CHAPTER

7

PERMUTATIONS AND COMBINATIONS

CONCEPT TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

1. If $^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$, then the value of r is
(a) 41 (b) 14 (c) 10 (d) 51

2. If $^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then the value of n is:
(a) 20 (b) 19 (c) 18 (d) 17

3. If $^{30}C_{r+2} = {}^{30}C_{r-2}$, then r equals:
(a) 8 (b) 15 (c) 30 (d) 32

4 . The sum of the digits in the unit place of all the numbers formed with the help of 3, 4, 5, 6 taken all at a time is
(a) 432 (b) 108 (c) 36 (d) 18

5. In an examination, there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is :
(a) 11 (b) 12 (c) 27 (d) 63

6. How many arrangements can be made out of the letters of the word " MOTHER" taken four at a time so that each arrangement contains the letter ´M´?
(a) 240 (b) 120 (c) 60 (d) 360

7. In how many ways can a bowler take four wickets in a single 6-ball over ?
(a) 6 (b) 15 (c) 20 (d) 30

8. There are four chairs with two chairs in each row. In how many ways can four persons be seated on the chairs, so that no chair remains unoccupied ?
(a) 6 (b) 12 (c) 24 (d) 48

9. If a secretary and a joint secretary are to be selected from a committee of 11 members, then in how many ways can they be selected ?
(a) 110 (b) 55 (c) 22 (d) 11

10. A bag contains 3 black, 4 white and 2 red balls, all the balls being different. Number of selections of atmost 6 balls containing balls of all the colours is
(a) 1008 (b) 1080 (c) 1204 (d) 1130

11. Number of ways in which 20 different pearls of two colours can be set alternately on a necklace, there being 10 pearls of each colour.
(a) $6 \times (9!)^2$ (b) 12!
(c) $4 \times (8!)^2$ (d) $5 \times (9!)^2$

12. Number of words each containing 3 consonants and 2 vowels that can be formed out of 5 consonants and 4 vowels is
(a) 3600 (b) 7200 (c) 6728 (d) 2703

13. Every body in a room shakes hands with every body else. If total number of hand-shaken is 66, then the number of persons in the room is
(a) 11 (b) 10 (c) 12 (d) 19

14. Number of different seven digit numbers that can be written using only the three digits 1, 2 and 3 with the condition that the digit 2 occurs twice in each number is
(a) 652 (b) 650 (c) 651 (d) 640

15. Total number of eight digit numbers in which all digits are different is
(a) $\frac{8.7!}{3}$ (b) $\frac{5.8!}{3}$ (c) $\frac{8.9!}{2}$ (d) $\frac{9.9!}{2}$

16. Number of words from the letters of the words BHARAT in which B and H will never come together is
(a) 210 (b) 240 (c) 422 (d) 400

17. Four couples (husband and wife) decide to form a committee of four members, then the number of different committees that can be formed in which no couple finds a place is
(a) 15 (b) 16 (c) 20 (d) 21

18. Number of different ways in which 8 persons can stand in a row so that between two particular person A and B there are always two person is
(a) 11 (b) 13 (c) 15 (d) 16

19. Total number of four digit odd numbers that can be formed using 0, 1, 2, 3, 5, 7 (using repetition allowed) are
(a) 216 (b) 375 (c) 400 (d) 720

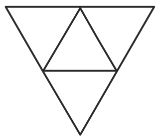
20. If the letters of the word SACHIN are arranged in all possible ways and these words are written out as in dictionary, then the word SACHIN appears at serial number
(a) 601 (b) 600 (c) 603 (d) 602

21. How many different words can be formed by jumbling the letters in the word MISSISSIPPI in which no two S are adjacent?
(a) $8.\,{}^6C_4.\,{}^7C_4$ (b) $6.7.\,{}^8C_4$
(c) $6.\,8.\,{}^7C_4.$ (d) $7.\,{}^6C_4.\,{}^8C_4$

22. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ?
(a) 16 (b) 36 (c) 60 (d) 180

23. The number of ways in which four letters of the word MATHEMATICS can be arranged is given by
(a) 136 (b) 192 (c) 1680 (d) 2454

24. In how many ways can this diagram be coloured subject to the following two conditions?
(i) Each of the smaller triangle is to be painted with one of three colours: red, blue or green.
(ii) No two adjacent regions have the same colour.



(a) 20 (b) 24 (c) 28 (d) 30

25. There are four bus routes between A and B; and three bus routes between B and C. A man can travel round-trip in number of ways by bus from A to C via B. If he does not want to use a bus route more than once, in how many ways can he make round trip?
(a) 72 (b) 144 (c) 14 (d) 19

26. In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together?
(a) 41472 (b) 41470 (c) 41400 (d) 41274

27. The number of ways of distributing 50 identical things among 8 persons in such a way that three of them get 8 things each, two of them get 7 things each, and remaining 3 get 4 things each, is equal to

(a) $\dfrac{(50!)(8!)}{(8!)^3(3!)^2(7!)^2(4!)^3(2!)}$

(b) $\dfrac{(50!)(8!)}{(8!)^3(7!)^3(4!)^3}$

(c) $\dfrac{(50!)}{(8!)^3(7!)^2(4!)^3}$

(d) $\dfrac{(8!)}{(3!)^2(2!)}$

28. If eleven members of a committee sit at a round table so that the President and Secretary always sit together, then the number of arrangements is
(a) $10!\times 2$ (b) $10!$ (c) $9!\times 2$ (d) $11!\times 2!$

29. ABC is a triangle. 4, 5, 6 points are marked on the sides AB, BC, CA, respectively, the number of triangles on different side is
(a) $(4+5+6)!$ (b) $(4-1)(5-1)(6-1)$
(c) $5!\,4!\,6!$ (d) $4\times 5\times 6$

30. Total number of words formed by 2 vowels and 3 consonants taken from 4 vowels and 5 consonants is equal to
(a) 60 (b) 120 (c) 7200 (d) 720

31. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines is
(a) 6 (b) 18 (c) 12 (d) 9

32. The number of ways in which 3 prizes can be distributed to 4 children, so that no child gets all the three prizes, are
(a) 64 (b) 62 (c) 60 (d) None of these

33. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then, what is the rank of the word RACHIT?
(a) 479 (b) 480 (c) 481 (d) 482

34. The number of chords that can be drawn through 21 points on a circle, is
(a) 200 (b) 190 (c) 210 (d) None of these

35. The number of ways a student can choose a programme out of 5 courses, if 9 courses are available and 2 specific courses are compulsory for every student is
(a) 35 (b) 40 (c) 24 (d) 120

36. The number of ways in which we can choose a committee from four men and six women so that the committee include at least two men and exactly twice as many women as men is
(a) 94 (b) 126 (c) 128 (d) None of these

37. A father with 8 children takes them 3 at a time to the zoological garden, as often as he can without taking the same 3 children together more than once. The number of times he will go to the garden is
(a) 56 (b) 100 (c) 112 (d) None of these

38. 4 buses runs between Bhopal and Gwalior. If a man goes from Gwalior to Bhopal by a bus and comes back to Gwalior by another bus, then the total possible ways are
(a) 12 (b) 16 (c) 4 (d) 8

39. Six identical coins are arranged in a row. The number of ways in which the number of tails is equal to the number of heads is
(a) 20 (b) 9 (c) 120 (d) 40

40. The figures 4, 5, 6, 7, 8 are written in every possible order. The number of numbers greater than 56000 is
(a) 72 (b) 96 (c) 90 (d) 98

41. There are 5 roads leading to a town from a village. The number of different ways in which a villager can go to the town and return back, is
(a) 25 (b) 20 (c) 10 (d) 5

42. The number of numbers that can be formed with the help of the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy odd places, is
(a) 24 (b) 18 (c) 12 (d) 30

43. In a circus, there are ten cages for accommodating ten animals. Out of these, four cages are so small that five out of 10 animals cannot enter into them. In how many ways will it be possible to accommodate ten animals in these ten cages?
(a) 66400 (b) 86400 (c) 96400 (d) None of these

44. On the occasion of Deepawali festival, each student of a class sends greeting cards to the others. If there are 20 students in the class, then the total number of greeting cards exchanged by the students is
(a) ${}^{20}C_2$ (b) $2\cdot{}^{20}C_2$
(c) $2\cdot{}^{20}P_2$ (d) None of these

45. To fill 12 vacancies, there are 25 candidates of which five are from scheduled caste. If 3 of the vacancies are reserved for scheduled caste candidates while the rest are open to all, then the number of ways in which the selection can be made

(a) ${}^5C_3 \times {}^{22}C_9$ (b) ${}^{22}C_9 - {}^5C_3$
(c) ${}^{22}C_3 + {}^5C_3$ (d) None of these

46. 12 persons are to be arranged to a round table. If two particular persons among them are not to be side by side, the total number of arrangements is

(a) 9(10!) (b) 2(10!) (c) 45(8!) (d) 10!

STATEMENT TYPE QUESTIONS

Directions : Read the following statements and choose the correct option from the given below four options.

47. The number of 3 letters words, with or without meaning which can be formed out of the letters of the word 'NUMBER'.

Statement I : When repetition of letters is not allowed is 120.

Statement II : When repetition of letters is allowed is 216.

Choose the correct option.

(a) Only Statement I is correct
(b) Only Statement II is correct
(c) Both I and II are correct
(d) Both I and II are false

48. The number of 4 letter words that can be formed from letters of the word 'PART', when:

Statement I : Repetition is not allowed is 24.

Statement II : Repetition is allowed is 256.

Which of the above statement(s) is/are true?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

49. Consider the following statements:

Statement I : The number of diagonals of n-sided polygon is ${}^nC_2 - n$.

Statement II : A polygon has 44 diagonals. The number of its sides are 10.

Choose the correct option from the choices given below.

(a) Only I is true (b) Only II is true
(c) Both I and II are true (d) Both I and II are false

50. A committee of 7 has to be formed from 9 boys and 4 girls.

I. In 504 ways, this can be done, when the committee consists of exactly 3 girls.

II. In 588 ways, this can be done, when the committee consists of at least 3 girls.

Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

51. Consider the following statements.

I. ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2} = {}^{n+2}C_r$

II. ${}^nC_p = {}^nC_q \Rightarrow p = q$ or $p + q = n$

Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

52. Consider the following statements.

I. Value of ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$ is zero.

II. The total number of 9 digit numbers which have all different digits is 9!

Choose the correct option.

(a) Only I is true (b) Only II is true.
(c) Both are true. (d) Both are false.

53. Consider the following statements.

I. Three letters can be posted in five letter boxes in 3^5 ways.

II. In the permutations of n things, r taken together, the number of permutations in which m particular things occur together is ${}^{n-m}P_{r-m} \times {}^rP_m$.

Choose the correct option.

(a) Only I is false. (b) Only II is false.
(c) Both are false. (d) Both are true.

54. Consider the following statements.

I. If some or all n objects are taken at a time, the number of combinations is $2^n - 1$.

II. An arrangement in a definite order which can be made by taking some or all of a number of things is called a permutation.

Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

55. Consider the following statements.

I. If there are n different objects, then ${}^nC_r = {}^nC_{n-r}$, $0 \le r \le n$.

II. If there are n different objects, then ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$, $0 \le r \le n$

Choose the correct option.

(a) Both are false. (b) Both are true.
(c) Only I is true. (d) Only II is true.

56. Consider the following statements.

I. If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the values of n and r are 3 and 2 respectively.

II. From a class of 32 students, 4 are to be chosen for a competition. This can be done in ${}^{32}C_2$ ways.

Choose the correct option.

(a) Only I is true. (b) Only II is true.
(c) Both are false. (d) Both are true.

57. Consider the following statements.

I. If n is an even natural number, then the greatest among ${}^nC_0, {}^nC_1, {}^nC_2, \ldots, {}^nC_n$ is ${}^nC_{n/2}$.

II. If n is an odd natural number, then the greatest among ${}^nC_0, {}^nC_1, {}^nC_2 \ldots, {}^nC_n$ is ${}^nC_{\frac{n-1}{2}}$ or ${}^nC_{\frac{n+1}{2}}$.

Choose the correct option.

(a) Only I is false. (b) Only II is true.
(c) Both are true. (d) Both are false.

58. Consider the following statements.
If n is a natural number and r is non-negative integer such that $0 \le r \le n$, then
I. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
II. ${}^nC_r = \frac{n}{r}.{}^{n-1}C_{r-1}$
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

59. Consider the following statements.
I. The continued product of first n natural numbers is called the permutation.
II. L.C.M of 4!, 5! and 6! is 720.
Choose the correct option.
(a) Only I is true. (b) Only II is true.
(c) Both are true. (d) Both are false.

MATCHING TYPE QUESTIONS

Directions : Match the terms given in column-I with the terms given in column-II and choose the correct option from the codes given below.

60.

Column - I	Column - II
A. $\frac{7!}{5!}$ equals	1. 28
B. $\frac{12!}{(10!)(2!)}$ equals	2. 42
C. $\frac{8!}{6! \times 2!}$ equals	3. 66

Codes:

	A	B	C
(a)	1	2	3
(b)	1	3	2
(c)	3	2	1
(d)	2	3	1

61. Using the digits 1, 2, 3, 4, 5, 6, 7, a number of 4 different digits is formed. Find :

Column - I	Column - II
A. How many numbers are formed?	1. 840
B. How many numbers are exactly divisible by 2?	2. 200
C. How many numbers are exactly divisible by 25?	3. 360
D. How many of these are exactly divisible by 4?	4. 40

Match the questions in column-I with column-II and choose the correct option from the codes given below.

Codes:

	A	B	C	D
(a)	1	2	3	4
(b)	3	1	4	2
(c)	1	3	4	2
(d)	4	2	3	1

62.

Column - I	Column - II
(A) Value of n, if $(n+2)! = 2550 \times n!$, is	(1) 5
(B) Value of n, if $(n+1)! = 12\,(n-1)!$, is	(2) 121
(C) Value of x, if $\frac{1}{9!} + \frac{1}{10!} = \frac{x}{11!}$, is	(3) 2730
(D) Value of P(15, 3) is	(4) 49
(E) Value of n, if 2. P(5, 3) = P(n, 4), is	(5) 3

Codes

	A	B	C	D	E
(a)	4	5	2	3	1
(b)	1	3	5	2	4
(c)	4	2	5	3	1
(d)	1	5	3	2	4

63.

Column-I	Column-II
(A) If P(n, 4) = 20 . P(n, 2) then the value of n is	(1) 28
(B) ${}^5P_r = 2\,{}^6P_{r-1}$	(2) 4
(C) ${}^5P_r = {}^6P_{r-1}$	(3) 7
(D) Value of $\frac{8!}{6! \times 2!}$ is	(4) 3

Codes

	A	B	C	D
(a)	4	3	2	1
(b)	3	4	1	2
(c)	4	2	3	1
(d)	3	4	2	1

64.

Column - I	Column - II
(A) If ${}^nC_8 = {}^nC_2$. Find ${}^nC_2$.	(1) 5
(B) Determine n if ${}^{2n}C_3 : {}^nC_2 = 12 : 1$	(2) 91
(C) Determine n if ${}^{2n}C_3 : {}^nC_3 = 11 : 1$	(3) 6
(D) If ${}^nC_8 = {}^nC_6$, then the value of ${}^nC_2$ is	(4) 45

Codes

	A	B	C	D
(a)	4	3	1	2
(b)	4	1	3	2
(c)	2	1	3	4
(d)	2	3	1	4

65.

Column - I	Column - II
(A) If ${}^nP_r = 720$ and ${}^nC_r = 120$, then the value of 'r' is	(1) 3
(B) If ${}^{2n}C_3 : {}^nC_3 = 11 : 1$, then the value of 'n' is	(2) 4950
(C) If ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, then the value of 'n' is	(3) 19
(D) Value of ${}^{100}C_{98}$ is	(4) 6

Codes

	A	B	C	D
(a)	1	4	3	2
(b)	1	3	4	2
(c)	2	4	3	1
(d)	2	3	4	1

66. How many words (with or without dictionary meaning) can be made from the letters of the word MONDAY, assuming that no letter is repeated, if

Column - I	Column - II
A. 4 letters are used at a time	1. 720
B. All letters are used at a time	2. 240
C. All letters are used but the first is a vowel	3. 360

Match the statements in column-I with column-II and choose the correct options from the codes given below.

Codes:

	A	B	C
(a)	1	2	3
(b)	3	1	2
(c)	2	1	3
(d)	3	2	1

INTEGER TYPE QUESTIONS

Directions : This section contains integer type questions. The answer to each of the question is a single digit integer, ranging from 0 to 9. Choose the correct option.

67. If ${}^nC_9 = {}^nC_8$, what is the value of ${}^nC_{17}$?
(a) 1 (b) 0 (c) 3 (d) 17

68. If ${}^{10}C_x = {}^{10}C_{x+4}$, then the value of x is
(a) 5 (b) 4 (c) 3 (d) 2

69. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals`
(a) 5 (b) 7 (c) 6 (d) 4

70. Total number of ways of selecting five letters from letters of the word INDEPENDENT is
(a) 70 (b) 72 (c) 75 (d) 80

71. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, then the value of $x = 2^m$. The value of m is
(a) 2 (b) 4 (c) 6 (d) 5

72. Value of $\frac{n!}{(n-r)!}$ when n = 6, r = 2 is 5 m. The value of m is
(a) 2 (b) 4 (c) 6 (d) 5

73. Find n if ${}^{n-1}P_3 : {}^nP_4 = 1 : 9$
(a) 2 (b) 6 (c) 8 (d) 9

74. Determine n if ${}^{2n}C_3 : {}^nC_2 = 12 : 1$
(a) 5 (b) 3 (c) 4 (d) 1

75. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n equals`
(a) 5 (b) 7 (c) 6 (d) 4

76. The number of values of r satisfying the equation ${}^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$ is
(a) 1 (b) 2 (c) 3 (d) 4

77. What is the value of ${}^nP_0$?
(a) 0 (b) 1 (c) ∞ (d) $\frac{1}{2}$

78. What is the value of ${}^nC_n$?
(a) 0 (b) ∞ (c) r (d) 1

79. What is the value of ${}^nC_0$?
(a) 0 (b) ∞
(c) 1 (d) None of these

80. If ${}^nC_9 = {}^nC_8$, what is the value of ${}^nC_{17}$?
(a) 1 (b) 0 (c) 3 (d) 17

81. If ${}^{10}C_x = {}^{10}C_{x+4}$, then the value of x is
(a) 5 (b) 4 (c) 3 (d) 2

82. If the ratio ${}^{2n}C_3 : {}^nC_3$ is equal to 11 : 1, n equals
(a) 2 (b) 6 (c) 8 (d) 9

83. The number of combinations of 4 different objects A, B, C, D taken 2 at a time is
(a) 4 (b) 6 (c) 8 (d) 7

84. If ${}^{12}P_r = {}^{11}P_6 + 6.\,{}^{11}P_5$, then r is equal to:
(a) 6 (b) 5
(c) 7 (d) None of these

85. $({}^8C_1 - {}^8C_2 + {}^8C_3 - {}^8C_4 + {}^8C_5 - {}^8C_6 + {}^8C_7 - {}^8C_8)$ equals:
(a) 0 (b) 1 (c) 70 (d) 256

ASSERTION - REASON TYPE QUESTIONS

Directions : Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
(b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
(c) Assertion is correct, reason is incorrect
(d) Assertion is incorrect, reason is correct.

86. **Assertion :** If the letters W, I, F, E are arranged in a row in all possible ways and the words (with or without meaning) so formed are written as in a dictionary, then the word WIFE occurs in the 24th position.
Reason : The number of ways of arranging four distinct objects taken all at a time is C(4, 4).

87. **Assertion :** A number of four different digits is formed with the help of the digits 1, 2, 3, 4, 5, 6, 7 in all possible ways. Then, number of ways which are exactly divisible by 4 is 200.
Reason : A number divisible by 4, if unit place digit is divisible by 4.

88. **Assertion :** Product of five consecutive natural numbers is divisible by 4!.
Reason : Product of n consecutive natural numbers is divisible by (n + 1)!

89. **Assertion :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Reason : The number of ways of choosing any 3 places, from 9 different places is ${}^9C_3$.

90. **Assertion :** A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3, 4 and 5 with repetition. The total number formed are 216.

Reason : If sum of digits of any number is divisible by 3 then the number must be divisible by 3.

CRITICALTHINKING TYPE QUESTIONS

Directions : This section contains multiple choice questions. Each question has four choices (a), (b), (c) and (d), out of which only one is correct.

91. ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$ is equal to:

(a) ${}^{n+2}C_r$ (b) ${}^nC_{r+1}$
(c) ${}^{n-1}C_{r+1}$ (d) None of these

92. If ${}^nC_r$ denotes the number of combination of n things taken r at a time, then the expression ${}^nC_{r+1} + {}^nC_{r-1} + 2 \times {}^nC_r$ equals

(a) ${}^{n+1}C_{r+1}$ (b) ${}^{n+2}C_r$
(c) ${}^{n+2}C_{r+1}$ (d) ${}^{n+1}C_r$

93. Given 12 points in a plane, no three of which are collinear. Then number of line segments can be determined, are:

(a) 76 (b) 66 (c) 60 (d) 80

94. There are 10 true-false questions in a examination. Then these questions can be answered in:

(a) 100 ways (b) 20 ways
(c) 512 ways (d) 1024 ways

95. The total number of ways of selecting six coins out of 20 one rupee coins, 10 fifty paise coins and 7 twenty five paise coins is:

(a) ${}^{37}C_6$ (b) 56 (c) 28 (d) 29

96. In a chess tournament where the participants were to play one game with one another, two players fell ill having played 6 games each, without playing among themselves. If the total number of games is 117, then the number of participants at the beginning was :

(a) 15 (b) 16 (c) 17 (d) 18

97. In how many ways can 10 lion and 6 tigers be arranged in a row so that no two tigers are together?

(a) $10! \times {}^{11}P_6$ (b) $10! \times {}^{10}P_6$
(c) $6! \times {}^{10}P_7$ (d) $6! \times {}^{10}P_6$

98. In how many ways can the letters of the word CORPORATION be arranged so that vowels always occupy even places ?

(a) 120 (b) 2700 (c) 720 (d) 7200

99. What is $\dfrac{(n+2)!+(n+1)!(n-1)!}{(n+1)!(n-1)!}$ equal to ?

(a) 1 (b) Always an odd integer
(c) A perfect square (d) None of these

100. The number of numbers of 9 different non-zero digits such that all the digits in the first four places are less than the digit in the middle and all the digits in the last four places are greater than the digit in the middle is

(a) $2(4!)$ (b) $(4!)^2$
(c) $8!$ (d) None of these

101. Number of 6 digit numbers that can be made with the digits 1, 2, 3 and 4 and having exactly two pairs of digits is

(a) 978 (b) 1801 (c) 1080 (d) 789

102. Number of 5 digit numbers that can be made using the digits 1 and 2 and in which at least one digit is different.

(a) 30 (b) 25 (c) 28 (d) 31

103. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls, Number or ways in which we can place the balls in the boxes (order is not considered in the box) so that no box remains empty is

(a) 150 (b) 160 (c) 12 (d) 19

104. In an examination, there are three multiple choice questions and each question has 4 choices. Number of ways in which a student can fail to get all answers correct is

(a) 11 (b) 12 (c) 27 (d) 63

105. Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

(a) 12 (b) 13 (c) 14 (d) 15

106. Four writers must write a book containing 17 chapters. The first and third writer must write 5 chapters each, the second writer must write 4 chapters and fourth writer must write three chapters. The number of ways that can be found to divide the book between four writers, is

(a) $\dfrac{17!}{(5!)^2\,4!\,3!\,2!}$ (b) $\dfrac{17!}{5!\,4!\,3!\,2!}$

(c) $\dfrac{17!}{(5!)^2\,4!\,3!}$ (d) $\dfrac{17!}{(5!)^2 \times 4 \times 3}$

107. A student has to answer 10 questions, choosing at least 4 from each of parts A and B. If there are 6 questions in Part A and 7 in Part B, in how many ways can the student choose 10 questions?

(a) 266 (b) 260 (c) 256 (d) 270

108. In a small village, there are 87 families, of which 52 families have at most 2 children. In a rural development programme 20 families are to be chosen for assistance, of which at least 18 families must have at most 2 children. In how many ways can the choice be made?

(a) ${}^{52}C_{18}\,{}^{35}C_2$

(b) ${}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$

(c) ${}^{52}C_{18} + {}^{35}C_2 + {}^{52}C_{19}$

(d) ${}^{52}C_{18} \times {}^{35}C_2 + {}^{35}C_1 \times {}^{52}C_{19}$

109. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II, unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed?
(a) 40 (b) 45 (c) 42 (d) 41

110. There were two women participants in a chess tournament. The number of games the men played between themselves exceeded by 52 the number of games they played with women. If each player played one game with each other, the number of men in the tournament, was
(a) 10 (b) 11 (c) 12 (d) 13

111. For a game in which two partners play against two other partners, six persons are available. If every possible pair must play with every other possible pair, then the total number of games played is
(a) 90 (b) 45 (c) 30 (d) 60

112. A house master in a vegetarian boarding school takes 3 children from his house to the nearby dhaba for non-vegetarian food at a time as often as he can, but he does not take the same three children more than once. He finds that he goes to the dhaba (road side hotel) 84 times more than a particular child goes with him. Then the number of children taking non-vegetarian food in his hostel, is
(a) 15 (b) 5 (c) 20 (d) 10

113. The number of circles that can be drawn out of 10 points of which 7 are collinear, is
(a) 120 (b) 113
(c) 85 (d) 86

114. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many ways can we place the balls so that no box remains empty?
(a) 50 (b) 100
(c) 150 (d) 200

115. The number of ways of dividing 52 cards amongst four players so that three players have 17 cards each and the fourth player have just one card, is
(a) $\frac{52!}{(17!)^3}$ (b) 52!
(c) $\frac{52!}{17!}$ (d) None of these

116. The number of 3 digit numbers having at least one of their digit as 5 are
(a) 250 (b) 251
(c) 252 (d) 253

117. The number of 4-digit numbers that can be formed with the digits 1, 2, 3, 4 and 5 in which at least 2 digits are identical, is
(a) 505 (b) $4^5 - 5!$
(c) 600 (d) None of these

118. If the letters of the word KRISNA are arranged in all possible ways and these words are written out as in a dictionary, then the rank of the word KRISNA is
(a) 324 (b) 341
(c) 359 (d) None of these

119. How many numbers lying between 999 and 10000 can be formed with the help of the digits 0, 2, 3, 6, 7, 8, when the digits are not repeated?
(a) 100 (b) 200
(c) 300 (d) 400

120. Eighteen guests are to be seated, half on each side of a long table. Four particular guests desire to sit on one particular side and three others on the other side of the table. The number of ways in which the seating arrangement can be done equals
(a) ${}^{11}C_4\,(9!)^2$ (b) ${}^{11}C_6\,(9!)^2$
(c) ${}^6P_0 \times {}^5P_0$ (d) None of these

121. At an election, a voter may vote for any number of candidates not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote, is
(a) 6210 (b) 385
(c) 1110 (d) 5040

122. A student is to answer 10 out of 13 questions in an examination such that he must choose at least 4 from the first five questions. The number of choices available to him is
(a) 140 (b) 196
(c) 280 (d) 346

123. Ten persons, amongst whom are A, B and C to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is
(a) $\frac{10!}{6}$ (b) 3! 7!
(c) ${}^{10}P_3 \cdot 7!$ (d) None of these

124. A car will hold 2 in the front seat and 1 in the rear seat. If among 6 persons 2 can drive, then the number of ways in which the car can be filled is
(a) 10 (b) 20
(c) 30 (d) None of these

HINTS AND SOLUTIONS

CONCEPT TYPE QUESTIONS

1. (a) $\frac{56!}{(50-r)!} = 30800\left(\frac{54!}{(51-r)!}\right)$

$\Rightarrow 56 \times 55 = \frac{30800}{51-r}$

$\Rightarrow 51 - r = \frac{30800}{56 \times 55} \Rightarrow 51 - r = 10 \Rightarrow 41 = r$

2. (b) Given ${}^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$

$\frac{{}^{n+2}C_8}{{}^{n-2}P_4} = \frac{57}{16}$ $\left[\because {}^nC_r = \frac{n!}{r!(n-r)!} \text{ and } {}^nP_r = \frac{n!}{(n-r)!}\right]$

$\Rightarrow \frac{(n+2)!}{8!(n+2-8)!} \times \frac{(n-2-4)!}{(n-2)!} = \frac{57}{16}$

$\Rightarrow \frac{(n+2)(n+1)n.(n-1)}{8.7.6.5.4.3.2.1} = \frac{57}{16}$

$\Rightarrow (n+2)(n+1)n(n-1) = 143640$

$\Rightarrow (n^2+n-2)(n^2+n) = 143640$

$\Rightarrow (n^2+n)^2 - 2(n^2+n) + 1 = 143640 + 1$

$\Rightarrow (n^2+n-1)^2 = (379)^2$

$\Rightarrow n^2+n-1 = 379$ $[\because n^2+n-1 > 0]$

$\Rightarrow n^2+n-1-379 = 0$

$\Rightarrow n^2+n-380 = 0$

$\Rightarrow (n+20)(n-19) = 0$

$\Rightarrow n = -20, n = 19$

$\because$ n is not negative.

$\therefore n = 19$

3. (b) Let ${}^{30}C_{r+2} = {}^{30}C_{r-2}$ ...(i)

We know, If ${}^nC_{r_1} = {}^nC_{r_2}$, then $r_1 + r_2 = n$

In above given equation (i), we have

$n = 30, r_1 = r+2, r-2 = r_2$

$\therefore r_1 + r_2 = r + 2 + r - 2 = 2r$

and $n = 30$

$\therefore 2r = 30 \Rightarrow r = 15$

4. (b) The total number of numbers that can be formed with the digits 3, 4, 5, 6 taken all at a time = ${}^4P_4 = 4! = 24$. Consider the digits at the unit places in all these number. Each of the digits 3, 4, 5, 6 occurs in 3! = 6 times in unit's place. So, total of the digits at the unit places

$= (3+4+5+6)6 = 108$.

[Similary, the sum of the digits in the other places will also be 108]

5. (d) Student has 4 choices to answer the question.

$\therefore$ Total no. of ways to answer the question

$= 4 \times 4 \times 4 = 64$ ($\because$ total choices = 4)

But out of these there is only one way such that all answers are correct.

$\therefore$ Required number of ways of (student can fail to get all answers correct = 1 – 64 = 63.

6. (a) There are six letters in MOTHER, all different, i.e. arrangement can be made out of the letters of the word MOTHER taken four at a time with M present in every arrangement.

So, rest 3 letters can be arrangement from 5 letters

So, total number of ways = $4 \times {}^5P_3$

$= 4 \times \frac{5!}{(5-3)!} = \frac{4 \times 5 \times 4 \times 3 \times 2}{2} = 240$

7. (b) There are 6 balls in one over and 4 wickets are to be taken. So, 4 balls are to succeed. This can be done in ${}^6C_4$ ways.

$\Rightarrow$ Required number of ways = ${}^6C_4$

$= \frac{6!}{4!2!} = \frac{6 \times 5}{2} = 15$

8. (c) First chair can be occupied in 4 ways and second chair can be occupied in 3 ways, third chair can be occupied in 2 ways and last chair can be occupied in one way only. So total number of ways = $4 \times 3 \times 2 \times 1 = 24$.

9. (b) Selection of 2 members out of 11 has ${}^{11}C_2$ number of ways

So, ${}^{11}C_2 = 55$

10. (a) The required number of selections

$= {}^3C_1 \times {}^4C_1 \times {}^2C_1 ({}^6C_3 + {}^6C_2 + {}^6C_0) = 42 \times 4! = 1008$

11. (d) Ten pearls of one colour can be arranged in $\frac{1}{2}.(10-1)!$ ways. The number of arrangements of 10 pearls of the other colour in 10 places between the pearls of the first colour = 10!

$\therefore$ Required number of ways $= \frac{1}{2} \times 9! \times 10! = 5(9!)^2$

12. (b) 3 consonants and 2 vowels from 5 consonants and 4 vowels can be selected in ${}^5C_3 \times {}^4C_2 = 60$ ways. But total number of words with 3 + 2 = 5 letters = 5! ways = 120

$\therefore$ The required number of words = $60 \times 120 = 7200$

13. (c) If number of persons = n.

Then total number of hand-shaken = ${}^nC_2 = 66$

$\Rightarrow n(n-1) = 132$

$\Rightarrow (n+11)(n-12) = 0$

$\therefore n = 12$ $(\because n \neq -11)$

14. (a) Other than 2 numbers, remaining five places are filled by 1 and 3 and for each place there is two conditions.
No. of ways for five places = $2 \times 2 \times 2 \times 2 \times 2 = 2^5$
For 2 numbers, selecting 2 places out of 7 = ${}^7C_2$
$\therefore$ Required no. of ways = ${}^7C_2 . 2^5 = 652$

15. (d) There are ten digits 0, 1, 2, ..., 9. Permutations of these digits taken eight at a time = ${}^{10}P_8$ which includes permutations having 0 at the first. When 0 is fixed at the first place, then number of such permutations = ${}^9P_7$.
So, required number

$$= {}^{10}P_8 - {}^9P_7 = \frac{10!}{2} - \frac{9!}{2} = \frac{9.9!}{2}$$

16. (b) There are 6 letters in the word BHARAT, 2 of them are identical. Hence total number of words = 6!/2! = 360
Number of words in which B and H come together

$$= \frac{5!2!}{2!} = 120$$

$\therefore$ The required number of words = 360 – 120 = 240

17. (b) The number of committees of 4 gentlemen = ${}^4C_4 = 1$
The number of committees of 3 gentlemen, 1 wife
$= {}^4C_3 \times {}^1C_1$
($\because$ after selecting 3 gentlemen only 1 wife is left who can be included)
The number of committees of 2 gentlemen, 2 wives
$= {}^4C_2 \times {}^2C_2$
The number of committees of 1 gentleman, 3 wives
$= {}^4C_1 \times {}^3C_3$
The number of committees of 4 wives = 1
$\therefore$ The required number of committees = 1 + 4 + 6 + 4 + 1 = 16

18. (d) The number of 4 persons including A and B = ${}^6C_2$
Considering these four as a group, number of arrangements with the other four = 5!
But in each group the number of arrangements = 2! × 2!
$\therefore$ Required number of ways = ${}^6C_2 \times 5! \times 2! \times 2! + 1 = 16$

19. (d) Required number of numbers
$= 5 \times 6 \times 6 \times 4 = 36 \times 20 = 720.$

20. (a) Alphabetical order is
A, C, H, I, N, S
No. of words starting with A = 5!
No. of words starting with C = 5!
No. of words starting with H = 5!
No. of words starting with I = 5!
No. of words starting with N = 5!
SACHIN – 1
$\therefore$ Sachin appears at serial no. 601

21. (d) First let us arrange M, I, I, I, P, P
Which can be done in $\frac{7!}{4!2!}$ ways
Now 4 S can be kept at any of the ticked places in ${}^8C_4$ ways so that no two S are adjacent.
Total required ways

$$= \frac{7!}{4!2!} {}^8C_4 = \frac{7!}{4!2!} {}^8C_4 = 7 \times {}^6C_4 \times {}^8C_4$$

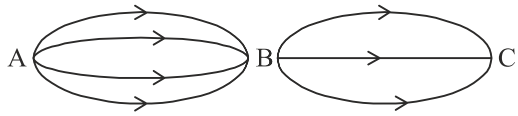
22. (c) X - X - X - X - X. The four digits 3, 3, 5,5 can be arranged at (-) places in $\frac{4!}{2!2!} = 6$ ways.
The five digits 2, 2, 8, 8, 8 can be arranged at (X) places in $\frac{5!}{2!3!}$ ways = 10 ways
Total no. of arrangements = $6 \times 10 = 60$ ways

23. (d) Two pairs of identical letters can be arranged in ${}^3C_2 \frac{4!}{2!2!}$ ways. Two identical letters and two different letters can be arranged in ${}^3C_1 \times {}^7C_2 \times \frac{4!}{2!}$ ways. All different letters can be arranged in ${}^8P_4$ ways
$\therefore$ Total no. of arrangements

$$= {}^3C_2 \frac{4!}{2!2!} + {}^3C_1 \times {}^7C_2 \times \frac{4!}{2!} + \frac{8!}{4!} = 2454.$$

24. (b) These conditions are satisfied exactly when we do as follows: First paint the central triangle in any one of the three colours. Next, paint the remaining 3 triangles, with any one of the remaining two colours. By the fundamental principle of counting, this can be done in 3 × 2 × 2 × 2 = 24 ways.

25. (a) In the following figure :



There are 4 bus routes from A to B and 3 routes from B to C. Therefore, there are 4 × 3 = 12 ways to go from A to C. It is round trip so the man will travel back from C to A via B. It is restricted that man cannot use same bus routes from C to B and B to A more than once. Thus, there are 2 × 3 = 6 routes for return journey. Therefore, the required number of ways = 12 × 6 = 72.

26. (a) First, we take books of a particular subject as one unit. Thus, there are 4 units which can be arranged in 4! = 24 ways. Now, in each of the arrangements, mathematics books can be arranged in 3! ways, history books in 4! ways, chemistry books in 3! ways and biology books in 2! ways. Thus, the total number of ways = 4! × 3! × 4! × 3! × 2! = 41472.

27. (d) Number of ways of dividing 8 persons in three groups, first having 3 persons, second having 2 persons and third having 3 persons = $\frac{8!}{3!2!3!}$. Since all the 50 things are identical.

So, required number = $\frac{8!}{(3!)^2 \cdot (2!)}$

28. (c) Since, out of eleven members, two members sit together, then the number of arrangements = 9! × 2 ($\because$ two members can sit in two ways).

29. (d) Required number of such triangles
$= {}^4C_1 \times {}^5C_1 \times {}^6C_1 = 4 \times 5 \times 6$

30. (c) Given 4 vowels and 5 consonants
$\therefore$ Total number of words = ${}^4C_2 \times {}^5C_3 \times 5!$
$= 6 \times 10 \times 120 = 7200$.

31. (b) Total number of parallelograms formed
$= {}^4C_2 \times {}^3C_2 = 6 \times 3 = 18$

32. (c) Each of the three prizes can be given to any of the four children.
$\therefore$ Total number of ways of distributing prizes
$= 4 \times 4 \times 4 = 64$
Number of ways in which one child gets all prizes = 4
$\therefore$ Number of ways in which no child gets all the three prizes = 64 – 4 = 60

33. (c) In the word 'RACHIT', the number of words beginning with A, C, H, I is 5! and the next word we get RACHIT.
$\therefore$ Required number of words
$= 4 \times 5! + 1 = 4 \times 120 + 1 = 481$

34. (c) Number of chords that can be drawn through 21 points on circle = Number of ways of selecting 2 points from 21 points on circle

$$= {}^{21}C_2 = \frac{21 \times 20}{2 \times 1} = 210$$

35. (a) Total number of available courses = 9
Out of these, 5 courses have to be chosen. But it is given that 2 courses are compulsory for every student, i.e. you have to choose only 3 courses, out of 7.

It can be done in ${}^7C_3$ ways $= \frac{7 \times 6 \times 5}{6} = 35$ ways.

36. (a) There are two possibilities :

	Men	Women
(i)	2	4
(ii)	3	6

(i) Number of ways of choosing a committee of 2 men and 4 women = ${}^4C_2 \times {}^6C_4$

$$= \frac{4 \times 3}{2 \times 1} \times \frac{6 \times 5}{2 \times 1} = 90$$

(ii) Number of ways of choosing a committee of 3 men and 6 women = ${}^4C_3 \times {}^6C_6$
$= 4 \times 1 = 4$

$\therefore$ Required number of ways = 94

37. (a) Number of times he will go to garden
= Number of ways of selecting 3 children from 8 children

$$= {}^8C_3 = \frac{8 \times 7 \times 6}{3 \times 2} = 56$$

38. (a) Since the man can go in 4 ways and can back in 3 ways.
Therefore, total number of ways are $4 \times 3 = 12$ ways.

39. (a) Required number of ways $= \frac{6!}{3!\,3!} = \frac{720}{6 \times 6} = 20$

[Number of heads = 3, number of tails = 3 and coins are identical]

40. (c) Required number of ways = 5! – 4! – 3!
= 120 – 24 – 6 = 90
[Number will be less than 56000 only if either 4 occurs on the first place or 5, 4 occurs on the first two places].

41. (a) The man can go in 5 ways and he can return in 5 ways. Hence, total number of ways are $5 \times 5 = 25$.

42. (b) The 4 odd digits 1, 3, 3, 1 can be arranged in the 4 odd places in $\frac{4!}{2!\,2!} = 6$ ways and 3 even digits 2, 4, 2 can be arranged in the three even places in $\frac{3!}{2!}$ = 3 ways
Hence, the required number of ways = $6 \times 3 = 18$.

43. (b) At first, we have to accommodate those 5 animals in cages which cannot enter in 4 small cages, therefore number of ways are ${}^6P_5$. Now, after accommodating 5 animals we left with 5 cages and 5 animals, therefore, number of ways are 5!. Hence, required number of ways = ${}^6P_5 \times 5! = 86400$.

44. (b) $2 \cdot {}^{20}C_2$ {Since two students can exchange cards each other in two ways}.

45. (a) The selection can be made in ${}^5C_3 \times {}^{22}C_9$ ways.
{Since 3 vacancies filled from 5 candidates in ${}^5C_3$ ways and now remaining candidates are 22 and remaining seats are 9}.

46. (a) 12 persons can be seated around a round table in 11! ways. The total number of ways in which 2 particular persons sit side by side is $10! \times 2!$. Hence, the required number of arrangements $= 11! - 10! \times 2! = 9 \times 10!$.

STATEMENT TYPE QUESTIONS

47. (c) I. Number of 3 letter words (repetition not allowed)
$= 6 \times 5 \times 4 = 120$
(as first place can be filled in 6 different ways, second place can be filled in 5 different ways and third place can be filled in 4 different ways)
II. Number of 3 letter words (repetition is allowed)
$= 6 \times 6 \times 6 = 216$
(as each of the place can be filled in 6 different ways)

48. (c) I. Number of 4 letter words that can be formed from alphabets of the word 'PART'
$= {}^4P_4 = 4! = 24$
II. Number of 4 letter words that can be formed when repetition is allowed = $4^4 = 256$

49. (a) I. In n-sided polygon, the number of vertices = n
$\therefore$ Number of lines that can be formed using n points = ${}^nC_2$.
Out of these, ${}^nC_2$ lines, n lines from the polygon.
$\therefore$ Number of diagonals = ${}^nC_2 - n$
II. Let the number of sides of a polygon = n
Number of diagonal = Number of line segment joining any two vertices of polygon – Number of sides
$= {}^nC_2 - n$

$$= \frac{n(n-1)}{2} - n = \frac{n(n-3)}{2}$$

Now, $\frac{n(n-3)}{2} = 44$

$\Rightarrow n^2 - 3n - 88 = 0$

$\Rightarrow (n - 11)(n + 8) = 0$

$\Rightarrow n = 11$

or $n = -8$ rejected.

50. (c) (I) A committee consisting of 3 girls and 4 boys can be formed in ${}^4C_3 \times {}^9C_4$ ways

$= {}^4C_1 \times {}^9C_4 = \frac{4}{1} \times \frac{9\times8\times7\times6}{1.2.3.4}$ ways

= 504 ways

(II) A committee having at least 3 girls will consists of (a) 3 girls 4 boys, (b) 4 girls 3 boys

This can be done in ${}^4C_3 \times {}^9C_4 + {}^4C_4 \times {}^9C_3$ ways

$= \frac{4}{1} \times \frac{9\times8\times7\times6}{1\times2\times3\times4} + 1 \times \frac{9\times8\times7}{1\times2\times3}$ ways

= 504 + 84 ways = 588 ways

51. (c) (I) ${}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$

$= \left[{}^nC_r + {}^nC_{r-1}\right] + \left[{}^nC_{r-1} + {}^nC_{r-2}\right]$

$= {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r.$

(II) If ${}^nC_p = {}^nC_q \Rightarrow {}^nC_p = {}^nC_{n-q}$

$\Rightarrow p = q$ or $p = n - q \left[\because {}^nC_r = {}^nC_{n-r}\right]$

52. (a) I. ${}^{15}C_8 + {}^{15}C_9 - {}^{15}C_6 - {}^{15}C_7$

$= {}^{15}C_7 + {}^{15}C_6 - {}^{15}C_6 - {}^{15}C_7 = 0$

II. Total number $= 10! - 9! = 9 \times 9!$

53. (c) Both are false

I. Correct is 5^3.

($\because$ each one of the three letters can be posted in anyone of the five letter boxes.)

II. Statement will be true if m particular things always occur.

54. (c) Both are true statements.

55. (b) Both are true statements.

I. ${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!\,[n-(n-r)]!}$

$= {}^nC_{n-r}$

II. ${}^nC_r + {}^nC_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!\,(n-r+1)!}$

$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{1}{r} + \frac{1}{n-r+1}\right]$

$= \frac{n!}{(r-1)!(n-r)!}\left[\frac{n-r+1+r}{r(n-r+1)}\right] = \frac{(n+1)!}{r!(n+1-r)!}$

56. (a) I. ${}^nP_r = {}^nP_{r+1}$

$\Rightarrow n - r = 1$ …(i)

and ${}^nC_r = {}^nC_{r-1}$

$\Rightarrow n - r + 1 = r \Rightarrow n - 2r = -1$ …(ii)

On solving (i) and (ii), we get

n = 3 and r = 2

II. Required no. of ways $= {}^{32}C_4 = \frac{32!}{4!\,28!}$

57. (c) Both are true.

58. (c) Both statements are true.

59. (b) I. The continued product of first n natural numbers is called the 'n factorial'.

II. $5! = 5 \times 4!$

$6! = 6 \times 5 \times 4!$

$\therefore$ L.C.M. of 4!, 5!, 6! = L.C.M. [4!, 5 × 4!, 6 × 5 × 4!]

$= 4! \times 5 \times 6 = 6! = 720$

MATCHING TYPE QUESTIONS

60. (d) A. $\frac{7!}{5!} = \frac{7\times6\times5!}{5!} = 42$

B. $\frac{12!}{10!\,2!} = \frac{12\times11\times10!}{10!\times2\times1} = 66$

C. $\frac{8!}{6!\,2!} = \frac{8\times7\times6!}{6!\times2\times1} = 28$

61. (c) A. The number of 4 different digits $= {}^7P_4$

$= \frac{7!}{(7-4)!}$

$= 7 \times 6 \times 5 \times 4 = 840$

B. The numbers exactly divisible by 2

= Number of ways of filling first 3 places × Number of ways of filling unit's place

$= {}^6P_3 \times 3$

$= \frac{6!}{(6-3)!} \times 3 = \frac{6!}{(3!)} \times 3$

$= 6 \times 5 \times 4 \times 3 = 360$

C. Number of 4-digit numbers divisible by 25

= Numbers ending with 25 or 75

5 × 4   25 or 75

= □□   □□

$= 5 \times 4 \times 2 = 40$

($\because$ when numbers end with 25 or 75, the other two places can be filled in 5 and 4 ways)

D. Number of 4-digit numbers divisible by 4

= Numbers ending with 12, 16, 24, 32, 36, 64, 72, 76, 52, 56

Now, number ending with 12

= □ □ □ □ = 20

$4 \times 5 \times 1 \times 1$

Similarly, numbers ending with other number (16, 24, ...) = 20 each

$\therefore$ Required numbers $= 10 \times 20 = 200$

62. (a) (A) $(n+2)(n+1)\,n! = 2550 \times n!$

$\Rightarrow n^2 + 3n - 2548 = 0$

$\Rightarrow (n+52)(n-49) = 0$

$\Rightarrow n = 49$

(B) $(n+1)\,n\,(n-1)! = 12(n-1)!$

$\Rightarrow n^2 + n - 12 = 0 \Rightarrow (n+4)(n-3) = 0$

$\Rightarrow n = 3$

(C) $\frac{1}{9!}\left[1+\frac{1}{10}\right]=\frac{x}{11\times10}\times\frac{1}{9!}$

$\Rightarrow \frac{11}{10}=\frac{x}{11\times10} \Rightarrow x=11\times11=121$

(D) $P(15,3)=\frac{15!}{12!}=\frac{15\times14\times13\times12!}{12!}=2730$

(E) $P(n,4)=2.\,P(5,3)$

$\Rightarrow \frac{n!}{(n-4)!}=2\cdot\left[\frac{5!}{(5-3)!}\right]$

$\Rightarrow n(n-1)(n-2)(n-3)=\frac{2(5!)}{2!}$

$\Rightarrow n(n-1)(n-2)(n-3)=5\times4\times3\times2\times1$

$\Rightarrow n(n-1)(n-2)(n-3)=5\times(5-1)\times(5-2)\times(5-3)$

$\Rightarrow n=5$

63. (d) (A) $\frac{n!}{(n-4)!}=20\cdot\frac{n!}{(n-2)!}$

$\Rightarrow (n-2)!=20(n-4)!$

$\Rightarrow (n-2)(n-3)=5\times4$

$\Rightarrow n-3=4 \Rightarrow n=7$

(B) We have,

${}^5P_r=2\,{}^6P_{r-1}$

or $\frac{5!}{(5-r)!}=2\left[\frac{6!}{(6-r+1)!}\right]$

or $\frac{5!}{(5-r)!}=2\left[\frac{6\times5!}{(7-r)!}\right]$

or $\frac{1}{(5-r)!}=\frac{12}{(7-r)(6-r)(5-r)!}$

or $(7-r)(6-r)=12$

or $42-7r-6r+r^2=12$

or $r^2-13r+30=0$

or $r^2-10r-3r+30=0$

or $r(r-10)-3(r-10)=0$

or $(r-10)(r-3)=0$

or $r=10$ or $r=3$

Hence, $r=3$

[$r=10 \Rightarrow {}^5P_{10}$ which is meaningless]

(C) We have,

${}^5P_r={}^6P_{r-1}$

or $\frac{5!}{(5-r)!}=2\left[\frac{6!}{[6-(r-1)]!}\right]$

or $\frac{5!}{(5-r)!}=\frac{6\times5!}{(6-r+1)!}$

or $\frac{5!}{(5-r)!}=\frac{6\times5!}{(7-r)!}$

or $\frac{5!}{(5-r)!}=\frac{6\times5!}{(7-r)(6-r)(5-r)!}$

or $(7-r)(6-r)=6$

or $r^2-13r+36=0$

or $r=4,9$

or $r=4$

[$r=9 \Rightarrow {}^5P_r$ which is meaningless]

(D) $\frac{8!}{6!\times2!}=\frac{8\times7\times6\times5\times4\times3\times2\times1}{(6\times5\times4\times3\times2\times1)\times(2\times1)}$

$=\frac{8\times7}{2\times1}=28$

64. (b) (A) We have

${}^nC_r={}^nC_{n-r}$

${}^nC_2={}^nC_{n-2}$

${}^nC_8={}^nC_{n-2} \Rightarrow n-2=8$ or $n=10$

${}^nC_2={}^{10}C_2=\frac{10\times9}{1\times2}=45$

(B) ${}^{2n}C_3:{}^nC_2=12:1$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{1.2.3}\div\frac{n(n-1)}{1.2}=\frac{12}{1}$

$\left[{}^nC_r=\frac{n(n-1)......(n-r+1)}{1.2.3........n}\right]$

or $\frac{2n(2n-1)2(n-1)}{6}\times\frac{2}{n(n-1)}=\frac{12}{1}$

or $\frac{4n(2n-1)(n-1)}{3}\times\frac{1}{n(n-1)}=12$

$2n-1=9, 2n=10$ or $n=5$

(C) ${}^{2n}C_3:{}^nC_3=11:1$

or $\frac{2n(2n-1)(2n-2)}{1.2.3}\div\frac{n(n-1)(n-2)}{1.2.3}=\frac{11}{1}$

or $\frac{4n(n-1)(2n-1)}{6}\times\frac{6}{n(n-1)(n-2)}=\frac{11}{1}$

or $\frac{4(2n-1)}{n-2}=11$

$4(2n-1)=11(n-2)$ or $8n-4=11n-22$

or $3n=18$ $\therefore n=6$

(D) ${}^nC_8={}^nC_6 \Rightarrow n=8+6=14$

$\therefore {}^nC_2={}^{14}C_2=\frac{14}{2}\times\frac{13}{1}\times{}^{12}C_0$

$=\frac{14}{2}\times\frac{13}{1}\times1=91$

65. (a) (A) $120=\frac{720}{r!} \Rightarrow r!=6 \Rightarrow r!=3! \Rightarrow r=3$

(B) $\frac{(2n)!}{(2n-3)!}\times\frac{(n-3)!}{n!}=\frac{11}{1}$

$\frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)}=\frac{11}{1}$

$\Rightarrow \frac{4(2n-1)}{n-2}=\frac{11}{1} \Rightarrow 3n=18 \Rightarrow n=6$

(C) $\frac{(n+2)!}{8!(n-6)!}\times\frac{(n-6)!}{(n-2)!}=\frac{57}{16}$

$\Rightarrow (n+2)(n+1)(n)(n-1)=\frac{57}{16}\times8!$

$\Rightarrow (n-1)n(n+1)(n+2)=18\times19\times20\times21$

$\Rightarrow n-1=18 \Rightarrow n=19$

(D) $^{100}C_{98} = {}^{100}C_{100-98} = {}^{100}C_2$

$= \frac{100}{2} \times \frac{99}{1} \times {}^{98}C_0 \left(\because {}^nC_r = \frac{n}{r} \cdot {}^{n-1}C_{r-1} \right)$

$= 4950$

66. **(b)** A. Number of words using 4 letters out of 6 letters

$= {}^6P_4 = \frac{6!}{2!} = 6 \times 5 \times 4 \times 3 = 360$

B. Number of words using all letters
$= {}^6P_6 = 6! = 720$

C. Number of words starting with vowel
= Number of ways of choosing first letter (out of O and A) × Number of ways of arranging 5 alphabets

$= 2 \times 5! = 2 \times 120 = 240$

INTEGER TYPE QUESTIONS

67. **(a)** $\frac{n!}{9!(n-9)!} = \frac{n!}{8!(n-8)!}$

$\Rightarrow \frac{1}{9 \times 8!(n-9)!} = \frac{1}{8!(n-8)(n-9)!}$

$\Rightarrow \frac{1}{9} = \frac{1}{(n-8)} \Rightarrow 9 = n - 8$

$\Rightarrow 9 + 8 = n \Rightarrow n = 17$

$\therefore {}^nC_{17} = {}^{17}C_{17} = 1 \qquad [\because {}^nC_n = 1]$

68. **(c)** We have $^{10}C_x = {}^{10}C_{x+4}$

$\Rightarrow x + x + 4 = 10 \Rightarrow 2x = 6 \Rightarrow x = 3$

69. **(b)** $^{n+1}C_3 - {}^nC_3 = 21 \quad \because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^nC_2 = 21 \Rightarrow n = 7$

70. **(b)** There are 11 letters in the given word which are as follows (NNN) (EEE) (DD)IPT

Five letters can be selected in the following manners :

(i) All letters different : $^6C_5 = 6$

(ii) Two similar and three different : $^3C_1 \cdot {}^5C_3 = 30$

(iii) Three similar and two different : $^2C_1 . {}^5C_2 = 20$

(iv) Three similar and two similar : $^2C_1 . {}^2C_1 = 4$

(v) Two similar, two similar and one different :
$^3C_2 . {}^4C_1 = 12$

$\therefore$ Total selections $= 6 + 30 + 20 + 4 + 12 = 72$

71. **(d)** $\frac{1}{6!} + \frac{1}{7!} = \frac{1}{6!} + \frac{1}{7.6!} = \frac{x}{8.7.6.!}$

$\Rightarrow \frac{1}{6!}\left(1 + \frac{1}{7}\right) = \frac{x}{8.7.6!}$

$\Rightarrow \frac{8}{7} = \frac{x}{8.7} \Rightarrow x = 64$

72. **(c)** n = 6, r = 2

$\frac{n!}{(n-r)!} = \frac{6!}{(6-2)!} = \frac{6!}{4!} = 6 \times 5$

73. **(d)** $\frac{^{n-1}P_3}{^nP_4} = \frac{1}{9} \Rightarrow \frac{^{n-1}P_3}{n.{}^{n-1}P_3} = \frac{1}{9}$

$\Rightarrow \frac{1}{n} = \frac{1}{9}$ or $n = 9$

74. **(a)** $^{2n}C_3 : {}^nC_2 = 12 : 1$

$\Rightarrow \frac{2n(2n-1)(2n-2)}{1.2.3} \div \frac{n(n-1)}{1.2} = \frac{12}{1}$

$\left[{}^nC_r = \frac{n(n-1)......(n-r+1)}{1.2.3........n} \right]$

or $\frac{2n(2n-1)2(n-1)}{6} \times \frac{2}{n(n-1)} = \frac{12}{1}$

or $\frac{4n(2n-1)(n-1)}{3} \times \frac{1}{n(n-1)} = 12$

$2n - 1 = 9, 2n = 10$ or $n = 5$

75. **(b)** $^{n+1}C_3 - {}^nC_3 = 21$

$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\Rightarrow {}^nC_2 = 21 \Rightarrow n = 7$

76. **(b)** $^{39}C_{3r-1} - {}^{39}C_{r^2} = {}^{39}C_{r^2-1} - {}^{39}C_{3r}$

$\Rightarrow {}^{39}C_{3r-1} + {}^{39}C_{3r} = {}^{39}C_{r^2-1} + {}^{39}C_{r^2}$

$\Rightarrow {}^{40}C_{3r} = {}^{40}C_{r^2}$

$\Rightarrow r^2 = 3r$ or $r^2 = 40 - 3r \Rightarrow r = 0, 3$ or $-8, 5$

3 and 5 are the values as the given equation is not defined by $r = 0$ and $r = -8$. Hence, the number of values of r is 2.

77. **(b)** $^nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$

78. **(d)** $^nC_n = \frac{n!}{n!(n-n)!} = \frac{n!}{n!0!} = \frac{n!}{n!} = 1$

79. **(c)** $^nC_0 = \frac{n!}{0!(n-0)} = \frac{n!}{0!n!} = \frac{n!}{n!} = 1$

$\therefore {}^nC_{17} = {}^{17}C_{17} = 1 \; [\because {}^nC_n = 1]$

80. **(a)** $\frac{n!}{9!(n-9)!} = \frac{n!}{8!(n-8)!}$

$\Rightarrow \frac{1!}{9 \times 8!(n-9)!} = \frac{1!}{8!(n-8)(n-9)!}$

$\Rightarrow \frac{1}{9} = \frac{1}{(n-8)} \Rightarrow 9 = n - 8$

$\Rightarrow 9 + 8 = n \Rightarrow n = 17$

$\therefore {}^nC_{17} = {}^{17}C_{17} = 1 \qquad [\because {}^nC_n = 1]$

81. **(c)** We have $^{10}C_x = {}^{10}C_{x+4}$

$\Rightarrow x + x + 4 = 10 \Rightarrow 2x = 6 \Rightarrow x = 3$

82. **(b)** We have,

$^{2n}C_3 : {}^nC_3 = 11 : 1$

$\Rightarrow \frac{^{2n}C_3}{^nC_3} = \frac{11}{1} \Rightarrow \frac{\frac{(2n)!}{(2n-3)!3!}}{\frac{n!}{(n-3)!(3!)}} = \frac{11}{1}$

$\Rightarrow \frac{(2n)!}{(2n-3)!} \times \frac{(n-3)!}{n!} = \frac{11}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!}$

$\times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = \frac{11}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{11}{1}$

$\Rightarrow \frac{4(2n-1)}{n-2} = \frac{11}{1}$

$\Rightarrow 8n-4 = 11n-22 \Rightarrow 3n = 18 \Rightarrow n = 6$

83. (b) The combination will be AB, AC, AD, BC, BD and CD.

84. (a) Given: ${}^{12}P_r = {}^{11}P_6 + 6.{}^{11}P_5$

We know that

${}^{n-1}P_r + r.{}^{n-1}P_{r-1} = r!\,{}^nC_r$

$\therefore$ ${}^{11}P_6 + 6.\ {}^{11}P_5 = 6!\ {}^{12}C_6$

$\Rightarrow$ ${}^{12}P_6 = 6!\ {}^{12}C_6$

$\therefore$ $\frac{12!}{6!} = 6!\frac{12!}{6!6!}$ which are equal

$\therefore$ r = 6

85. (b) Let

$A = {}^8C_1 - {}^8C_2 + {}^8C_3 - {}^8C_4 + {}^8C_5 - {}^8C_6 + {}^8C_7 - {}^8C_8$

$= \frac{8!}{1!7!} - \frac{8!}{2!6!} + \frac{8!}{3!5!} - \frac{8!}{4!4!} + \frac{8!}{5!3!} - \frac{8!}{6!2!} + \frac{8!}{7!1!} - \frac{8!}{0!8!}$

Note: ${}^nC_r = \frac{n!}{r!(n-r)!}$

Thus,

$A = 8 - \frac{8\times7}{2} + \frac{8\times7\times6}{3\times2} - \frac{8\times7\times6\times5}{4\times3\times2\times1}$

$+ \frac{8\times7\times6}{3\times2} - \frac{8\times7}{2} + 8 - 1$

And A = 8 – 28 + 56 – 70 + 56 – 28 + 8 – 1 = 1

ASSERTION - REASON TYPE QUESTIONS

86. (c) Number of ways of arranging four distinct objects in a line is ${}^4P_4 = 4! = 24$.

Hence, Statement II is false.

Again, when W, I, F, E are arranged in all possible ways, then number of words formed is 4! = 24 and WIFE occurs last of all as its letters are against alphabetical order.

87. (c) For the number exactly divisible by 4, then last two digits must be divisible by 4, the last two digits are viz.

12, 16, 24, 32, 36, 52, 56, 64, 72, 76

Total 10 ways. Now, the remaining two first places on the left of 4-digit numbers are to be filled from the remaining 5-digits and this can be done in ${}^5P_2$ = 20 ways.

$\therefore$ Required number of ways = 20 × 10 = 200.

88. (c) Product of n consecutive natural numbers

= (m + 1) (m + 2) (m + 3) ..... (m + n), m ∈ whole number

$= \frac{(m+n)!}{m!} = n! \times \frac{(m+n)!}{m!\ n!}$

$= n! \times {}^{m+n}C_m$

$\Rightarrow$ Product is divisible by n!, then it is always divisible by (n – 1)! but not by (n + 1)!

89. (a) Let the number of ways of distributing n identical objects among r persons such that each person gets at least one object is same as the number of ways of selecting (r – 1) places out of (n – 1) different places, i.e. ${}^{n-1}C_{r-1}$.

90. (d) Number form by using 1, 2, 3, 4, 5 = 5! = 120

Number formed by using 0, 1, 2, 4, 5

4	4	3	2	1

= 4.4.3.2.1 = 96

Total number formed, divisible by 3 (taking numbers without repetition) = 216

Statement 1 is false and statement 2 is true.

CRITICALTHINKING TYPE QUESTIONS

91. (a) Let $A = {}^nC_r + 2\,{}^nC_{r-1} + {}^nC_{r-2}$

$= \frac{n!}{r!(n-r)!} + \frac{2n!}{(r-1)!(n-r+1)!} + \frac{n!}{(r-2)!(n-r+2)!}$

$= \frac{n![(n-r+2.n-r+1)+2(n-r+2)r+r(r-1)]}{r!(n-r+2)!}$

$= \frac{\left[\begin{matrix} n![(n^2-nr+n-nr+r^2-r+2n-2r+2 \\ +2nr-2r^2+4r+r^2-r] \end{matrix}\right]}{r!(n-r+2)!}$

$= \frac{(n^2+3n+2)n!}{r!(n-r+2)!} = \frac{(n+1)(n+2)n!}{r!(n-r+2)!}$

$= \frac{(n+2)!}{r!(n+2-r)!} = {}^{n+2}C_r.$

92. (c) ${}^nC_{r+1} + {}^nC_{r-1} + 2\,{}^nC_r$

$= {}^nC_{r-1} + {}^nC_r + {}^nC_r + {}^nC_{r+1}$

$= {}^{n+1}C_r + {}^{n+1}C_{r+1} = {}^{n+2}C_{r+1}$

93. (b) To find number of line segment we will have to draw the line segments joining two points. If n is the number of such lines segments, then

$n = {}^{12}C_2 = \frac{12!}{2!(12-2)!} = \frac{12\times11\times10!}{2\times10!} = 66.$

94. (d) There are 10 questions with options of false/ true. It means each question has two options. Thus the number of ways that these questions can be answered

$= 2^{10} = 1024$ ways.

95. (c) Since we know that the total number of selections of r things from n things where each thing can be repeated as many times as one can, is ${}^{n+r-1}C_r$

Here r = 6 ($\because$ we have to select 6 coins)

and n = 3 ($\because$ it is repeated 3 times)

$\therefore$ Required number $= {}^{3+6-1}C_6 = 28$

96. (a) Let the no. of participants at the beginning was n.

Now, we have 6 games and each participant will play 2 games.

$\therefore$ Total no. of games played by 2 persons

$= 6 \times 2 = 12$

Since, two players fell ill having played 6 games each, without playing among them selves and total no. of games = 117

$\therefore \frac{n(n-1)}{2} = 117 - 12$

$\Rightarrow n(n-1) = 2(105) = 210$

$\Rightarrow n^2 - n - 210 = 0$

$\Rightarrow n^2 - 15n + 14n - 210 = 0$

$\Rightarrow n(n-15) + 14(n-15) = 0$

$\Rightarrow n = -14, 15$

But no. of participants can not be –ve

$\therefore n = 15.$

97. (a) There are 10 lions and there is no restrictions on arranging lions. They can be arranged in 10! ways. But there is a restriction in arrangements of tigers that no two tigers come together. So two tiger are to be arranged on the either side of a lion. This gives 11 places for tigers and there are 6 tigers. So, tigers can be arranged in ${}^{11}P_6$ ways.
So, total arrangemns are $10! \times {}^{11}P_6$

98. (d) In the word CORPORATION, there are 11 positions, there are 3 vowels O, A and I and they can occupy even places only (2nd 4th, 6th, 8th and 10th positions), total 5 positions : This can be done in ${}^5C_3$ ways.
There are remaining 6 positions for odd numbered places (i.e. 1, 3, 5, 7, 9, 11) and these are be occupied by 5 consonants, namely, C, R, P, T, N.
This can be done in ${}^6C_5$ ways.
Total number of ways $= {}^5C_3 \times {}^6C_5 = 7200$

99. (c) Given expression is :

$$\frac{(n+2)! + (n+1)!(n-1)!}{(n+1)!(n-1)!} = x \text{ (let)}$$

$$\Rightarrow x = \frac{(n+2)(n+1)n(n-1)! + (n+1)(n-1)!}{(n+1)(n-1)!}$$

$= (n+2)n + 1 = n^2 + 2n + 1 = (n+1)^2$

Which is a perfect square.

100. (b) $x_1\, x_2\, x_3\, x_4\, (x_5)\, x_6\, x_7\, x_8\, x_9$. Under the given situation x_5 can be 5 only. The selection for x_1, x_2, x_3, x_4 must be from 1, 2, 3, 4, so they can be arranged 4 ! ways. Again the selection of x_6, x_7, x_8, x_9 must be from 6, 7, 8, 9 so they can be arranged in 4! ways.
Desired number of ways $= (4!)(4!) = (4!)^2$

101. (c) The number will have 2 pairs and 2 different digits.
The number of selections $= {}^4C_2 \times {}^2C_2$, and for each selection, number of arrangements $= \frac{6!}{2!2!}$.

Thus, the required number $= {}^4C_2 \times {}^2C_2 \times \frac{6!}{2!2!} = 1080$

102. (a) Total number of numbers without restriction $= 2^5$
Two numbers have all the digits equal. So,
The required number $= 2^5 - 2 = 30$

103. (a) One possible arrangement = | 2 | 2 | 1 |
Three such arrangements are possible. Therefore, the number of ways $= ({}^5C_2)({}^3C_2)({}^1C_1)(3) = 90$
The other possible arrangements = | 1 | 1 | 3 |
Three such arrangements are possible.
Thus, the number of ways $= ({}^5C_1)({}^4C_1)({}^3C_3)(3) = 60$
Hence, the total number of ways = 90 + 60 = 150.

104. (d) There are three multiple choice questions, each has four possible answers. Therefore, the total number of possible answers will be $4 \times 4 \times 4 = 64$. Out of these, possible answers only one will be correct and hence the number of ways in which a student can fail to get all correct answers is $64 - 1 = 63$.

105. (a) There will be as many signals as there are ways of filling in 2 vacant places in succession by the 4 flags of different colours. The upper vacant place can be filled in 4 different ways by anyone of the 4 flags; following which, the lower vacant place can be filled in 3 different ways by anyone of the remaining 3 different flags. Hence, by the multiplication principle, the required number of signals $= 4 \times 3 = 12$.

106. (c) Evidently, (c) is correct option because we have to divide 17 into four groups each distinguishable into groups of 5, 5, 4 and 3.

107. (a) The possibilities are:
4 from Part A and 6 from Part B
or 5 from Part A and 5 from Part B
or 6 from Part A and 4 from Part B
Therefore, the required number of ways is
$= {}^6C_4 \times {}^7C_6 + {}^6C_5 \times {}^7C_5 + {}^6C_6 \times {}^7C_4$
$= 105 + 126 + 35 = 266.$

108. (b) The following are the number of possible choices:
${}^{52}C_{18} \times {}^{35}C_2$ (18 families having atmost 2 children and 2 selected from other type of families)
${}^{52}C_{19} \times {}^{35}C_1$ (19 families having atmost 2 children and 1 selected from other type of families)
${}^{52}C_{20}$ (All selected 20 families having atmost 2 children). Hence, the total number of possible choices is : $= {}^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}$

109. (d) Let us make the following cases :
Case I : Boy borrows Mathematics Part II, then he borrows Mathematics Part I also. So, the number of possible choices is ${}^6C_1 = 6$.
Case II : Boy does not borrow Mathematics Part II, then the number of possible choices is ${}^7C_3 = 35$.
Hence, the total number of possible choices is $= 35 + 6 = 41$.

110. (d) Let there were n men playing in the tournament with 2 women. According to the given condition,
${}^nC_2 - {}^nC_1 \times {}^2C_1 = 52$

$\Rightarrow \frac{n(n-1)}{2} - 2n = 52$

$\Rightarrow n^2 - n - 4n = 104$

$\Rightarrow n^2 - 5n - 104 = 0$

$\Rightarrow n = 13.$

111. (b) For one game four persons are required.
This can be done in ${}^6C_4 = 15$ ways.
Once a set of 4 persons are selected, number of games possible will be $\frac{{}^4C_2}{2} = 3$ games.

$\therefore$ Total number of possible games $= 3 \times 15 = 45$.

112. (d) The number of times the house master goes to dhaba is ${}^nC_3$. Let n be the number of children taking non-vegetarian food.
Now, ${}^nC_3 - {}^{n-1}C_2 = 84$
$\Rightarrow \frac{n(n-1)(n-2)}{6} - \frac{(n-1)(n-2)}{2} = 84$
$\Rightarrow (n-1)(n-2)\left[\frac{n}{6} - \frac{1}{2}\right] = 84$
$\Rightarrow (n-1)(n-2)(n-3) = 6 \times 6 \times 14$
$\Rightarrow (n-1)(n-2)(n-3) = 3 \times 2 \times 3 \times 2 \times 7 \times 2$
$= 7 \times 8 \times 9$
$\Rightarrow (n-1) = 9 \Rightarrow n = 10.$

113. (c) Required number
$= {}^3C_3 + {}^3C_2 \times {}^7C_1 + {}^7C_2 \times {}^3C_1$
$= 1 + 3 \times 7 + 21 \times 3 = 1 + 21 + 63 = 85.$

114. (c) Let the boxes be marked as A, B and C. We have to ensure that no box remains empty and all five balls have to put in. There will be two possibilities :
(i) Any two box containing one ball each and 3rd box containing 3 balls. Number of ways
$= A(1)\, B(1)\, C(3)$
$= {}^5C_1 \cdot {}^4C_1 \cdot {}^3C_3 = 5 \cdot 4 \cdot 1 = 20$
(ii) Any two box containing 2 balls each and third containing 1 ball, the number of ways
$= A(2)\, B(2)\, C(1) = {}^5C_2 \cdot {}^3C_2 \cdot {}^1C_1$
$= 10 \times 3 \times 1 = 30$
Since, the box containing 1 ball could be any of the three boxes A, B, C. Hence, the required number of ways = 30 × 3 = 90.
Hence, total number of ways = 60 + 90 = 150.

115. (a) For the first player, distribute the cards in ${}^{52}C_{17}$ ways. Now, out of 35 cards left, 17 cards can be put for second player in ${}^{35}C_{17}$ ways. Similarly, for third player put them in ${}^{18}C_{17}$ ways. One card for the last player can be put in ${}^1C_1$ way. Therefore, the required number of ways for the proper distribution
$= {}^{52}C_{17} \times {}^{35}C_{17} \times {}^{18}C_{17} \times {}^1C_1$
$= \frac{52!}{35!17!} \times \frac{35!}{18!17!} \times \frac{18!}{17!1!} \times 1! = \frac{52!}{(17!)^3}.$

116. (c) Total number of 3-digit numbers having at least one of their digits as 5 = Total number of 3-digit numbers – (Total number of 3-digit numbers in which 5 does not appear at all)
$= 9 \times 10 \times 10 - 8 \times 9 \times 9$
$= 900 - 648 = 252$

117. (a) Total number of 4-digit numbers = 5 × 5 × 5 × 5 = 625 (as each place can be filled by anyone of the numbers 1, 2, 3, 4 and 5)
Numbers in which no two digits are identical
= 5 × 4 × 3 × 2 = 120 (i.e. repetition not allowed)
(as 1st place can be filled in 5 different ways, 2nd place can be filled in 4 different ways and so on)
Number of 4-digits numbers in which at least 2 digits are identical = 625 – 120 = 505

118. (a) The number of words starting from A are 5! = 120
The number of words starting from I are 5! = 120
The number of words starting from KA are 4! = 24
The number of words starting from KI are 4! = 24
The number of words starting from KN are 4! = 24
The number of words starting from KRA are 3! = 6
The number of words starting from KRIA are 2! = 2
The number of words starting from KRIN are 2! = 2
The number of words starting from KRISA are 1! = 1
The number of words starting from KRISNA are 1! = 1
Hence, rank of word 'KRISNA
= 2(120) + 3(24) + 6 + 2(2) + 2(1) = 324

119. (c) The numbers between 999 and 10000 are all 4-digit numbers. The number of 4-digit numbers formed by digits 0, 2, 3, 6, 7, 8 is ${}^6P_4 = 360$.
But here those numbers are also involved which begin from 0. So, we take those numbers as three-digit numbers.
Taking initial digit 0, the number of ways to fill remaining 3 places from five digits 2, 3, 6, 7, 8 are ${}^5P_3$ = 60
So, the required numbers = 360 – 60 = 300.

120. (b) After sending 4 to one side and 3 to other side. We have to select 5 for one side and 6 for other side from remaining.
This can be done in ${}^{11}C_5 \times {}^6C_6$ ways $= {}^{11}C_5$
Now, there are 9 on each side of the long table and each can be arranged in 9! ways.
$\therefore$ Required number of ways $= {}^{11}C_5 \times 9! \times 9!$
$= {}^{11}C_6 \times (9!)^2$ $[\because {}^nC_r = {}^nC_{n-r}]$

121. (b) Total number of ways
$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$
$= 10 + 45 + 120 + 210 = 385$

122. (b) The number of choices available to him
$= {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$
$= \frac{5!}{4!\,1!} \times \frac{8!}{6!\,2!} + \frac{5!}{5!\,0!} \times \frac{8!}{5!\,3!}$
$= 5 \times \frac{8 \times 7}{2} + 1 \times \frac{8 \times 7 \times 6}{3 \times 2}$
$= 5 \times 4 \times 7 + 8 \times 7$
$= 140 + 56 = 196$

123. (a) For A, B, C to speak in order of alphabets, 3 places out of 10 may be chosen first in ${}^{10}C_3$ ways.
The remaining 7 persons can speak in 7! ways. Hence, the number of ways in which all the 10 persons can speak is ${}^{10}C_3 \cdot 7! = \frac{10!}{3!} = \frac{10!}{6}.$

124. (d) Since 2 persons can drive the car, therefore we have to select 1 from these two. This can be done in ${}^2C_1$ ways. Now from the remaining 5 persons we have to select 2 which can be done in ${}^5C_2$ ways. But the front seat and the rear seat person can interchange among themselves. Therefore, the required number of ways in which the car can be filled is ${}^5C_2 \times {}^2C_1 \times 2!$ = 20 × 2 = 40.