Chapter 2

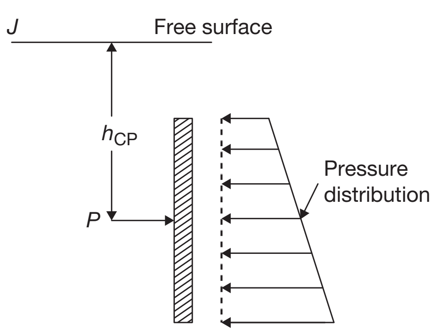
Fluid Pressure and Buoyancy

CHAPTER HIGHLIGHTS

- *Fluid pressure*
- *Hydrostatic forces on a submerged inclined plane surface*
- *Buoyancy*

Fluid Pressure

A surface exposed to a static fluid will be subjected to a distribution of fluid pressure over the exposed area; the pressure distribution is called *hydrostatic pressure distribution.* The hydrostatic pressure distribution gives rise to a system of hydrostatic forces that act on the surface's exposed area. The determination of the hydrostatic forces along with their locations are important in the design of structures such as storage tanks, dams, ships, etc., which have surfaces exposed to fluids at rest.



P is total pressure (N)
h_{CP} is centre of pressure (m)

Total pressure:

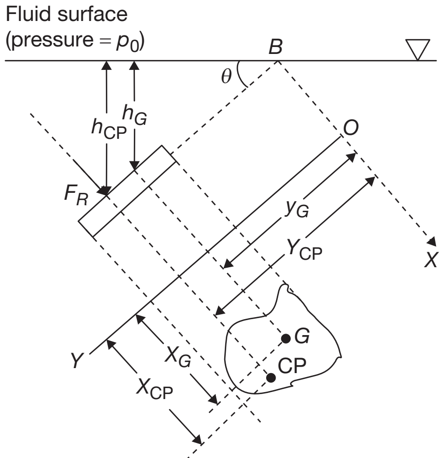
Total pressure is defined as the force exerted by a static fluid on a surface that is in contact with the fluid. The total pressure, i.e., the resultant hydrostatic force, always acts normal to the surface.

Center of pressure:

The point of intersection of the line of action of the resultant hydrostatic force and the corresponding surface is called *centre of pressure*. Centre of pressure is also defined as the point of application of the total pressure on the corresponding surface.

Hydrostatic Forces on a Submerged Inclined Plane Surface

Consider the top plane surface of an inclined plate (of arbitrary shape) submerged in a fluid as illustrated in the figure below, where the fluid is assumed to have a constant density ρ and the area of the top surface exposed to the fluid is A.



The plane, containing the top surface, intersects the fluid surface (having a pressure P_0) at B making an angle θ with it. The orthogonal X and Y axes with the origin are defined such that the top surface lies in the XY-plane generated by the axes and the X-axis lies in the intersection of the fluid surface and the plane containing the top surface.

The points $G(x_G, y_G)$ and $CP(x_{CP}, y_{CP})$ represent the centroid of the top surface area exposed to the fluid and the centre of pressure respectively. If F_R is the resultant hydrostatic force or the total pressure, then

$$F_R = (p_0 + \rho g h_G)\,A$$
$$= (p_0 + \rho g y_G \sin\theta)\,A$$

Where, $h_G (= y_G \sin\theta)$ is the vertical distance of the centroid G from the fluid surface. 'p_0' being the pressure acting at the fluid surface usually atmospheric pressure. The coordinates of the centre of pressure are given by

$$X_{CP} = X_G + \frac{I_{XYG}}{\left(Y_G + \dfrac{p_0}{\rho g \sin\theta}\right)A}$$

$$Y_{CP} = Y_G + \frac{I \times G}{\left(Y_G + \dfrac{p_0}{pg \sin\theta}\right)A}$$

Where, I_{XYG} is the product of inertia with respect to an orthogonal coordinate system passing through the centroid G and formed by a translation of the x-y coordinate system and I_{XG} is the moment of inertia about an axis passing through the centroid G and parallel to the X axis.

$$I_{XYG} = I_{XY} - AX_G Y_G$$

$$I_{XY} = \int_A^{xy} dA$$

Where, I_{XY} is the product of inertia with respect to the X- and Y-axes and I_X is the moment of inertia with respect to the X-axis.

NOTE

The inertia terms I_{XYG}, I_{XG}, I_{XY} and I_X are defined with respect to the area of the surface that is exposed to the fluid.

The centre of pressure is generally expressed only in terms of its vertical distance (h_{CP}) from the fluid surface, where

$$H_{CP} = y_{CP} \sin\theta$$

$$= Y_G \sin\theta + \frac{I_{XG}\sin\theta}{\left(Y_G + \dfrac{p_0}{\rho g \sin\theta}\right)A}$$

$$= h_G + \frac{I_{XG}\sin^2\theta}{\left(h_G + \dfrac{p_0}{\rho g}\right)A}$$

NOTE

If the fluid surface pressure P_0 also acts at the bottom surface of the inclined plate. Then the variable P_0 can be ignored (i.e., P_0 can be set to zero) in the equation for total pressure and centre of pressure to yield the following equations:

$$F_R = \rho g h_G A = \rho g y_G \sin\theta A$$

$$X_{CP} = X_G + \frac{I_{XYG}}{Y_G A}$$

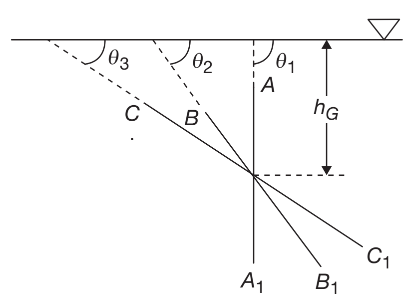
$$Y_{CP} = Y_G + \frac{I_{XG}}{Y_G A}$$

$$h_{CP} = Y_G \sin\theta + \frac{I_{XG}\sin\theta}{Y_G A} = h_G + \frac{I_{XG}\sin^2\theta}{h_G A}$$

NOTE

Consider an inclined plate in different orientations, i.e., along AA_1', BB_1' and CC_1' as shown in the following figure, such that the centre of gravity depth h_G is the same for all the orientations.

For all the orientations, the total pressure acting at the top surface remains the same, i.e.,

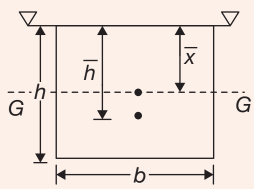
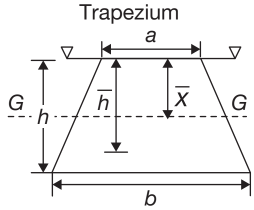
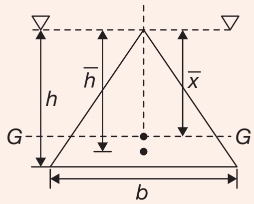
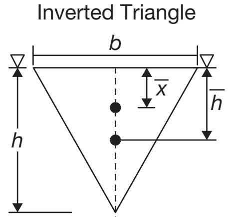
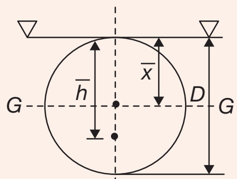


For an inclined plate, the total pressure does not depend on the angle of inclination θ as long as the depth of centre of gravity h_G does not change.

NOTE

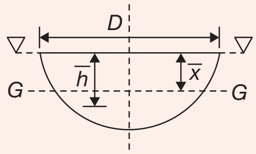
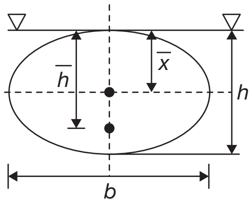
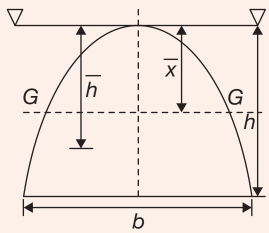
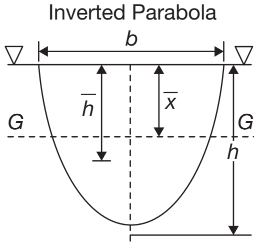
As the inclined plate is submerged deeper and deeper from the fluid surface, the distance between the centre of pressure and the centre of gravity decreases hyperbolically. At very large depths, for practical calculations, the centre of pressure and the centre of gravity are the same. It should be however noted that the centre of pressure can coincide or to be below the centre of gravity but can never be above it.

Geometric Properties of Some Important Plane Surfaces

Plane Surface	CG From Free Surface ($\bar{x}$)	CP From Free Surface ($\bar{h}$)	Area	Moment of Inertia About an Axis Passing Through CG and Parallel to Free Surface (I_G)
Rectangle 	$\frac{h}{2}$	$\frac{2h}{3}$	bh	$\frac{bh^3}{12}$
Trapezium 	$\left(\frac{a+2b}{a+b}\right)\frac{h}{3}$	$\left(\frac{a+3b}{a+2b}\right)\frac{h}{2}$	$\frac{(a+b)}{2}h$	$\frac{(a^2+4ab+b^2)}{(a+b)}\frac{h^3}{36}$
Triangle 	$\frac{2h}{3}$	$\frac{3h}{4}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$
Inverted Triangle 	$\frac{h}{3}$	$\frac{h}{2}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$
Circle 	$\frac{D}{2}$	$\frac{5D}{8}$	$\frac{\pi D^2}{4}$	$\frac{\pi D^4}{64}$

(continued)

(continued)

Plane Surface	CG From Free Surface ($\bar{x}$)	CP From Free Surface ($\bar{h}$)	Area	Moment of Inertia About an Axis Passing Through CG and Parallel to Free Surface (I_G)
Semi-circle 	$\frac{2D}{3\pi}$	$\frac{3\pi D}{32}$	$\frac{\pi D^2}{8}$	$0.1098\left(\frac{D}{2}\right)^4 = 0.11R^4$ $\left(R = \frac{D}{2}\right)$
Ellipse 	$\frac{h}{2}$	$\frac{5h}{8}$	$\frac{\pi bh}{4}$	$\frac{\pi bh^3}{64}$
Parabola 	$\frac{3}{5}h$	$\frac{5h}{7}$	$\frac{2}{3}bh$	$\left(\frac{8}{175}\right)bh^3$
Inverted Parabola 	$\frac{2}{5}h$	$\frac{4h}{7}$	$\frac{2}{3}bh$	$\left(\frac{8}{175}\right)bh^3$

SOLVED EXAMPLES

Direction for solved examples 1 and 2:

A rectangular flat thin plate (length = 4 m, breadth = 2 m) is submerged in water such that the plate's largest and smallest depths from the water surface are 5 m ad 3 m respectively.

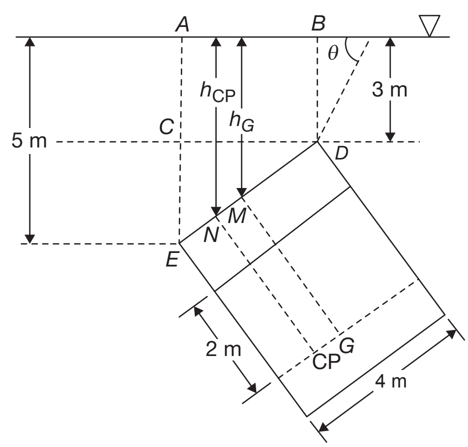
The breadth of the plate is parallel to the water surface

Example 1

The total pressure force on the top surface of the plate is

(A) 1125 kN
(B) 314 kN
(C) 141 kN
(D) 40 kN

Solution



Area of the plate, $A = 4 \times 2 = 8\ m^2$

$$\sin\theta = \frac{CE}{ED} = \frac{5-3}{4} = 0.5$$

Vertical distance of the centroid G from the water surface, $h_G = BD + MD\sin\theta = 3 + 2 \times 0.5 = 4$ m

Since the fluid surface pressure p_0 will act at the top and bottom surfaces, total pressure force, $F_R = \rho g h_G A$ (ρ is density of water = 1000 kg/m^3)

$= 1000 \times 9.81 \times 4 \times 8$

$= 313.92$ kN

Hence, the correct answer is option (B).

Example 2

The vertical distance of the centre of pressure, on the top surface of the plate, from the water surface is

(A) 3.917 m (B) 4.333 m
(C) 4 m (D) 4.083 m

Solution

The vertical distance of the centre of manure from the water surface,

$$h_{CP} = h_G + \frac{I_{xG}\sin^2\theta}{h_G A}$$

$$\text{Moment of inertia, } I_{XG} = \frac{b \times l^3}{12}$$

(b is breadth, l is length)

$$= \frac{2 \times 4^3}{12} = 10.667\ m^4$$

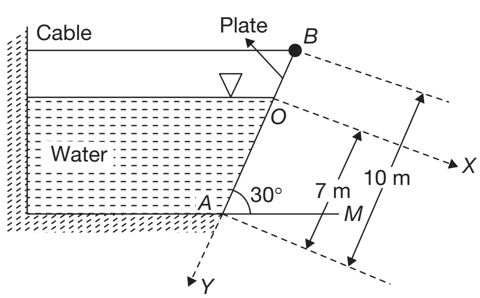
$$\therefore h_{CP} = 4 + \frac{10.667 \times (0.5)^2}{4 \times 8}$$

$$= 4.083 \text{ m.}$$

Hence, the correct answer is option (D).

Example 3

On a homogenous rectangular plate of weight 7.5 kN, length 10 m, width 5 m, and hanged at point A, a body of water acts as shown in the figure



The plate is held in place, inclined at an angle of 30° to the horizontal, by a horizontal flexible cable of negligible weight attached to the plate at point B. If the friction in the hinge is negligible, then the tension in the cable is

(A) 286.5 kN (B) 847.7 kN
(C) 280 kN (D) 284.55 kN

Solution

Vertical distance of the centroid of the plate surface area exposed to the fluid, $h_G = \dfrac{l_e \sin 30°}{2}$.

Where, l_e (= 7 m) is the length of the plate enposed to the fluid.

$$\therefore h_G = 1.75 \text{ m}$$

Area of the plate exposed to the fluid,

$$A = l_e \times w \ (w \text{ is the width})$$

$$= 7 \times 5 = 35\ m^2$$

Since the fluid surface pressure p_0(= atmospheric pressure) acts on both the surface of the plate,

Total pressure force, $F_R = h_G\ \rho g A$ (ρ is the density of water = 1000 Kg/M^3)

$$= 1.75 \times 1000 \times 9.81 \times 35$$

$$= 600862.5 \text{ N}$$

y-coordinate of the centroid of the plate surface area exposed to the fluid, $y_G = 3.5$ m

$$\text{Moment of inertia, } I \times G = \frac{wl_e^3}{12}$$

$$= \frac{1}{12} \times 5 \times 7^3 = 142.92\ m^4$$

y-coordinate of the centre of pressure,

$$Y_{CP} = y_G + \frac{I_{XG}}{y_G A}$$

$$= 3.5 + \frac{142.92}{3.5 \times 35} = 4.67 \text{ m}$$

Taking moment of the forces about the point A for equilibrium, we have

$$T \times l \times \sin 30°$$

$$= W \times \frac{l \times \cos 30°}{2} + F_R \times (l_e - y_{CP})$$

Where, T is the tension in the cable, W(= 7.5 kN) and l (= 10 m) are the weight and length of the plate

$$\therefore T \times 10 \times \sin 30°$$

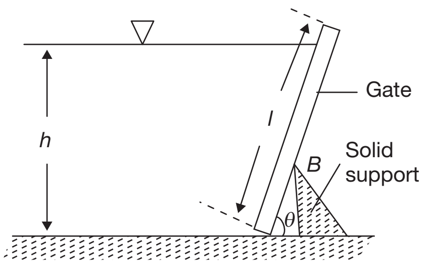
$$= \frac{7.5 \times 10^3 \times 10 \times \cos 30°}{2} + 600862.5 \times (7 - 4.67)$$

$$\therefore T = 286.5 \text{ kN.}$$

Hence, the correct answer is option (A).

Example 4

A rectangular gate, of length l metres, width w metres and negligible weight, is inclined at an angle θ to the horizontal and supports a water body of height h metres as shown in the figure



The height of the water body is such that the gate tips about the point B at which it is hinged against a solid support. If the vertical distance of the centre of pressure, on the rectangular gate, from the water surface is h_{CP}, then the ratio $\dfrac{h}{h_{CP}}$ is equal to

(A) $\dfrac{2}{3}$ (B) $\dfrac{2\sin\theta}{3}$

(C) $\dfrac{\sin\theta}{3}$ (D) $\dfrac{1}{3}$

Solution

At water body height h, the gate just tips about the point B, i.e., the total pressure force is acting at point B.

Area of the gate exposed to water,

$$A = w \times \frac{h}{\sin\theta}$$

Vertical distance of the centroid, of the area exposed to water, from the water surface,

$$h_G = \frac{h}{2}$$

Since the fluid surface pressure P_0 (= atmospheric pressure) acts on the top and bottom surface of the gate, vertical distance of the centre of pressure from the water surface,

$$h_{CP} = h_G + \frac{I_{XG}\sin^2\theta}{Ah_G}$$

$$\text{Moment of inertia, } I_{XG} = \frac{\omega \times \left(\dfrac{h}{\sin\theta}\right)^3}{12}$$

$$\therefore h_{CP} = \frac{h}{2} + \frac{\omega h^3 \times \sin^2\theta \times \sin\theta}{12\sin^3\theta \times \omega \times h \times h} \times 2$$

$$= \frac{2h}{3}$$

$$\therefore \frac{h_{CP}}{h_G} = \frac{2}{3}.$$

Hence, the correct answer is option (A).

Hydrostatic Forces on a Submerged Vertical Plane Surface

The total pressure and centre of pressure equations are

$$\boxed{F_R = (\rho_0 + \rho g h_G)A}$$

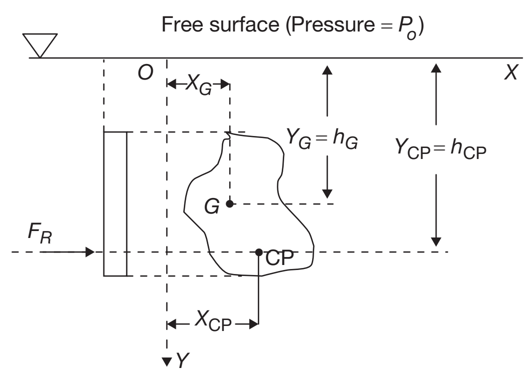
$$\boxed{X_{CP} = X_G + \frac{I_{XYG}}{\left(h_G + \dfrac{P_0}{\rho g}\right)A}}$$

$$\boxed{h_{CP} = h_G + \frac{I_{XG}}{\left(h_G + \dfrac{P_0}{\rho g}\right)A}}$$

Here also, the fluid surface P_0 can be set to zero in the above equations if it acts on both sides of the submerged vertical plane surface.

Example 5

For a horizontal surface of negligible thickness submerged in a fluid of constant density, let points G and CP represent the centroid and centre of pressure of the area exposed to the fluid. Then, which one of the following statement is ONLY correct about the points G and CP?



(A) Point G is always above point CP.
(B) Point CP is always above point G.
(C) Points G and CP will always coincide
(D) Points G and CP may coincide.

Solution

Considering that the fluid surface pressure P_0 is acting on both the sides of the submerged surface, we have

$$h_{CP} = h_G + \frac{I_{XG}\sin^2\theta}{h_G A}$$
$$(0 < \theta \le 90°)$$

Since $\frac{I_{XG}\sin^2\theta}{h_G A}$ is always greater than zero, h_{CP} is always greater than h_G, i.e., point G is always above point CP.

However, one would find the points G and CP coinciding for non-horizontal surfaces when one considers the fluid contacting the surface to be a gas at constant pressure. Hence, the correct answer is option (A).

Example 6

An equilateral triangular thin plate of side b metres in length is immersed vertically in a liquid such that one side of the plate coincides with the free surface of the liquid. The vertical distance between the centre of pressure on a surface of the plate from the corner of the plate, that away from the free surface, is h metres. The ratio h: b is equal to

(A) $\sqrt{3}:4$ (B) $1:2$

(C) $13\sqrt{3}:36$ (D) $\sqrt{3}:12$

Solution

Since the fluid surface pressure P_0 acts on both sides of the immersed surface,

$$h_{CP} = h_G + \frac{I_{XG}}{h_{GA}} \tag{1}$$

Here, $$h_G = \frac{b\sin 60°}{3}$$

$$A = \frac{1}{2}\times b\times b\sin 60° \text{ and}$$

$$I_{XG} = \frac{b\times(b\sin 60°)^3}{36} \tag{2}$$

Now, $h_{CP} + h = b\sin 60°$

That is, $$h_{CP} = b\sin 60° - h \tag{3}$$

From Eqs. (1), (2) and (3), one could write

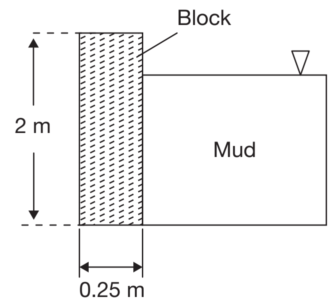
$$b\sin 60° - h$$
$$= \frac{b\sin 60°}{3} + \frac{b\times b^3\times\sin^3 60^0\times 3\times 2}{36\times b\times\sin 60°\times b\times b\sin 60°}$$

i.e., $\frac{h}{b} = \frac{\sqrt{3}}{4}$.

Hence, the correct answer is option (A).

Example 7

A concrete block 2 m high, 0.25 m wide and 1 m long, is used for holding mud at one side of the block as shown in the figure.



The density of concrete is 2600 kg/m^3 while the density of the mud is 1700 kg/m^3. If the coefficient of friction between the ground and the concrete block is 0.4, then the mud height at which the block will start to slide is

(A) 1.237 m (B) 5 m

(C) 0.782 m (D) 0.553 m

Solution

Weight of the concrete block,

$$W = (1\times 0.25\times 2)\times 2600\times 9.81$$
$$= 12753 \text{ N}$$

Frictional force acting between the block and the ground,

$$F_{\text{fric}} = \mu W$$
$$= 0.4\times 12753$$
$$= 5101.2 \text{ N}$$

Let 'h' be the mud height at which the block will start to slide.

Here, $$A = h\times 1 = h \text{ m}^2$$

$$h_G = \frac{h}{2}$$

Total pressure force exerted by the mud on the block,

$$F_R = h_G\times\rho\times g\times A \ (\rho \text{ is the density of the mud})$$

$$= \frac{h^2}{2}\times 1700\times 9.81 = 8338.5h^2$$

Just before the blocks starts to slide,

$$F_R = F_{\text{fric}}$$

i.e., $$8338.5\,h^2 = 5101.2$$

i.e., $$h = 0.782 \text{ m.}$$

Hence, the correct answer is option (C).

Direction for solved examples 8 and 9:

A tank 12 m high contains a liquid (specific gravity = 0.8) upto a height of 11 m. The air space above the free surface of the liquid is at a pressure of 1.2 atm. A circular opening (diameter = 2 m) present in the vertical side of the tank is closed by a disc of 2 m diameter. The disc can rotate about a horizontal diameter that is at a height of 7 m from the bottom of the tank.

Example 8

The total pressure force on the disc is

(A) 98.7 kN (B) 162.2 kN
(C) 480.36 kN (D) 186.85 kN

Solution

Diameter of the circular opening, $d = 2$ m

Area of the circular opening, $A = \frac{\pi}{4} \times d^2$

$$= \frac{\pi}{4} \times (2)^2 = 3.14 \text{ m}^2$$

Assuming that atmospheric pressure acts on the outside of the disc,

$$P_0 = (1.2 - 1) \times 101325 = 20265 \text{ Pa}$$

Total pressure force on the disc,

$$F_R = (P_0 + h_G \rho g)A \ (\rho \text{ is density of the liquid}).$$

h_G = Liquid height – Height of the disc centre from the bottom of the tank.

That is, $h_G = 11 - 7 = 4$ m

$$\therefore F_R = (20265 + 4 \times 0.8 \times 1000 \times 9.8) \times 3.14$$

$$= 162202.98 \text{ N} \approx 162.2 \text{ kN}.$$

Hence, the correct answer is option (B).

Example 9

The torque required to maintain the disc in equilibrium in the vertical position is

(A) 7100.3 Nm (B) 18253.7 Nm
(C) 3745.7 Nm (D) 6163.7 Nm

Solution

Here, $$h_{CP} = h_G + \frac{I_{XG}}{\left(h_G + \frac{P_0}{\rho g}\right)A}$$

Here, $$I_{XG} = \frac{\pi}{64} \times d^4$$

$$= \frac{\pi}{64} \times (2)^4$$

$$= 0.7854 \text{ m}^4$$

$$\therefore h_{CP} = 4 + \frac{0.7854}{\left(4 + \frac{20265}{0.8 \times 1000 \times 9.81}\right) \times 3.14}$$

$$= 4.038 \text{ m}$$

Moment of the total pressure force about the horizontal diameter.

$$= F_R \times (h_{CP} - h_G)$$

$$= 162202.98 \times (4.038 - 4)$$

$$= 6163.713 \text{ N/m (anticlockwise)}$$

Here the torque required

$$= 6163.713 \text{ N/m (clockwise)}$$

Hydrostatic force on a Submerged Horizontal Plane Surface

If the horizontal surface is at a distance of h^* from the liquid surface, then the total pressure and centre of pressure equations are:

$$\boxed{F_R = (P_0 + \rho g h^*)\, A}$$

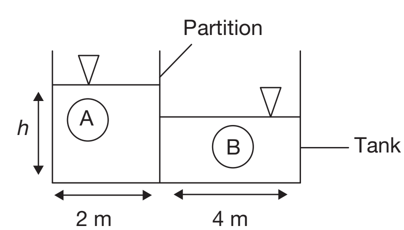
$$\boxed{\begin{array}{c} X_{CP} = x_G \\ Y_{CP} = y_G \end{array}}$$

$$\boxed{h_{CP} = h_G = h^*}$$

From the above equation, one can see that for horizontal surfaces submerged in a fluid of constant density, the centre of pressure always coincides with the centroid of the horizontal surface area. This is because the fluid pressure is constant and uniformly distributed over the surface. The fluid surface pressure P_0 can be set to zero in the above equation if it acts on both sides of the horizontal surface.

Example 10

A rectangular tank has a rotatable bottom PR that is hinged at Q. The tank is partitioned into two volumes as shown in the figure



The volume of the tank to the right of the partition is filled to a height of 2 m with liquid B of density 1500 kg/m^3. When the volume of the tank to the left of the partition is filled to a height of h metres with liquid A of density 900 kg/m^3, the bottom of the tank remains in a stationary horizontal position, If the hinge is assumed to be frictionless, then the value of h for the bottom just to tilt is

(A) 13.33 m (B) 3.33 m
(C) 6.67 m (D) 1.67 m

Solution

Let the length of the tank into the plane of the paper be l metres.

Total pressure force on the bottom of the tank due to liquid A,

$$F_{RA} = h \times \rho_A \times g \times A_{pq}$$
$$= h \times 900 \times g \times 2 \times l$$

Similarly, the total pressure force on the bottom of the tank due to liquid B

$$F_{RB} = 2 \times 1500 \times g \times 4 \times l$$

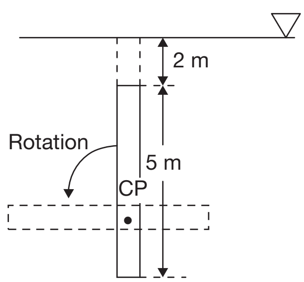
For equilibrium, taking moments about B we have

$$F_{RA} \times 1 = F_{RB} \times 2$$
$$h \times 900 \times g \times 2 \times l \times 1$$
$$= 2 \times 1500 \times g \times 4 \times l \times 2$$
$$\therefore h = 6.67 \text{ m.}$$

Hence, the correct answer is option (C).

Example 11

A rectangular thin plate of height (h) 5 m and width (w) 2 m is immersed in water, vertically along its height such that the plate is at a distance of 2 m from the water surface as shown in the figure below. Alter determining the centre of pressure for the right surface of the plate, if the plate is rotated 90° anticlockwise about an axis parallel to the water surface and on which the centre of pressure lies, then the resultant hydrostatic force acting on the plate would be



(A) 441.45 kN (B) 686.70 kN
(C) 486. 47 kN (D) 327 kN

Solution

Since the fluid surface pressure P_0 acts on both the surfaces.

$$h_{CP} = h_G + \frac{I_{XG}}{h_G A}$$

Area of the plate, $A = h \times w = 5 \times 2$
$$= 10 \text{ m}^2$$

Moment of inertia,

$$I_{XG} = \frac{w \times h^3}{12} = \frac{2 \times 5^3}{12}$$
$$= 20.833 \text{ m}^4$$

Now, $$h_G = 2 + \frac{5}{2} = 4.5 \text{ m}$$

$$\therefore h_{CP} = 4.5 + \frac{20.833}{4.5 \times 10} = 4.963 \text{ m}$$

The rotated surface will be a horizontal surface located at a distance h_{CP} from the water surface.

$$\therefore h^* = h_{CP} = 4.963 \text{ m}$$

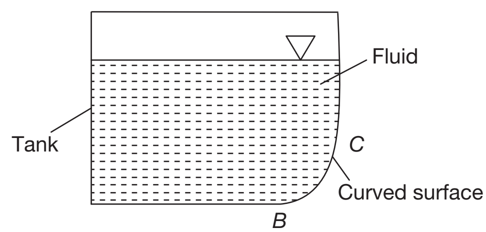
Resultant hydrostatic force,

$$F_R = \rho g h^* A \text{ (}\rho\text{ is density of water}$$
$$= 1000 \text{ kg/m}^3\text{)}$$
$$1000 \times 9.81 \times 4.963 \times 10$$
$$= 486.87 \text{ kN.}$$

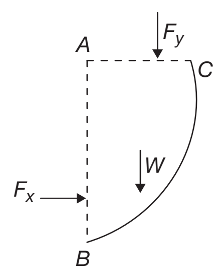
Hence, the correct answer is option (C).

Hydrostatic Force on a Submerged Curved Surface

Consider the curved surface BC of a tank filled with a fluid as shown in the figure below.



Let f_x and f_y be the respective hydrostatic forces acting on the planar surfaces AB and AC which form the respective vertical and horizontal projections of the curved surface BC is illustrated in the free body diagram I (FBD I).

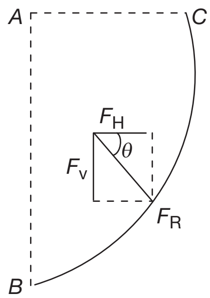


FBD I

In FBD I, W corresponds to the weight of the fluid block enclosed by the curved surface and the two planer surfaces. Let F_R (horizontal component = F_{HS}, vertical component = F_v) be the total pressure force or the resultant hydrostatic force acting on the curved surface BC as illustrated in free body diagram I (FBD).

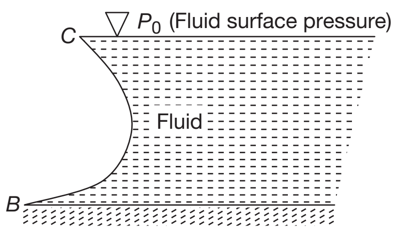
$$\boxed{\begin{aligned} F_R &= \sqrt{F_H{}^2 + F_v{}^2} \\ F_H &= F_x \\ F_v &= F_y + w \\ \tan\theta &= \frac{F_v}{F_H} \end{aligned}}$$

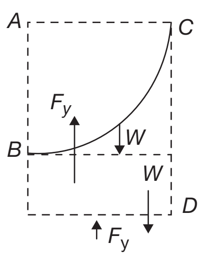
The location of the line of action of the total pressure force (for example, with respect to any of the ends of the curved surface) can be determined by taking moments about an appropriate point.



NOTE

When the fluid is present on the convex side of a curved surface BC as shown below, then the vertical component (F_v) of the total pressure force F_R acting on the curved surface is given as follows.





$$F_v = F_y - W$$

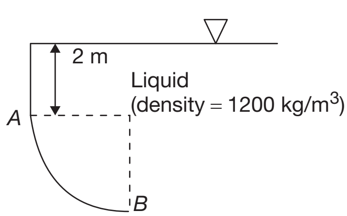
$$P_0 + W_{IV},$$

Where, W_{IV} is the weight of the fluid enclosed by the imaginary volume $ABCA$.

Here, W_{IV} = Weight of the fluid enclosed by the imaginary volume $ABCDA$ – Weight of the fluid enclosed by the volume $CBDC$.

Example 12

A gate has a curved surface AB in the form of a quadrant of a circle of radius 3 m as shown in the figure. If the width of the gate is 2.5 m, then the total pressure force acting on the curved surface AB is



(A) 384608.4 N
(B) 309015 N
(C) 493370 N
(D) Cannot be determined

Solution

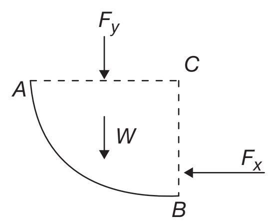
Given, $\rho = 1200$ Kg /m^3

$R = 3$ m

$h = 2$ m and

$W = 2.5$ m

The free body diagram is,



Let F_R be the total pressure force action on the curved surface AB.

Area of the planar surface BC,

$$A_{BC} = r \times w$$

$$= 3 \times 2.5 = 7.5 \text{ m}^2$$

Area of the planar surface AC,

$$A_{AC} = r \times w = 7.5 \text{ m}^2$$

Horizontal component of F_R,

$$F_A = P_0 + \left(h + \frac{x}{2}\right)\rho \times g \times A_{BC}$$

Vertical component of F_R,

$$F_v = F_y + W$$

Where, $F_y = P_0 + h\rho g \times A_{AC}$ and

$$W = \frac{\pi}{4} r^2 \times w \times \rho \times g$$

Since the fluid surface pressure P_0 also acts on the bottom surface of the gate, it can be ignored.

$$\therefore F_H = \left(2 + \frac{3}{2}\right) \times 1200 \times 9.81 \times 7.5$$

$$= 309015 \text{ N}$$

$$\text{and } F_v = 2 \times 1200 \times 9.81 \times 7.5$$

$$+ \frac{\pi}{4} \times 3^2 \times 2.5 \times 1200 \times 9.81$$

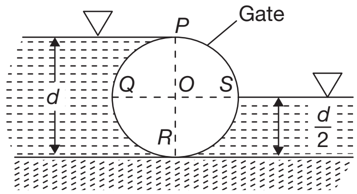
$$= 384608.4 \text{ N}$$

$$F_R = \sqrt{F_H{}^2 + F_v{}^2}$$
$$= \sqrt{(309015)^2 + (384608.4)^2}$$
$$= 493370 \text{ N.}$$

Hence, the correct answer is option (C).

Example 13

A cylindrical gate 5 m long has a liquid, of density 900 kg/m^3, on both sides as shown in the figure. The gate has a diameter of d metres and width of W metres. If it is to be ensured that the gate is in contact with the floor, then the density of the material making up the gate should be at least



(A) 169 kg/m^3 (B) 675 kg/m^3
(C) 1200 kg/m^3 (D) 900 kg/m^3

Solution

Let ρ_f (= 900 kg/m^3) be the density of the fluid.

Here the pressures at the surfaces of the fluid are ignored, i.e., P_0 is ignored.

Vertical force acting upward on QO.

F_{v1} = Weight of the fluid enclosed by the imaginary volume $PQRP$

$$= \frac{\pi}{8} d^2 \times w \times \rho_f \times g$$

Vertical force acting upward on OS,

F_{v2} = Weight of the fluid enclosed by the imaginary volume $SORS$

$$= \frac{\pi}{16} d^2 \times w \times \rho_f \times g$$

Total vertical force acting upward,

$$F_v = F_{v1} + F_{v2}$$
$$= \pi d^2 w \rho fg\left(\frac{1}{8} + \frac{1}{16}\right) = \frac{3}{16}\pi d^2 w \rho jg$$

Weight of the gate,

$$W = \frac{\pi d^2}{4} \times w \times \rho_s \times g$$

Where, ρ_s is the density of the material making up the gate. For the gate to remain in contact with the floor.

$$W \geq F_v$$

That is, $$\frac{\pi d^2}{4} w \rho_s g \geq \frac{3}{16}\pi d^2 w \rho fg$$

That is, $$\rho_s \geq \frac{3}{4}\rho$$
$$\geq \frac{3}{4} \times 900$$
$$\geq 675 \text{ kg/m}^3.$$

Hence, the correct answer is option (B).

Buoyancy

Buoyancy is the tendency of a body to be lifted (or buoyed) up in a fluid in which it is immersed wholly or partially. The force, acting opposite to the gravity force (i.e., acting vertically upward), that tends to lift the body is called the *buoyant force* or *force of buoyancy* or *upthrust*.

Archimedes principle: The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body.

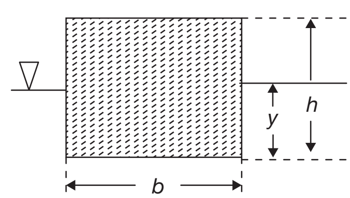
Centre of buoyancy: The point of application of the buoyant force on a body is known as the *centre of buoyancy*. It is always located at the centroid of the fluid volume displaced by the body.

NOTE

In the above definition, the fluid is always assumed to be of constant specific weight.

Buoyant Force—Single Fluid

Consider a rectangular block (density = ρ_s) of length l metres in to the plane of the paper, immersed in a fluid of density ρ_j as shown in the figure



Force of buoyancy,

$$\boxed{F_B = V_s \times \rho_j \times g}$$

Where, V_s is the volume of the body.

Submerged in the fluid, i.e., the displaced volume $= b \times y \times l$.

Here, $F_B = b \times y \times l \times \rho_j \times g$

For the body to be in static equilibrium,

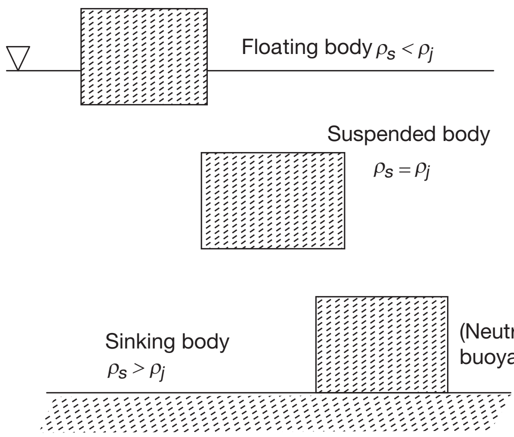
$$F_B = W,$$

Where, $W = b \times h \times l \times \rho_s \times g$, is the weight of the block.

$$\therefore\ b \times y \times l \times \rho_j \times g = b \times h \times l \times \rho_s \times g$$

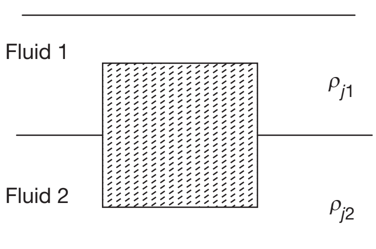
$$\text{or} \quad \boxed{\frac{V_s}{V_t} = \frac{\rho_s}{\rho_j}}$$

Where, $V_t (= b \times h \times l)$ is the total volume of the block.



Buoyant Force—Layered Fluid

If the rectangular block considered above is present in a layered fluid (as shown in the figure). Where the ith layer of fluid has the density ρ_{ji}, then



Force of buoyancy,

$$\boxed{F_B = \left(\sum_i V_{si}\rho_{ji}\right)g}$$

Example 14

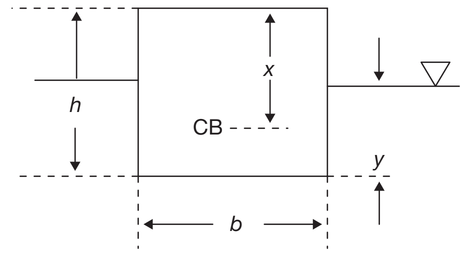
A rectangular block of width = 5 m, height = 3 m and length = 10 m (in to the plane of the paper) is floating in a liquid of density 1500 kg/m^3. If the centre of buoyancy is located at a vertical distance of 2.5 metres from the top edge of the block, then the density of the material making up the block is

(A) 1500 kg/m^3 (B) 500 kg/m^3
(C) 750 kg/m^3 (D) 1700 kg/m^3

Solution

CB $\rightarrow$ Centre of buoyancy

Let l be the length of the block into the plane of the paper.



Given, $l = 10$ m

$H = 3$ m

$b = 5$ m

$x = 2.5$ m

Since point CB form the centroid of the displaced fluid volume,

$$x + \frac{y}{2} = h$$

That is, $y = 2 \times (3 - 2.5) = 1$ m.

Let, ρl and ρ_s be the densities of the liquid and the material of the block respectively.

Given, $\rho l = 1500$ Kg/m^3

Now, $$\frac{V_s}{V_t} = \frac{\rho_s}{\rho_l}$$

i.e, $$\frac{y}{h} = \frac{\rho_s}{\rho_l}$$

or $$\rho_s = \frac{1500 \times 1}{3} = 500 \text{ kg/m}^3.$$

Hence, the correct answer is option (B).

Example 15

A spherical object (density = ρ_s) is submerged in a tank of liquid (density = ρ_j). The object, which does not touch the tank's bottom, is held in place by chaining it to the bottom of the tank. The tension in the chain would be zero when

(A) $\rho_s = 2\rho_j$ (B) $2\rho_s = \rho_j$
(C) $\rho_s = \rho_j$ (D) $\rho_{j >>>} \rho_s$

Solution

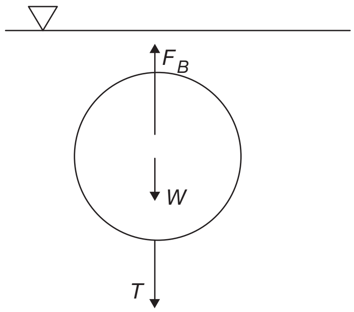
Let, the radius of the spherical object be r

Weight of the object,

$$W = \frac{4}{3}\pi r^3 \times \rho_s \times g$$

Buoyant force acting on the object,

$$F_B = \frac{4}{3}\pi r^3 \times \rho_g \times g$$



If T is the tension in the chain, then from the equilibrium of force we have,

$$T = W - F_B = \frac{4}{3}\pi r^3 g \times (\rho_s - \rho_j)$$

So when, $\rho_s = \rho_j$, $T = 0$

This problem can also be solved without the above mathematical steps. If the tension in the chain is zero, then the object would be a suspended body in the tank. For suspended body, $\rho_s = \rho_j$.

Hence, the correct answer is option (C).

Example 16

A body of unknown shape and density ρ kg/m^3 floats at the interface of two immiscible liquids A and B having the respective densities of ρ_A and ρ_B ($\rho_A < \rho_B$). The ratio of the volume of the block submerged in liquid B to the total volume of the block is equal to

(A) $(\rho - \rho_A) / (\rho_B - \rho_A)$
(B) $(\rho_A - \rho) / (\rho_B - \rho_A)$
(C) $(\rho_B - \rho_A) / (\rho - \rho_A)$
(D) $(\rho_B - \rho_A) / (\rho_A - \rho)$

Solution

Let V_{SA} and V_{SB} be the volumes of the block submerged respectively in liquid A and B. Force of buoyancy,

$$F_B = (V_{SA}\,\rho_A + V_{SB}\,\rho_B)g$$

If V_t is the total volume of the block then, $V_t = V_{SA} + V_{SB}$

$$\therefore\ F_B = (V_t\rho_A + V_{SB}\,(\rho_B - \rho_A)\,g$$

Under static equilibrium,

$$F_B = W = V_t\,\rho g\ (W \text{ is weight of the block})$$

$$\text{i.e., } [V_t\rho_A + V_{SB}\,(\rho_B - \rho_{A)}] = V_t\,\rho$$

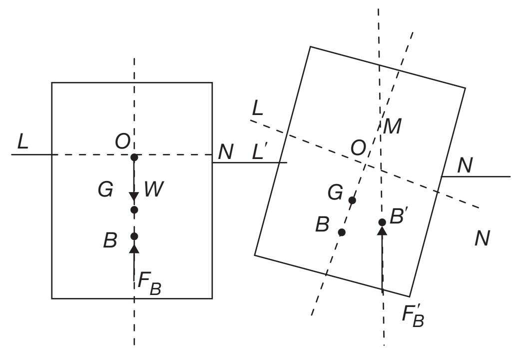
$$\text{or } \frac{V_{SB}}{V_t} = \frac{\rho - \rho_A}{\rho_B - \rho_A}.$$

Hence, the correct answer is option (A).

Stability (rotational stability) of a submerged body: A body is said to be *stable*, i.e., in *stable* equilibrium if the centre of gravity is directly below the centre of buoyancy for the body. However if the centre of gravity is directly above the centre of buoyancy, then the submerged body is said to be *unstable*, i.e., in unstable *equilibrium*. When the centre of gravity and the centre of buoyancy coincide, the submerged body is said to be *neutrally stable*, i.e., in *neutral equilibrium.*

Stability (rotational stability) of a floating body: A floating body is always stable when the centre of gravity is directly below the centre of buoyancy. When the centre of gravity is directly above the centre of buoyancy, metacentre plays a role in determining the stability of the floating body.

Consider a body being rotated by a small angle, along an axis that passes through the point O and that is perpendicular to the plane of the paper as shown in the following figures.



Metacentre: (Point M) is the point of intersection of the line passing through the centre of gravity (point G) and the original centre of buoyancy (point B) and a vertical line passing through the centre of buoyancy of the rotated position of the body (point B')

Metacentric height: The distance between the metacentre and the centre of gravity of a floating body is called as the *metacentric height* (GM).

If BM and BG represent the distance between the centre of buoyancy (point B) and the metacentre and the centre of gravity points respectively, then

$$\boxed{GM = BM - BG}$$

$$\boxed{BM = \frac{\rho g I}{W} = \frac{I}{V_s}}$$

$$\frac{\rho g I}{W} = \frac{I}{\frac{W}{\rho g}} = \frac{I}{V_s}$$

Where ρ is the density of the fluid, W is the weight of the body, I is the moment of inertia of the sectional area of the body at the fluid surface (i.e., LN) about the axis at point O and V_s is the volume of the body submerged in the fluid.

A floating body is said to be *stable* if point M is above point G (i.e., GM is positive) and *unstable* if point M is below point G. The floating body is *neutrally stable* if point M coincides with point G. Larger the metacentric height, more stable the floating body will be.

Example 17

A rectangular block of width W, height h and length l (perpendicular to the plane of the paper) is floating in a liquid. The height of the block submerged in the liquid is b.

If the centre of gravity of the body is located at the liquid surface, then which one of the following condition when satisfied will ensure that the block is stable?

(A) $W > \sqrt{\frac{3}{2}}b$ (B) $l > \sqrt{6}b$

(C) $W > \sqrt{6}b$ (D) $l > \sqrt{\frac{3}{2}}b$

Solution

Volume of the block that is submerged in the liquid

$$V_S = b \times W \times l$$

The cross-sectional area of the block at the water surface will have a base l and height W. Relative to the axis (perpendicular to the plane of the paper) at point O.

∴ Moment of inertia,

$$I = \frac{1}{12}w^3 l$$

Metacentric height, $GM = BM - BG$

Here, $$BM = \frac{I}{V_s}$$

$$\frac{\frac{1}{12}w^3 l}{bwl} = \frac{w^2}{12b}$$

Also, $$BG = \frac{b}{2}$$

$$\therefore GM = \frac{w^2}{12b} - \frac{b}{2}$$

For a stable body GM should be positive, i.e., $\frac{w^2}{12b} > \frac{b}{2}$

or $$w > \sqrt{6}b.$$

Hence, the correct answer is option (C).

Example 18

Cubes A, B, C and D all have the same side length of l metres and are floating in the same water body. The cubes A, B, C and D have the respective constant specific gravities of 0.152, 0.561, 0.789 and 0.923 respectively. Which one of the following statement is ONLY not correct?

(A) Cube C is instable (B) Cube B is unstable

(C) Cube A is stable (D) Cube D is stable

Solution

Let us consider a floating cube in general with specific gravity V. Let the height of the floating cube submerged in water be h.

For the floating cube,

Buoyant force experienced = Weight of the cube

That is, $h \times l^2 \times 1000 \times g = l^3 \times r \times 1000 \times g$

or $$h = lr \qquad (1)$$

Volume of the cube submerged,

$$V_s = h \times l^2$$

Moment of inertia,

$$I = \frac{1}{12} \times l^3 \times l$$

$$BM = \frac{I}{V} = \frac{\frac{1}{12}l^4}{hl^2} = \frac{l^2}{12h}$$

Here, $BG = \frac{l}{2} - \frac{h}{2}$ (point B is below point G)

$$GM = BM - BG$$

$$GM = \frac{l^2}{12h} - \frac{l}{2} + \frac{h}{2} \qquad (2)$$

Substituting Eq. (1) in Eq. (2), we get

$$GM = \frac{l}{12r}(1 - 6\gamma + 6\gamma^2)$$

When, $GM = 0$, we have

$$6\gamma^2 - 6\gamma + 1 = 0$$

That is, $$\gamma = 0.789 \text{ or } 0.211$$

It can be shown that when

$$0.211 < \gamma < 0.789, GM \text{ is negative}$$

∴ Cube C is neutrally stable (or stable if higher precision is considered in the values of γ).

Hence, the correct answer is option (A).

Oscillation of a Floating Body

When a floating body is given a small angular displacement, the body will oscillate about its metacentre. If T denotes the time period of oscillation or rolling (i.e., time for one complete oscillation) of the floating body, then

$$\boxed{T = 2\pi\sqrt{\frac{k^2}{GM \cdot g}}}$$

Where GM is the metacentric height, k is the radius of gyration and T is in seconds.

Exercises

1. **Assertion (A):** Depth of centre of pressure of any immersed surface is independent of the density of liquid.

Reason (R): Center of area of immersed surface lies below the centre of pressure.

(A) Both A and R are individually true and R is the correct explanation of A.
(B) Both A and R are individually true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

2. A circular annular plate bounded by two concentric circles of diameter 1.2 m and 0.8 m is immersed in water with its plane making an angle of 45° with the horizontal. The center of the circles is 1.625 m below the free surface. What will be the total pressure force on the face of the plate?

(A) 7.07 kN (B) 10.00 kN
(C) 14.14 kN (D) 18.00 kN

3. **Assertion (A):** At great depth, the vertical distance between the centre of pressure and the centre of area of immersed surface becomes negligible.

Reason (R): The depth of centre of pressure of any immersed surface is independent of the density of the liquid.

(A) Both A and R are individually true and R is the correct explanation of A.
(B) Both A and R are individually true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

4. The following terms relate to floating bodies Centre of gravity—G, Metacentre—M, Weight of floating body—W, Buoyant force—F_B

Match List I (Condition) with List II (Result) and select the correct answer using the codes given

	List I		List II
a.	G is above M	**1.**	Stable equilibrium
b.	G and M coincide	**2.**	Unstable equilibrium
c.	G is below M	**3.**	Floating body
d.	$F_B > W$	**4.**	Neutral equilibrium

Codes:

	a	b	c	d		a	b	c	d
(A)	1	3	2	4	(B)	3	2	1	4
(C)	2	3	4	1	(D)	2	4	1	3

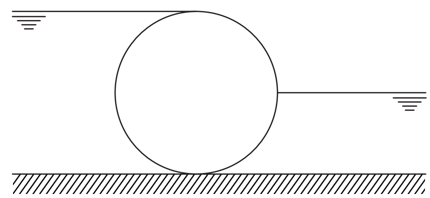
5. A vertical triangular plane area, submerged in water, with one side in the free surface, vertex downward and latitude 'h' has the pressure centre below the free surface by

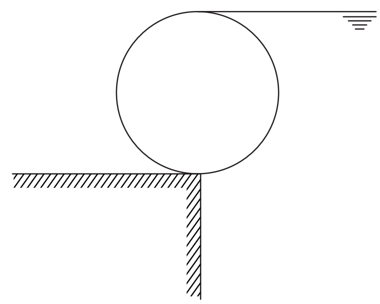
(A) $h/4$ (B) $h/3$
(C) $2h/3$ (D) $h/2$

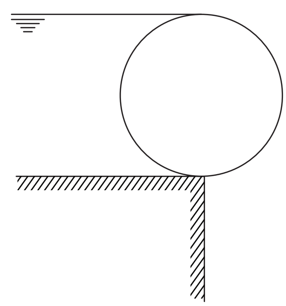
6. The centre of pressure of a liquid on a plane surface immersed vertically in a static body of liquid, always lies below the centroid of the surface area, because

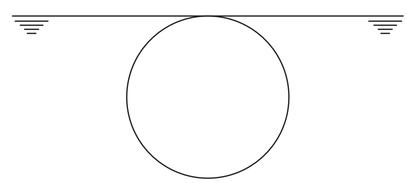
(A) in liquids the pressure acting is same in all directions.
(B) there is no shear stress in liquids at rest.
(C) the liquid pressure is constant over depth.
(D) the liquid pressure increases linearly with depth.

7. In which one of the following arrangement would the vertical force on the cylinder due to water be the maximum?

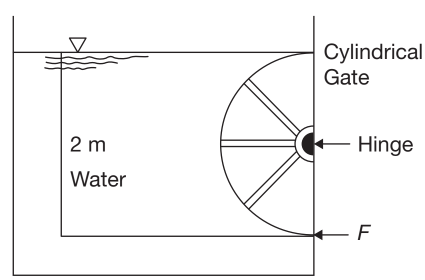
(A)


(B)


(C)


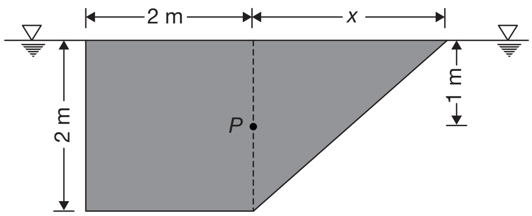
(D)


8. The force 'F' required at equilibrium on the semi-cylindrical gate shown in the figure is



(A) 9.81 kN
(B) 0.0 kN
(C) 19.62 kN
(D) None of these

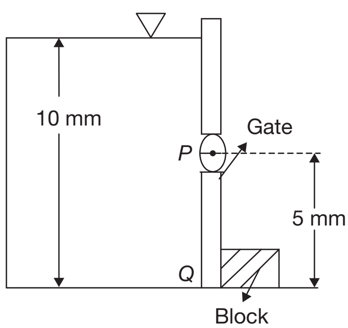
9. Cross-section of an object (having same section normal to the paper) submerged into a fluid consists of a square of sides 2 m and triangle as shown in the figure. The object is hinged at point P that is one metre below the fluid free surface. If the object is to be kept in the position as shown in the figure, the value of 'x' should be



(A) $2\sqrt{3}$ (B) $4\sqrt{3}$
(C) 4 m (D) 8 m

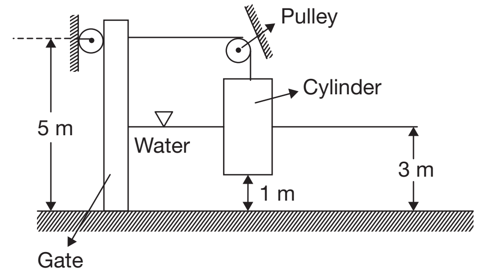
10. The necessary and sufficient condition for a surface to be called as a 'free surface' is
(A) no stress should be acting on it.
(B) tensile stress acting on it must be zero.
(C) shear stress acting on it must be zero.
(D) no point it should be under any stress.

11. A rectangular gate PQ has a length of 7 m into the plane of the paper and a height of 5 m. The gate is hinged at P and is prevented from opening by a block at Q as shown in the figure



When the height of the body of liquid (present on the left side of the gate) reaches 10 m, the hinge breaks as the reaction at the hinge becomes 1029 kN. The density of the liquid is
(A) 257 kg/m^3
(B) 900 kg/m^3
(C) 2700 kg/m^3
(D) 400 kg/m^3

12. A cylinder of diameter 2 m and height 4 m is connected to a gate of width 3 m as shown in the following figure. When the water level drops to a height of 3 m, the gate opens up. If the friction of the gate and pulley can be neglected, then the density of the cylinder is



(A) 500 kg/m^3
(B) 715 kg/m^3
(C) 822 kg/m^3
(D) 965 kg/m^3

13. Consider the following statements:

The metacentric height of a floating body depends
I. directly on the shape of its water-line area.
II. on the volume of liquid displaced by the body.
III. on the distance between the metacentre and the centre of gravity.
IV. on the second moment of water-line area.

Which of these statements are correct?
(A) I and II
(B) II and III
(C) III and IV
(D) I and IV

14. Consider the following statements:

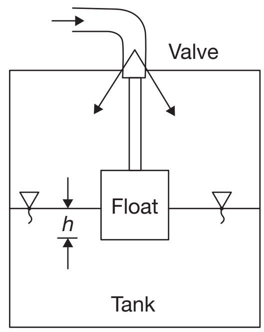
Filling up a part of the empty hold of a ship with ballasts will
I. reduce the metacentric height.
II. lower the position of the centre of gravity.
III. elevate the position of centre of gravity.
IV. elevate the position of centre of buoyancy.

Which of these statements are correct?
(A) I, III and IV
(B) I and II
(C) III and IV
(D) II and IV

15. **Assertion (A):** If a boat, built with sheet metal on wooden frame, has an average density which is greater than that of water, then the boat can float in water with its hollow face upward but will sink once it overturns.

Reason (R): Buoyant force always acts in the upward direction.
(A) Both A and R are individually true and R is the correct explanation of A.
(B) Both A and R are individually true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

16. A float of cubical shape has sides of 10 cm. The float valve just touches the valve seat to have a flow area of 0.5 cm^2 as shown in the figure



If the pressure of water in the pipeline is 1 bar, the rise of water level 'h' in the tank to just stop the water flow will be

(A) 7.5 cm (B) 5.0 cm
(C) 2.5 cm (D) 0.5 cm

17. A homogeneous solid of any arbitrary shape floats upright in a homogeneous liquid with immersed volume V and is in stable equilibrium. If the solid is overturned and made to float upside down in a different homogeneous liquid with exactly same volume V above the liquid surface, then the equilibrium

(A) would be stable.
(B) would be neutral.
(C) would be unstable.
(D) may or may not be stable.

18. The time period of rolling of a ship of weight 25000 kN in sea water is half a minute. Along a line joining the metacentre (point M), centre of gravity (point G) and centre of buoyancy (point B), the distance between points M and B is 3.5 m while the distance between points B and G is 1.5 m. Point M is above point G, which is above point B along this line. If the specific weight of sea water is 10.1 kN/m^3. Then the radius of gyration of the ship is

(A) 21.5 m (B) 10.63 m
(C) 0.352 m (D) 8.42 m

19. One of the following statements is true with regards to bodies that float or are submerged in liquids:

(A) For a body wholly submerged in a liquid the stability is ensured if the centre of buoyancy is below the centre of gravity of the body.
(B) For a body floating in a liquid the stability is ensured if the centre of buoyancy is below the centre of gravity of the body.
(C) For a body floating in a liquid the stability is ensured if the centre of buoyancy and the centre of gravity, regardless of the relative positions of the centre of buoyancy and gravity.
(D) For a body floating in a liquids the stability is ensured if the centre of buoyancy is below the centre of gravity and the metacentre is above both the centers of gravity and buoyancy.

20. In an iceberg, 15% of the volume projects above the sea surface. If the specific weight of sea water is 10.5 kN/m^3, the specific weight of iceberg in kN/m^3 is

(A) 12.52 (B) 9.81
(C) 8.93 (D) 7.83

21. A 15 cm length of steel rod with relative density of 7.4 is submerged in a two layer fluid. The bottom layer is mercury and the top layer is water. The height of top surface of the rod above the liquid interface in 'cm' is

(A) 8.24 (B) 7.82
(C) 7.64 (D) 7.38

22. Stability of a floating body can be improved by which of the following?

I. Making its width large
II. Making the draft large
III. Keeping the centre of mass low
IV. Reducing its density

Select the correct answer from the given options

(A) I, II, III and IV (B) I, II and III only
(C) I and II only (D) III and IV only

23. Consider the following statements related to the stability of floating bodies:

I. The metacentre should be above the centre of gravity of the floating body for stable equilibrium during small oscillations.
II. For a floating body, stability is not determined simply by the relative positions of centre of gravity and centre buoyancy.
III. The position of metacentre of a floating body is fixed irrespective of the axis of oscillations.
IV. Large value of metacentric height reduces the period of roll of the vessel.

Which of these statements are correct?

(A) I and III only (B) II and IV only
(C) I, II and IV only (D) I, II, III and IV

24. **Statement I:** When a given body floats in different liquids, the volume displaced will decrease with increase in the specific gravity of the fluid.

Statement II: The weight of the floating body is equal to the weight of the volume displaced.

(A) Both I and II are individually true and II is the correct explanation of I.
(B) Both I and II are individually true but II is not the correct explanation of I.
(C) I is true but II is false.
(D) I is false but II is true.

25. A vertical wall is holding a liquid of specific weight 'w' and height 'h' on one side. The total pressure on the wall per unit length is

(A) wh (B) $wh/2$
(C) $\frac{wh^2}{2}$ (D) $\frac{2}{3}wh$

26. If the surface of liquid is concave, then cohesive pressure is

(A) increased.
(B) decreased.
(C) absent.
(D) negligible or does not matter.

27. A body weight 4 kg in air was found to weigh 3.5 kg when submerged in water. Its specific gravity is

(A) 1 (B) 3
(C) 6 (D) 8

28. For stability of a floating body

(A) the metacentre '*M*' should lie between the centre of gravity '*C*' and centre of buoyancy '*B*'.
(B) *M* should lie above *B* and *C*.
(C) *M* should coincide with *B* and *C*.
(D) *M* should lie below *B* and *C*.

29. Match List I with List II and select the correct answer:

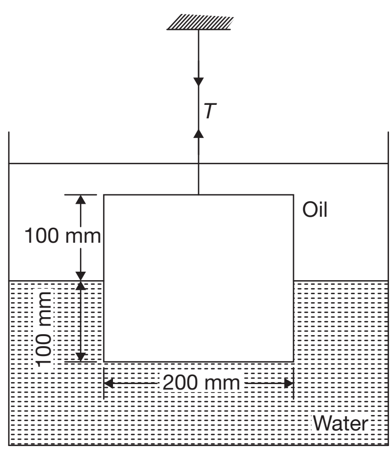
	List I (Condition of Floating Bodies)		List II (Result)
a.	*M* below *G*	**1.**	Floating body
b.	*M* above *G*	**2.**	Unstable equilibrium
c.	*M* and *G* coincides	**3.**	Stable equilibrium
d.	*B* below *G*	**4.**	Neutral equilibrium

[*M* – Metacentre, *G* – Centre of gravity, *B* – Centre of buoyancy]

Codes:

	a	b	c	d		a	b	c	d
(A)	3	4	1	2	(B)	2	3	4	1
(C)	1	2	3	4	(D)	4	1	2	3

30. A metallic cube of side 200 mm and specific weight 26 kN/m^3 is suspended by a string in oil and water as shown in the figure. Half of the cube is submerged in water and the remaining half is submerged in oil. If specific gravity of the oil is 0.8, determine the tension in the string.



(A) 98.2 N (B) 176.6 N
(C) 156.7 N (D) 192.4 N

31. A curved surface is submerged in a fluid. Consider the following statements relating to it.

I. Vertical component of the hydrostatic force acting on the surface is equal to the weight of the fluid vertically above the surface.
II. Horizontal component of the force acting on the curved surface is the hydrostatic force acting on the vertical projection of the curved surface.
III. Horizontal component of the force acts through the centre of gravity of the vertical projection of the curved surface.

(A) I and II are correct
(B) I and III are correct
(C) II and III are correct
(D) I, II and III are correct

Direction for questions 32 to 34:

An 80 mm diameter composite solid cylinder consists of a 20 mm thick metallic plate and 650 mm long wooden cylinder of specific gravity 4 and 0.8 respectively. The cylinder floats in water its axis vertical.

32. The position of centre of gravity from bottom is

(A) 0.3 m (B) 0.35 m
(C) 0.4 m (D) 0.45 m

33. Position of centre of buoyancy from bottom is

(A) 0.2 m (B) 0.25 m
(C) 0.3 m (D) 0.35 m

34. Metacentric height is

(A) 0.667 mm (B) 6.67 mm
(C) 3.33 mm (D) 8.63 mm

35.

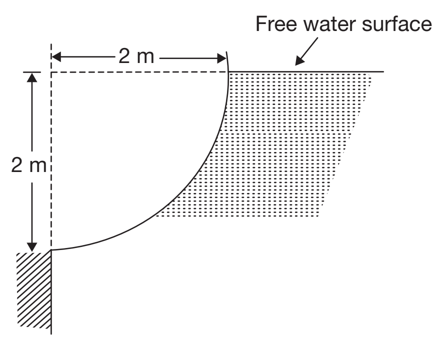


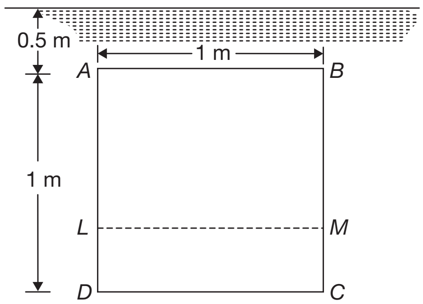
Figure shows a gate having quadrant shape. Vertical component of the resultant force acting per metre length on the curved surface (in kN) ______.

(A) 30.8 kN
(B) 30.4 kN
(C) 30.38 kN
(D) 30.96 kN

36. A triangular plate of 90 cm height and 60 cm base is submerged vertically in water such that base is horizontal and the upper point is at a depth of 9.4 m from water surface. Total hydrostatic pressure (in kN) acting on one side of the plate is______.

(A) 26.48 kN
(C) 26.2 kN
(C) 24 kN
(D) 25.2 kN

37. A square plate *ABCD* of size 1 m × 1 m is submerged vertically in water such that the upper edge is horizontal and is at a depth of 0.5 m from water surface as shown in the figure.



Horizontal line LM on the plate is such that the total pressure force above it is equal to the total pressure force below. Then the distance BM (in cm) is equal to

(A) 51.6 (B) 54.2
(C) 61.8 (D) 66.7

Previous Years' Questions

1. For the stability of a floating body, under the influence of gravity alone, which of the following is TRUE? **[GATE, 2010]**

(A) Metacentre should be below centre of gravity.
(B) Metacentre should be above centre of gravity.
(C) Metacentre and centre of gravity must lie on the same horizontal line.
(D) Metacentre and centre of gravity must lie on the same vertical line.

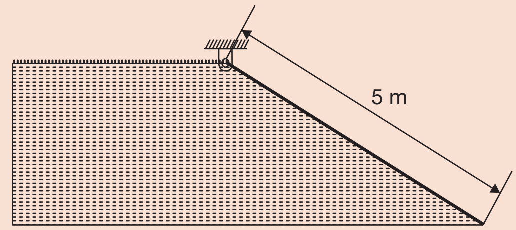
2. For a body completely submerged in a fluid, the centre of gravity (G) and centre of Buoyancy (O) are known. The body is considered to be in stable equilibrium if **[GATE, 2011]**

(A) O does not coincide with the centre of mass of the displaced fluid.
(B) G coincides with the centre of mass of the displaced fluid.
(C) O lies below G.
(D) O lies above G.

3. If a small concrete cube is submerged deep in still water in such a way that the pressure exerted on all faces of the cube is p, then the maximum shear stress developed inside the cube is **[GATE, 2012]**

(A) 0 (B) $\frac{p}{2}$
(C) p (D) $2p$

4. A hinged gate of length 5 m, inclined at 30° with the horizontal and with water mass on its left, is shown in the figure. Density of water is 1000 kg/m^3. The minimum mass of the gate in kg per unit width (perpendicular to the plane of paper), required to keep it closed is **[GATE, 2013]**



(A) 5000 (B) 6600
(C) 7546 (D) 9623

5. For a completely submerged body with centre of gravity 'G' and centre of buoyancy 'B', the condition of stability will be **[GATE, 2014]**

(A) G is located below B.
(B) G is located above B.
(C) G and B are coincident.
(D) independent of the locations of G and B.

6. A flow field which has only convective acceleration is **[GATE, 2014]**

(A) a steady uniform flow.
(B) an unsteady uniform flow.
(C) a steady non-uniform flow.
(D) an unsteady non-uniform flow.

7. A triangular gate with a base width of 2 m and a height of 1.5 m lies in a vertical plane. The top vertex of the gate is 1.5 m below the surface of a tank which contains oil of specific gravity 0.8. Considering the density of water and acceleration due to gravity to be 1000 kg/m^3 and 9.81 m/s^2, respectively, the hydrostatic force (in kN) exerted by the oil on the gate is ______. **[GATE, 2015]**

Answer Keys

Exercises

1. C	**2.** B	**3.** B	**4.** D	**5.** D	**6.** D	**7.** D	**8.** B	**9.** A	**10.** C
11. B	**12.** B	**13.** B	**14.** D	**15.** B	**16.** B	**17.** D	**18.** A	**19.** D	**20.** C
21. D	**22.** B	**23.** C	**24.** A	**25.** C	**26.** B	**27.** D	**28.** B	**29.** B	**30.** B
31. A	**32.** A	**33.** C	**34.** A	**35.** A	**36.** A	**37.** C			

Previous Years' Questions

1. B	**2.** D	**3.** A	**4.** D	**5.** A	**6.** C	**7.** 29.43