}, T, \frac{3T}{2} ...$ and maximum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4} ...$

- **Kinetic Energy (K)**

(i) In terms of displacement

If mass of the particle executing S.H.M. is m and its velocity is v then kinetic energy at any instant.



$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$$

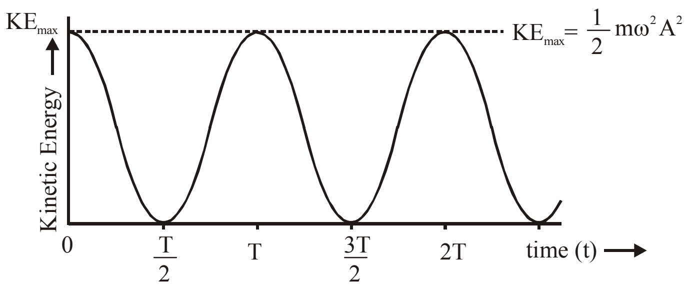
(ii) In terms of time

$$v = A\omega\cos(\omega t + \phi)$$

$$K = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$$

If initial phase ϕ is zero

$$K = \frac{1}{2}m\omega^2A^2\cos^2\omega t$$



Note :

(i) In S.H.M. the kinetic energy is a inverted parabolic function of displacement. The kinetic energy is maximum ($\frac{1}{2}kA^2$) at mean position ($x = 0$) and minimum (zero) at extreme position ($x = \pm A$)

(ii) The kinetic energy is the periodic function of time. For $x = A \sin(\omega t)$, it is maximum at $t = 0$, T, 2T, 3T.............and minimum at $t = \frac{T}{2}, \frac{3T}{2}, \frac{5T}{2} ...$

- **Total energy (E)**

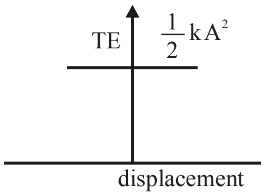
  Total energy in S.H.M. is given by ; E = potential energy + kinetic energy = U + K

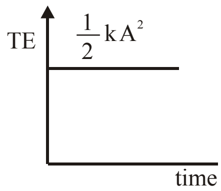
(i) w.r.t. position $E = \frac{1}{2}kx^2 + \frac{1}{2}k(A^2 - x^2) \Rightarrow E = \frac{1}{2}kA^2 = \text{constant}$

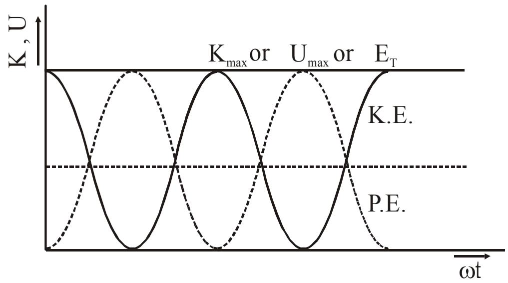
(ii) w.r.t. time

$$E = \frac{1}{2}m\omega^2A^2\sin^2\omega t + \frac{1}{2}m\omega^2A^2\cos^2\omega t = \frac{1}{2}m\omega^2A^2\left(\sin^2\omega t + \cos^2\omega t\right) = \frac{1}{2}m\omega^2A^2$$

$$= \frac{1}{2}kA^2 = \text{constant}$$







Note :

(i) Total energy of a particle in S.H.M. is same at all instant and at all displacement.

(ii) Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

SIMPLE PENDULUM

If a heavy point mass is suspended by a weightless, inextensible string from a rigid support, then this arrangement is called a simple pendulum

- **Second's pendulum**

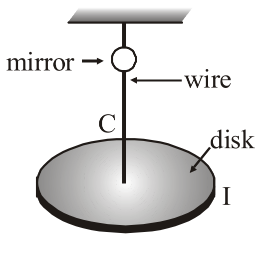
  If the time period of a simple pendulum is 2 second then it is called second's pendulum. Second's pendulum take one second to go from one extreme position to other extreme position.

COMPOUND PENDULUM

Any rigid body which is free to oscillate in a vertical plane about a horizontal axis passing through a point, is define compound pendulum

- **Torsional Oscillator :** (Angular SHM)

$$T = 2\pi\sqrt{\frac{I}{C}} \qquad \text{where } C = \frac{\eta\pi r^4}{2\ell}$$



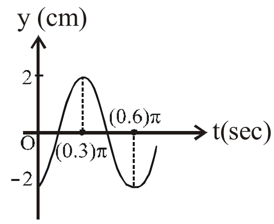
η = modulus of elasticity of the wire ; r = radius of the wire

L = length of the wire ; I = Moment of inertia of the disc

EXERCISE (S-1)

Kinematics of SHM :

1. Part of a simple harmonic motion is graphed in the figure, where y is the displacement from the mean position. The correct equation describing this S.H.M is :-



SH0001

2. The displacement of a body executing SHM is given by $x = A \sin(2\pi t + \pi/3)$. The first time from $t = 0$ when the velocity is maximum is.

SH0002

3. A body undergoing SHM about the origin has its equation given by $x = 0.2 \cos 5\pi t$. Find its average speed from $t = 0$ to $t = 0.7$ sec.

SH0003

Energy of SHM :

4. An object of mass 0.2 kg executes SHM along the x-axis with frequency of $(25/\pi)$ Hz. At the point $x = 0.04$m the object has KE 0.5 J and PE 0.4 J. The amplitude of oscillation is ________.

SH0004

5. A point particle of mass 0.1kg is executing SHM with amplitude of 0.1m. When the particle passes through the mean position, its K.E. is 8×10^{-3}J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.

SH0005

6. Potential Energy (U) of a body of unit mass moving in a one-dimension conservative force field is given by $U = (x^2 - 4x + 3)$. All units are in S.I.

(i) Find the equilibrium position of the body.

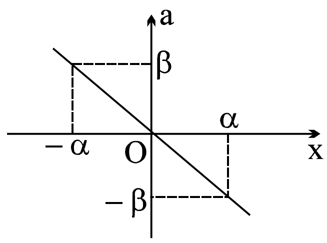
(ii) Show that oscillations of the body about this equilibrium position is simple harmonic motion & find its time period.

(iii) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}$ m/s.

SH0006

Time Period :

7. The acceleration-displacement (a – x) graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.



SH0007

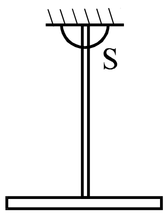
8. A small body of mass m is fixed to the middle of a stretched string of length 2ℓ. In the equilibrium position the string tension is equal to T_0. Find the angular frequency of small oscillations of the body in the transverse direction. The mass of the string is negligible, the gravitational field is absent. Assume tension in string to be constant.

SH0008

9. A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1 : 3 & again oscillated from the two parts separately, then the periods are T_1 & T_2 then find T_1/T_2.

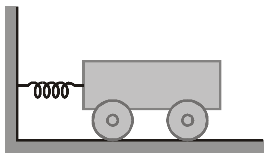
SH0009

10. Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of suspension). Find the time period of such small oscillations.



SH0010

11. A cart consists of a body and four wheels on frictionless axles. The body has a mass m. The wheels are uniform disks of mass M and radius R. The cart rolls, without slipping, back and forth on a horizontal plane under the influence of a spring attached to one end of the cart (figure). The spring constant is k. Taking into account the moment of inertia of the wheels, find a formula for the frequency of the back and forth motion of the cart.



SH0011

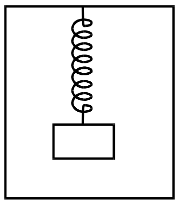
12. A mass M attached to a spring, oscillates with a period of 2 sec. If the mass is increased by 2 kg the period increases by one sec. Find the initial mass M assuming that Hooke's Law is obeyed.

SH0012

Complex situations :

13. A spring mass system is hanging from the ceiling of an elevator in equilibrium. Elongation of spring is l. The elevator suddenly starts accelerating downwards with acceleration g/3, find

(a) the frequency and (b) the amplitude of the resulting SHM.



SH0013

14. (a) Find the time period of oscillations of a torsional pendulum, if the torsional constant of the wire is $K = 10\pi^2$J/rad. The moment of inertia of rigid body is 10 kg m^2 about the axis of rotation.

(b) A simple pendulum of length $l = 0.5$ m is hanging from ceiling of a car. The car is kept on a horizontal plane. The car starts accelerating on the horizontal road with acceleration of 5 m/s^2. Find the time period of oscillations of the pendulum for small amplitudes about the mean position.

SH0014

15. The motion of a simple pendulum is given by

$$\theta = A\cos\left(\sqrt{\frac{g}{\ell}}\,t\right) \text{ (symbols have their usual meaning)}$$

(a) Find the tension in the string of this pendulum as a function of time assume that $\theta << \ell$.

(b) At what time is the tension maximum? What is the value of this maximum tension?

SH0015

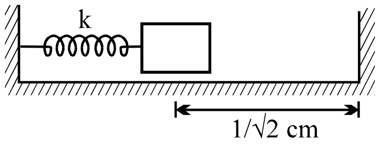
16. A physical pendulum has the shape of a disk of radius R. The pendulum swings about an axis perpendicular to the plane of the disk and at distance ℓ from the center of the disk.

(a) Show that the frequency of the oscillations of this pendulum is $\omega = \sqrt{\dfrac{g\ell}{\frac{1}{2}R^2 + \ell^2}}$

(b) For what value of ℓ is this frequency at a maximum ?

SH0016

17. A block of mass 0.9 kg attached to a spring of force constant k is lying on a frictionless floor. The spring is compressed to $\sqrt{2}$ cm and the block is at a distance $1/\sqrt{2}$ cm from the wall as shown in the figure. When the block is released, it makes elastic collision with the wall and its period of motion is 0.2 sec. Find the approximate value of k.

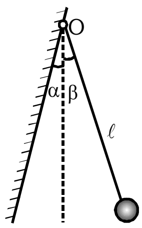


SH0017

18. A body of mass 1 kg is suspended from a weightless spring having force constant 600N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3.0m/s and get embedded in it. Find the frequency of oscillations and amplitude of motion.

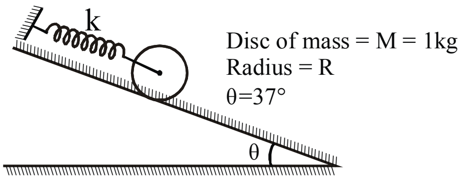
SH0018

19. A ball is suspended by a thread of length ℓ at the point O on the wall, forming a small angle α with the vertical as shown in figure. Then the thread with the ball was deviated through a small angle β ($\beta > \alpha$) and set free. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.



SH0019

20. A disc of mass m is connected to an ideal spring of force constant 'k'. If disc is released from rest, then what is maximum friction force on disc (in N). Assuming friction is sufficient for pure rolling

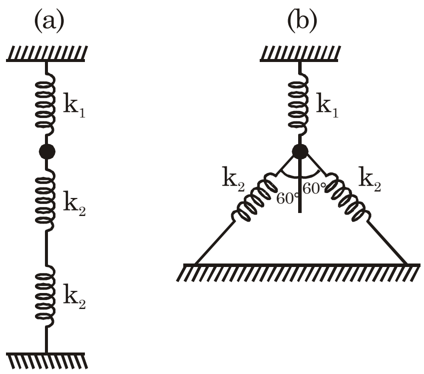


SH0020

21. A particle of mass 5×10^{-5} kg is placed at the lowest point of a smooth parabola having the equation $20x^2 = y$ (x,y in m). Here y is the vertical height. If it is displaced slightly and it moves such that it is constrained to move along the parabola, the angular frequency of oscillation will be, (in rad/s). If your answer is N fill value N/4.

SH0021

22. Mass m is suspended by ideal massless springs in two different ways, indicated by (a) and (b) in the figure. The mass is displaced upwards by a small amount from equilibrium position and is then released resulting in a SHM of the mass in the vertical direction. We denote the oscillation frequencies associated with the two cases (a) and (b) by f_a and f_b respectively. Find $\frac{f_a}{f_b}$. Given $k_2 = 2k_1$.



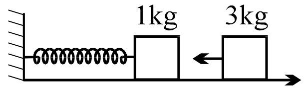
SH0022

23. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[IIT-JEE 2004]**

SH0023

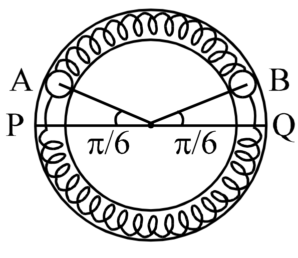
EXERCISE (S-2)

1. One end of an ideal spring is fixed to a wall at origin O and the axis of spring is parallel to x-axis. A block of mass $m = 1$ kg is attached to free end of the spring and it is performing SHM. Equation of position of block in coordinate system shown is $x = 10 + 3\sin 10t$, t is in second and x in cm. Another block of mass M = 3kg, moving towards the origin with velocity 30cm/s collides with the block performing SHM at t = 0 and gets struck to it, calculate :
(i) new amplitude of oscillations.
(ii) new equation for position of the combined body.
(iii) loss of energy during collision. Neglect friction.



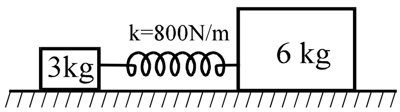
SH0024

2. Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe in the form of a circle as in fig. The pipe is fixed in a horizontal plane. The centres of the ball can move in a circle of radius 0.06m. Each spring has a natural length 0.06π m and force constant 0.1N/m. Initially both the balls are displaced by an angle of $\theta = \pi/6$ radian with respect to diameter PQ of the circle and released from rest
(a) Calculate the frequency of oscillation of the ball B.
(b) What is the total energy of the system.
(c) Find the speed of the ball A when A and B are at the two ends of the diameter PQ.



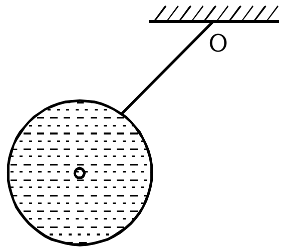
SH0025

3. The system shown in the figure can move on a smooth surface. The spring is initially compressed by 6 cm and then released. Find
(a) time period;
(b) amplitude of 3 kg block and
(c) maximum momentum of 6 kg block



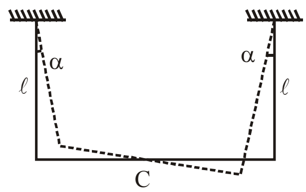
SH0026

4. A pendulum is constructed as a light thin-walled sphere of radius R filled up with water and suspended at the point O from a light rigid rod. The distance between the point O and the centre of the sphere is equal to ℓ. How many times will the small oscillations of such a pendulum change after the water freezes? The viscosity of water and the change of its volume on freezing are to be neglected.



SH0027

5. A uniform rod of mass m = 1.5 kg suspended by two identical threads ℓ = 90 cm in length was turned through a small angle about the vertical axis passing through its middle point C. The threads deviated in the process through an angle $\alpha = 5.0°$. Then the rod was released to start performing small oscillations. Find:
(a) the oscillation period;
(b) the rod's oscillation energy.

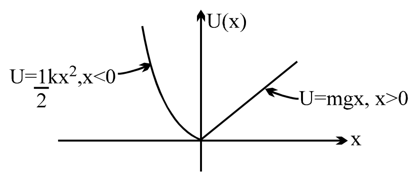


SH0028

6. Two physical pendulums perform small oscillations about the same horizontal axis with frequencies ω_1 and ω_2. Their moments of inertia relative to the given axis are equal to I_1 and I_2 respectively. In a state of stable equilibrium the pendulums were fastened rigidly together. What will be the frequency of small oscillations of the compound pendulum?

SH0029

7. A particle of mass *m* moves in the potential energy U shown above. Find the period of the motion when the particle has total energy E.



SH0030

8. A particle of mass 4 kg moves between two points A and B on a smooth horizontal surface under the action of two forces such that when it is at a point P, the forces are $2\overrightarrow{PA}$ N and $2\overrightarrow{PB}$ N. If the particle is released from rest at A, find the time it takes to travel a quarter of the way from A to B.

SH0031

9. Two masses m_1 and m_2 connected by a light spring of natural length ℓ_0 is compressed completely and tied by a string. This system while moving with a velocity v_0 along +ve *x*-axis pass through the origin at $t = 0$. At this position the string snaps. Position of mass m_1 at time *t* is given by the equation

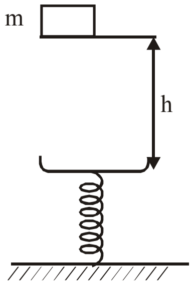
$x_1(t) = v_0 t - A(1 - \cos\omega t)$

Calculate : **[IIT JEE 2003]**

(i) Position of the particle m_2 as a function of time. (ii) ℓ_0 in terms of A.

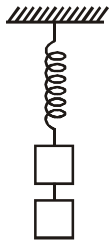
SH0032

10. A body of mass m fell from a height h onto the pan of a spring balance. The masses of the pan and the spring are negligible, the stiffness of the latter is x. Having stuck to the pan, the body starts performing harmonic oscillations in the vertical direction. Find the amplitude and the energy of these oscillations.



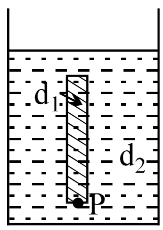
SH0033

11. Two small bodies of mass 2kg each hang on a spring of force constant of 200 N/m, attached to each other using a thread with a length of 10 cm, as shown in the figure. We burn the thread. Find the distance between the two bodies (in cm) when the top body first arrives at its highest position? If the distance comes out to be 16 × n. Fill n [Take: $g = 10\ m/s^2$ and $\pi^2 = 10$]



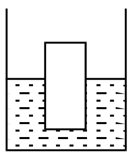
SH0034

12. A thin rod of length L & area of cross-section S is pivoted at its lowest point P inside a stationary, homogeneous & non-viscous liquid (Figure). The rod is free to rotate in a vertical plane about a horizontal axis passing through P. The density d_1 of the material of the rod is smaller than the entity d_2 of the liquid. The rod is displaced by a small angle θ from its equilibrium position and then released. Show that the motion of the rod is simple harmonic and determine its angular frequency in terms of the given parameters.



SH0035

13. A liquid of density 2ρ is filled in a cylindrical vessel, whose cross-sectional area is 2A. A wooden cylinder of height H, cross-sectional area A and density ρ is floating in the liquid at equilibrium with its axis vertical. The cylinder is pushed down by a small distance x from its equilibrium position and released. Find its initial acceleration.



SH0036

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

Kinematics of SHM :

1. The equation of motion of a particle is $x = a\cos(\alpha t)^2$. The motion is
(A) periodic but not oscillatory. (B) periodic and oscillatory.
(C) oscillatory but not periodic. (D) neither periodic nor oscillatory.

SH0037

2. **Statement 1** : Position–time equation of a particle moving along x–axis is x = 4 + 6 sinωt. Under this situation, motion of particle is not simple harmonic.
and

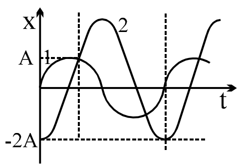
Statement 2 : $\frac{d^2x}{dt^2}$ for the given equation is not proportional to –x.
(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1.
(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1.
(C) Statement–1 is True, Statement–2 is False.
(D) Statement–1 is False, Statement–2 is True.

SH0038

3. A simple harmonic motion having an amplitude A and time period T is represented by the equation : y = 5 sin π(t + 4) m. Then the values of A (in m) and T (in sec) are :
(A) A = 5; T = 2 (B) A = 10 ; T = 1 (C) A = 5 ; T = 1 (D) A = 10 ; T = 2

SH0039

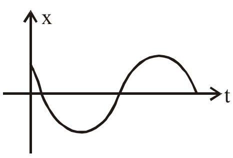
4. The oscillations represented by curve 1 in the graph are expressed by equation x = A sinωt. The equation for the oscillations represented by curve 2 is expressed as:

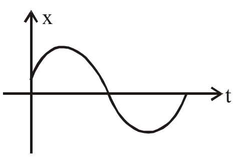


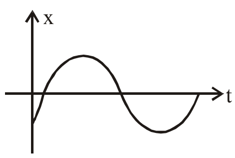
(A) x = 2A sin (ωt – π/2) (B) x = 2A sin (ωt + π/2)
(C) x = – 2A sin (ωt – π/2) (D) x = A sin (ωt – π/2)

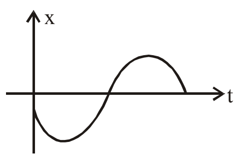
SH0040

5. A particle performing S.H.M. about mean position x = 0 and at t = 0 it is at position $x = \frac{A}{\sqrt{2}}$ and moving towards the origin. Then which of the following is its possible graph between position (x) and time (t)

(A) 

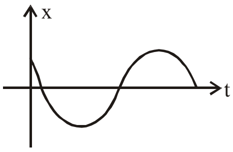
(B) 

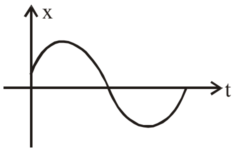
(C) 

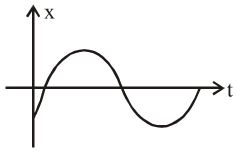
(D) 

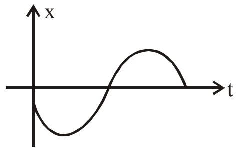
SH0041

6. A particle is performing S.H.M. and at $t = \frac{3T}{4}$, is at position $= \frac{A}{\sqrt{2}}$ and moving towards the origin. Equilibrium position of the particle is at $x = 0$. After $t = \frac{3T}{2}$ what will be the graph of the particle :-

(A) 

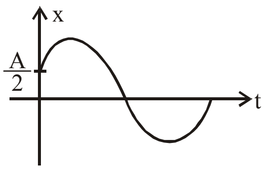
(B) 

(C) 

(D) 

SH0042

7. Following graph shows a particle performing S.H.M. about mean position x = 0. The equation of particle if $t = \frac{T}{4}$ is taken as starting time is (Notations have usual meanings)



(A) $A\sin\left(\omega t + \frac{2\pi}{3}\right)$ (B) $A\sin\left(\omega t + \frac{\pi}{3}\right)$ (C) $A\sin\left(\omega t + \frac{\pi}{6}\right)$ (D) $A\cos\left(\omega t + \frac{2\pi}{3}\right)$

SH0043

8. A particle is performing S.H.M. and at $t = \frac{3T}{4}$, is at position $\frac{\sqrt{3}A}{2}$ and moving towards the origin. Equilibrium position of the particle is at $x = 0$. Then what was the position and direction of particle at $t = 0$?

(A) $-\frac{A}{2}$, away from mean position (B) $\frac{A}{2}$, away from mean position

(C) $\frac{A}{2}$, towards mean position (D) $-\frac{A}{2}$, towards mean position

SH0044

9. The time taken by a particle performing SHM to pass from point A to B where its velocities are same is 2 seconds. After another 2 seconds it returns to B. The time period of oscillation is (in seconds)

(A) 2 (B) 8 (C) 6 (D) 4

SH0045

10. A bob is attached to a long, light string. The string is deflected by 3° initially with respect to vertical. The length of the string is 1 m. The value of θ at any time t after the bob released can be approximately written as (Use : $g = \pi^2$)

(A) $3° \cos \pi t$ (B) $3° \sin \pi t$ (C) $3° \sin (\pi t + \frac{\pi}{6})$ (D) $3° \cos (\pi t + \frac{\pi}{6})$

SH0046

11. A particle performing SHM is found at its equilibrium at t = 1sec. and it is found to have a speed of 0.25 m/s at t = 2 sec. If the period of oscillation is 6 sec. Calculate amplitude of oscillation

(A) $\frac{3}{2\pi}$ m (B) $\frac{3}{4\pi}$ m (C) $\frac{6}{\pi}$ m (D) $\frac{3}{8\pi}$

SH0047

12. A particle performs SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance a/2 from the extreme position is

(A) a/T (B) 2a/T (C) 3a/T (D) a/2T

SH0048

13. Two particles execute SHM of same amplitude of 20 cm with same period along the same line about the same equilibrium position. The maximum distance between the two is 20 cm. Their phase difference in radians is :-

(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

SH0049

14. Two particles are in SHM on same straight line with amplitude A and 2A and with same angular frequency ω. It is observed that when first particle is at a distance $A/\sqrt{2}$ from origin and going toward mean position, other particle is at extreme position on other side of mean position. Find phase difference between the two particles

(A) 45° (B) 90° (C) 135° (D) 180°

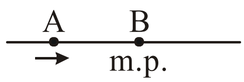
SH0050

15. Two pendulums have time periods T and 5T/4. They start SHM at the same time from the mean position. After how many oscillations of the smaller pendulum they will be again in the same phase:

(A) 5 (B) 4 (C) 11 (D) 9

SH0051

16. A particle is oscillating simple harmonically with angular frequency ω and amplitude A . It is at a point (A) at certain instant (shown in figure). At this instant it is moving towards mean position (B). It takes time t to reach mean position (B). If time period of oscillation is T, the average speed between A and B is :-



(A) $\frac{A\sin\omega t}{t}$ (B) $\frac{A\cos\omega t}{t}$ (C) $\frac{A\sin\omega t}{T}$ (D) $\frac{A\cos\omega t}{T}$

SH0052

17. A particle executes SHM on a straight line path. The amplitude of oscillation is 2 cm. When the displacement of the particle from the mean position is 1 cm, the numerical value of magnitude of acceleration is equal to the numerical value of magnitude of velocity. The frequency of SHM (in second^{-1}) is:

(A) $2\pi\sqrt{3}$ (B) $\frac{2\pi}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2\pi}$ (D) $\frac{1}{2\pi\sqrt{3}}$

SH0053

18. Speed v of a particle moving along a straight line, when it is at a distance x from a fixed point on the line is given by $v^2 = 108 - 9x^2$ (all quantities in S.I. unit). Then

(A) The motion is uniformly accelerated along the straight line

(B) The magnitude of the acceleration at a distance 3 cm from the fixed point is 0.27 m/s^2.

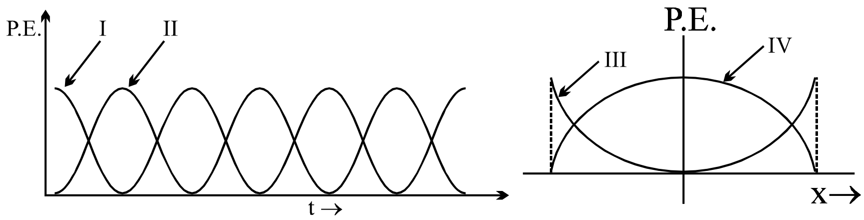
(C) The motion is simple harmonic about $x = \sqrt{12}$ m.

(D) The maximum displacement from the fixed point is 4 cm.

SH0054

Energy of SHM :

19. A particle is executing SHM according to $x = a \cos \omega t$. Then which of the graphs represents variations of potential energy : **[IIT JEE (Scr) 2003]**



(A) (I) & (III) (B) (II) & (IV) (C) (I) & (IV) (D) (II) & (III)

SH0055

20. Potential energy of a particle is given as $U(x) = 2x^3 - 9x^2 + 12x$ where U is in joule and x is in metre. If the motion of a particle is S.H.M., then find the potential energy of the particle at mean position :-

(A) – 36 J (B) 4 J (C) 5 J (D) None of these

SH0056

21. If the potential energy of a harmonic oscillator of mass 2 kg on its equilibrium position is 5 joules and the total energy is 9 joules. If the amplitude is one meter then period of the oscillator (in sec) is:

(A) 1.5 (B) 3.14 (C) 6.28 (D) 4.67

SH0057

22. Kinetic energy of a particle executing simple harmonic motion in straight line is pv^2 and potential energy is qx^2, where v is speed at distance x from the mean position. It time period is given by the expression :-

(A) $2\pi\sqrt{\frac{q}{p}}$ (B) $2\pi\sqrt{\frac{p}{q}}$ (C) $2\pi\sqrt{\frac{q}{p+q}}$ (D) $2\pi\sqrt{\frac{p}{p+q}}$

SH0058

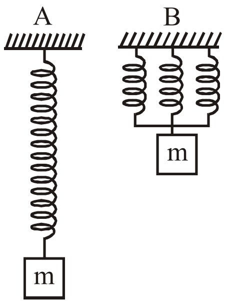
Time Period :

23. The angular frequency of motion whose equation is $4\frac{d^2y}{dt^2} + 9y = 0$ is (y = displacement and t = time)

(A) $\frac{9}{4}$ (B) $\frac{4}{9}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

SH0059

24. The springs in figure A and B are identical but length of spring in A is three times than the length of each spring in B. The ratio of period T_A/T_B is :-



(A) $\sqrt{3}$ (B) 1/3 (C) 3 (D) $1/\sqrt{3}$

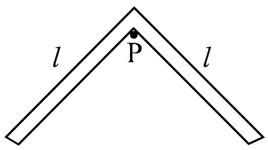
SH0060

25. A rod whose ends are A & B of length 25 cm is hanged in vertical plane. When hanged from point A and point B the time periods calculated are 3 sec & 4 sec respectively. Given the moment of inertia of rod about axis perpendicular to the rod is in ratio 9 : 4 at points A and B. Find the distance of the centre of mass from point A.

(A) 9 cm (B) 5 cm (C) 25 cm (D) 20 cm

SH0061

26. A system of two identical rods (L-shaped) of mass m and length l are resting on a peg P as shown in the figure. If the system is displaced in its plane by a small angle θ, find the period of oscillations:



(A) $2\pi\sqrt{\frac{\sqrt{2}l}{3g}}$ (B) $2\pi\sqrt{\frac{2\sqrt{2}l}{3g}}$ (C) $2\pi\sqrt{\frac{2l}{3g}}$ (D) $3\pi\sqrt{\frac{l}{3g}}$

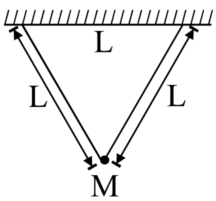
SH0062

27. A ring of diameter 2m oscillates as a compound pendulum about a horizontal axis passing through a point at its rim. It oscillates such that its centre move in a plane which is perpendicular to the plane of the ring. The equivalent length of the simple pendulum is :-

(A) 2m (B) 4m (C) 1.5m (D) 3m

SH0063

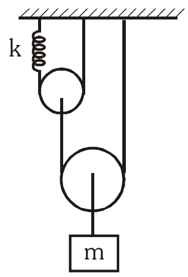
28. A man is swinging on a swing made of 2 ropes of equal length L and in direction perpendicular to the plane of paper. The time period of the small oscillations about the mean position is :-



(A) $2\pi\sqrt{\frac{L}{2g}}$ (B) $2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$ (C) $2\pi\sqrt{\frac{L}{2\sqrt{3}g}}$ (D) $\pi\sqrt{\frac{L}{g}}$

SH0064

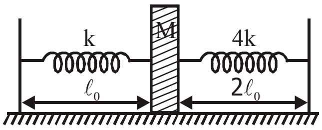
29. What is the period of small oscillations of the block of mass m if the springs are ideal and pulleys are massless ?



(A) $\frac{\pi}{2}\sqrt{\frac{m}{k}}$ (B) $\frac{\pi}{2}\sqrt{\frac{m}{2k}}$ (C) $\frac{\pi}{2}\sqrt{\frac{2m}{k}}$ (D) $\pi\sqrt{\frac{m}{2k}}$

SH0065

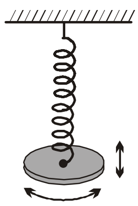
30. A block of mass M is kept in gravity free space and touches the two springs as shown in the figure. Initially springs are in their natural lengths. Now, the block is shifted ($\ell_0/2$) from the given position in such a way that it compresses a spring and is released. The time-period of oscillation of mass will be:-



(A) $\frac{\pi}{2}\sqrt{\frac{M}{K}}$ (B) $2\pi\sqrt{\frac{m}{5K}}$ (C) $\frac{3\pi}{2}\sqrt{\frac{M}{K}}$ (D) $\pi\sqrt{\frac{M}{2K}}$

SH0066

31. A solid disk of radius R is suspended from a spring of linear spring constant k and torsional constant c, as shown in figure. In terms of k and c, what value of R will give the same period for the vertical and torsional oscillations of this system?



(A) $\sqrt{\frac{2c}{k}}$ (B) $\sqrt{\frac{c}{2k}}$ (C) $2\sqrt{\frac{c}{k}}$ (D) $\frac{1}{2}\sqrt{\frac{c}{k}}$

SH0067

Superposition :

32. The amplitude of the vibrating particle due to superposition of two SHMs,

$y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = \sin \omega t$ is :

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

SH0068

33. Two simple harmonic motions $y_1 = A \sin \omega t$ and $y_2 = A \cos \omega t$ are superimposed on a particle of mass m. The total mechanical energy of the particle is:

(A) $\frac{1}{2} m\omega^2 A^2$ (B) $m\omega^2 A^2$ (C) $\frac{1}{4} m\omega^2 A^2$ (D) zero

SH0069

34. The displacement of a particle varies with time according to the relation $y = a \sin \omega t + b \cos \omega t$.

(A) The motion is oscillatory but not S.H.M.

(B) The motion is S.H.M. with amplitude $a + b$.

(C) The motion is S.H.M. with amplitude $a^2 + b^2$.

(D) The motion is S.H.M. with amplitude $\sqrt{a^2 + b^2}$

SH0070

35. Equations $y = 2A \cos^2 \omega t$ and $y = A (\sin \omega t + \sqrt{3} \cos \omega t)$ represent the motion of two particles.

(A) Only one of these is S.H.M. (B) Ratio of maximum speeds is 2 : 1

(C) Ratio of maximum speeds is 1 : 1 (D) Ratio of maximum accelerations is 1 : 4

SH0071

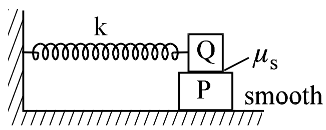
Complex situations :

36. A plank with a small block on top of it is under going vertical SHM. Its period is 2 sec. The minimum amplitude at which the block will separate from plank is :

(A) $\frac{10}{\pi^2}$ (B) $\frac{\pi^2}{10}$ (C) $\frac{20}{\pi^2}$ (D) $\frac{\pi}{10}$

SH0072

37. A block P of mass m is placed on a frictionless horizontal surface. Another block Q of same mass is kept on P and connected to the wall with the help of a spring of spring constant k as shown in the figure. μ_s is the coefficient of friction between P and Q. The blocks move together performing SHM of amplitude A. The maximum value of the friction force between P and Q is :- **[IIT JEE 2004]**



(A) kA (B) $\frac{kA}{2}$ (C) zero (D) $\mu_s mg$

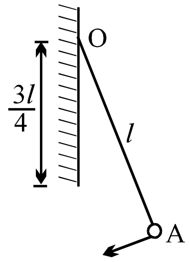
SH0073

38. Two bodies P & Q of equal mass are suspended from two separate massless springs of force constants k_1 & k_2 respectively. If the maximum velocity of them are equal during their motion, the ratio of amplitude of P to Q is :

(A) $\frac{k_1}{k_2}$ (B) $\sqrt{\frac{k_2}{k_1}}$ (C) $\frac{k_2}{k_1}$ (D) $\sqrt{\frac{k_1}{k_2}}$

SH0074

39. A small bob attached to a light inextensible thread of length l has a periodic time T when allowed to vibrate as a simple pendulum. The thread is now suspended from a fixed end O of a vertical rigid rod of length $\frac{3l}{4}$ (as in figure). If now the pendulum performs periodic oscillations in this arrangement, the periodic time will be



(A) $\frac{3T}{4}$ (B) $\frac{T}{2}$ (C) T (D) 2T

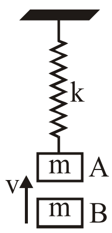
SH0075

40. Vertical displacement of a plank with a body of mass 'm' on it is varying according to law $y = \sin \omega t + \sqrt{3} \cos \omega t$. The minimum value of ω for which the mass just breaks off the plank and the moment it occurs first after t = 0 are given by: (y is positive vertically upwards)

(A) $\sqrt{\frac{g}{2}}, \frac{\sqrt{2}}{6}\frac{\pi}{\sqrt{g}}$ (B) $\frac{g}{\sqrt{2}}, \frac{2}{3}\sqrt{\frac{\pi}{g}}$ (C) $\sqrt{\frac{g}{2}}, \frac{\pi}{3}\sqrt{\frac{2}{g}}$ (D) $\sqrt{2g}, \sqrt{\frac{2\pi}{3g}}$

SH0076

41. Block A is hanging from a vertical spring and is at rest. Block B strikes the block A with velocity v and sticks to it. Then the value of v for which the spring just attains natural length is



(A) $\sqrt{\frac{60mg^2}{k}}$ (B) $\sqrt{\frac{6mg^2}{k}}$ (C) $\sqrt{\frac{10mg^2}{k}}$ (D) $\sqrt{\frac{8mg^2}{k}}$

SH0077

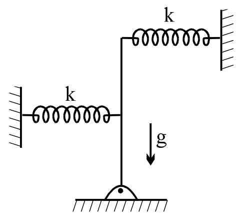
42. A particle of mass 10 gm moves in a field where potential energy per unit mass is given by expression $v = 8 \times 10^4 x^2$ erg/gm . If the total energy of the particle is 8×10^7 erg then the relation between x and time t is :[ϕ = constant]

(A) $x = 10 \sin (400\, t + \phi)$ cm (B) $x = \sin (400\, t + \phi)$ m

(C) $x = 10 \sin (40\, t + \phi)$ cm (D) $x = 100 \sin (4\, t + \phi)$ m

SH0078

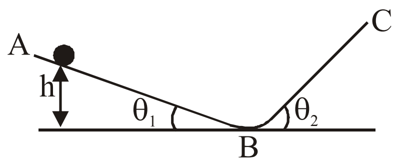
43. In the figure shown, the spring are connected to the rod at one end and at the midpoint. The rod is hinged at its lower end. Rotational SHM of the rod (Mass m, length L) will occur only if



(A) k > mg/3L (B) k > 2mg/3L (C) k > 2mg/5L (D) k > 0

SH0079

44. A small glass bead of mass m initially at rest starts from a point at height h above the horizontal and rolls down the inclined plane AB as shown. Then it rises along the inclined plane BC. Assuming no loss of energy and sufficient friction for pure rolling, the time period of oscillation of the glass bead is:



(A) $\sqrt{\frac{8h}{g}}(\sin\theta_1+\sin\theta_2)$

(B) $2\sqrt{\frac{14h}{5g}}\left(\frac{1}{\sin\theta_1}+\frac{1}{\sin\theta_2}\right)$

(C) $\sqrt{\frac{8h}{g}}\left(\frac{1}{\sin\theta_1}+\frac{1}{\sin\theta_2}\right)$

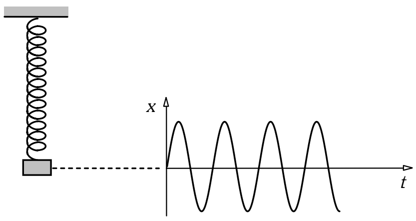
(D) $\sqrt{\frac{8h}{5g}}\left(\frac{1}{\sin\theta_1}+\frac{1}{\sin\theta_2}\right)$

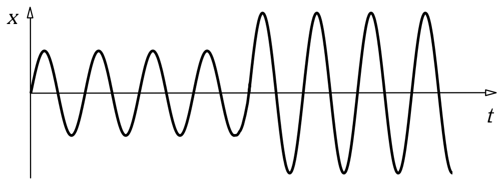
SH0080

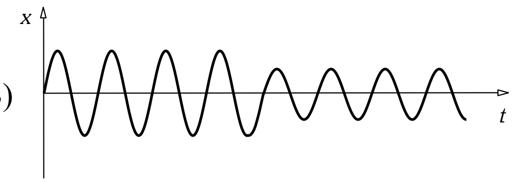
EXERCISE (O-2)

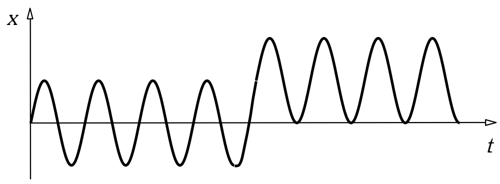
SINGLE CORRECT TYPE QUESTIONS

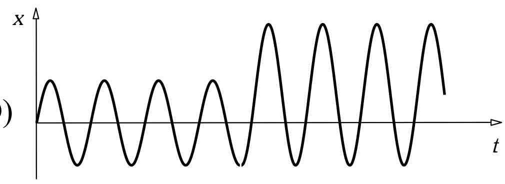
1. In the figure is shown a spring-mass system oscillating in uniform gravity. If we neglect all dissipative forces, it will keep on oscillating endlessly with constant amplitude and frequency. Accompanying graph shows how displacement x of the block from the equilibrium position varies with time t. Now at a certain instant $t = t_o$ when the block reaches its lowest position, gravity is switched off by some unknown mechanism. Which of the following graphs would correctly describes the changes taking place due to switching off the gravity?



(A) 

(B) 

(C) 

(D) 

SH0081

2. Time period of a particle executing SHM is 8 sec. At t = 0 it is at the mean position. The ratio of the distance covered by the particle in the 1st second to the 2nd second is :

(A) $\frac{1}{\sqrt{2}+1}$ (B) $\sqrt{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}+1$

SH0082

3. A particle executes SHM with time period T and amplitude A. The maximum possible average velocity in time $\frac{T}{4}$ is

(A) $\frac{2A}{T}$ (B) $\frac{4A}{T}$ (C) $\frac{8A}{T}$ (D) $\frac{4\sqrt{2}A}{T}$

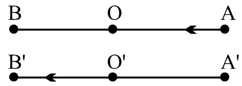
SH0083

4. A particle moves along the x-axis according to : $x = A.[1 + \sin \omega t]$. What distance does it travel between $t = 0$ and $t = 2.5\pi/\omega$?

(A) 4A (B) 6A (C) 5A (D) None

SH0084

5. Two particles undergo SHM along parallel lines with the same time period (T) and equal amplitudes. At a particular instant, one particle is at its extreme position while the other is at its mean position. They move in the same direction. They will cross each other after a further time



(A) T/8 (B) 3T/8 (C) T/6 (D) 4T/3

SH0085

6. Two particles are in SHM in a straight line about same equilibrium position. Amplitude A and time period T of both the particles are equal. At time t=0, one particle is at displacement $y_1 = +A$ and the other at $y_2 = -A/2$, and they are approaching towards each other. After what time they cross each other ?

(A) T/3 (B) T/4 (C) 5T/6 (D) T/6

SH0086

7. Two particles A and B perform SHM along the same straight line with the same amplitude 'a', same frequency 'f' and same equilibrium position 'O'. The greatest distance between them is found to be $\frac{3a}{2}$. At some instant of time they have the same displacement from mean position. What is the displacement?

(A) a/2 (B) $a\sqrt{7}/4$ (C) $\sqrt{3}\,a/2$ (D) $3a/4$

SH0087

8. A wire frame in the shape of an equilateral triangle is hinged at one vertex so that it can swing freely in a vertical plane, with the plane of the Δ always remaining vertical. The side of the frame is $1/\sqrt{3}$ m. The time period in seconds of small oscillations of the frame will be

(A) $\frac{\pi}{\sqrt{2}}$ (B) $\pi\sqrt{2}$ (C) $\frac{\pi}{\sqrt{6}}$ (D) $\frac{\pi}{\sqrt{5}}$

SH0088

9. For a particle acceleration is defined as $\vec{a} = \frac{-5x\hat{i}}{|x|}$ for $x \neq 0$ and $\vec{a} = 0$ for $x = 0$. If the particle is initially at rest at (a, 0), what is period of motion of the particle.

(A) $4\sqrt{2a/5}$ sec. (B) $8\sqrt{2a/5}$ sec. (C) $2\sqrt{2a/5}$ sec. (D) cannot be determined

SH0089

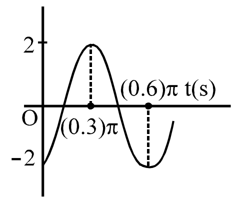
10. Part of a simple harmonic motion is graphed in the figure, where y is the displacement from the mean position. The correct equation describing this S.H.M is

(A) $y = 4\cos(0.6t)$

(B) $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$

(C) $y = 4\sin\left(\frac{10}{3}t + \frac{\pi}{2}\right)$

(D) $y = 2\cos\left(\frac{10}{3}t + \frac{\pi}{2}\right)$



SH0090

11. The angular frequency of a spring block system is ω_0. This system is suspended from the ceiling of an elevator moving downwards with a constant speed v_0. The block is at rest relative to the elevator. Lift is suddenly stopped. Assuming the downwards as a positive direction, choose the wrong statement:

(A) The amplitude of the block is $\frac{v_0}{\omega_0}$

(B) The initial phase of the block is π.

(C) The equation of motion for the block is $\frac{v_0}{\omega_0}\sin\omega_0 t$.

(D) The maximum speed of the block is v_0.

SH0091

12. Two particles are performing SHM with same angular frequency and amplitudes A and 2A respectively along same straight line with same mean position. They cross each other at position A/2 distance from mean position in opposite direction. The phase between them is :

(A) $\frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (B) $\frac{\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (C) $\frac{5\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$ (D) $\frac{\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$

SH0092

13. A particle executing a simple harmonic motion of period 2s. When it is at its extreme displacement from its mean position, it receives an additional energy equal to what it had in its mean position. Due to this, in its subsequent motion,

(A) its amplitude will change and become equal to $\sqrt{2}$ times its previous amplitude

(B) its periodic time will become doubled i.e. 4s

(C) its potential energy will be decreased

(D) it will continue to execute simple harmonic motion of the same amplitude and period as before receiving the additional energy.

SH0093

14. A particle is executing SHM of amplitude A, about the mean position x = 0. Which of the following cannot be a possible phase difference between the positions of the particle at $x = +A/2$ and $x = -A/\sqrt{2}$.

(A) 75° (B) 165° (C) 135° (D) 195°

SH0094

15. A particle free to move along the x-axis has potential energy given by $U(x) = k[1-\exp(-x^2)]$ for $-\infty < x < +\infty$, where k is a positive constant of appropriate dimensions. Then
(A) at point away from the origin, the particle is in unstable equilibrium.
(B) for any finite non-zero value of x, there is a force directed away from the origin.
(C) if its total mechanical energy is k/2, it has its minimum kinetic energy at the origin.
(D) for small displacements from x = 0, the motion is simple harmonic.

SH0095

16. The time period of a bar pendulum when suspended at distances 30 cm and 50 cm from its centre of gravity comes out to be the same. If the mass of the body is 2kg. Find out its moment of inertia about an axis passing through first point.
(A) 0.24 kg-m^2 (B) 0.72 kg-m^2 (C) 0.48 kg-m^2 (D) Data insufficient

SH0096

MULTIPLE CORRECT TYPE QUESTIONS

17. For a SHM with given angular frequency, two arbitrary initial conditions are necessary and sufficient to determine the motion completely. These initial conditions may be-
(A) Amplitude and initial phase
(B) Amplitude and total energy of oscillation
(C) Initial phase and total energy of oscillation
(D) Initial position and initial velocity

SH0097

18. A particle is executing SHM between points $-X_m$ and X_m, as shown in figure-I. The velocity V(t) of the particle is partially graphed and shown in figure-II. Two points A and B corresponding to time t_1 and time t_2 respectively are marked on the V(t) curve.

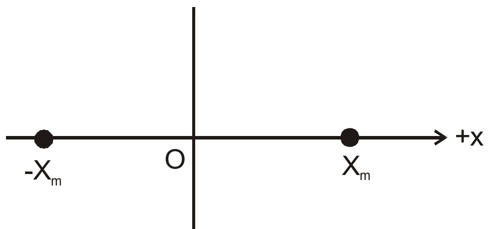


Figure-I

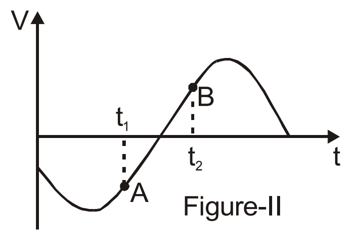


Figure-II

(A) At time t_1 , it is going towards X_m.
(B) At time t_1, its speed is decreasing.
(C) At time t_2, its position lies in between $-X_m$ and O.
(D) The phase difference $\Delta\phi$ between points A and B must be expressed as $90° < \Delta\phi < 180°$.

SH0098

19. For a body executing SHM with amplitudes A, time period T, max velocity v_{max} and phase constant zero, which of the following statements are correct?
(A) At $y = (A/2)$, $v > (v_{max} / 2)$ (B) $v = (v_{max}/2)$ for $|y| > (A/2)$
(C) For $t = (T/8)$, $y > (A/2)$ (D) For $y = (A/2)$, $t < (T/8)$

SH0099

20. The amplitude of a particle executing SHM about O is 10 cm. Then :

(A) When the K.E. is 0.64 of its max. K.E. its displacement is 6cm from O.

(B) When the displacement is 5 cm from O its K.E. is 0.75 of its max.P.E.

(C) Its total energy at any point is equal to its maximum K.E.

(D) Its velocity is half the maximum velocity when its displacement is half the maximum displacement.

SH0100

21. The position vector of a particle that is moving in space is given by

$$\vec{r} = (1+2\cos 2\omega t)\hat{i} + (3\sin^2 \omega t)\hat{j} + (3)\hat{k}$$

in the ground frame. All units are in SI. Choose the correct statement (s) :

(A) The particle executes SHM in the ground frame about the mean position $\left(1, \frac{3}{2}, 3\right)$

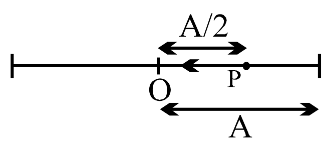
(B) The particle executes SHM in a frame moving along the z-axis with a velocity of 3 m/s.

(C) The amplitude of the SHM of the particle is $\frac{5}{2}$ m.

(D) The direction of the SHM of the particle is given by the vector $\left(\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right)$

SH0101

22. A particle starts from a point P at a distance of A/2 from the mean position O & travels towards left as shown in the figure. If the time period of SHM, executed about O is T and amplitude A then the equation of motion of particle is :



(A) $x = A \sin\left(\frac{2\pi}{T}t + \frac{\pi}{6}\right)$

(B) $x = A \sin\left(\frac{2\pi}{T}t + \frac{5\pi}{6}\right)$

(C) $x = A \cos\left(\frac{2\pi}{T}t + \frac{\pi}{6}\right)$

(D) $x = A \cos\left(\frac{2\pi}{T}t + \frac{\pi}{3}\right)$

SH0102

23. A spring has natural length 40 cm and spring constant 500 N/m. A block of mass 1 kg is attached at one end of the spring and other end of the spring is attached to ceiling. The block released from the position, where the spring has length 45 cm.

(A) the block will perform SHM of amplitude 5 cm.

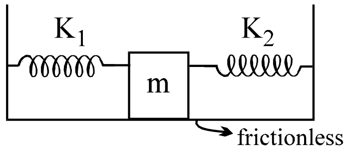
(B) the block will have maximum velocity $30\sqrt{5}$ cm/sec.

(C) the block will have maximum acceleration 15 m/s^2

(D) the minimum potential energy of the spring will be zero.

SH0103

24. Two springs with negligible masses and force constant of K_1 = 200 Nm^{-1} and K_2 = 160 Nm^{-1} are attached to the block of mass m = 10 kg as shown in the figure. Initially the block is at rest, at the equilibrium position in which both springs are neither stretched nor compressed. At time t = 0, a sharp impulse of 50 Ns is given to the block with a hammer.



(A) Period of oscillations for the mass m is $\frac{\pi}{3}$ s.
(B) Maximum velocity of the mass m during its oscillation is 5 ms^{-1}.
(C) Data are insufficient to determine maximum velocity.
(D) Amplitude of oscillation is 0.42 m.

SH0104

25. A mass of 0.2kg is attached to the lower end of a massless spring of force-constant 200 N/m, the upper end of which is fixed to a rigid support. Which of the following statements is/are true?
(A) In equilibrium, the spring will be stretched by 1cm.
(B) If the mass is raised till the spring is unstretched state and then released, it will go down by 2cm before moving upwards.
(C) The frequency of oscillation will be nearly 5 Hz.
(D) If the system is taken to the moon, the frequency of oscillation will be the same as on the earth.

SH0105

26. A particle moves in x-y plane according to equation $\vec{S} = \left(2\hat{i} + \hat{j}\right)\cos\omega t$. The motion of particle is:-
(A) On straight line (B) On ellipse (C) Periodic (D) SHM

SH0106

27. Two particles are in SHM with same amplitude A and same angular frequency ω. At time t = 0, one is at $x = +\frac{A}{2}$ and other is at $x = -\frac{A}{2}$. Both are moving in same direction.

(A) Phase difference between the two particle is $\frac{\pi}{3}$

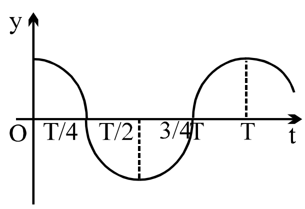
(B) Phase difference between the two particle is $\frac{2\pi}{3}$

(C) They will collide after time $t = \frac{\pi}{2\omega}$

(D) They will collide after time $t = \frac{3\pi}{\omega}$

SH0107

28. The displacement-time graph of a particle executing SHM is shown. Which of the following statements is/are true?



(A) The velocity is maximum at t = T/2
(B) The acceleration is maximum at t = T
(C) The force is zero at t = 3T/4
(D) The potential energy equals the oscillation energy at t = T/2.

SH0108

29. A particle is executing SHM with amplitude A, time period T, maximum acceleration a_o and maximum velocity v_0. Its starts from mean position at t = 0 and at time t, it has the displacement A/2, acceleration a and velocity v then :-

(A) t = T/12 (B) $a = a_o/2$ (C) $v = v_o/2$ (D) t = T/8

SH0109

30. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then :- **(1999S)**

(A) the resultant amplitude is $(1 + \sqrt{2})$ a

(B) the phase of the resultant motion relative to the first is 90°

(C) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion

(D) the resulting motion is not simple harmonic

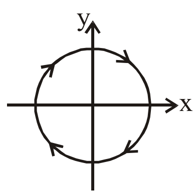
SH0110

COMPREHENSION TYPE QUESTIONS

Paragraph for question no. 31 and 32

Lissajous figures are produced by superposition of 2 SHM's in mutually perpendicular directions. If both SHM have same frequency, the lissajous figure is simple. It may be a circle, ellipse or a straight line depending on the phase difference between two SHMs. Interesting case occurs when one of the SHM is at double frequency of another. Suppose a body executes SHM vertically with frequency f_0, but horizontally with a frequency $2f_0$ and is initially at A, mean position of vertical as well as horizontal SHM. It can be seen to trace out a figure of 8 in space as shown. Since horizontal SHM has half the time period, it executes two horizontal oscillations by the time it completes a vertical oscillation.

31. A lissajous figure as shown here can be produced by

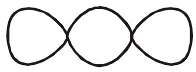


(A) x = A sin ωt ; y = A cos ωt (B) x = A cos ωt ; y = A sin ωt

(C) x = A sin ωt ; y = A sin $\left(\omega t + \frac{\pi}{4}\right)$ (D) x = A sin ωt ; y = A sin $\left(\omega t + \frac{3\pi}{4}\right)$

SH0111

32. For the Lissajous figure shown here, the frequency in vertical direction is _______ times the frequency in horizontal direction :-

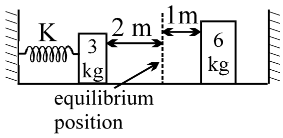


(A) 3 (B) $\frac{1}{3}$ (C) 2 (D) $\frac{1}{2}$

SH0111

Paragraph for Question No. 33 and 34

Two blocks of masses 3 kg and 6 kg rest on on a horizontal frictionless surface. The 3 kg block is attached to a spring with a force constant K = 900 N/m which is compressed 2 m initially from its equilibrium position. When 3 kg mass is released, it strikes the 6 kg mass and the two stick together.



33. The common velocity of the blocks after collision is :-

(A) 10 m/s (B) 30 m/s (C) 15 m/s (D) 2 m/s

SH0112

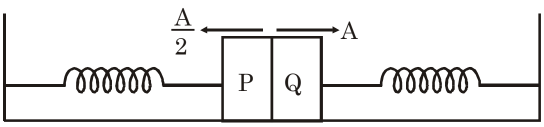
34. The amplitude of resulting oscillation after the collision is :-

(A) $\frac{1}{\sqrt{2}}$ m (B) $\frac{1}{\sqrt{3}}$ m (C) $\sqrt{2}$ m (D) $\sqrt{3}$ m

SH0112

Paragraph for Question no. 35 and 36

Two identical blocks P and Q have mass m each. They are attached to two identical springs initially unstretched. Now the left spring (alongwith P) is compressed by $\frac{A}{2}$ and the right spring (alongwith Q) is compressed by A. Both the blocks are released simultaneously. They collide perfectly inelastically. Initially time period of both the block was T.



35. The time period of oscillation of combined mass is :

(A) $\frac{T}{\sqrt{2}}$ (B) $\sqrt{2}T$ (C) T (D) $\frac{T}{2}$

SH0113

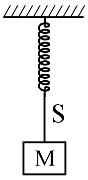
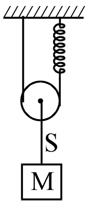
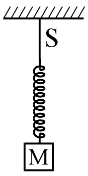
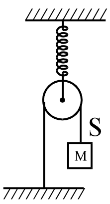
36. The amplitude of combined mass is :

(A) $\frac{A}{4}$ (B) $\frac{A}{2}$ (C) $\frac{2A}{3}$ (D) $\frac{3A}{4}$

SH0113

MATRIX MATCH TYPE QUESTION

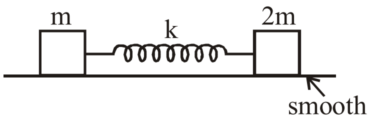
37. In the following four situations, mass M of 1kg is kept in equilibrium. k = 100 N/m in all cases. What speed can be given to mass M vertically so that inextensible string S does not become slack in subsequent motion. Consider that pulley is ideal and string is massless :

Column-I		**Column-II**	
(A)  (the block is attached to spring by an inextensible thread)		(P)	1 ms^{-1}
(B) 		(Q)	0.5 ms^{-1}
(C)  (block is attached to the spring)		(R)	0.25 ms^{-1}
(D) 		(S)	2 ms^{-1}

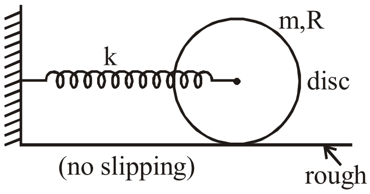
SH0114

38. In list-I, the systems are performing SHM and in list-II, the time period of SHM is shown then match list-I with list-II.

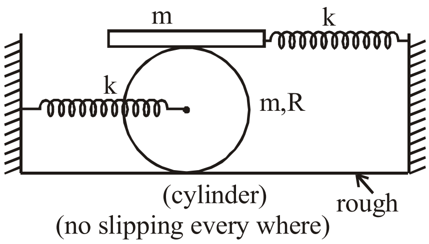
List-I | **List-II**

(P) 

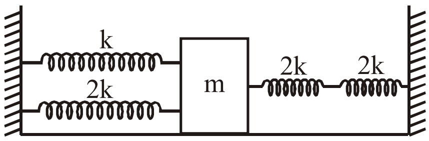
(1) $\left[2\pi\sqrt{\frac{11m}{10k}}\right]$

(Q) 

(2) $2\pi\sqrt{\frac{m}{4k}}$

(R) 

(3) $2\pi\sqrt{\frac{3m}{2k}}$

(S) 

(4) $2\pi\sqrt{\frac{2m}{3k}}$

Codes :

	P	Q	R	S
(A)	3	2	1	4
(B)	4	3	2	1
(C)	3	4	2	1
(D)	4	3	1	2

SH0115

EXERCISE (JM)

1. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is :- **[AIEEE-2011]**

(1) $\left(\frac{M}{M+m}\right)^{1/2}$ (2) $\left(\frac{M+m}{M}\right)^{1/2}$ (3) $\frac{M}{M+m}$ (4) $\frac{M+m}{M}$

SH0116

2. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :- **[AIEEE-2011]**

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

SH0117

3. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'. Then, 'T' is equal to :- **[AIEEE-2011]**

(1) $2\pi\sqrt{\frac{\ell\rho}{(\rho-d)g}}$ (2) $2\pi\sqrt{\frac{\ell d}{\rho g}}$

(3) $2\pi\sqrt{\frac{\ell\rho}{dg}}$ (4) $2\pi\sqrt{\frac{\ell d}{(\rho-d)g}}$

SH0118

4. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then : **[JEE Mains-2014]**

(1) Amplitude of motion is 4a

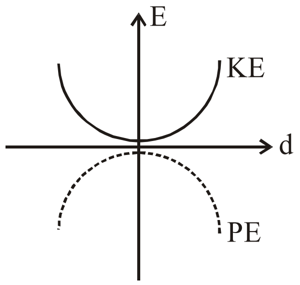
(2) Time period of oscillation is 6τ

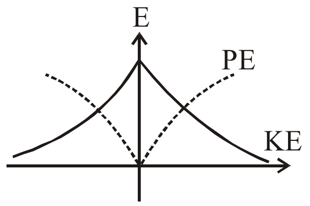
(3) Amplitude of motion is 3a

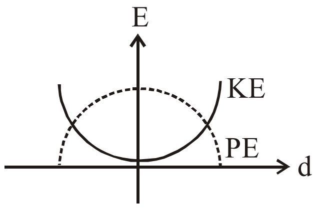
(4) Time period of oscillation is 8τ

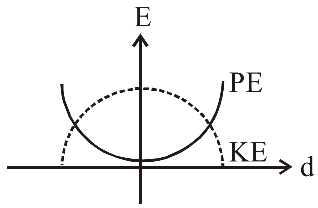
SH0119

5. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ? (graphs are schematic and not drawn to scale) **[JEE Mains-2015]**

(1) 

(2) 

(3) 

(4) 

SH0120

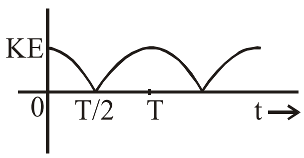
6. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is :-

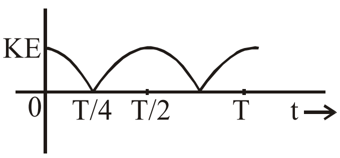
[JEE Mains-2016]

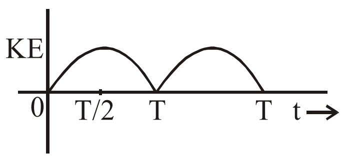
(1) $\frac{7A}{3}$ (2) $\frac{A}{3}\sqrt{41}$ (3) 3A (4) $A\sqrt{3}$

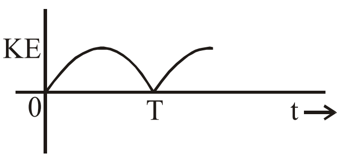
SH0121

7. A particle is executing simple harmonic motion with a time period T. AT time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like **[JEE Main-2017]**

(1) 

(2) 

(3) 

(4) 

SH0122

8. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver =108 and Avagadro number = 6.02×10^{23} gm mole^{-1}) **[JEE Main-2018]**

(1) 7.1 N/m (2) 2.2 N/m (3) 5.5 N/m (4) 6.4 N/m

SH0123

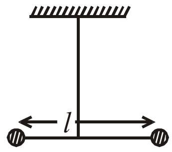
SELECTIVE PROBLEMS FROM JEE-MAINS ONLINE PAPERS

9. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to : **[JEE-Main-2019_Jan]**

(1) 0.17 (2) 0.37 (3) 0.57 (4) 0.77

SH0142

10. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system(see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be: **[JEE-Main-2019_Jan]**



(1) $\frac{3k\theta_0^2}{l}$ (2) $\frac{k\theta_0^2}{2l}$ (3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{k\theta_0^2}{l}$

SH0143

11. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is : **[JEE-Main-2019_Jan]**

(1) $\frac{7}{3}\pi$ (2) $\frac{3}{8}\pi$ (3) $\frac{4\pi}{3}$ (4) $\frac{8\pi}{3}$

SH0144

12. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by :- **[JEE-Main-2019_Jan]**

(1) 10^{-3} rad/s (2) 10^{-1} rad/s (3) 1 rad/s (4) 10^{-5} rad/s

SH0145

13. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then :- **[JEE-Main-2019_Jan]**

(1) $K_2 = \frac{K_1}{4}$ (2) $K_2 = \frac{K_1}{2}$ (3) $K_2 = 2K_1$ (4) $K_2 = K_1$

SH0146

14. A simple harmonic motion is represented by:

$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm

The amplitude and time period of the motion are: **[JEE-Main-2019_Jan]**

(1) 5cm, $\frac{3}{2}$s (2) 5cm, $\frac{2}{3}$s (3) 10cm, $\frac{3}{2}$s (4) 10cm, $\frac{2}{3}$s

SH0147

15. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is : **[JEE-Main-2019_April]**

(1) $4T\sqrt{\frac{1}{15}}$ (2) $2T\sqrt{\frac{1}{10}}$ (3) $4T\sqrt{\frac{1}{14}}$ (4) $2T\sqrt{\frac{1}{14}}$

SH0148

16. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. the ratio $\frac{T_1}{T_2}$ will be :

[JEE-Main-2020_Sep]

(1) $\frac{2}{\sqrt{3}}$ (2) $\frac{\sqrt{2}}{3}$ (3) $\frac{2}{3}$ (4) $\frac{3}{\sqrt{2}}$

SH0149

17. When a particle of mass m is attached to a vertical spring of spring constant k and released, its motion is described by $y(t) = y_0\sin^2\omega t$, where 'y' is measured from the lower end of unstretched spring. Then ω is :

[JEE-Main-2020_Sep]

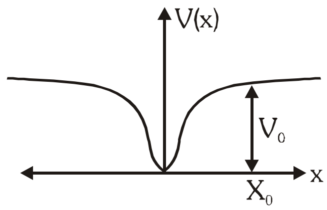
(1) $\sqrt{\frac{g}{y_0}}$ (2) $\sqrt{\frac{g}{2y_0}}$ (3) $\frac{1}{2}\sqrt{\frac{g}{y_0}}$ (4) $\sqrt{\frac{2g}{y_0}}$

SH0150

EXERCISE (JA)

Paragraph for Question No. 1 to 3

When a particle of mass m moves on the x–axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x–axis. Its potential energy is $V(x) = \alpha x^4$ $(\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure) **[IIT-JEE 2010]**



1. If the total energy of the particle is E, it will perform periodic motion only if :-

(A) $E < 0$ (B) $E > 0$ (C) $V_0 > E > 0$ (D) $E > V_0$

SH0124

2. For periodic motion of small amplitude A, the time period T of this particle is proportional to :-

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

SH0124

3. The acceleration of this particle for $|x| > X_0$ is :-

(A) proportional to V_0 (B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (D) Zero

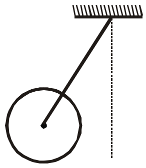
SH0124

4. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are :- **[IIT-JEE 2011]**

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

SH0125

5. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (< L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its center and (case B) the disc is free to rotated about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is(are) true? **[IIT-JEE 2011]**

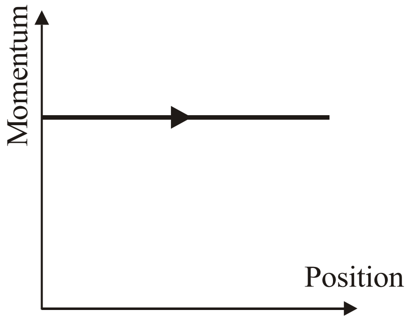


(A) Restoring torque in case A = Restoring torque in case B
(B) Restoring torque in case A < Restoring torque in case B
(C) Angular frequency for case A> Angular frequency for case B
(D) Angular frequency for case A< Angular frequency for case B

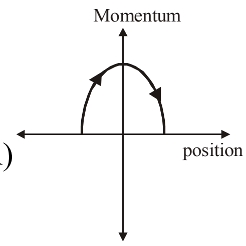
SH0126

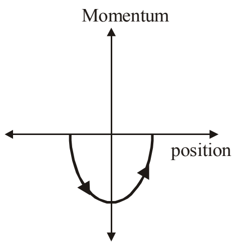
Paragraph for Questions Nos. 6 to 8

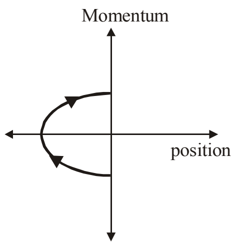
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum upwards (or to right) is positive and downwards (or to left) is negative. **[IIT-JEE 2011]**

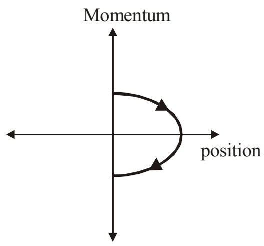


6. The phase space diagram for a ball thrown vertically up from ground is

(A) 

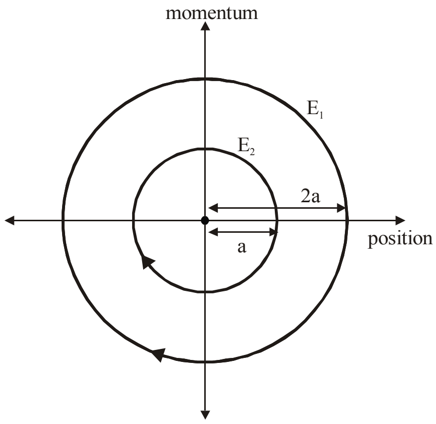
(B) 

(C) 

(D) 

SH0127

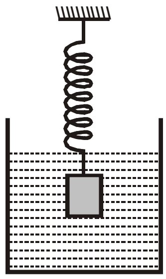
7. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

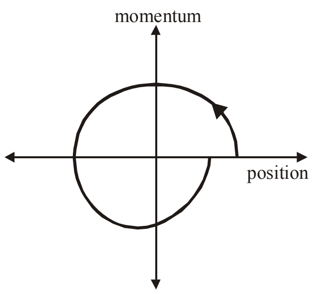


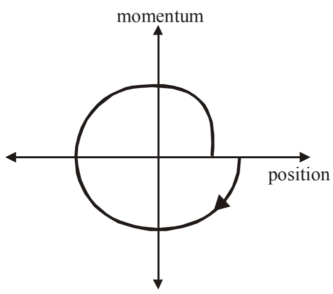
(A) $E_1 = \sqrt{2}E_2$ (B) $E_1=2E_2$ (C) $E_1=4E_2$ (D) $E_1=16E_2$

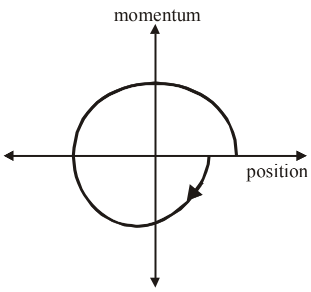
SH0127

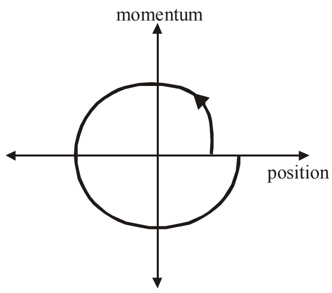
8. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is



(A)


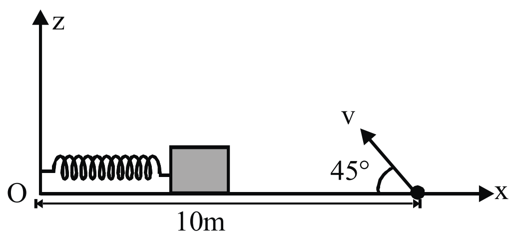
(B)


(C)


(D)


SH0127

9. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. They system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10m from O. If the pebble hits the block at t = 1s, the value of v is:- (take g = 10 m/s^2) **[IIT-JEE 2012]**



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s

(C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

SH0128

10. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision :- **[JEE-Advanced-2013]**

(A) the speed of the particle when it returns to its equilibrium position is u_0

(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

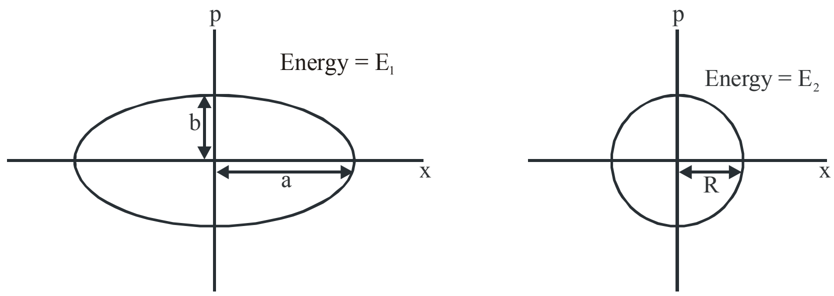
(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(D) the time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

SH0129

11. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation (s) is (are)

[JEE-Advanced-2015]



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

SH0130

12. A particle of unit mass is moving along the x-axis under the influence of a force and its total enegy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column-II **[JEE-Advanced-2015]**

Column-I	**Column-II**
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) The force acting on the particle is zero at x = a.
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero at x = 0.
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R) The force acting on the particle is zero at x =–a.
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S) The particle experiences an attractive force towards x = 0 in the region $\|x\| < a$.
	(T) The particle with total enegy $\frac{U_0}{4}$ can oscillate about the point x = –a.

SH0131

13. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M? **[JEE-Advanced-2016]**

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

SH0132

ANSWER KEY

EXERCISE (S-1)

1. Ans. $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$ **2. Ans.** 0.33 sec **3. Ans.** 2 m/s **4. Ans.** 0.06m

5. Ans. $y = 0.1\sin(4t + \pi/4)$ **6. Ans.** (i) $x_0 = 2m$; (ii) $T = \sqrt{2}\,\pi$ sec.; (iii) $2\sqrt{3}$

7. Ans. $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$ **8. Ans.** $\omega = \sqrt{\frac{2T_0}{m\ell}}$ **9. Ans.** $1/\sqrt{3}$ **10. Ans.** $2\pi\sqrt{\frac{17L}{18g}}$

11. Ans. $\omega = \sqrt{\frac{k}{(m+6M)}}$ **12. Ans.** M = 1.6 kg **13. Ans.** (a) $\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$, (b) $\frac{L}{3}$

14. Ans. (a) 2 sec, (b) $T = \frac{2}{5^{1/4}}$ sec

15. Ans. (a) $T = mg + mgA^2 \sin^2 \sqrt{\frac{g}{\ell}}\, t$ (b) $t = (2n+1)\frac{\pi}{2}\sqrt{\frac{\ell}{g}}$; $n \in I$ $T_{max} = mg + mgA^2$

16. Ans. (b) $\frac{R}{\sqrt{2}}$ **17. Ans.** 100 Nm^{-1} **18. Ans.** $10/\pi$ Hz, $\frac{5\sqrt{37}}{6}$ cm

19. Ans. $T = 2\sqrt{\ell/g}\,[\pi/2 + \sin^{-1}(\alpha/\beta)]$ **20. Ans.** 2 **21. Ans.** 5

22. Ans. 1 **23. Ans.** $f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$

EXERCISE (S-2)

1. Ans. 3cm, $x = 10 - 3\sin 5t$; $\Delta E = 0.135J$ **2. Ans.** $f = \frac{1}{\pi}$; $E = 4\pi^2 \times 10^{-5}J$; $v = 2\pi \times 10^{-2}m/s$

3. Ans. (a) $\frac{\pi}{10}$ sec, (b) 4 cm, (c) 2.40 kg m/s. **4. Ans.** Will increase $\sqrt{1+\frac{2}{5}\left(\frac{R}{\ell}\right)^2}$ times.

5. Ans. (a) $T = 2\pi\sqrt{\ell/3g} = 1.1$ s; (b) $E = \frac{1}{2}mg\ell\alpha^2 = 0.05$ J

6. Ans. $\omega = \sqrt{(I_1\omega_1^2 + I_2\omega_2^2)/(I_1 + I_2)}$ **7. Ans.** $\pi\sqrt{m/k} + 2\sqrt{2E/mg^2}$

8. Ans. $\frac{\pi}{3}$ s **9. Ans.** (i) $x_2 = v_0 t + \frac{m_1}{m_2}A(1-\cos\omega t)$ (ii) $\ell_0 = \left(\frac{m_1}{m_2}+1\right)A$

10. Ans. $a = (mg/x)\sqrt{1+2hx/mg}$, $\frac{1}{2}xA^2$ **11. Ans.** 5

12. Ans. $\omega = \sqrt{\frac{3g}{2L}\left(\frac{d_2 - d_1}{d_1}\right)}$ **13. Ans.** $\left(\frac{4g}{H}\right)x$

EXERCISE (O-1)

1. Ans. (C)	2. Ans. (D)	3. Ans. (A)	4. Ans. (A)	5. Ans. (A)	6. Ans. (B)
7. Ans. (A)	8. Ans. (A)	9. Ans. (B)	10. Ans. (A)	11. Ans. (A)	12. Ans. (C)
13. Ans. (C)	14. Ans. (C)	15. Ans. (A)	16. Ans. (A)	17. Ans. (C)	18. Ans. (B)
19. Ans. (A)	20. Ans. (B)	21. Ans. (B)	22. Ans. (B)	23. Ans. (C)	24. Ans. (C)
25. Ans. (D)	26. Ans. (B)	27. Ans. (C)	28. Ans. (B)	29. Ans. (A)	30. Ans. (C)
31. Ans. (A)	32. Ans. (C)	33. Ans. (B)	34. Ans. (D)	35. Ans. (C)	36. Ans. (A)
37. Ans. (B)	38. Ans. (B)	39. Ans. (A)	40. Ans. (A)	41. Ans. (B)	42. Ans. (A)
43. Ans. (C)	44. Ans. (B)				

EXERCISE (O-2)

1. Ans. (D)	2. Ans. (D)	3. Ans. (D)	4. Ans. (C)	5. Ans. (B)	6. Ans. (D)
7. Ans. (B)	8. Ans. (D)	9. Ans. (A)	10. Ans. (B)	11. Ans. (B)	12. Ans. (A)
13. Ans. (A)	14. Ans. (C)	15. Ans. (D)	16. Ans. (C)	17. Ans. (A,D)	18. Ans. (B, C)

19. Ans. (A,B,C)	20. Ans. (A,B,C)	21. Ans. (A,C,D)	22. Ans. (B,D)
23. Ans. (B,C,D)	24. Ans. (A,B)	25. Ans. (A,B,C,D)	26. Ans. (A,C,D)
27. Ans. (A, C)	28. Ans. (B,C,D)	29. Ans. (A,B)	30. Ans. (A, C)

31. Ans. (A)	32. Ans. (A)	33. Ans. (A)	34. Ans. (C)	35. Ans. (C)	36. Ans. (A)

37. Ans. (A)-Q,R ; (B)-R ; (C)-Q,R ; (D)-P,Q,R

38. Ans. (D)

EXERCISE (JM)

1. Ans. (2)	2. Ans. (4)	3. Ans. (2)	4. Ans. (2)	5. Ans. (4)	6. Ans. (1)
7. Ans. (2)	8. Ans. (1)	9. Ans. (2)	10. Ans. (4)	11. Ans. (4)	12. Ans. (1)
13. Ans. (2)	14. Ans. (4)	15. Ans. (1)	16. Ans. (1)	17. Ans. (2)	

EXERCISE (JA)

1. Ans. (C)	2. Ans. (B)	3. Ans. (D)	4. Ans. (B)	5. Ans. (A,D)	6. Ans. (D)
7. Ans. (C)	8. Ans. (B)	9. Ans. (A)	10. Ans. (A,D)	11. Ans. (B,D)	

12. Ans. (A)-P,Q,R,T; (B)-Q,S; (C)-P,Q,R,S; (D)-P,R,T

13. Ans. (A,B,D)