2 TRIGONOMETRY

EXERCISE

151. If k_1 = tan 27θ – tan θ and $k_2 = \frac{\sin\theta}{\cos 3\theta} + \frac{\sin 3\theta}{\cos 9\theta} + \frac{\sin 9\theta}{\cos 27\theta}$, then

(A) $k_1 = 2k_2$ (B) $k_1 = k_2 + 4$ (C) $k_1 = k_2$ (D) none of these

152. If x = sin θ|sin θ|, y = cos θ | cos θ|, where $\frac{99\pi}{2} \le \theta \le 50\pi$, then

(A) x – y = 1 (B) x + y = –1 (C) x + y = 1 (D) x – x = 1

153. If $\frac{\sec^8\theta}{a} + \frac{\tan^8\theta}{b} = \frac{1}{a+b}$, then for every real value of $\sin^2\theta$

(A) ab ≤ 0 (B) ab ≥ 0 (C) a + b = 0 (D) none of these

154. If sin x + cos x = $\sqrt{\left(y+\frac{1}{y}\right)}$, x ∈ [0, π], then

(A) $x = \frac{\pi}{4}$, y = 1 (B) y = 0 (C) y = 2 (D) $x = \frac{3\pi}{4}$

155. The values of x between 0 and 2π which satisfy the equation sin x $\sqrt{(8\cos^2 x)}$ = 1 are in AP with common difference

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{3\pi}{8}$ (D) $\frac{5\pi}{8}$

156. If 0 < θ < 2π and $2\sin^2\theta - 5\sin\theta + 2 > 0$, then the range of θ is

(A) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$ (B) $\left(0, \frac{5\pi}{6}\right) \cup (\pi, 2\pi)$ (C) $\left(0, \frac{\pi}{6}\right) \cup (\pi, 2\pi)$ (D) none of these

157. The sum of the infinite ters of the series $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\left(\frac{4}{33}\right) + \ldots..$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

158. The greatest of tan 1, $\tan^{-1} 1$, $\sin^{-1} 1$, sin 1, cos 1, is

(A) sin 1 (B) tan 1 (C) $\tan^{-1} 1$ (D) noen of these

159. If [$\sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1} x$] = 1, where [*] denotes the greatest integer function, then x belongs to the interval

(A) [tan sin cos 1, tan sin cos sin 1] (B) (tan sin cos 1, tan sin cos sin 1)
(C) [−1, 1] (D) [sin cos tan 1, sin cos sin tan 1]

160. In a triangle ABC, $r^2 + r_1^2 + r_2^2 + r_3^2 + a^2 + b^2 + c^2$ is equal to (where r is inradius and r_1, r_2, r_3 are exradii a, b, c are the sides of ΔABC)

(A) $2R^2$ (B) $4R^2$ (C) $8R^2$ (D) $16R^2$

161. In a Δ ABC, angles A, B, C are in AP. Then $\lim_{x \to c} \frac{\sqrt{(3 - 4\sin A \sin C)}}{|A - C|}$ is

(A) 1 (B) 2 (C) 3 (D) 4

162. If r and R are respectively the radii of the inscribed and circumscribed circles of a regular polygon of n sides such that $\frac{R}{r} = \sqrt{5} - 1$, then n is equal to

(A) 5 (B) 10 (C) 6 (D) 18

163. If A = cos (cos x) + sin (cos x) the least and greatest value of A are

(A) 0 and 2 (B) −1 and 1 (C) $-\sqrt{2}$ and $\sqrt{2}$ (D) 0 and $\sqrt{2}$

164. Let n be a fixed positive integer such that $\sin \frac{\pi}{2n} + \cos \frac{\pi}{2n} = \frac{\sqrt{n}}{2}$, then

(A) n = 4 (B) n = 5 (C) n = 6 (D) none of these

165. If tan α, tan β, tan γ are the roots of the equation $x^3 - px^2 - r = 0$, then the value of $(1 + \tan^2\alpha)(1 + \tan^2\beta)(1 + \tan^2\gamma)$ is equal to

(A) $(p - r)^2$ (B) $1 + (p - r)^2$ (C) $1 - (P - r)^2$ (D) none of these

166. The number of solutions of the equation tan x + sec x = 2 cos x lying in the interval $[0, 2\pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3

167. The solution set of (2 cos x − 1) (3 + 2 cos x) = 0 in the interval $0 \le x \le 2\pi$ is

(A) $\left\{\frac{\pi}{3}\right\}$ (B) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}\right\}$ (C) $\left\{\frac{\pi}{3}, \frac{5\pi}{3}, \cos^{-1}\left(-\frac{3}{2}\right)\right\}$ (D) none of these

168. The number of solutions of the equation $\sin^5 x - \cos^5 x = \frac{1}{\cos x} - \frac{1}{\sin x}$ $(\sin x \ne \cos x)$ is

(A) 0 (B) 1 (C) infinite (D) none of these

169. The number of solutions of the equation $\cos^{-1}(1 - x) + m\cos^{-1} x = \frac{n\pi}{2}$, where $m > 0$, $n \le 0$, is

(A) 0 (B) 1 (C) 2 (D) infinite

170. $\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right) - \frac{\pi}{4}\right\}$ is equal to

(A) $\frac{5}{4}$ (B) $\frac{5}{16}$ (C) $-\frac{7}{17}$ (D) $\frac{7}{17}$

171. The value of $\sin^{-1}\left[\cot\left\{\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)}+\cos^{-1}\left(\frac{\sqrt{12}}{4}\right)+\sec^{-1}\left(\sqrt{2}\right)\right\}\right]$ is equal to

(A) 0 (B) $\pi/4$ (C) $\pi/6$ (D) $\pi/2$

172. In a triangle ABC, $2a^2 + 4b^2 + c^2 = 4ab + 2ac$, then the numerical value of cos B is equal to

(A) 0 (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{7}{8}$

173. If G is the centroid of a Δ ABC, then $GA^2 + GB^2 + GC^2$ is equal to

(A) $(a^2 + b^2 + c^2)$ (B) $\frac{1}{3}(a^2 + b^2 + c^2)$ (C) $\frac{1}{2}(a^2 + b^2 + c^2)$ (D) $\frac{1}{3}(a + b + c)^2$

174. In an isosceles triangle ABC, AB = AC. If vertical angle a is 20º, then $a^3 + b^3$ is equal to

(A) $3a^2b$ (B) $3b^2c$ (C) $3c^2a$ (D) abc

175. The least value of $\text{cosec}^2 x + 25\sec^2 x$ is

(A) 0 (B) 26 (C) 28 (D) 36

176. If θ is an acute angle and $\tan\theta = \frac{1}{\sqrt{7}}$, then the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$ is

(A) 3/4 (B) 1/2 (C) 2 (D) 5/4

177. If A + C = B, then tan A tan B tan C is equal to

(A) tan A + tan B + tan C (B) tan B – tan C – tan A
(C) tan A + tan C – tan B (D) – (tan A tan B + tan C)

178. If $1 + \sin\theta + \sin^2\theta + \ldots\ldots \infty = 4 + 2\sqrt{3}$, $0 < \theta < \pi$, $\theta \neq \frac{\pi}{2}$, then

(A) $\theta = \frac{\pi}{6}$ (B) $\theta = \frac{\pi}{3}$ (C) $\theta = \frac{\pi}{3}$ or $\frac{\pi}{6}$ (D) $\theta = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

179. The number of roots of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is

(A) 1 (B) 2 (C) 3 (D) infinite

180. The general value of θ such that $\sin 2\theta = \frac{\sqrt{3}}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$ is given by

(A) $n\pi + \frac{7\pi}{6}$, $n \in I$ (B) $n\pi \pm \frac{7\pi}{6}$, $n \in I$ (C) $2n\pi + \frac{7\pi}{6}$, $n \in I$ (D) none of these

181. The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ is equal to

(A) $\frac{\pi}{4}$ (B) $\frac{5\pi}{12}$ (C) $\frac{3\pi}{4}$ (D) $\frac{13\pi}{12}$

182. If $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is independent of x, then

(A) $x \in [1, \infty)$ (B) $x \in [-1, 1]$ (C) $x \in (-\infty, -1]$ (D) none of these

183. The principal value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$ is

(A) π (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{4\pi}{3}$

184. In a triangle ABC, (a + b + c) (b + c − a) = kbc if

(A) $k < 0$ (B) $k > 6$ (C) $0 < k < 4$ (D) $k > 4$

185. If the area of a triangle ABC is given by $\Delta = a^2 - (b - c)^2$, then $\tan\left(\frac{A}{2}\right)$ is equal to

(A) −1 (B) 0 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

186. In a Δ ABC, if $r = r_2 + r_3 - r_1$ and $\angle A > \frac{\pi}{3}$, then the range of $\frac{s}{a}$ is equal to

(A) $\left(\frac{1}{2}, 2\right)$ (B) $\left(\frac{1}{2}, \infty\right)$ (C) $\left(\frac{1}{2}, 3\right)$ (D) $(3, \infty)$

187. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x$ is equal to

(A) −1 (B) 0 (C) 1 (D) 2

188. If $0 < \alpha < \pi/6$ and $\sin\alpha + \cos\alpha = \sqrt{7}/2$, then $\tan\alpha/2$ is equal to

(A) $\frac{\sqrt{7}-2}{3}$ (B) $\frac{\sqrt{7}+2}{3}$ (C) $\frac{\sqrt{7}}{3}$ (D) none of these

189. If $\alpha,\beta,\gamma \in \left(0, \frac{\pi}{2}\right)$, then the value of $\frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma}$ is

(A) < 1 (B) > 1 (C) = 1 (D) none of these

190. Solutions of the equation $|\cos x| = 2[x]$ are (where [*] denotes the greatest integer function)

(A) nil (B) $x = \pm 1$ (C) $x = \frac{\pi}{3}$ (D) none of these

191. The set of all x in $(-\pi, \pi)$ satisfying $|4\sin x - 1| < \sqrt{5}$ is given by

(A) $x \in \left(-\frac{\pi}{10}, \pi\right)$ (B) $x \in \left(-\frac{\pi}{10}, \frac{3\pi}{10}\right)$ (C) $x \in \left(-\pi, \frac{3\pi}{10}\right)$ (D) $x \in (-\pi, \pi)$

192. The number of solutions of the equation $1 + \sin x \sin^2\left(\frac{x}{2}\right) = 0$ in $[-\pi, \pi]$ is

(A) 0 (B) 1 (C) 2 (D) 3

193. The solution of the inequality $(\cot^{-1} x)^2 - 5\cot^{-1} x + 6 > 0$ is

(A) (cot 3, cot 2) (B) $(-\infty, \cot 3) \cup (\cot 2, \infty)$
(C) $(\cot 2, \infty)$ (D) None of these

194. If in a Δ ABC, $a^2 + b^2 + c^2 = 8R^2$, where R = circumradius, the the triangle is

(A) equilateral (B) isosceles (C) right angled (D) none of these

195. In any triangle ABC, $\sum \frac{\sin^2 A + \sin A + 1}{\sin A}$ is always greater than

(A) 9 (B) 3 (C) 27 (D) none of these

196. If in a triangle ABC, $2\frac{\cos A}{a} + \frac{\cos B}{b} + 2\frac{\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$, then the value of the angle A is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

197. If in a triangle, R and r are the circumradius and inradius respectively, then the Harmonic mean of the exradii of the triangle is

(A) 3r (B) 2R (C) R + r (D) none of these

198. For what and only what values of α lying between 0 and π is the inequality $\sin\alpha \cos^3\alpha > \sin^3\alpha \cos\alpha$ valid ?

(A) $a \in (0, \pi/4)$ (B) $a \in (0, \pi/2)$ (C) $a \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$ (D) none of these

199. The minimum and maximum values of $ab \sin x + b\sqrt{(1 - a^2)} \cos x + c$ ($|a| < 1$, $b > 0$) respectively are

(A) {b – c, b + c} (B) {b + c, b – c} (C) {c – b, b + c} (D) none of these

200. If $2\cos\theta + \sin\theta = 1$, then $7\cos\theta + 6\sin\theta$ equals

(A) 1 or 2 (B) 2 or 3 (C) 2 or 4 (D) 2 or 6

201. If $\sin\alpha = \frac{336}{625}$ and $450° < \alpha < 540°$, then $\sin(\alpha/4)$ is equal to

(A) $\frac{1}{5\sqrt{2}}$ (B) $\frac{7}{25}$ (C) $\frac{4}{5}$ (D) $\frac{3}{5}$

202. Minimum value of $4x^2 - 4x|\sin\theta| - \cos^2\theta$ is equal to

(A) –2 (B) –1 (C) –1/2 (D) 0

203. If $\cos^4\theta \sec^2\alpha$, $\frac{1}{2}$ and $\sin^4\theta \operatorname{cosec}^2\alpha$ are in AP, then $\cos^8\theta \sec^6\alpha$, $\frac{1}{2}$ and $\sin^8\theta \operatorname{cosec}^6\alpha$ are in

(A) AP (B) GP (C) HP (D) none of these

204. The maximum value of the expression $\left|\sqrt{(\sin^2 x + 2a^2)} - \sqrt{(2a^2 - 1 - \cos^2 x)}\right|$, where a and x are real numbers is

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) 1 (D) $\sqrt{5}$

205. The real roots of the equation $\cos^7 x + \sin^4 x = 1$ in the interval $(-\pi, \pi)$ are

(A) $-\frac{\pi}{2}, 0$ (B) $-\frac{\pi}{2}, 0, \frac{\pi}{2}$ (C) $\frac{\pi}{2}, 0$ (D) $0, \frac{\pi}{4}, \frac{\pi}{2}$

Questions based on statements (Q. 206 - 210)

Each of the questions given below consist of Statement – I and Statement – II. Use the following Key to choose the appropriate answer.

(A) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.

(B) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.

(C) If Statement-I is true but Statement - II is false.

(D) If Statement-I is false but Statement - II is true.

206. Statement–I : If $\sin^2\theta_1 + \sin^2\theta_2 + \ldots\ldots + \sin^2\theta_n = 0$, then the different sets of values of $(\theta_1, \theta_2, \ldots, \theta_n)$ for which $\cos\theta_1 + \cos\theta_2 + \ldots + \cos\theta_n = n - 4$ is $n(n-1)$.

Statement–II : If $\sin^2\theta_1 + \sin^2\theta_2 + \ldots\ldots + \sin^2\theta_n = 0$, then $\cos\theta_1 \cdot \cos\theta_2 \cdot \cos\theta_n = \pm 1$.

207. Let α, β and γ satisfy $0 < \alpha < \beta < \gamma < 2\pi$ and $\cos(x + \alpha) + \cos(x + \beta) + \cos(x + \gamma) = 0 \;\forall\; x \in R$.

Statement–I : $\gamma - \alpha = \frac{2\pi}{3}$.

Statement–II : $\cos\alpha + \cos\alpha + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$.

208. **Statement–I :** The equation $\sin(\cos x) = \cos(\sin x)$ has no real solution.

Statement–II : $\sin x \pm \cos x \in \left[-\sqrt{2}, \sqrt{2}\right]$.

209. **Statement–I :** $\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$.

Statement–II : $\sin^{-1} x > \tan^{-1} y$ for $x > y$, $\forall\; x, y \in (0, 1)$.

210. **Statement–I :** In any ΔABC, the maximum value of $r_1 + r_2 + r_3 = 9R/2$.

Statement–II : In any ΔABC, $R \geq 2r$.

TRIGONOMETRY

151. A	**152.** D	**153.** A	**154.** A	**155.** B	**156.** A
157. B	**158.** D	**159.** A	**160.** D	**161.** A	**162.** A
163. C	**164.** C	**165.** B	**166.** C	**167.** B	**168.** A
169. A	**170.** C	**171.** A	**172.** D	**173.** B	**174.** C
175. D	**176.** A	**177.** B	**178.** D	**179.** **C**	**180.** D
181. C	**182.** A	**183.** A	**184.** C	**185.** C	**186.** A
187. C	**188.** A	**189.** A	**190.** A	**191.** B	**192.** A
193. B	**194.** C	**195.** A	**196.** C	**197.** A	**198.** A
199. C	**200.** D	**201.** A	**202.** B	**203.** A	**204.** B
205. B	**206.** D	**207.** D	**208.** A	**209.** A	**210.** A

HINTS & SOLUTIONS : TRIGONOMETRY

151. A

we have $k_1 = \tan 27\theta - \tan\theta$

$= \tan(27\theta - \tan 9\theta) + (\tan 9\theta - \tan 3\theta) + (\tan 3\theta - \tan\theta)$

Now, $\tan 3\theta - \tan\theta = \dfrac{\sin 2\theta}{\cos 3\theta \cos\theta} = \dfrac{2\sin\theta}{\cos 3\theta}$

Similarly, $\tan 9\theta - \tan 3\theta = \dfrac{2\sin 3\theta}{\cos 9\theta}$

and $\tan 27\theta - \tan 9\theta = \dfrac{2\sin 9\theta}{\cos 27\theta}$

$\therefore k_1 = 2\left[\dfrac{\sin 9\theta}{\cos 27\theta} + \dfrac{\sin 3\theta}{\cos 9\theta} + \dfrac{\sin\theta}{\cos 3\theta}\right] = 2k_2$

152. D

$\dfrac{99\pi}{2} \le \theta \le 50\pi$ is equivalent to $\dfrac{3\pi}{2} \le \theta \le 2\pi$

$\therefore x = -\sin^2\theta$ and $y = \cos^2\theta$

$\therefore y - x = 1$

153. A

$\dfrac{\sec^8\theta}{a} + \dfrac{\tan^8\theta}{b} = \dfrac{1}{a+b}$

or $a^2\sin^8\theta + ab + b^2 = ab(\cos^8\theta - \sin^8\theta)$

$= ab(1 - 2\sin^2\theta\cos^2\theta)\cos 2\theta$

$= ab(1 - 2\sin^2\theta)\left(1 - \dfrac{1}{2}\sin^2 2\theta\right)$

$\Rightarrow a^2\sin^8\theta - 2ab\sin^4\theta + b^2 = -2ab\sin^2\theta$

$+ ab\sin^2\theta\sin^2 2\theta - \dfrac{ab}{2}\sin^2 2\theta - 2ab\sin^4\theta$

$(a\sin^4\theta - b)^2 = 2ab\sin^2\theta(-\sin^2\theta - 1 + 2\sin^2\theta\cos^2\theta - \cos^2\theta)$

$\Rightarrow 2ab\sin^2\theta(-2 + 2\sin^2\theta\cos^2\theta) \ge 0$

$\Rightarrow 4ab\sin^2\theta(\sin^4\theta - \sin^2\theta + 1) \ge 0$

$\Rightarrow ab \le 0$

154. A

$\because \dfrac{y + \frac{1}{y}}{2} \ge \sqrt{y.\dfrac{1}{y}} \Rightarrow \sqrt{\left(y + \dfrac{1}{y}\right)} \ge \sqrt{2}$

But $|\sin x + \cos x| < \sqrt{2}$

which is possible only when

$y + \dfrac{1}{y} = 2 \;\therefore y = 1$ and $x = \dfrac{\pi}{4}$

155. B

we have, $\sin x\sqrt{(8\cos^2 x)} = 1$

$\Rightarrow \sin x.\, 2\sqrt{2}\,|\cos x| = 1$

or $\sin x\,|\cos x| = \dfrac{1}{2\sqrt{2}}$

Case I : when $\cos x > 0$

$\Rightarrow \sin 2x = \dfrac{1}{\sqrt{2}} = \sin\dfrac{\pi}{4}$

$\Rightarrow 2x = n\pi + (-1)^n\dfrac{\pi}{4}$

$\Rightarrow x = \dfrac{n\pi}{2} + (-1)^n\dfrac{\pi}{8}$

$\therefore x = \dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{9\pi}{8}, \dfrac{11\pi}{8}$ $(\because 0 < x < 2\pi)$

But $\cos x > 0$, $\therefore x = \dfrac{\pi}{8}, \dfrac{3\pi}{8}$

Case II : when $\cos x < 0$

$\Rightarrow \sin 2x = -\dfrac{1}{\sqrt{2}} = -\sin\dfrac{\pi}{4}$

$\therefore 2x = n\pi - (-1)^n\dfrac{\pi}{4}$

$\Rightarrow x = \dfrac{n\pi}{2} - (-1)^n\dfrac{\pi}{8}$

$\therefore x = \dfrac{5\pi}{8}, \dfrac{7\pi}{8}, \dfrac{13\pi}{8}, \dfrac{15\pi}{8}$

But $\cos x < 0$ $\therefore x = \dfrac{5\pi}{8}, \dfrac{7\pi}{8}$

Hence, the values of x satisfying the given equation which lies between 0 and 2π are

$\dfrac{\pi}{8}, \dfrac{3\pi}{8}, \dfrac{5\pi}{8}, \dfrac{7\pi}{8}$ these are in AP with

common difference $\dfrac{\pi}{4}$.

156. A

$2\sin^2\theta - 5\sin\theta + 2 > 0$

$\Rightarrow (2\sin\theta - 1)(\sin\theta - 2) > 0$

$\because \sin\theta - 2 < 0$ (always)

$\therefore 2\sin\theta - 1 < 0 \Rightarrow \sin\theta < \dfrac{1}{2}$

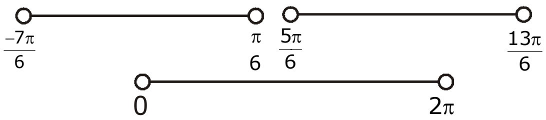
$\Rightarrow -\cos\theta > -\frac{1}{2} \Rightarrow \cos(\pi + \theta) > \cos\frac{2\pi}{3}$

or $2n\pi - \frac{2\pi}{3} < \frac{\pi}{2} + \theta < 2n\pi + \frac{2\pi}{3}$

$\Rightarrow 2n\pi - \frac{7\pi}{6} < \theta < 2n\pi + \frac{\pi}{6}$

For, n = 0, $\frac{7\pi}{6} < \theta < \frac{\pi}{6}$

For n = 1, $\frac{5\pi}{6} < \theta < \frac{13\pi}{6}$



$\therefore\ \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

Alternative Method :

$\sin\theta < \frac{1}{2} = \sin\frac{\pi}{6} = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{5\pi}{6}$

$\therefore\ \theta \in \left(0, \frac{\pi}{6}\right)$ or $\theta \in \left(\frac{5\pi}{6}, 2\pi\right)$

$\therefore\ \theta \in \left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

157. B

$\because\ T_r = \tan^{-1}\left(\frac{2^r - 1}{1 + 2^{2r-1}}\right) = \tan^{-1}\left(\frac{2^r - 2^{r-1}}{1 + 2^r . 2^{r-1}}\right)$

$= \tan^{-1}(2^r) - \tan^{-1}(2^{r-1})$

$\therefore\ S_n = \sum_{r=1}^{n} T_r = \sum_{r=1}^{n} \tan^{-1}(2^r) - \tan^{-1}(2^{r-1})$

$= \tan^{-1}(2^n) - \tan^{-1}(2^0)$

$= \tan^{-1}(2^n) - \tan^{-1}(1)$

$= \tan^{-1}(2^n) - \frac{\pi}{4}$

Hence, $S_\infty = \tan^{-1}(2^\infty) - \frac{\pi}{4}$

$= \tan^{-1}(\infty) - \frac{\pi}{4} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

158. D

$\because$ 1 Radian = 57º17′ 44.8″

$\therefore$ $\sin^{-1} 1 = \frac{\pi}{2}$ is the greatest.

159. A

We have,

$1 < \sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1} x \le \frac{\pi}{2}$

$\Rightarrow \sin 1 \le \cos^{-1}\sin^{-1}\tan^{-1} x \le 1$

$\Rightarrow \cos\sin 1 \ge \sin^{-1}\tan^{-1} x \ge \cos 1$

$\Rightarrow \sin\cos\sin 1 \ge \tan^{-1} x \ge \sin\cos 1$

$\Rightarrow \tan\sin\cos\sin 1 \ge x \ge \tan\sin\cos 1$

$\therefore\ x \in [\tan\sin\cos 1, \tan\sin\cos\sin 1]$

160. D

$\because\ (r_1 + r_2 + r_3 - r)^2$

$= r_1^2 + r_2^2 + r_3^2 + r^2$

$+ 2(r_1r_2 + r_2r_3 + r_3r_1) - 2r(r_1 + r_2 + r_3)$

$\Rightarrow (4R)^2 = r^2 + r_1^2 + r_2^2 + r_3^2 + 2s^2$

$-2\{(s-a)(s-b)+(s-b)(s-c)+(s-c)(s-a)\}$

$\Rightarrow 16R^2 = r^2 + r_1^2 + r_2^2 + r_3^2 + 2s^2$

$-2\{(3s^2 - 2s(a + b + c) + ab + bc + ca\}$

$\Rightarrow 16R^2 = r^2 + r_1^2 + r_2^2 + r_3^2 + 4s^2$

$-2(ab + bc + ca)$

$= r^2 + r_1^2 + r_2^2 + r_3^2 + (a + b + c)^2$

$- 2(ab + bc + ca)$

$= r^2 + r_1^2 + r_2^2 + r_3^2 + a^2 + b^2 + c^2$

Hence, $r^2 + r_1^2 + r_2^2 + r_3^2 + a^2 + b^2 + c^2$

$= 16R^2$

161. A

$\because$ A, B, C are in AP. $\therefore$ 2B = A + C

and A + B + C = 180° $\Rightarrow$ 3B = 180° $\therefore$ B = 60°

$\therefore \cos B = \frac{1}{2} = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow a^2 + c^2 = b^2 + ac \Rightarrow (a - c)^2 = b^2 - ac$

$\Rightarrow |a - c| = \sqrt{(b^2 - ac)}$

or $|\sin A - \sin C| = \sqrt{(\sin^2 B - \sin A \sin C)}$

$\Rightarrow 2\cos\left(\frac{A + C}{2}\right)\left|\sin\left(\frac{A - C}{2}\right)\right|$

$= \sqrt{\left(\frac{3}{4} - \sin A \sin C\right)}$

$\Rightarrow 2\left|\sin\left(\frac{A - C}{2}\right)\right| = \sqrt{\left(\frac{3}{4} - \sin A \sin C\right)}$

$\therefore\ \lim_{x \to c} \frac{\sqrt{(3 - 4\sin A \sin C)}}{|A - C|} = \lim_{x \to c} \frac{2\left|\sin\left(\frac{A - C}{2}\right)\right|}{|A - C|}$

$$= \lim_{x\to c}\left|\frac{\sin\left(\frac{A-C}{2}\right)}{\left(\frac{A-C}{2}\right)}\right| = |1| = 1$$

162. A

Let a be length of side of regular polygon,

then $R = \frac{a}{2}\operatorname{cosec}\left(\frac{\pi}{n}\right)$ and $r = \frac{a}{2}\cot\left(\frac{\pi}{n}\right)$

$$\therefore \frac{R}{r} = \frac{\operatorname{cosec}\left(\frac{\pi}{n}\right)}{\cot\left(\frac{\pi}{n}\right)} = \frac{1}{\cos\left(\frac{\pi}{n}\right)} = \sqrt{5} - 1$$

$$\therefore \cos\left(\frac{\pi}{n}\right) = \frac{1}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{4} = \cos\left(\frac{\pi}{5}\right)$$

$\therefore n = 5$

163. C

A = cos (cos x) + sin (cos x)

$$= \sqrt{2}\left\{\cos(\cos x)\cos\frac{\pi}{4} + \sin(\cos x)\sin\frac{\pi}{4}\right\}$$

$$= \sqrt{2}\left\{\cos\left(\cos x - \frac{\pi}{4}\right)\right\}$$

$$\because -1 \le \cos\left(\cos x - \frac{\pi}{4}\right) \le 1$$

$$\therefore -\sqrt{2} \le A \le \sqrt{2}$$

164. C

$$\sin\frac{\pi}{2n} + \cos\frac{\pi}{2n} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$$

$$\Rightarrow \frac{\sqrt{n}}{2} = \sqrt{2}\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right)$$

So for $n > 1$, $\frac{\sqrt{n}}{2\sqrt{2}} = \sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) > \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$ or $n > 4$

Since, $\sin\left(\frac{\pi}{4} + \frac{\pi}{2n}\right) < 1$ for all $n > 2$,

we get $\frac{\sqrt{n}}{2\sqrt{2}} < 1$ or $n < 8$

So that 4<n<8. By actual verification we find that only n = 6 satisfies the given relation.

165. B

From the given equations we have $\Sigma \tan\alpha = p$,

$\Sigma \tan\alpha\tan\beta = 0$ and $\tan\alpha\tan\beta\tan\gamma = r$

So that $(1 + \tan^2\alpha)(1 + \tan^2\beta)(1 + \tan^2\gamma)$

$= 1 + \Sigma\tan^2\alpha + \Sigma\tan^2\alpha\tan^2\beta + \tan^2\alpha\tan^2\beta\tan^2\gamma$

$= 1 + (\Sigma\tan\alpha)^2 - 2\Sigma\tan\alpha\tan\beta + (\Sigma\tan\alpha\tan\beta)^2$

$-2\tan\alpha\tan\beta\tan\gamma\Sigma\tan\alpha + \tan^2\alpha\tan^2\beta\tan^2\gamma$

$= 1 + p^2 - 2pr + r^2 = 1 + (p - r)^2$

166. C

Given, tan x + sec x = 2 cos x

Multiplying by $\cos x \ne 0$

$\therefore \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin x)(1 + \sin x)$

$\Rightarrow (\sin x + 1)(2\sin x - 1)$

$\therefore \sin x = -1$ and $\sin x = \frac{1}{2}$

$\sin x \ne -1$ $\quad(\because \cos x \ne 0)$

$\therefore \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

167. B

Since, $(2\cos x - 1)(3 + 2\cos x) = 0$

$\because \cos x \ne -\frac{3}{2}$

$\therefore \cos x = \frac{1}{2} \Rightarrow x = 2n\pi \pm \frac{\pi}{3}, n \in I$

and given $0 \le x \le 2\pi$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ (for n = 0, 1)

168. A

Given that

$$\sin^5 x - \cos^5 x = \frac{\sin x - \cos x}{\sin x\cos x}$$

$$\sin x\cos x\left[\frac{\sin^5 x - \cos^5 x}{\sin x - \cos x}\right] = 1$$

$\sin x \cos x \{\sin^4 x+\sin^3 x \cos x+\sin^2 x \cos^2 x + \sin x \cos^3 x + \cos^4 x\} = 1$

$\sin x \cos x [(\sin^4 x+\cos^4 x)+\sin^2 x \cos^2 x + \sin x \cos x (\sin^2 x + \cos^2 x)] = 1$

$\sin x \cos x \{(\sin^2 x+\cos^2 x)^2-\sin^2 x \cos^2 x + \sin x \cos x\} = 1$

$$\Rightarrow \frac{1}{2} \sin 2x \left[1-\frac{1}{4}\sin^2 2x+\frac{1}{2}\sin 2x\right] = 1$$

$\sin 2x (\sin^2 2x - 2 \sin 2x - 4) = -8$

$\sin^3 2x - 2 \sin^2 x - 4 \sin 2x + 8 = 0$

$(\sin 2x - 2)^2 (\sin 2x + 2) = 0$

$\Rightarrow \sin 2x = \pm 2$ which is impossible

169. A

For $\cos^{-1}(1 - x)$

$\Rightarrow -1 < 1 - x < 1 \Rightarrow 1 \geq -1 + x \geq -1$

or $2 \geq x \geq 0$ or $x \in [0, 2]$ .....(i)

For $\cos^{-1} x \Rightarrow -1 \leq x \leq 1$ . ....(ii)

From Eqs. (i) and (ii), we get $0 \leq x \leq 1$

$\therefore$ L.H.S. ≥ 0, but R.H.S. ≤ 0 ($\because x \leq 0$)

Equality holds if L.H.S. = 0 and R.H.S.=0

$\therefore \cos^{-1}(1 - x) + m \cos^{-1} x = 0$

$\Rightarrow \cos^{-1}(1 - x) = 0$ and $\cos^{-1} x = 0$

which is impossible i.e. no solution

Hence, number of solution = 0

170. C

$$\tan\left\{2\tan^{-1}\left(\frac{1}{5}\right)-\frac{\pi}{4}\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{2\times\frac{1}{5}}{1-\frac{1}{25}}\right)-\tan^{-1}\right\}$$

$$= \tan\left\{\tan^{-1}\left(\frac{10}{24}\right)-\tan^{-1}1\right\}$$

$$= \tan\tan^{-1}\left\{\frac{\frac{10}{24}-1}{1+\frac{10}{24}}\right\} = -\frac{14}{34} = -\frac{7}{17}$$

171. A

$$\sin^{-1}\left[\cot\left\{\sin^{-1}\sqrt{\left(\frac{2-\sqrt{3}}{4}\right)}+\cos^{-1}\left(\frac{\sqrt{12}}{4}\right)+\sec^{-1}\left(\sqrt{2}\right)\right\}\right]$$

$$= \sin^{-1}\left[\cot\left\{\sin^{-1}\sqrt{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}+\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\sec^{-1}\left(\sqrt{2}\right)\right\}\right]$$

$= \sin^{-1} [\cot \{15° + 30° + 45°\}]$

$= \sin^{-1} [\cot \pi/2] = \sin^{-1}(0) = 0$

172. D

$\because 2a^2 + 4b^2 + c^2 = 4ab + 2ac$

$\Rightarrow (a - 2b)^2 + (a - c)^2 = 0$

Which is possible only when

$a - 2b = 0$ and $a - c = 0$

or $\dfrac{a}{1} = \dfrac{b}{\frac{1}{2}} = \dfrac{c}{1} = \lambda$ (say)

$\therefore a = \lambda, b = \dfrac{\lambda}{2}, c = \lambda$

$$\therefore \cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{\lambda^2+\lambda^2-\frac{\lambda^2}{4}}{2\lambda^2}$$

$$= 1 - \frac{1}{8} = \frac{7}{8}$$

173. B

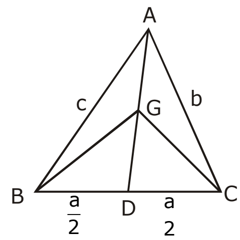
By appolloneous theorem

$(GB)^2 + (GC)^2$

$= 2 \{(GD)^2 + (DC)^2\}$

$\Rightarrow (GB)^2 + (GC)^2$

$$= 2\left\{\left(\frac{1}{2}GA\right)^2+\left(\frac{a}{2}\right)^2\right\}$$



$$\Rightarrow (GB)^2 + (GC)^2 = \frac{(GA)^2}{2} + \frac{a^2}{2} \quad ....(i)$$

Similarly,

$$(GC)^2 + (GA)^2 = \frac{(GB)^2}{2} + \frac{b^2}{2} \quad ...(ii)$$

$$\text{and } (GA)^2 + (GB)^2 = \frac{(GC)^2}{2} + \frac{c^2}{2} \quad ....(iii)$$

Adding Eqs. (i), (ii) and (iii), we get

$2 \{(GA)^2 + (GB)^2 + (GC)^2\}$

$$= \frac{1}{2}\{(GA)^2+(GB)^2+(GC)^2\}+\frac{(a^2+b^2+c^2)}{2}$$

$$\text{or } \frac{3}{2}\{(GA)^2+(GB)^2+(GC)^2\}=\frac{(a^2+b^2+c^2)}{2}$$

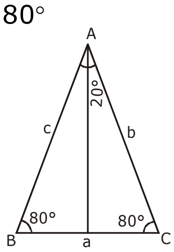
$$\Rightarrow (GA)^2 + (GB)^2 + (GC)^2 = \left(\frac{a^2+b^2+c^2}{3}\right)$$

174. **C**

$\because \angle A = 20°$ $\quad \therefore \angle B = \angle C = 80°$

Then, b = c

$\therefore \dfrac{a}{\sin 20°} = \dfrac{b}{\sin 80°} = \dfrac{c}{\sin 80°}$



or $\dfrac{a}{\sin 20°} = \dfrac{b}{\cos 10°}$

$\Rightarrow a = 2b \sin 10°$ ....(i)

$\therefore a^3 + b^3$

$= 8b^3 \sin^3 10° + b^3$

$= b^3 \{2(4 \sin^3 10°) + 1\}$

$= b^3 \{2 (3 \sin 10° - \sin 30°) + 1\}$

$= b^3 \{6 \sin 10°\} = 3b^2 (2b \sin 10°)$

$= 3b^2 a = 3 ac^2$ [from Eq. (i)] ($\because$ b = c)

175. **D**

$\text{cosec}^2 x + 25 \sec^2 x = 26 + \cot^2 x + 25 \tan^2 x$

$= 26 + 10 + (\cot x - 5 \tan x)^2 \geq 36$

176. **A**

$\because \tan\theta = \dfrac{1}{\sqrt{7}}$

$\therefore \dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \dfrac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$

$= \dfrac{(\cot^2\theta - \tan^2\theta)}{2+\tan^2\theta+\cot^2\theta}$

$= \dfrac{7-\frac{1}{7}}{2+\frac{1}{7}+7} = \dfrac{48}{14+1+49} = \dfrac{48}{64} = \dfrac{3}{4}$

177. **B**

$A + C = B \Rightarrow \tan(A + C) = \tan B$

$\Rightarrow \dfrac{\tan A + \tan C}{1 - \tan A \tan C} = \tan B$

$\Rightarrow \tan A \tan B \tan C = \tan B - \tan A - \tan C$

178. **D**

$1 + \sin\theta + \sin^2\theta + \ldots\ldots = 4 + 2\sqrt{3}$

$\dfrac{1}{1-\sin\theta} = 4 + 2\sqrt{3}$

$\Rightarrow 1 - \sin\theta = \dfrac{1}{4+2\sqrt{3}} = \dfrac{4-2\sqrt{3}}{4}$

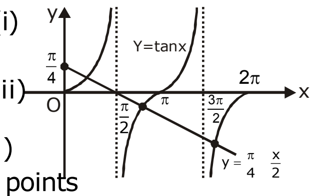
$\therefore \sin\theta = \dfrac{\sqrt{3}}{2}$ $\therefore \theta = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

179. **C**

Since, $x + 2\tan x = \dfrac{\pi}{2}$ or $\tan x = \dfrac{\pi}{4} - \dfrac{x}{2} = y$ (say)

$\therefore y = \tan x$ ...(i)

and $y = \dfrac{\pi}{4} - \dfrac{x}{2}$ ...(ii)



Graphs (i) and (ii) intersect at three points

$\therefore$ No. of solutions is 3.

180. **D**

$\sin 2\theta = \dfrac{\sqrt{3}}{2} = \sin\left(\dfrac{\pi}{3}\right), \sin\left(\pi - \dfrac{\pi}{3}\right)$

$\Rightarrow 2\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ or $\theta = \dfrac{\pi}{6}, \dfrac{\pi}{3}$ .....(i)

and $\tan\theta = \dfrac{1}{\sqrt{3}}$ $\therefore \theta = \dfrac{\pi}{6}, \pi + \dfrac{\pi}{6}$

$\theta = \dfrac{\pi}{6}, \dfrac{7\pi}{6}$ . ....(ii)

From Eqs. (i) and (ii) common value of θ is $\dfrac{\pi}{6}$

Hence, general value of θ is $2n\pi + \dfrac{\pi}{6}$, $n \in I$

181. **C**

$\tan^{-1} 1 + \cos^{-1}\left(-\dfrac{1}{2}\right) + \sin^{-1}\left(-\dfrac{1}{2}\right)$

$= \dfrac{\pi}{4} + \dfrac{2\pi}{3} - \dfrac{\pi}{6} = \dfrac{3\pi}{4}$

182. **A**

Let $x = \tan\theta$

Then, $2\tan^{-1} x + \sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

$= 2\theta + \sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right) = 2\theta + \sin^{-1}(\sin 2\theta)$

If $-\dfrac{\pi}{2} \leq 2\theta \leq \dfrac{\pi}{2}$ Then,

$2\tan^{-1} x + \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) = 2\theta + 2\theta = 4\theta$

$= 4\tan^{-1} x$

Which is not independent of x

and if $-\frac{\pi}{2} \le \pi - 2\theta \le \frac{\pi}{2}$

Then, $2\tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$= 2\theta + \sin^{-1}\sin(\pi - 2\theta) = 2\theta + \pi - 2\theta$
$= \pi$ = independent of x

$\therefore \theta \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ Principal value of

$\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \therefore \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$ Hence, $x \in [1, \infty)$

183. **A**

$\because \cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right)$

$= \frac{2\pi}{3} + \left(\pi - \frac{2\pi}{3}\right) = \pi$

184. **C**

$(a + b + c)(b + c - a) = kbc$

$\Rightarrow 2s(2s - 2a) = kbc \Rightarrow \frac{s(s-a)}{bc} = \frac{k}{4}$

$\cos^2\left(\frac{A}{2}\right) = \frac{\ }{4} \qquad \because 0 < \cos^2\left(\frac{A}{2}\right) < 1$

$\therefore 0 < \frac{k}{4} < 4 \Rightarrow 0 < k < 4$

185. **C**

$\because \Delta = a^2 - (b - c)^2 = (a + b - c)(a - b + c)$
$= 2(s - c) \,.\, 2(s - b)$

$\sqrt{s(s-a)(s-b)(s-c)} = 4(s - b)(s - c)$

$\Rightarrow \frac{1}{4} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \tan\frac{A}{2}$

$\therefore \tan\frac{A}{2} = \frac{1}{4}$

186. **A**

$r = r_2 + r_3 - r_1$

$\Rightarrow \frac{\Delta}{s} = \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s-a}$

$\Rightarrow \frac{1}{s} + \frac{1}{s-a} = \frac{1}{s-b} + \frac{1}{s-c}$

$\Rightarrow \frac{2s-a}{2s-b-c} = \frac{s(s-a)}{(s-b)(s-c)}$

$\Rightarrow \frac{2s-a}{a} = \cot^2\frac{A}{2}$

$\Rightarrow \frac{s}{a} = \frac{1}{2}\left(\cot^2\frac{A}{2} + 1\right) \Rightarrow \frac{s}{a} \in \left(\frac{1}{2}, 2\right)$

187. **C**

$\because \sin x + \sin^2 x = 1 \Rightarrow \sin x = 1 - \sin^2 x$
$\Rightarrow \sin^2 x = \cos^4 x \Rightarrow 1 - \cos^2 x = \cos^4 x$
$\Rightarrow \cos^4 x + \cos^2 x = 1$
Squaring both sides,
then $\cos^8 x + \cos^4 x + 2\cos^6 x = 1$
Hence, $\cos^8 x + 2\cos^6 x + \cos^4 x = 1$

188. **A**

$\because 0 < \alpha < \frac{\pi}{6}$

$\Rightarrow 0 < \frac{\alpha}{2} < \frac{\pi}{12}$ (in first quadrant),

then $\tan \alpha/2$ is always positive

$\because \sin\alpha + \cos\alpha = \frac{\sqrt{7}}{2}$

$\Rightarrow \frac{2\tan\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} + \frac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} = \frac{\sqrt{7}}{2}$

$\Rightarrow$

$2\left(2\tan\frac{\alpha}{2} + 1 - \tan^2\frac{\alpha}{2}\right) = \sqrt{7}\left(1 + \tan^2\frac{\alpha}{2}\right)$

$\Rightarrow (\sqrt{7} + 2)\tan^2\frac{\alpha}{2} - 4\tan\frac{\alpha}{2} + (\sqrt{7} - 2) = 0$

$\therefore \tan\frac{\alpha}{2} = \frac{4 \pm \sqrt{16 - 4(\sqrt{7}+2)(\sqrt{7}-2)}}{2(\sqrt{7}+2)}$

$= \frac{4 \pm \sqrt{16-12}}{2(\sqrt{7}+2)} = \frac{4 \pm 2}{2(\sqrt{7}+2)} = \frac{2 \pm 1}{(\sqrt{7}+2)}$

$= \frac{1}{\sqrt{7}+2}$ or $\frac{3}{\sqrt{7}+2} = \frac{\sqrt{7}-2}{3}$ or $\sqrt{7} - 2$

Hence, $\tan(\alpha/2) = \frac{\sqrt{7}-2}{3}\left(\because 0 < \left(\frac{\alpha}{2}\right) < \frac{\pi}{12}\right)$

189. A

$\because \sin(\alpha+\beta) < \sin\alpha + \sin\beta$ in $(0, \pi/2)$

$\therefore \sin(\alpha+\beta+\gamma) < \sin\alpha + \sin\beta + \sin\gamma$

$$\Rightarrow \frac{\sin(\alpha+\beta+\gamma)}{\sin\alpha+\sin\beta+\sin\gamma} < 1$$

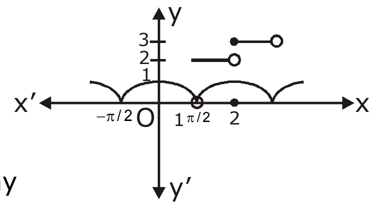
190. A

We have, $|\cos x| = 2[x] = y$ (say)

$\therefore$ $y = |\cos x|$ and

$y = 2[x]$

From graph $|\cos x|$ and $2[x]$ don't cut each other for any real value of x.



Hence, number of solution is nil.

191. B

we have, $|4\sin x - 1| < \sqrt{5}$

$$\Rightarrow -\sqrt{5} < 4\sin x - 1 < \sqrt{5}$$

$$\Rightarrow -\left(\frac{\sqrt{5}-1}{4}\right) < \sin x < \left(\frac{\sqrt{5}+1}{4}\right)$$

$$\Rightarrow -\sin\frac{\pi}{10} < \sin x < \cos\frac{\pi}{5}$$

$$\Rightarrow \sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\left(\frac{\pi}{2}-\frac{\pi}{5}\right)$$

$$\Rightarrow \sin\left(-\frac{\pi}{10}\right) < \sin x < \sin\left(\frac{3\pi}{10}\right)$$

$$x \in \left(-\frac{\pi}{10}, \frac{3\pi}{10}\right) \quad \{\because\ x \in (-\pi, \pi)\}$$

192. A

$$1 + \sin x \sin^2\frac{x}{2} = 0$$

$$1 + \sin x\left(\frac{1-\cos x}{2}\right) = 0$$

$\Rightarrow 2 + \sin x - \sin x \cos x = 0$

$\Rightarrow 4 + 2\sin x = \sin 2x$

LHS $\in [2, 6]$ but RHS $\in [-1, 1]$

Hence, no solution

i.e., Number of solutions = zero

193. B

$(\cot^{-1} x)^2 - 5\cot^{-1} x + 6 > 0$

$\Rightarrow (\cot^{-1} x - 3)(\cot^{-1} x - 2) > 0$

Then, $\cot^{-1} x < 2$ and $\cot^{-1} x > 3$

$\Rightarrow x > \cot 2$ and $x < \cot 3$

hence, $x \in (-\infty, \cot 3) \cup (\cot 2, \infty)$

194. C

Since, $a^2 + b^2 + c^2 = 8R^2$

$\Rightarrow (2R\sin A)^2 + (2R\sin B)^2 + (2R\sin C)^2 = 8R^2$

$\Rightarrow \sin^2 A + \sin^2 B + \sin^2 C = 2$

$\Rightarrow \cos^2 A - \sin^2 B + \cos^2 C = 0$

$\Rightarrow \cos(A+B)\cos(A-B) + \cos^2 C = 0$

$\Rightarrow \cos(\pi - C)\cos(A-B) + \cos^2 C = 0$

$\Rightarrow -\cos C\{\cos(A-B) - \cos C\} = 0$

$\Rightarrow -\cos C\{\cos(A-B) + \cos(A+B)\} = 0$

$\Rightarrow -2\cos A\cos B\cos C = 0$

$\therefore \cos A = 0$ or $\cos B = 0$ or $\cos C = 0$

$$\Rightarrow A = \frac{\pi}{2} \text{ or } B = \frac{\pi}{2} \text{ or } C = \frac{\pi}{2}$$

195. A

$$\frac{\sin^2 A + \sin A + 1}{\sin A} = \sin A + \frac{1}{\sin A} + 1$$

$$= \left(\sqrt{\sin A} - \frac{1}{\sqrt{\sin A}}\right)^2 + 3 \geq 3$$

Minimum value of $\dfrac{\sin^2 A + \sin A + 1}{\sin A} = 3$

$\therefore$ Minimum value of $\Sigma \dfrac{\sin^2 A + \sin A + 1}{\sin A}$

$= 3 + 3 + 3 = 9$

196. C

We have,

$$2\frac{\cos A}{a} + \frac{\cos B}{b} + 2\frac{\cos C}{c} = \frac{a}{bc} + \frac{b}{ca}$$

$$\Rightarrow 2\left(\frac{b^2+c^2-a^2}{2abc}\right) + \left(\frac{c^2+a^2-b^2}{2abc}\right)$$

$$+ 2\left(\frac{a^2+b^2-c^2}{2abc}\right) = \frac{a^2+b^2}{abc}$$

$$\Rightarrow b^2 + c^2 = a^2 \Rightarrow \angle A = \frac{\pi}{2}$$

197. A

HM of exradii = $\frac{1}{\frac{\Sigma\frac{1}{r_1}}{3}} = \frac{3}{\frac{1}{r}} = 3r$

198. A

We have, $\sin\alpha\cos^3\alpha > \sin^3\alpha\cos\alpha$
$\Rightarrow \sin\alpha\cos\alpha(\cos^2\alpha - \sin^2\alpha) > 0$
$\Rightarrow \cos^3\alpha\sin\alpha(1 - \tan^2\alpha) > 0$
$(\because \sin\alpha > 0 \text{ for } 0 < \alpha < \pi)$
$\Rightarrow \cos\alpha(1 - \tan^2\alpha) > 0$
$\Rightarrow \cos\alpha > 0$ and $1 - \tan^2\alpha > 0$
or $\cos\alpha < 0$ and $1 - \tan^2\alpha < 0$

$\Rightarrow \alpha \in (0, \pi/4)$ or $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$

199. C

$ab\sin x + b\sqrt{(1-a^2)}\cos x$

Now, $\sqrt{(ab)^2 + (b\sqrt{(1-a^2)})^2}$

$= \sqrt{a^2b^2 + b^2(1-a^2)} = b\sqrt{(a^2+1-a^2)} = b$

$\Rightarrow b\{(a\sin x + \sqrt{(1-a^2)}\cos x)\}$

Let $a = \cos\alpha$,

$\therefore \sqrt{(1-a^2)} = \sin\alpha \Rightarrow b\sin(x + a)$

$\because -1 \le \sin(x+\alpha) \le 1$
$\therefore c - b \le b\sin(x+\alpha) + c \le b + c$
$\therefore b\sin(x+\alpha) + c \in [c - b, c + b]$

200. D

$\because 2\cos\theta + \sin\theta = 1$

$\therefore \frac{2(1-\tan^2\theta/2)}{(1+\tan^2\theta/2)} + \frac{(2\tan\theta/2)}{(1+\tan^2\theta/2)} = 1$

$\Rightarrow 2 - 2\tan^2\theta/2 + 2\tan\theta/2 = 1 + \tan^2\theta/2$
$\Rightarrow 3\tan^2(\theta/2) - 2\tan(\theta/2) - 1 = 0$

$\therefore \tan\theta/2 = 1, \tan\theta/2 = -\frac{1}{3}$

$\Rightarrow \theta = 90°, \tan\theta/2 = -\frac{1}{3}$

$\therefore 7\cos\theta + 6\sin\theta$ at $\theta = 90°$ is $0+6 = 6$
and $7\cos\theta + 6\sin\theta$

$= \frac{7(1-\tan^2\theta/2)}{(1+\tan^2\theta/2)} + \frac{12\tan\theta/2}{(1+\tan^2\theta/2)}$

$= \frac{7\left(1-\frac{1}{9}\right)+12\left(-\frac{1}{3}\right)}{1+\frac{1}{9}} = 2 \left(\because \tan\theta/2 = -\frac{1}{3}\right)$

201. A

$\because \sin\alpha = \frac{336}{625}$ and $450° < \alpha < 540°$

$\therefore \sin(\alpha/4) = \sqrt{\frac{1-\cos(\alpha/2)}{2}}$

$= \sqrt{\frac{1}{2}\left\{1-\sqrt{\left(\frac{1+\cos\alpha}{2}\right)}\right\}} = \sqrt{\frac{1}{2}\left\{1-\sqrt{\frac{1+\frac{527}{625}}{2}}\right\}}$

$= \sqrt{\frac{1}{2}\left\{1-\frac{24}{25}\right\}} = \sqrt{\frac{1}{50}} = \frac{1}{5\sqrt{2}}$

202. B

$4x^2 - 4x|\sin\theta| - (1 - \sin^2\theta)$
$= -1 + (2x - |\sin\theta|)^2 \therefore$ Minimum value $= -1$

203. A

$\because \cos^4\theta\sec^2\alpha, \frac{1}{2}$ & $\sin^4\theta\,\text{cosec}^2\alpha$ are in AP

$1 = \cos^4\theta\sec^2\alpha + \sin^4\theta\,\text{cosec}^2\alpha$

$\Rightarrow 1 = \frac{\cos^4\theta}{\cos^2\alpha} + \frac{\sin^4\theta}{\sin^2\alpha}$

$\Rightarrow (\sin^2\theta + \cos^2\theta)^2 = \frac{\cos^4\theta}{\cos^2\alpha} + \frac{\sin^4\theta}{\sin^2\alpha}$

$\Rightarrow \cos^4\theta\left(\frac{1}{\cos^2\alpha} - 1\right) + \sin^4\theta\left(\frac{1}{\sin^2\alpha} - 1\right)$
$- 2\sin^2\theta\cos^2\theta = 0$

$\Rightarrow \sin^4\alpha\cos^4\theta + \sin^4\theta\cos^4\alpha$
$- 2\sin^2\theta\cos^2\theta\sin^2\alpha\cos^2\alpha = 0$
$\Rightarrow (\sin^2\alpha\cos^2\theta - \cos^2\alpha\sin^2\theta)^2 = 0$
$\Rightarrow \tan^2\theta = \tan^2\alpha \quad \therefore \theta = n\pi \pm \alpha, n \in I$

Now, $\cos^8\theta\sec^6\alpha = \cos^8\alpha\sec^6\alpha = \cos^2\alpha$
and $\sin^8\theta\,\text{cosec}^6\alpha = \sin^8\alpha.\,\text{cosec}^6\alpha = \sin^2\alpha$

Hence, $\cos^8\theta\sec^6\alpha, \frac{1}{2}, \sin^8\theta, \text{cosec}^6\alpha$

ie, $\cos^2\alpha, \frac{1}{2}, \sin^2\alpha$ are in AP.

204. B

For maximum value $2a^2 - 1 - \cos^2 x = 0$

$\therefore \cos^2 x = 2a^2 - 1$

$\Rightarrow \sin^2 x = 1 - \cos^2 x = (2 - 2a^2)$

$\Rightarrow 2a^2 + \sin^2 x = 2$

$\therefore$ Maximum value of

$$\left| \sqrt{(\sin^2 x + 2a^2)} - \sqrt{(2a^2 - 1 - \cos^2 x)} \right|$$

$$= |\sqrt{2} - 0| = \sqrt{2}$$

205. B

$\because \quad \cos^7 x \le \cos^2 x \quad .....(i)$

and $\quad \sin^4 x \le \sin^2 x \quad .....(ii)$

Adding Eqs. (i) and (ii), then

$\cos^7 x + \sin^4 x \le 1$

But given $\cos^7 x + \sin^4 x = 1$

Equality holds only, if

$\cos^7 x = \cos^2 x$ and $\sin^4 x = \sin^2 x$

Both are satisfied by $x = \pm \frac{\pi}{2}, 0$.

206. D

$\sin^2 \theta_1 + \sin^2 \theta_2 + ...+ \sin^2 \theta_n = 0$

$\Rightarrow \sin \theta_1 = \sin \theta_2 = ... = \sin \theta_n = 0$

$\Rightarrow \cos^2 \theta_1, \cos^2 \theta_2, . ., \cos^2 \theta_n = 1$

$\Rightarrow \cos \theta_1, \cos \theta_2 . ..., \cos \theta_n = \pm 1$

Now $\cos \theta_1 + \cos \theta_2 +$.

.........................$+ \cos \theta_n = n-4$

means two of $\cos \theta_1, \cos \theta_2$,.

.................................., $\cos \theta_n$

must be –1 and the other are 1. Now two values from $\cos \theta_1, \cos \theta_2$.

............................., $\cos \theta_n$ can $\frac{n(n-1)}{}$.

Hence, statement 1 is false,

but statement 2 is correct.

207. D

Given $\cos x \sum \cos \alpha - \sin x \sum \sin \alpha = 0, \forall x \in R$

Hence, $\cos \alpha + \cos \beta + \cos \gamma = 0$

and $\sin \alpha + \sin \beta + \sin \gamma = 0$

Hence, statement 2 is true.

Now $(\cos \alpha + \cos \beta)^2 = (-\cos \gamma)^2$

and $(\sin \alpha + \sin \beta)^2 = (-\sin \gamma)^2$

Adding, we get

$2 + 2\cos(\alpha-\beta) = 1 \Rightarrow \cos(\alpha-\beta) = -1/2$

Similarly, $\cos(\beta - \gamma) = -1/2$ and

$\cos(\gamma - \alpha) = -1/2$

Now $0 < \alpha < \beta < \gamma < 2\pi \Rightarrow \beta - \alpha < \gamma - \alpha$

Hence, $\beta - \alpha = \frac{2\pi}{3}$ and $\gamma - \alpha = \frac{4\pi}{3}$

Statement 1 is false.

208. A

$\cos(\sin x) = \sin(\cos x)$

$\Rightarrow \cos(\sin x) = \cos[(\pi/2) - \cos x]$

$\Rightarrow \sin x = 2n\pi \pm (\pi/2 - \cos x), n \in Z$

Taking + ve sign, we get

$\sin x = 2n\pi + \pi/2 - \cos x$

or $(\cos x + \sin x) = \frac{1}{2}(4n + 1)\pi$

Now L.H.S. $\in [-\sqrt{2}, \sqrt{2}]$,

hence $-\sqrt{2} \le \frac{1}{2}(4n+1)\pi \le \sqrt{2}$.

$$\Rightarrow \frac{-2\sqrt{2} - \pi}{4\pi} \le n \le \frac{2\sqrt{2} - \pi}{4\pi},$$

which is not satisfied by any integer n.

Similary, taking –ve sign, we have

$\sin x - \cos x = (4n - 1)\pi/2$, which is also not satisfied for any integer n. Hence, there is no solution.

209. A

$$\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}} > \tan^{-1} x > \tan^{-1} y$$

$$\left[\because\ x > y, \frac{x}{\sqrt{1-x^2}} > x\right]$$

Therefore, statement 2 is true.

Now, $e < \pi \Rightarrow \frac{1}{\sqrt{e}} > \frac{1}{\sqrt{\pi}}$

By statement 2, we have

$$\sin^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{e}}\right) > \tan^{-1}\left(\frac{1}{\sqrt{\pi}}\right)$$

Therefore, statement 1 is true.

210. A

In any ΔABC, we have $r_1 + r_2 + r_3$

$= 4R + r \le 9(R/2)$