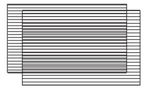
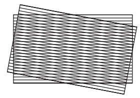
2.6 Mensuration

Moiré fringes[a]

Parallel pattern	$d_{\mathrm{M}} = \left\lvert \frac{1}{d_1} - \frac{1}{d_2} \right\rvert^{-1}$	(2.244)	d_{M} Moiré fringe spacing $d_{1,2}$ grating spacings
Rotational pattern[b]	$d_{\mathrm{M}} = \frac{d}{2\lvert\sin(\theta/2)\rvert}$	(2.245)	d common grating spacing θ relative rotation angle ($\lvert\theta\rvert \le \pi/2$)

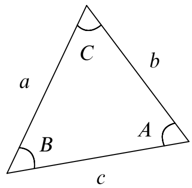




[a]From overlapping linear gratings.
[b]From identical gratings, spacing d, with a relative rotation θ.

Plane triangles

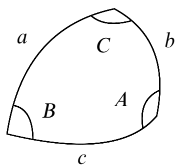
Sine formula[a]	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	(2.246)
Cosine formulas	$a^2 = b^2 + c^2 - 2bc\cos A$	(2.247)
	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$	(2.248)
	$a = b\cos C + c\cos B$	(2.249)
Tangent formula	$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$	(2.250)
Area	$\text{area} = \frac{1}{2}ab\sin C$	(2.251)
	$= \frac{a^2}{2}\frac{\sin B\sin C}{\sin A}$	(2.252)
	$= [s(s-a)(s-b)(s-c)]^{1/2}$	(2.253)
	where $s = \frac{1}{2}(a+b+c)$	(2.254)



[a]The diameter of the circumscribed circle equals $a/\sin A$.

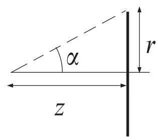
Spherical triangles[a]

Sine formula	$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$	(2.255)
Cosine formulas	$\cos a = \cos b\cos c + \sin b\sin c\cos A$	(2.256)
	$\cos A = -\cos B\cos C + \sin B\sin C\cos a$	(2.257)
Analogue formula	$\sin a\cos B = \cos b\sin c - \sin b\cos c\cos A$	(2.258)
Four-parts formula	$\cos a\cos C = \sin a\cot b - \sin C\cot B$	(2.259)
Area[b]	$E = A + B + C - \pi$	(2.260)



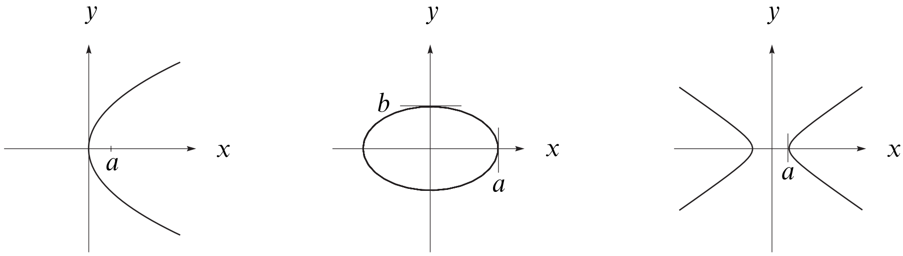
[a]On a unit sphere.
[b]Also called the "spherical excess."

Perimeter, area, and volume

Perimeter of circle	$P = 2\pi r$	(2.261)	P r	perimeter radius
Area of circle	$A = \pi r^2$	(2.262)	A	area
Surface area of sphere[a]	$A = 4\pi R^2$	(2.263)	R	sphere radius
Volume of sphere	$V = \frac{4}{3}\pi R^3$	(2.264)	V	volume
Perimeter of ellipse[b]	$P = 4a\,\mathrm{E}(\pi/2, e)$	(2.265)	a b E	semi-major axis semi-minor axis elliptic integral of the second kind (p. 45)
	$\simeq 2\pi \left(\frac{a^2+b^2}{2}\right)^{1/2}$	(2.266)	e	eccentricity $(= 1 - b^2/a^2)$
Area of ellipse	$A = \pi ab$	(2.267)		
Volume of ellipsoid[c]	$V = 4\pi \frac{abc}{3}$	(2.268)	c	third semi-axis
Surface area of cylinder	$A = 2\pi r(h + r)$	(2.269)	h	height
Volume of cylinder	$V = \pi r^2 h$	(2.270)		
Area of circular cone[d]	$A = \pi r l$	(2.271)	l	slant height
Volume of cone or pyramid	$V = A_\mathrm{b} h/3$	(2.272)	A_b	base area
Surface area of torus	$A = \pi^2 (r_1 + r_2)(r_2 - r_1)$	(2.273)	r_1 r_2	inner radius outer radius
Volume of torus	$V = \frac{\pi^2}{4}(r_2^2 - r_1^2)(r_2 - r_1)$	(2.274)		
Area[d] of spherical cap, depth d	$A = 2\pi R d$	(2.275)	d	cap depth
Volume of spherical cap, depth d	$V = \pi d^2 \left(R - \frac{d}{3}\right)$	(2.276)	Ω z α	solid angle distance from centre half-angle subtended
Solid angle of a circle from a point on its axis, z from centre	$\Omega = 2\pi \left[1 - \frac{z}{(z^2 + r^2)^{1/2}}\right]$	(2.277)		
	$= 2\pi(1 - \cos\alpha)$	(2.278)		

[a]Sphere defined by $x^2 + y^2 + z^2 = R^2$.
[b]The approximation is exact when $e = 0$ and $e \simeq 0.91$, giving a maximum error of 11% at $e = 1$.
[c]Ellipsoid defined by $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$.
[d]Curved surface only.

Conic sections



	parabola	*ellipse*	*hyperbola*
equation	$y^2=4ax$	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$
parametric form	$x=t^2/(4a)$ $y=t$	$x=a\cos t$ $y=b\sin t$	$x=\pm a\cosh t$ $y=b\sinh t$
foci	$(a,0)$	$(\pm\sqrt{a^2-b^2},0)$	$(\pm\sqrt{a^2+b^2},0)$
eccentricity	$e=1$	$e=\frac{\sqrt{a^2-b^2}}{a}$	$e=\frac{\sqrt{a^2+b^2}}{a}$
directrices	$x=-a$	$x=\pm\frac{a}{e}$	$x=\pm\frac{a}{e}$

Platonic solids[a]

solid *(faces,edges,vertices)*	*volume*	*surface area*	*circumradius*	*inradius*
tetrahedron (4,6,4)	$\frac{a^3\sqrt{2}}{12}$	$a^2\sqrt{3}$	$\frac{a\sqrt{6}}{4}$	$\frac{a\sqrt{6}}{12}$
cube (6,12,8)	a^3	$6a^2$	$\frac{a\sqrt{3}}{2}$	$\frac{a}{2}$
octahedron (8,12,6)	$\frac{a^3\sqrt{2}}{3}$	$2a^2\sqrt{3}$	$\frac{a}{\sqrt{2}}$	$\frac{a}{\sqrt{6}}$
dodecahedron (12,30,20)	$\frac{a^3(15+7\sqrt{5})}{4}$	$3a^2\sqrt{5(5+2\sqrt{5})}$	$\frac{a}{4}\sqrt{3}(1+\sqrt{5})$	$\frac{a}{4}\sqrt{\frac{50+22\sqrt{5}}{5}}$
icosahedron (20,30,12)	$\frac{5a^3(3+\sqrt{5})}{12}$	$5a^2\sqrt{3}$	$\frac{a}{4}\sqrt{2(5+\sqrt{5})}$	$\frac{a}{4}\left(\sqrt{3}+\sqrt{\frac{5}{3}}\right)$

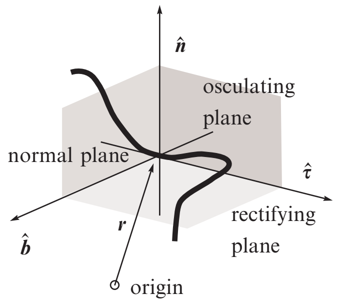
[a]Of side a. Both regular and irregular polyhedra follow the Euler relation, faces $-$ edges $+$ vertices $=2$.

Curve measure

Length of plane curve	$$l=\int_a^b\left[1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{1/2}\mathrm{d}x$$	(2.279)	a start point b end point $y(x)$ plane curve l length
Surface of revolution	$$A=2\pi\int_a^b y\left[1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{1/2}\mathrm{d}x$$	(2.280)	A surface area
Volume of revolution	$$V=\pi\int_a^b y^2\,\mathrm{d}x$$	(2.281)	V volume
Radius of curvature	$$\rho=\left[1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^{3/2}\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^{-1}$$	(2.282)	ρ radius of curvature

Differential geometry[a]

Unit tangent	$$\hat{\boldsymbol{\tau}}=\frac{\dot{\boldsymbol{r}}}{\lvert\dot{\boldsymbol{r}}\rvert}=\frac{\dot{\boldsymbol{r}}}{v}$$	(2.283)	τ tangent $\boldsymbol{r}$ curve parameterised by $\boldsymbol{r}(t)$ v $\lvert\dot{\boldsymbol{r}}(t)\rvert$
Unit principal normal	$$\hat{\boldsymbol{n}}=\frac{\ddot{\boldsymbol{r}}-\dot{v}\hat{\boldsymbol{\tau}}}{\lvert\ddot{\boldsymbol{r}}-\dot{v}\hat{\boldsymbol{\tau}}\rvert}$$	(2.284)	$\boldsymbol{n}$ principal normal
Unit binormal	$$\hat{\boldsymbol{b}}=\hat{\boldsymbol{\tau}}\times\hat{\boldsymbol{n}}$$	(2.285)	$\boldsymbol{b}$ binormal
Curvature	$$\kappa=\frac{\lvert\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}\rvert}{\lvert\dot{\boldsymbol{r}}\rvert^3}$$	(2.286)	κ curvature
Radius of curvature	$$\rho=\frac{1}{\kappa}$$	(2.287)	ρ radius of curvature
Torsion	$$\lambda=\frac{\dot{\boldsymbol{r}}\cdot(\ddot{\boldsymbol{r}}\times\dddot{\boldsymbol{r}})}{\lvert\dot{\boldsymbol{r}}\times\ddot{\boldsymbol{r}}\rvert^2}$$	(2.288)	λ torsion
Frenet's formulas	$$\dot{\hat{\boldsymbol{\tau}}}=\kappa v\hat{\boldsymbol{n}}$$	(2.289)	
	$$\dot{\hat{\boldsymbol{n}}}=-\kappa v\hat{\boldsymbol{\tau}}+\lambda v\hat{\boldsymbol{b}}$$	(2.290)	
	$$\dot{\hat{\boldsymbol{b}}}=-\lambda v\hat{\boldsymbol{n}}$$	(2.291)	



[a]For a continuous curve in three dimensions, traced by the position vector $\boldsymbol{r}(t)$.