CHAPTER 7

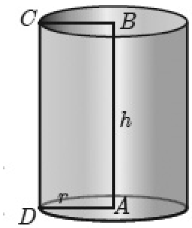
MENSURATION

I. SURFACE AREA

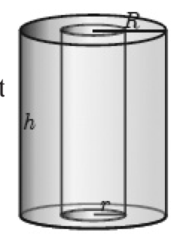
Key Points

Right Circular and Hollow Cylinder

- ✓ A right circular cylinder is a solid generated by the revolution of a rectangle about one of its sides as axis.

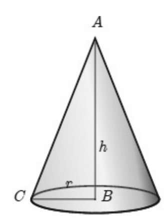


- ✓ If the axis is perpendicular to the radius then the cylinder is called a right circular cylinder.
- ✓ A solid cylinder is an object bounded by two circular plane surfaces and a curved surface.
- ✓ C.S.A. of a right circular cylinder = $2\pi rh$ sq. units.
- ✓ T.S.A. of a right circular cylinder = $2\pi r(h + r)$ sq. units

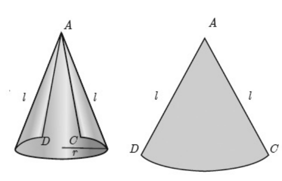


- ✓ An object bounded by two co-axial cylinders of the same height and different radii is called a 'hollow cylinder'.
- ✓ C.S.A of a hollow cylinder= $2\pi(R + r)h$ sq. units
- ✓ T.S.A. of a hollow cylinder = $2\pi(R + r)(R - r + h)$ sq. units

Right Circular and Hollow Cone

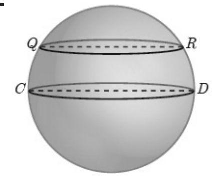


- ✓ A right circular cone is a solid generated by the revolution of a right angled triangle about one of the sides containing the right angle as axis.
- ✓ If the right triangle ABC revolves about AB as axis, the hypotenuse AC generates the curved surface of the cone.
- ✓ The height of the cone is the length of the axis AB, and the slant height is the length of the hypotenuse AC.

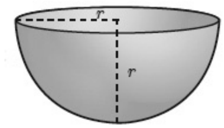


- ✓ Curved Surface Area of the cone = Area of the Sector = $\pi r l$ sq. units.
- ✓ T.S.A. of a right circular cone= $\pi r(l + r)$ sq. units, where $l = \sqrt{h^2 + r^2}$.

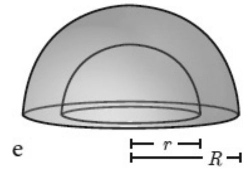
Sphere & Hemisphere



- ✓ A sphere is a solid generated by the revolution of a semicircle about its diameter as axis.
- ✓ Surface area of a sphere = $4\pi r^2$ sq.units.

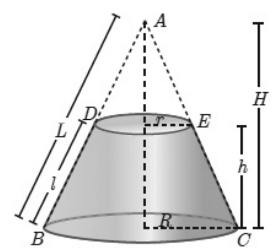


- ✓ A section of the sphere cut by a plane through any of its great circle is a hemisphere.
- ✓ C.S.A. of a hemisphere = $2\pi r^2$ sq.units.
- ✓ T.S.A. of a hemisphere = $3\pi r^2$ sq.units.



- ✓ C.S.A. of a hollow hemisphere = $2\pi(R^2 + r^2)$ sq. units.
- ✓ T.S.A. of a hollow hemisphere = $\pi(3R^2 + r^2)$ sq. units.
- ✓ Thickness = R− r

Frustum of a Cone



- ✓ When a cone ABC is cut through by a plane parallel to its base, the portion of the cone DECB between the cutting plane and the base is called a frustum of the cone.
- ✓ C.S.A. of a frustum = $\pi(R + r)l$ sq. units, where, $l = \sqrt{h^2 + (R - r)^2}$.
- ✓ T.S.A. of a frustum = $\pi(R + r)l + \pi R^2 + \pi r^2$ sq. units.

Example 7.1

A cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area.

Solution :

Given that, height of the cylinder h = 20 cm ; radius r =14 cm

Now, C.S.A. of the cylinder = $2\pi rh$ sq. units

C.S.A. of the cylinder

$$= 2 \times \frac{22}{7} \times 14 \times 20 = 2\times 22\times 2\times 20 = 1760 \text{ cm}^2$$

T.S.A. of the cylinder = $2\pi r (h + r)$ sq. units

$$= 2 \times \frac{22}{7} \times 14 \times (20 + 14) = 2 \times \frac{22}{7} \times 14 \times 34$$

$$= 2992 \text{ cm}^2$$

Therefore, C.S.A.

= 1760 cm^2 and T.S.A. = 2992 cm^2

Example 7.2

The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the cylinder.

Solution :

Given that, C.S.A. of the cylinder =88 sq. cm

$2\pi rh = 88$

$$= 2 \times \frac{22}{7} \times r \times 14 = 88 \text{ (given } h = 14 \text{ cm)}$$

$$2r = \frac{88 \times 7}{22 \times 14} = 2$$

Therefore, diameter = 2 cm

Example 7.3

A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions?

Solution :

Given that, diameter d = 2.8 m and height = 3 m

radius r = 1.4 m

Fig. 7.6

Area covered in one revolution = curved surface area of the cylinder

= $2\pi rh$ sq. units

$$= 2 \times \frac{22}{7} \times 1.4 \times 3 = 26.4$$

Area covered in 1 revolution = 26.4 m^2

Area covered in 8 revolutions

= 8 × 26.4 = 211.2

Therefore, area covered is 211.2 m^2

Example 7.4

If one litre of paint covers 10 m^2, how many litres of paint is required to paint the internal and external surface areas of a cylindrical tunnel whose thickness is 2 m, internal radius is 6 m and height is 25 m.

Solution :

Given that, height h = 25 m; thickness = 2 m.

Internal radius r = 6 m

Now, external radius R = 6 + 2 = 8m

C.S.A. of the cylindrical tunnel

= C.S.A. of the hollow cylinder

C.S.A. of the hollow cylinder

= 2π (R + r) h sq. units

$$= 2 \times \frac{22}{7}(8+6) \times 25$$

Hence, C.S.A. of the cylindrical tunnel

= 2200 m^2

Area covered by one litre of paint = 10 m^2

Number of litres required to paint the tunnel

$$= \frac{2200}{10} = 220$$

Therefore, 220 litres of paint is needed to paint the tunnel.

Example 7.5

The radius of a conical tent is 7 m and the height is 24 m. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is 4 m?

Solution :

Let r and h be the radius and height of the cone respectively.

Given that, radius r =7 m and height h = 24 m

$$\text{Hence, } l = \sqrt{r^2 + h^2}$$

$$= \sqrt{49 + 576}$$

$$l = \sqrt{625} = 25\text{m}$$

C.S.A. of the conical tent = πrl sq. units

Area of the canvas = $\frac{22}{7} \times 7 \times 25 = 550\ m^2$

Now, length of the canvas =

$$\frac{\text{Area of canvas}}{\text{Width}} = \frac{550}{4} = 137.5\ m$$

Therefore, the length of the canvas is137.5 m

Example 7.6

If the total surface area of a cone of radius 7cm is 704 cm^2, then find its slant height.

Solution :

Given that, radius r = 7 cm

Now, total surface area of the cone

$= \pi r\ (l + r)$ sq. units

T.S.A. = 704 cm^2

$$704 = \frac{22}{7} \times 7\ (l + 7)$$

$32 = l + 7$ implies $l = 25$ cm

Therefore, slant height of the cone is 25 cm.

Example 7.7

From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and base is hollowed out (Fig.7.13). Find the total surface area of the remaining solid.

Solution :

Let h and r be the height and radius of the cone and cylinder.

Let l be the slant height of the cone.

Given that, h = 2.4 cm and d = 1.4 cm ;

r = 0.7 cm

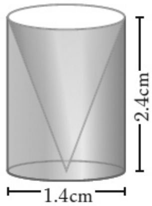


Fig. 7.13

Here, total surface area of the remaining solid

= C.S.A. of the cylinder + C.S.A. of the cone + area of the bottom

$= 2\pi rh + \pi rl + \pi r^2$ sq. units

Now,

$$l = \sqrt{r^2 + h^2} = \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5cm$$

$l = 2.5$ cm

Area of the remaining solid

$= 2\pi rh + \pi rl + \pi r^2$ sq. units

$= \pi r\ (2h + l + r)$

$= \frac{22}{7} \times 0.7 \times \left[(2 \times 2.4) + 2.5 + 0.7\right]$

$= 17.6$

Therefore, total surface area of the remaining solid is 17.6 m^2

Example 7.8

Find the diameter of a sphere whose surface area is 154 m^2.

Solution :

Let r be the radius of the sphere.

Given that, surface area of sphere = 154 m^2

$4\pi r^2 = 154$

$$4 \times \frac{22}{7} \times r^2 = 154$$

gives $\quad r^2 = 154 \times \frac{1}{4} \times \frac{7}{22}$

hence, $\quad r^2 = \frac{49}{4}$ we get $r = \frac{7}{2}$

Therefore, diameter is 7 m

Example 7.9

The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases.

Solution :

Let r_1 and r_2 be the radii of the balloons.

Given that,

$$\frac{r_1}{r_2} = \frac{12}{16} = \frac{3}{4}$$

Now, ratio of C.S.A. of balloons =

$$\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Therefore, ratio of C.S.A. of balloons is 9:16.

Example 7.10

If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area?

Solution :

Let r be the radius of the hemisphere.

Given that, base area = πr^2 = 1386 sq. m

T.S.A. = $3\pi r^2$ sq.m

= 3 × 1386 = 4158

Therefore, T.S.A. of the hemispherical solid is 4158 m^2.

Example 7.11

The internal and external radii of a hollow hemispherical shell are 3 m and 5 m respectively. Find the T.S.A. and C.S.A. of the shell.

Solution :

Let the internal and external radii of the hemispherical shell be r and R respectively.

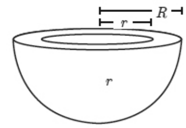


Fig. 7.19

Given that, R = 5 m, r = 3 m

C.S.A. of the shell = $2\pi(R^2 + r^2)$ sq. units

$$= 2 \times \frac{22}{7} \times (25 + 9) = 213.71$$

T.S.A. of the shell = $\pi(3R^2 + r^2)$ sq. units

$$= \frac{22}{7}(75 + 9) = 264$$

Therefore, C.S.A. = 213.71 m^2 and

T.S.A. = 264 m^2.

Example 7.12

A sphere, a cylinder and a cone (Fig.7.20) are of the same radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas.

Solution :

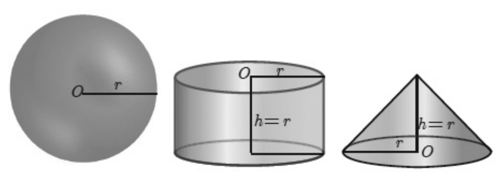


Fig. 7.20

Required Ratio = C.S.A. of the sphere: C.S.A. of the cylinder : C.S.A. of the cone

$$= 4\pi r^2 : 2\pi rh : \pi rl,$$

$$(l = \sqrt{r^2 + h^2} = \sqrt{2r^2} = \sqrt{2}r \text{ units})$$

$$= 4 : 2 : \sqrt{2} = 2\sqrt{2} : \sqrt{2} : 1$$

Example 7.13

The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area.

Solution :

Let l, R and r be the slant height, top radius and bottom radius of the frustum.

Given that, l = 5 cm, R = 4 cm, r = 1 cm

Now, C.S.A. of the frustum

$$= \pi(R + r)\, l \text{ sq. units}$$

$$= \frac{22}{7} \times (4+1) \times 5$$

$$= \frac{550}{7}$$

Therefore, C.S.A. = 78.57 cm^2

Example 7.14

An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket.

Solution :

Let h, l, R and r be the height, slant height, outer radius and inner radius of the frustum.

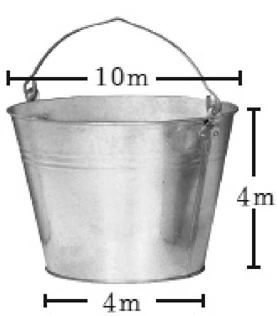


Fig. 7.24

Given that, diameter of the top =10 m; radius of the top R = 5 m. diameter of the bottom = 4 m; radius of the bottom r = 2 m, height h = 4 m

Now, $l = \sqrt{h^2 + (R-r)^2}$

$$= \sqrt{4^2 + (5-2)^2}$$

$$l = \sqrt{16+9} = \sqrt{25} = 5\text{m}$$

Here, C.S.A $= \pi\,(R+r)l$ sq.units

$$= \frac{22}{7}(5+2) \times 5 = 110 m^2$$

T.S.A. $= \pi\,(R+r)l + \pi R^2 + \pi r^2$ sq.units

$$= \frac{22}{7}[(5+2)\,5 + 25 + 4]$$

$$= \frac{1408}{7} = 201.14$$

Therefore, C.S.A. = 110 m^2 and

T.S.A. = 201.14 m^2

EXERCISE 7.1

1. **The radius and height of a cylinder are in the ratio 5 : 7 and its curved surface area is 5500 sq.cm. Find its radius and height.**

Solution:

Given r : h = 5 : 7

$$\Rightarrow \frac{r}{h} = \frac{5}{7}$$

$$\Rightarrow 7r = 5h$$

$$\Rightarrow h = \frac{7r}{5}$$

CSA of Cylinder = 5500

$$\Rightarrow 2\pi rh = 5500$$

$$\Rightarrow 2 \times \frac{22}{7} \times r \times \frac{7r}{5} = 5500$$

$$\Rightarrow \frac{44}{5} r^2 = 5500$$

$$\Rightarrow \quad r^2 = \frac{5500 \times 5}{44}$$

$$\Rightarrow \quad r = \frac{500 \times 5}{4}$$

$$= 125 \times 5$$

$$= 625$$

$$\therefore r = 25$$

$$\therefore h = \frac{7}{5} \times 25 = 35$$

$\therefore$ Radius = 25 cm, Height = 35 cm

2. A solid iron cylinder has total surface area of 1848 sq.m. Its curved surface area is five – sixth of its total surface area. Find the radius and height of the iron cylinder.

Solution :

Given total surface area of cylinder

$= 1848\ m^2$ &

$$CSA = \frac{5}{6}(TSA)$$

$$\Rightarrow 2\pi rh = \frac{5}{6} \times 1848$$

$$= 5 \times 308$$

$$2\pi rh = 1540 \quad \text{..........} (1)$$

$$2\pi r\,(h + r) = 1848$$

$$\Rightarrow 2\pi rh + 2\pi r^2 = 1848$$

$$\Rightarrow 1540 + 2\pi r^2 = 1848$$

$$\Rightarrow 2\pi r^2 = 308$$

$$\Rightarrow 2 \times \frac{22}{7} \times r^2 = 308$$

$$\Rightarrow r^2 = \frac{\overset{7}{\cancel{\overset{14}{\cancel{308}}}} \times 7}{\cancel{2} \times \underset{1}{\cancel{22}}}$$

$$\Rightarrow r^2 = 49$$

$$r = 7\ m$$

Sub $r = 7$ in (1)

$$2 \times \frac{22}{7} \times 7 \times h = 1540$$

$$\Rightarrow h = \frac{1540}{2 \times 22}$$

$$h = 35$$

$\therefore$ Radius = 7 m, Height = 35 m.

3. The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A.

Solution :

Given, external radius of

hollow cylinder = 16 cm = R

length of log = 13 cm = R

thickness = 4 cm = t

$$\therefore \quad t = R - r$$

$$\therefore \quad r = R - t$$

$$= 16 - 4$$

$$r = 12$$

$\therefore$ TSA of hollow cylinder

$$= 2\pi\,(R + r)\,(R - r + h)$$

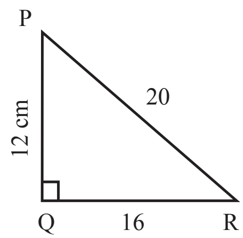
$$= 2 \times \frac{22}{7}(28)\,(4 + 13)$$

$$= 44 \times 4 \times 17$$

$$= 2992\ cm^2$$

4. A right angled triangle PQR where $\angle Q = 90^0$ is rotated about QR and PQ. If QR = 16 cm and PR = 20 cm, compare the curved surface areas of the right circular cones so formed by the triangle.

Solution :



$\therefore PQ = \sqrt{400 - 256}$

$= \sqrt{144} = 12$

i) When the Δ is rotated about PQ,

h = 12 cm, r = 16 cm

$\therefore$ CSA of cone $= \pi r l$

$= \pi \times 16 \times 20$

$= 320\pi$ cm^2

ii) When the Δ is rotated about QR,

h = 16 cm, r = 12 cm

$\therefore$ CSA of cone $= \pi r l$

$= \pi \times 16 \times 12$

$= 192\pi$ cm^2

$\therefore$ CSA of the cone when rotated about PQ is larger than that of QR.

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5. 4 persons live in a conical tent whose slant height is 19 cm. If each person require 22 cm² of the floor area, then find the height of the tent.

Solution :

Given slant height of the cone l = 19 cm

Total floor area of 4 persons = 88 cm^2

$\Rightarrow \quad \pi r^2 = 88$

$\Rightarrow \quad \frac{22}{7} \times r^2 = 88$

$\Rightarrow \quad r^2 = 28$

$\therefore h = \sqrt{l^2 - r^2}$

$= \sqrt{19^2 - 28}$

$= \sqrt{361 - 28}$

$= \sqrt{333}$

height of cone $\simeq$ 18.25 cm.

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6. A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is 5720 cm², how many caps can be made with radius 5 cm and height 12 cm.

Solution :

Given r = 5 cm, h = 12 cm in a cone

$\therefore l = \sqrt{h^2 + r^2}$

$= \sqrt{144 + 25}$

$= \sqrt{169}$

$= 13$

$\therefore$ CSA of cone $= \pi r l$

$= \frac{22}{7} \times 5 \times 13$

$= \frac{110 \times 13}{7}$ cm^2

Given, area of sheet of paper = 5720 cm^2

$\therefore \text{Number of caps} = \frac{5720 \times 7}{110 \times 3}$

= 28 caps

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7. The ratio of the radii of two right circular cones of same height is 1 : 3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.

Solution :

Given $r_1 : r_2 = 1 : 3$ $h_1 = 3r_1, h_2 = 3r_1$

Let $r_1 = x$ $= 3x$, $= 3x$

$r_2 = 3x$

$$l_1 = \sqrt{h_1^2 + r_1^2}\ ,\quad l_2 = \sqrt{h_2^2 + r_2^2}$$

$$= \sqrt{9x^2 + x^2} \qquad = \sqrt{9x^2 + 9x^2}$$

$$= \sqrt{10}x \qquad = 3\sqrt{2}x$$

∴ Ratio of their CSA

$$= \frac{\pi r_1 l_1}{\pi r_2 l_2}$$

$$= \frac{1}{3} \times \frac{\sqrt{10}}{3\sqrt{2}} = \frac{\sqrt{5}}{9}$$

∴ Ratio of their CSA $= \sqrt{5} : 9$

8. The radius of a sphere increases by 25%. Find the percentage increase in its surface area.

Solution :

Le 'r' be the original radius of sphere

∴ Its surface area = $4\pi r^2$

If the radius increases by 25%

$$\text{New radius} = r + \frac{25}{100}r$$

$$= r + \frac{1}{4}r$$

$$= \frac{5}{4}r$$

$$\text{New surface area} = 4\pi\left(\frac{5}{4}r\right)^2$$

$$= 4\pi \times \frac{25}{16}r^2$$

$$= \frac{25\pi r^2}{4}$$

$$\therefore \text{Increment in SA} = \frac{25\pi r^2}{4} - 4\pi r^2$$

$$= \frac{9\pi r^2}{4}$$

$$\therefore \text{Percentage inc. in SA} = \frac{\frac{9\pi r^2}{4}}{4\pi r^2} \times 100$$

$$= \frac{9}{16} \times 100$$

$$= \frac{225}{4}$$

$$= 56.25\%$$

9. The internal and external diameters of a hollow hemispherical vessel are 20 cm and 28 cm respectively. Find the cost to paint the vessel all over at ₹ 0.14 per cm².

Solution :

Given in a hollow hemisphere

D = 28 cm,   d = 20 cm

⇒ R = 14 cm,   r = 10 cm

∴ TSA of hollow hemisphere

$$= \pi(3R^2 + r^2)$$

$$= \frac{22}{7}(588 + 100)$$

$$= \frac{22}{7} \times 688\ cm^2$$

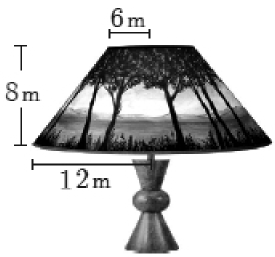
Given cost of painting = 0.14 / cm²

∴ Total cost of painting

$$= \frac{22}{7} \times 688 \times 0.14$$

= ₹ 302.72

10. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2.



Solution :

Given in a frustum shaped lamp

R = 12m, r = 6m, h = 8m

$l = \sqrt{(R-r)^2 + h^2}$

$= \sqrt{36+64}$

$= \sqrt{100} = 10$ cm

$\therefore$ Required portion to be painted =

CSA of frustum + πr^2

$= \pi\,(R+r)\,l + \pi r^2$

$= \pi\,[18(10) + 36]$

$= \frac{22}{7} \times 216$

$= \frac{4752}{7}$

$= 678.86\ m^2$

Given cost of painting = Rs.2/m^2

$\therefore$ Total cost = 678.86 × 2

= Rs.1357.72/-

II. VOLUME

Key Points

Right Circular and Hollow Cylinder

✓ Volume of a cylinder = $\pi r^2 h$ cu. units.

✓ Volume of a hollow cylinder = $\pi(R^2 - r^2)h$ cu. units.

✓ Volume of a cone = $\frac{1}{3}\pi r^2 h$ cu. units.

Sphere and Hemi-sphere

✓ Volume of a sphere = $\frac{4}{3}\pi r^3$ cu. units.

✓ Volume of a hollow sphere = $\frac{4}{3}\pi(R^3 - r^3)$ cu. units.

✓ Volume of a solid hemisphere = $\frac{2}{3}\pi r^3$ cu. units.

✓ Volume of a hollow hemisphere = $\frac{2}{3}\pi(R^3 - r^3)$

Frustum of a Cone

✓ Volume of a frustum = $\frac{\pi h}{3}(R^2 + Rr + r^2)$ cu. units.

Example 7.15

Find the volume of a cylinder whose height is 2 m and whose base area is 250 m^2.

Solution :

Let r and h be the radius and height of the cylinder respectively.

Given that, height h = 2 m,

base area = 250 m^2

Now, volume of a cylinder = $\pi r^2 h$ cu. units

= base area × h

= 250 × 2 = 500 m^3

Therefore, volume of the cylinder = 500 m^3

Example 7.16

The volume of a cylindrical water tank is 1.078×10^6 litres. If the diameter of the tank is 7 m, find its height.

Solution :

Let r and h be the radius and height of the cylinder respectively.

Given that, volume of the tank

= 1.078×10^6 = 1078000 litre

$$= 1078\ m^3 \quad \left(\text{since } 1\, l = \frac{1}{1000}\ m^3\right)$$

diameter = 7m gives radius = $\frac{7}{2}$ m

volume of the tank = $\pi r^2 h$ cu. units

$$1078 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h$$

Therefore, height of the tank is 28 m.

Example 7.17

Find the volume of the iron used to make a hollow cylinder of height 9 cm and whose internal and external radii are 21 cm and 28 cm respectively.

Solution :

Let r, R and h be the internal radius, external radius and height of the hollow cylinder respectively.

Given that, r = 21cm, R = 28 cm, h = 9 cm

Now, volume of hollow cylinder

= $\pi(R^2 - r^2)$ h cu. units

$$= \frac{22}{7}(28^2 - 21^2) \times 9$$

$$= \frac{22}{7}(784 - 441) \times 9 = 9702$$

Therefore, volume of iron used = 9702 cm^3.

Example 7.18

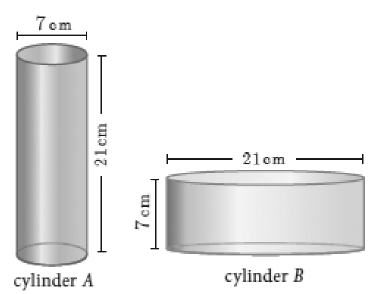
For the cylinders A and B

(i) find out the cylinder whose volume is greater.

(ii) verify whether the cylinder with greater volume has greater total surface area.

(iii) find the ratios of the volumes of the cylinders A and B.

Solution :



(i) Volume of cylinder = $\pi r^2 h$ cu. units

Volume of cylinder A

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 21$$
$$= 808.5 \text{ cm}^3$$

Volume of cylinder B

$$= \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 7$$
$$= 2425.5 \text{ cm}^3$$

Therefore, volume of cylinder B is greater than volume of cylinder A.

(ii) T.S.A. of cylinder = $2\pi r (h + r)$ sq. units

T.S.A. of cylinder

$$A = 2 \times \frac{22}{7} \times \frac{7}{2} \times (21 + 3.5) = 539 \text{ cm}^2$$

T.S.A. of cylinder

$$B = 2 \times \frac{22}{7} \times \frac{21}{2} \times (7 + 10.5) = 1155 \text{ cm}^2$$

Hence verified that cylinder B with greater volume has a greater surface area.

(*iii*) $\dfrac{\text{Volume of cylinder A}}{\text{Volume of cylinder B}} = \dfrac{808.5}{2425.5} = \dfrac{1}{3}$

Therefore, ratio of the volumes of cylinders A and B is 1:3.

Example 7.19

The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone.

Solution :

Let r and h be the radius and height of the cone respectively.

Given that,

Volume of the cone = 11088 cm^3

$$\frac{1}{3}\pi r^2 h = 11088$$
$$\frac{1}{3} \times \frac{22}{7} \times r^2 \times 24 = 11088$$
$$r^2 = 441$$

Therefore, radius of the cone r = 21 cm

Example 7.20

The ratio of the volumes of two cones is 2 : 3. Find the ratio of their radii if the height of second cone is double the height of the first..

Solution :

Let r_1 and h_1 be the radius and height of the cone-I and let r_2 and h_2 be the radius and height of the cone-II.

$$\text{Given } h_2 = 2h_1 \text{ and } \frac{\text{Volume of the cone I}}{\text{Volume of the cone II}} = \frac{2}{3}$$

$$\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{2}{3}$$

$$\frac{r_1^2}{r_2^2} \times \frac{h_1}{2h_1} = \frac{2}{3}$$

$$\frac{r_1^2}{r_2^2} = \frac{4}{3} \text{ gives } \frac{r_1}{r_2} = \frac{2}{\sqrt{3}}$$

Therefore, ratio of their radii = 2 : $\sqrt{3}$

Example 7.21

The volume of a solid hemisphere is 29106 cm^3. Another hemisphere whose volume is two-third of the above is carved out. Find the radius of the new hemisphere.

Solution :

Let r be the radius of the hemisphere.
Given that, volume of the hemisphere

$= 29106\ cm^3$

Now, volume of new hemisphere

$$= \frac{2}{3}\text{(Volume of original sphere)}$$

$$= \frac{2}{3} \times 29106$$

Volume of new hemisphere = 19404 cm^3

$$\frac{2}{3}\pi r^3 = 19404$$

$$r^3 = \frac{19404 \times 3 \times 7}{2 \times 22} = 9261$$

$$r = \sqrt[3]{9261} = 21\,\text{cm}$$

Therefore, r = 21cm.

Example 7.22

Calculate the weight of a hollow brass sphere if the inner diameter is 14 cm and thickness is 1mm, and whose density is 17.3 g/ cm^3.

Solution :

Let r and R be the inner and outer radii of the hollow sphere.

Given that, inner diameter d = 14 cm; inner radius r = 7 cm ;

$$\text{thickness} = 1\text{ mm} = \frac{1}{10}\text{ cm}$$

$$\text{Outer radius R} = 7 + \frac{1}{10} = \frac{71}{10} = 7.1\,\text{cm}$$

$$\text{Volume of hollow sphere} = \frac{4}{3}\pi\ (R^3 - r^3)\text{ cu.cm}$$

$$= \frac{4}{3} \times \frac{22}{7}(357.91 - 343)$$

$$= 62.48\,\text{cm}^3$$

But, weight of brass in 1 cm^3 = 17.3 gm

Total weight = 17.3 × 62.48 = 1080.90 gm

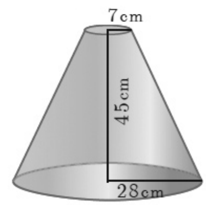
Therefore, total weight is 1080.90 grams.

Example 7.23

If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum.

Solution :

Let h, r and R be the height, top and bottom radii of the frustum.



Given that, h = 45 cm, R = 28 cm, r = 7 cm

Now,

$$\text{Volume} = \frac{1}{3}\pi\,[R^2 + Rr + r^2]\,h\text{ cu.units}$$

$$= \frac{1}{3} \times \frac{22}{7} \times [28^2 + (28 \times 7) + 7^2] \times 45$$

$$= \frac{1}{3} \times \frac{22}{7} \times 1029 \times 45 = 48510$$

Therefore, volume of the frustum is 48510 cm^3

EXERCISE 7.2

1. **A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.**

Solution :

Given radius of well, r = 5m

height of well, h = 14m

∴ Volume of earth taken out = $\pi r^2 h$

$$= \frac{22}{7} \times 25 \times 14$$

$$= 1100\ m^3$$

Since it is spread to form embankment which is the form of hollow cylinder, Inner radius = 5 cm,

Outer radius = 5 + 5 (Given width = 5m)

= 10 m

$$\therefore \quad \pi h_1 (R^2 - r^2) = 1100$$

$$\Rightarrow \frac{22}{7} \times h_1 (10^2 - 5^2) = 1100$$

$$\Rightarrow \quad \frac{22}{7} \times h_1 (75) = 1100$$

$$\Rightarrow \quad h_1 = \frac{1100 \times 7}{22 \times 75}$$

$$h_1 = 4.67\ m$$

$\therefore$ height of embankment = 4.67 m

2. A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass?

Solution :

Cylindrical Glass	Cylindrical Metal
R = 10 cm	r = 5 cm
H = 9 cm	h = 4 cm

When cylindrical metal is immersed completely in glass,

Total volume = Vol. of glass

+ Vol. of metal

$= \pi[100 \times 9 + 25 \times 4] = 1000\pi\ cm^3$

Let h_1 be the height of water level

$\therefore \pi R^2 h_1 = 1000\pi$

$\Rightarrow \quad 100\ h_1 = 1000$

$\therefore \quad h_1 = 10$ cm

$\therefore$ Rise in water level $= h_1 - H$

$= - 10 - 9 = 1$ cm

3. If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm.

Solution :

Given circumference of a cone = 484 cm

ie $\quad 2\pi r = 484$

$$\Rightarrow 2 \times \frac{22}{7} \times r = 484$$

$$\Rightarrow \quad r = \frac{484 \times 7}{2 \times 22}$$

$$r = 77 \text{ cm}$$

Also given h = 105 cm

$$\therefore \text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{\not{3}} \times \frac{22}{\not{7}} \times 77 \times 77 \times \overset{5}{\not{105}}$$

$$= 652190\ cm^3$$

4. A conical container is fully filled with petrol. The radius is 10m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu. meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute.

Solution :

Given r = 10m, h = 15m in a cone

$$\therefore \text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{\not{3}} \times \frac{22}{\not{7}} \times 100 \times \overset{5}{\not{15}}$$

$$= \frac{11000}{7}\ cm^3$$

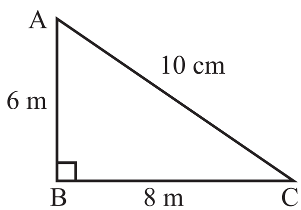
Rate of releasing the petrol = 25 cm^3/min

$\therefore$ Total time taken to empty the can

$$= \frac{11000}{7 \times 25}$$
$$= \frac{440}{7}$$
$$= 62.851 \text{ min}.$$
$$\simeq 63 \text{ min}.$$

5. A right angled triangle whose sides are 6 cm, 8 cm and 10 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two solids so formed.

Solution :



i) When it revolves about AB = 6 cm

h = 6 cm, r = 8 cm

$\therefore$ Volume of the cone

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 64 \times 6$$
$$= \frac{22 \times 64 \times 2}{7}$$
$$= \frac{2816}{7} \text{ cm}^3$$

i) When it revolves about BC = 8 cm

h = 8 cm, r = 6 cm

$\therefore$ Volume of the cone

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 36 \times 8$$
$$= \frac{22 \times 12 \times 8}{7}$$
$$= \frac{2112}{7} \text{ cm}^3$$

$\therefore$ Difference in volumes

$$= \frac{2816}{7} - \frac{2112}{7}$$
$$= \frac{704}{7}$$
$$= 100.58 \text{ cm}^3$$

6. The volumes of two cones of same base radius are 3600 cm³ and 5040 cm³. Find the ratio of heights.

Solution :

Given volumes of 2 cones

= 3600 cm^3 & 5040 cm^3

& base radius are equal

$$\therefore \text{Ratio of volumes} = \frac{V_1}{V_2} = \frac{3600}{5040}$$
$$\Rightarrow \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{3600}{5040}$$
$$\Rightarrow \frac{h_1}{h_2} = \frac{40}{56}$$
$$= \frac{5}{7}$$

$\therefore h_1 : h_2 = 5 : 7$

7. If the ratio of radii of two spheres is 4 : 7, find the ratio of their volumes.

Solution :

Given ratio of radii of 2 spheres = 4 : 7

$$ie \frac{r_1}{r_2} = \frac{4}{7}$$

$$\therefore \text{Ratio of their volumes} = \frac{V_1}{V_2}$$

$$= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$$

$$= \left(\frac{r_1}{r_2}\right)^3$$

$$= \left(\frac{4}{7}\right)^3$$

$$= \frac{64}{343}$$

$\therefore$ Ratio of the volumes = 64 : 343

8. A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3}$: 4.

Solution :

Given TSA of a solid sphere

= TSA of a solid hemisphere

$$\Rightarrow \quad 4\pi R^2 = 3\pi r^2$$

$$\Rightarrow \quad \therefore \frac{R^2}{r^2} = \frac{3}{4}$$

$$\therefore \frac{R}{r} = \frac{\sqrt{3}}{2}$$

$$\therefore \text{Ratio of their volumes} = \frac{\frac{4}{3}\pi R^3}{\frac{2}{3}\pi r^3}$$

$$= \frac{2R^3}{r^3}$$

$$= 2\left[\frac{R}{r}\right]^3$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= 2 \times \frac{3\sqrt{3}}{8}$$

$$= \frac{3\sqrt{3}}{4}$$

$\therefore$ Ratio of the volumes $= 3\sqrt{3} : 4$

9. The outer and the inner surface areas of a spherical copper shell are 576π cm² and 324π cm² respectively. Find the volume of the material required to make the shell.

Solution :

Given $4\pi R^2 = 576\pi$	$4\pi r^2 = 324\pi$
$R^2 = 144$	$r^2 = 81$
R = 12 cm	r = 9 cm

$\therefore$ Volume of the material

$$= \frac{4}{3}\pi\left(R^3 - r^3\right)$$

$$= \frac{4}{3} \times \frac{22}{7}(1728 - 729)$$

$$= \frac{4}{3} \times \frac{22}{7} \times 999$$

$$= \frac{88 \times 333}{7} = 4186.29 \text{ cm}^3$$

10. A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of ₹ 40 per litre.

Solution :

Given R = 20cm, r = 8cm, h = 16cm

in frustum of a cone

$\therefore$ Volume of frustum of a cone

$$= \frac{\pi h}{3}(R^2 + Rr + r^2)$$

$$= \frac{22}{7} \times \frac{16}{3}(400 + 160 + 64)$$

$$= \frac{22}{7} \times \frac{16}{3} \times \overset{208}{624}$$

$$= \frac{73216}{7}$$

$$= 10,459.42 \text{ cm}^3$$

$$= \frac{10459.42}{1000} \text{ litres}$$

$$= 10.459 \text{ litres}$$

$\therefore$ Cost of milk at ₹ 40 / lr,

= 10.459 × 40

= ₹ 418.36

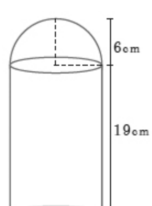
III. Volume and Surface Area of Combined Solids

Example 7.24

A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm. Find the total surface area of the toy if its common diameter is 12 cm.

Solution :

Let r and h be the radius and height of the cylinder respectively.



Given that, diameter d = 12 cm, radius r = 6 cm

Total height of the toy is 25 cm

Therefore, height of the cylindrical portion

= 25 − 6 = 19 cm

T.S.A. of the toy = C.S.A. of the cylinder + C.S.A. of the hemisphere + Base Area of the cylinder

$$= 2\pi rh + 2\pi r^2 + \pi r^2$$

$$= \pi r(2h + 3r) \text{ sq.units}$$

$$= \frac{22}{7} \times 6 \times (38 + 18)$$

$$= \frac{22}{7} \times 6 \times 56 = 1056$$

Therefore, T.S.A. of the toy is 1056 cm^2.

Example 7.25

A jewel box (Fig. 7.39) is in the shape of a cuboid of dimensions 30 cm × 15 cm × 10 cm surmounted by a half part of a cylinder as shown in the figure. Find the volume and T.S.A. of the box.

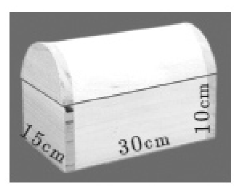


Fig. 7.39

Solution :

Let l, b and h_1 be the length, breadth and height of the cuboid. Also let us take r and h_2 be the radius and height of the cylinder.

Now, Volume of the box =

Volume of the cuboid +

$\frac{1}{2}$ (Volume of cylinder)

$$= (l \times b \times h_1) + \frac{1}{2}(\pi r^2 h_2) \text{ cu. units}$$

$$= (30 \times 15 \times 10) + \frac{1}{2}\left(\frac{22}{7} \times \frac{15}{2} \times \frac{15}{2} \times 30\right)$$

$$= 4500 + 2651.79 = 7151.79$$

Therefore, Volume of the box = 7151.79 cm^3

Now, T.S.A. of the box = C.S.A. of the cuboid + $\frac{1}{2}$(C.S.A. of the cylinder)

$$= 2(l + b)\, h_1 + \frac{1}{2}(2\pi r h_2)$$

$$= 2\,(45 \times 10) + \left(\frac{22}{7} \times \frac{15}{2} \times 30\right)$$

$$= 900 + 707.14 = 1607.14$$

Therefore, T.S.A. of the box = 1607.14 cm^2

Example 7.26

Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq. m of the space on ground and 40 cu. meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m?.

Solution :

Let h_1 and h_2 be the height of cylinder and cone respectively.

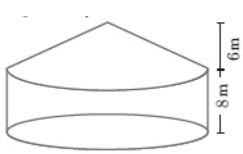


Fig. 7.40

Area for one person = 4 sq. m

Total number of persons = 150

Therefore total base area = 150 × 4

πr^2 = 600

$$r^2 = 600 \times \frac{7}{22} = \frac{2100}{11} \quad ....(1)$$

Volume of air required for 1 person = 40 m^3

Total Volume of air required for 150 persons

= 150 × 40 = 6000 m^3

$$\pi r^2 h_1 + \frac{1}{3}\pi r_2 h_2 = 6000$$

$$\pi r^2\left(h_1 + \frac{1}{3}h_2\right) = 6000$$

$$\frac{22}{7} \times \frac{2100}{11}\left(8 + \frac{1}{3}h_2\right) = 6000 \quad \text{[using (1)]}$$

$$8 + \frac{1}{3}h_2 = \frac{6000 \times 7 \times 11}{22 \times 2100}$$

$$\frac{1}{3}h_2 = 10 - 8 = 2$$

Therefore, the height of the conical tent h_2 is 6 m

Example 7.27

A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface area of the funnel.

Solution :

Let R, r be the top and bottom radii of the frustum.

Let h_1, h_2 be the heights of the frustum and cylinder respectively.

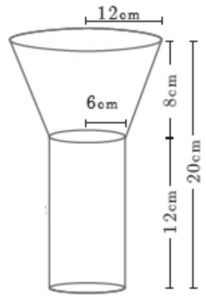


Fig. 7.41

Given that, R = 12 cm, r = 6 cm, h_2 = 12 cm

Now, h_1 = 20 – 12 = 8 cm

Here, Slant height of the frustum

$$l = \sqrt{(R-r)^2 + h_1^2} \text{ units}$$

$$= \sqrt{36+64}$$

$$l = 10 \text{ cm}$$

Outer surface area

$= 2\pi r h_2 + \pi(R + r)\, l$ sq. units

$= \pi[2rh_2 + (R + r)\, l]$

$= \pi[(2 \times 6 \times 12) + (18 \times 10)]$

$= \pi[144 + 180]$

$$= \frac{22}{7} \times 324 = 1018.28$$

Therefore, outer surface area of the funnel is 1018.28 cm^2

Example 7.28

A hemispherical section is cut out from one face of a cubical block (Fig.7.42) such that the diameter l of the hemisphere is equal to side length of the cube. Determine the surface area of the remaining solid.

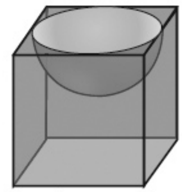


Fig. 7.42

Solution :

Let r be the radius of the hemisphere.

Given that, diameter of the hemisphere = side of the cube = l

Radius of the hemisphere = $\frac{l}{2}$

TSA of the remaining solid =

Surface area of the cubical part +
C.S.A. of the hemispherical part −
Area of the base of the hemispherical part

$= 6 \times (\text{Edge})^2 + 2\pi r^2 - \pi r^2$

$= 6 \times (\text{Edge})^2 + \pi r^2$

$$= 6 \times (l)^2 + \pi\left(\frac{l}{2}\right)^2 = \frac{1}{4}(24+\pi)\, l^2$$

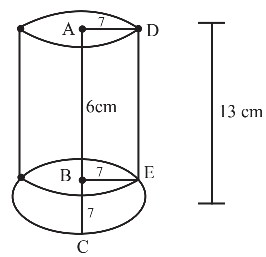
Total surface area of the remaining solid =

$$\frac{1}{4}(24+\pi)\, l^2 \text{ sq.units}$$

EXERCISE 7.3

1. **A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel.**

Solution:



Given AD = BE = 7 cm = radius of the vessel = BC

AC = 13 cm = height of the vessel

$\therefore$ AB = 13 – 7

= 6cm = height of cylindrical part

$\therefore$ Capacity of the vessel

= Capacity of cylinder + Capacity of HS

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi r^2 \left[h + \frac{2}{3} r \right]$$

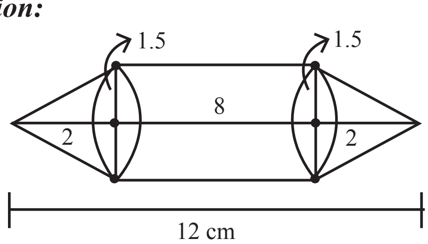
$$= \frac{22}{7} \times 49 \left[6 + \frac{14}{3} \right]$$

$$= 154 \times \frac{32}{3}$$

$$= 1642.67 \text{ cm}^3$$

2. Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.

Solution:



Cone	**Cylinder**
h = 2cm	H = 8cm
$r = 1.5 \text{ cm} = 3/2$	$r = 1.5 \text{ cm} = 3/2$

$\therefore$ Volume of the model =

2 (Vol. of Cone) + Vol. of Cylinder

$$= \frac{2}{3}\pi r^2 h + \pi r^2 H$$

$$= \pi r^2 \left[\frac{2h}{3} + H \right]$$

$$= \frac{22}{7} \times \frac{9}{4} \left[\frac{4}{3} + 8 \right]$$

$$= \frac{11 \times 9}{7 \times 2} \left[\frac{28}{3} \right]$$

$$= \frac{11 \times 3 \times \overset{2}{\cancel{14}}}{\cancel{7}}$$

$$= 66 \text{ cm}^3$$

3. From a solid cylinder whose height is 2.4 cm and the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm^3.

Solution:

Volume of the remaining solid

= Vol. of Cylinder – Vol. of Cone

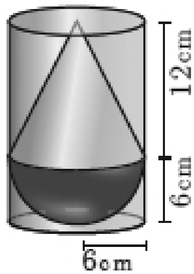
$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times 0.7 \times 0.7 \times 2.4$$

$$= 2.46 \text{ cm}^3$$

4. A solid consisting of a right circular cone of height 12 cm and radius 6 cm standing on a hemisphere of radius 6 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water displaced out of the cylinder, if the radius of the cylinder is 6 cm and height is 18 cm.



Solution:

Cone	Hemisphere	Cylinder
r = 6cm	r = 6cm	r = 6cm
h = 12cm		H = 18cm

Volume of water displaced out of cylinder =

Volumme of cone + Volume of HS

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 [h + 2r]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 36(12 + 12)$$

$$= \frac{22}{7} \times 12 \times 24$$

$$= 905.14\text{cm}^3$$

Note : When the conical hemisphere is completely submerged in water inside the cylinder,

Volume of water left in the cylinder.

= Volume of cylinder – [Volume of cone + Vol. of Hemi sphere)

$$= \pi r^2 H - \left[\frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3\right]$$

$$= \pi\left[36 \times 18 - \frac{1}{3} \times 36 \times 12 - \frac{2}{3} \times 216\right]$$

$$= \frac{22}{7}[648 - 144 - 144]$$

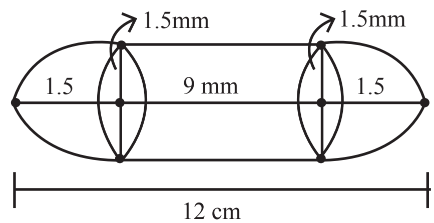
$$= \frac{22}{7} \times 360$$

$$= \frac{7920}{7}$$

$$= 1131.42\text{ cm}^3$$

5. A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold?

Solution:



Cylinder	Hemisphere
H = 9 mm	r = 1.5 mm = $3/2$
r = 1.5 mm = $3/2$	

∴ Volume of the Capsule =

Vol. of Cylinder + 2 (Vol. of hemisphere)

$$= \pi r^2 H + 2\left(\frac{2}{3}\pi r^3\right)$$

$$= \frac{22}{7}\left[\frac{9}{4} \times 9 + \frac{4}{3} \times \frac{27}{8}\right]$$

$$= \frac{22}{7}\left[\frac{81}{4} + \frac{9}{2}\right]$$

$$= \frac{22}{7}\left[\frac{81 + 18}{4}\right]$$

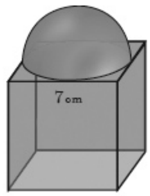
$= \frac{22 \times 99}{28}$

$= \frac{11 \times 99}{14}$

$= 77.78 \text{ mm}^3$

---

6. As shown in figure a cubical block of side 7 cm is surmounted by a hemisphere. Find the surface area of the solid.



Solution:

Given side of cube = 7 cm

radius of hemisphere = $7/2$ cm

Surface area of the solid = CSA of cube + CSA of hemisphere – Base area of HS

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6(49) + \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{2}$

$= 294 + \frac{77}{2}$

$= 294 + 38.5$

$= 332.5 \text{ cm}^2$

---

7. A right circular cylinder just enclose a sphere of radius r units. Calculate

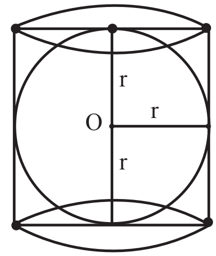
(i) the surface area of the sphere

(ii) the curved surface area of the cylinder

(iii) the ratio of the areas obtained in (i) and (ii).

Solution:

From the given fig, it is clear that h = 2r



i) Surface area of sphere

$= 4\pi r^2$ sq.units

ii) CSA of the cylinder $= 2\pi rh$

$= 2\pi r\,(2r)$

$= 4\pi r^2$ sq. units

iii) Ratio of areas obtained in (i) & (ii)

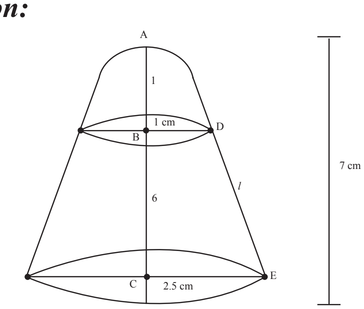
$= 4\pi r^2 : 4\pi r^2$

$= 1 : 1$

---

8. A shuttle cock used for playing badminton has the shape of a frustum of a cone is mounted on a hemisphere. The diameters of the frustum are 5 cm and 2 cm. The height of the entire shuttle cock is 7 cm. Find its external surface area.

Solution:



AB = BD = radius of hemisphere = 1 cm

radius of frustum = r

AC = 7cm = Total length of cock

$\therefore$ BC = 7 – 1 = 6 cm = height of frustum

CE = 2.5 cm = R

$$\therefore l = \sqrt{h^2 + (R-r)^2} = \sqrt{26 + (1.5)^2} = 6.18$$

$\therefore$ External Surface Area =

CSA of Frustum + CSA of HS

$$= \pi\,(R+r)l + 2\pi r^2$$
$$= \pi\,[(2.5+1)\,6.18 + 2\times 1]$$
$$= \frac{22}{7}\left[\frac{7}{2}(6.1)+2\right]$$
$$= \frac{22}{7}[21.35+2]$$
$$= \frac{22\times 23.35}{7}$$
$$= \frac{513.7}{7}$$
$$= 73.39 \text{ cm}^2$$

IV. Conversion of Solids

Example 7.29

A metallic sphere of radius 16 cm is melted and recast into small spheres each of radius 2 cm. How many small spheres can be obtained?

Solution :

Let the number of small spheres obtained be n.
Let r be the radius of each small sphere and R be the radius of metallic sphere.
Here, R = 16 cm, r = 2 cm
Now, n´(Volume of a small sphere)
= Volume of big metallic sphere

$$n\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$
$$n\left(\frac{4}{3}\pi\times 2^3\right) = \frac{4}{3}\pi\times 16^3$$

8n = 4096 gives n = 512

Therefore, there will be 512 small spheres.

Example 7.30

A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder.

Solution :

Let h_1 and h_2 be the heights of a cone and cylinder respectively.

Also, let r be the radius of the cone.

Given that, height of the cone h_1 = 24 cm; radius of the cone and cylinder r = 6 cm

Since, Volume of cylinder = Volume of cone

$$\pi^2 h_2 = \frac{1}{3}\pi r^2 h_1$$
$$h_2 = \frac{1}{3}\times h_1 \text{ gives } h_2 = \frac{1}{3}\times 24 = 8$$

Therefore, height of cylinder is 8 cm.

Example 7.31

A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container.

Solution :

Let h and r be the height and radius of the cylinder respectively.

Given that, h = 15 cm, r = 6 cm

Volume of the container V = $\pi r^2 h$ cubic units.

$$= \frac{22}{7}\times 6\times 6\times 15$$

Let, r_1 = 3 cm, h_1 = 9 cm be the radius and height of the cone.

Also, r_1 = 3 cm is the radius of the hemispherical cap.

Volume of one ice cream cone = (Volume of the cone + Volume of the hemispherical cap)

$$= \frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3$$

$$= \frac{1}{3}\times\frac{22}{7}\times 3\times 3\times 9 + \frac{2}{3}\times\frac{22}{7}\times 3\times 3\times 3$$

$$= \frac{22}{7}\times 9(3+2) = \frac{22}{7}\times 45$$

Number of cones =

$$\frac{\text{Volume of the cylinder}}{\text{Volume of one ice cream cone}}$$

Number of ice cream cones needed =

$$\frac{\frac{22}{7}\times 6\times 6\times 15}{\frac{22}{7}\times 45} = 12$$

Thus 12 ice cream cones are required to empty the cylindrical container.

EXERCISE 7.4

1. An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder.

Solution:

Given radius of sphere = 12 cm = R &

radius of cylinder = 8 cm = r

By the data given,

Volume of sphere = Volume of Cylinder

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = \pi r^2 h$$

$$\Rightarrow \quad \frac{4}{3}\times 12\times 12\times 12 = 8\times 8\times h$$

$$\Rightarrow \quad h = 36 \text{ cm}$$

$\therefore$ Height of the cylinder = 36 cm

2. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm.

Solution:

Cylindrical Pipe

Given, Speed of water in the pipe

= 15 Km/hr

H = 15000 m

Radius of pipe r = 7 cm = $\frac{7}{100}$ m

Rectangular Tank

$l = 50$ m $\quad b = 44\ m$

$$h = 21\text{cm} = \frac{21}{100} m$$

$$\therefore \text{Required time} = \frac{\text{Volume of tank}}{\text{Volume of pipe}}$$

$$= \frac{lbh}{\pi r^2 H}$$

$$= \frac{50\times 44\times {}^{21}\!/_{100}}{\frac{22}{7}\times\frac{7}{100}\times\frac{7}{100}\times 15000}$$

= 2 hrs

3. A conical flask is full of water. The flask has base radius r units and height h units, the water poured into a cylindrical flask of base radius xr units. Find the height of water in the cylindrical flask.

Solution:

By the data given,

Volume of Cylindrical Flask =

Volume of Conical Flask

$$\Rightarrow \quad \pi(xr)^2 H = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad x^2 r^2 H = \frac{1}{3} r^2 h$$

$$\Rightarrow \quad H = \frac{h}{3x^2}$$

$\therefore$ Height of the Cylindrical Flask = $\frac{h}{3x^2}$ cm

4. A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, find the internal diameter.

Solution:

Right Circular Cone	**Hollow Sphere**
r = 7 cm	R = 5 cm
h = 8 cm	r = ?

By the problem,

Volume of Hollow Sphere = Vol. of Right Circular Cone

$$\Rightarrow \frac{4}{3}\pi\left(R^3 - r^3\right) = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow 4\,(125 - r^3) = 49 \times 8$$

$$\Rightarrow 125 - r^3 = 49 \times 2$$

$$\Rightarrow r^3 = 125 - 98$$

$$r^3 = 27$$

$$\therefore \quad r = 3$$

$\therefore$ Internal diameter of hollow sphere = 6 cm

5. Seenu's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid. The sump has dimensions 2 m × 1.5 m × 1 m. The overhead tank has its radius of 60 cm and height 105 cm. Find the volume of the water left in the sump after the overhead tank has been completely filled with water from the sump which has been full, initially.

Solution:

Over head tank	**Sump**
(Cylinder)	(Cuboid)
R = 60 cm	l = 2 m = 200 cm
H = 105 cm	b = 1.5 m = 150 cm
	h = 1 m = 100 cm

Volume of water left

= Volume of Sump – Volume of tank

= $l$bh – $\pi R^2 H$

$$= 200 \times 150 \times 100 - \frac{22}{\cancel{7}} \times 60 \times 60 \times \overset{15}{\cancel{105}}$$

$$= 3000000 - 1188000$$

$$= 2812000 \text{ cm}^3$$

6. The internal and external diameter of a hollow hemispherical shell are 6 cm and 10 cm respectively. If it is melted and re-cast into a solid cylinder of diameter 14 cm, then find the height of the cylinder.

Solution:

Hollow Hemisphere	**Solid Cylinder**
R = 5 cm	r = 7 cm
r = 3 cm	h = ?

$\therefore$ By the problem given,

Volume of Solid Cylinder = Volume of Hollow Hemisphere

$$\Rightarrow \quad \pi r^2 h = \frac{2}{3}\pi\,(R^3 - r^3)$$

$$\Rightarrow 49 \times h = \frac{2}{3}(125 - 27)$$

$$\Rightarrow \quad h = \frac{2}{3} \times \frac{98}{49}$$

$$\therefore \; h = \frac{4}{3} = 1.33\ cm$$

$\therefore$ Height of Solid Cylinder = 1.33 cm

7. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, then find the thickness of the cylinder.

Solution:

Solid Sphere	**Hollow Cylinder**
r = 6 cm	R = 5 cm
	H = 32 cm
	t = ?

By the problem given,

Volume of Hollow Cylinder = Volume of Solid Sphere

$$\Rightarrow \quad \pi(R^2 - r^2)\,H = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad (25 - r^2)32 = \frac{4}{\cancel{3}} \times \overset{2}{\cancel{6}} \times 6 \times 6$$

$$\Rightarrow \quad 25 - r^2 = \frac{\cancel{4} \times \cancel{2} \times \overset{3}{\cancel{6}} \times \overset{3}{\cancel{6}}}{\underset{\underset{2}{\cancel{4}}}{\cancel{32}}}$$

$$\Rightarrow \quad 25 - r^2 = 9$$

$$r^2 = 16$$

$$r^2 = 4$$

$\therefore$ Thickness = R – r

= 5 – 4

= 1 cm

8. A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.

Solution:

Hemisphere	**Cylinder**
Radius = r	Radius = r
	$= h + \frac{1}{2}h$
	$r = \frac{3}{2}h$

$\therefore$ Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3}\pi \times \left(\frac{3}{2}h\right)^3$$

$$= \frac{2}{3}\pi \times \frac{27}{8}h^3$$

$$= \frac{9}{4}\pi h^3$$

Volume of Cylinder $= \pi r^2 h$

$$= \pi \times \left(\frac{3}{2}h\right)^2 h$$

$$= \pi \times \frac{9}{4}h^2\, h$$

$$= \frac{9}{4}\pi h^3$$

$\therefore$ Vol. of Hemisphere = Vol. of Cylinder

$\therefore$ % of juice that can be trasnferred to the cylindrical vessel = 100 %

EXERCISE 7.5

Multiple choice questions

1. The curved surface area of a right circular cone of height 15 cm and base diameter 16 cm is

(1) 60π cm^2 (2) 68π cm^2

(3) 120π cm^2 (4) 136π cm^2

Hint : Ans : (4)

h = 15 cm, r = 8 cm

$$\Rightarrow\ l = \sqrt{h^2 + r^2}$$
$$= \sqrt{225 + 64}$$
$$= \sqrt{289}$$
$$= 17$$

$\therefore$ CSA of Cone = $\pi r l$

$= \pi \times 8 \times 17$

$= 136\ \pi cm^2$

2. If two solid hemispheres of same base radius r units are joined together along their bases, then curved surface area of this new solid is

(1) $4\pi r^2$ sq. units (2) $6\pi r^2$ sq. units

(3) $3\pi r^2$ sq. units (4) $8\pi r^2$ sq. units

Hint : Ans : (1)

The CSA of the new solid is nothing but the CSA of a sphere = $4\pi r^2$ sq.units

3. The height of a right circular cone whose radius is 5 cm and slant height is 13 cm will be

(1) 12 cm (2) 10 cm

(3) 13 cm (4) 5 cm

Hint : Ans : (1)

r = 5cm, l = 13cm

$$\therefore\ h = \sqrt{l^2 - r^2}$$
$$= \sqrt{169 - 25}$$
$$= \sqrt{144}$$
$$= 12 \text{ cm}$$

4. If the radius of the base of a right circular cylinder is halved keeping the same height, then the ratio of the volume of the cylinder thus obtained to the volume of original cylinder is

(1) 1 : 2 (2) 1 : 4

(3) 1 : 6 (4) 1 : 8

Hint : Ans : (2)

$$\frac{\text{Volume of New Cylinder}}{\text{Volume of Original Cylinder}} = \frac{\pi R^2 h}{\pi r^2 h}$$

where $R = \dfrac{r}{2}$

$$= \frac{R^2}{r^2}$$
$$= \frac{\frac{r^2}{4}}{r^2}$$
$$= \frac{1}{4}$$

$\therefore V_1 : V_2 = 1 : 4$

5. The total surface area of a cylinder whose radius is $\dfrac{1}{3}$ of its height is

(1) $\dfrac{9\pi h^2}{8}$ sq.units (2) $24\pi h^2$ sq.units

(3) $\dfrac{8\pi h^2}{9}$ sq.units (4) $\dfrac{56\pi h^2}{9}$ sq.units

Hint : Ans : (3)

TSA of Cylinder = $2\pi r\,(h + r)$

where $r = \dfrac{1}{3}h$

$$= 2\pi \times \frac{h}{3}\left(h + \frac{h}{3}\right)$$
$$= 2\pi\,\frac{h}{3} \times \frac{4h}{3}$$
$$= \frac{8\pi h^2}{9} \text{ Sq. units}$$

6. In a hollow cylinder, the sum of the external and internal radii is 14 cm and the width is 4 cm. If its height is 20 cm, the volume of the material in it is

(1) $5600 \pi \text{ cm}^3$ (2) $11200 \pi \text{ cm}^3$

(3) $56\pi \text{ cm}^3$ (4) $3600 \pi \text{ cm}^3$

Hint : Ans : (2)

R + r = 14 cm, h= 20 cm, W = 4 cm

R – r = 4 cm

Volume of hollow cylinder

$= \pi h (R^2 - r^2)$

$= \pi h (R + r)(R - r)$

$= \pi \times 20 \times 14 \times 4$

$= 1120\pi \text{ cm}^3$

---

7. If the radius of the base of a cone is tripled and the height is doubled then the volume is

(1) made 6 times (2) made 18 times

(3) made 12 times (4) unchanged

Hint : Ans : (2)

Volume of cone = $\frac{1}{3}\pi r^2 h$

When $r \to 3r$, $h \to 2h$

Volume of new cone

$= \frac{1}{3}\pi \times 9r^2 \times 2h$

$= 18\left(\frac{1}{3}\pi r^2 h\right)$

$= 18$ times

---

8. The total surface area of a hemi-sphere is how much times the square of its radius.

(1) π (2) 4π (3) 3π
(4) 2π

Hint : Ans : (3)

TSA of a hemisphere = $3\pi r^2$

= 3π (square of its radius)

= 3π times r^2

---

9. A solid sphere of radius x cm is melted and cast into a shape of a solid cone of same radius. The height of the cone is

(1) 3x cm (2) x cm

(3) 4x cm (4) 2x cm

Hint : Ans : (3)

Volume of sphere = Volume of Cone

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad \frac{4}{3}\pi x^3 = \frac{1}{3}\pi x^2 \times h$$

$$\Rightarrow \quad h = \frac{\frac{4}{3}\pi x^3}{\frac{1}{3}\pi x^2} = 4x$$

---

10. A frustum of a right circular cone is of height 16cm with radii of its ends as 8cm and 20cm. Then, the volume of the frustum is

(1) $3328\pi \text{ cm}^3$ (2) $3228\pi \text{ cm}^3$

(3) $3240\pi \text{ cm}^3$ (4) $3340\pi \text{ cm}^3$

Hint : Ans : (1)

Volume of Frustum of a Cone

$$= \frac{\pi h}{3}(R^2 + Rr + r^2)$$

$$= \frac{\pi}{3} \times 16\,[400 + 160 + 64]$$

$$= \frac{16\pi}{3} \times 624$$

$$= 16\pi \times 208$$

$$= 3328\pi \text{ cm}^3$$

---

11. A shuttle cock used for playing badminton has the shape of the combination of

(1) a cylinder and a sphere

(2) a hemisphere and a cone

(3) a sphere and a cone

(4) frustum of a cone and a hemisphere

Hint : Ans : (4)

Frustum of a cone & a hemisphere

12. A spherical ball of radius r_1 units is melted to make 8 new identical balls each of radius r_2 units. Then $r_1 : r_2$ is

(1) 2:1 (2) 1:2

(3) 4:1 (4) 1:4

Hint : Ans : (1)

Volume of a sphere = 8 (Volume of new identical balls)

$$\frac{4}{3}\pi r_1^3 = 8\left(\frac{4}{3}\pi r_2^3\right)$$

$$\Rightarrow \frac{r_1^3}{r_2^3} = \frac{8}{1}$$

$\therefore r_1 : r_2 = 2 : 1$

13. The volume (in cm^3) of the greatest sphere that can be cut off from a cylindrical log of wood of base radius 1 cm and height 5 cm is

(1) $\frac{4}{3}\pi$ (2) $\frac{10}{3}\pi$

(3) 5π (4) $\frac{20}{3}\pi$

Hint : Ans : (1)

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 \text{ where } r = 1$$

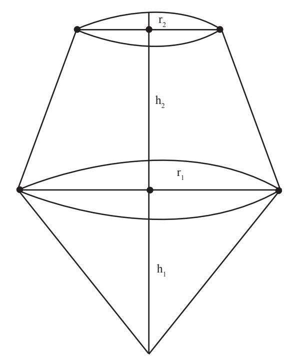
$$= \frac{4}{3}\pi$$

14. The height and radius of the cone of which the frustum is a part are h_1 units and r_1 units respectively. Height of the frustum is h_2 units and radius of the smaller base is r_2 units. If $h_2 : h_1 = 1 : 2$ then $r_2 : r_1$ is

(1) 1 : 3 (2) 1 : 2

(3) 2 : 1 (4) 3 : 1

Hint : Ans : (2)



Given $h_2 : h_1 = 1 : 2$

$$\Rightarrow h_2 = \frac{1}{2}h_1 \qquad \therefore \frac{r_2}{r_1} = \frac{1}{2}$$

15. The ratio of the volumes of a cylinder, a cone and a sphere, if each has the same diameter and same height is

(1) 1:2:3 (2) 2:1:3

(3) 1:3:2 (4) 3:1:2

Hint : Ans : (4)

Ratio of volumes of Cylinder, Cone, Sphere

$$= \pi r^2 h : \frac{1}{3}\pi r^2 h : \frac{4}{3}\pi r^3 h$$

with same height & same radius.

Since each of them has same diameter and same height, h = 2r

$$V_1 = \pi r^2(2r) = 2\pi r^3$$

$$V_2 = \frac{1}{3}\pi r^2(2r) = \frac{2}{3}\pi r^3$$

$$V_2 = \frac{4}{3}\pi r^3$$

$$\therefore V_1 : V_2 : V_3 = 2 : \frac{2}{3} : \frac{4}{3}$$

= 6 : 2 : 4

= 3 : 1 : 2

UNIT EXERCISE - 7

1. The barrel of a fountain-pen cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen will be used for writing 330 words on an average. How-many words can be written using a bottle of ink containing one fifth of a litre?

Solution :

Given height of the pen = 7 cm = 70 mm

$$\text{radius} = \frac{5}{2} \text{ mm}$$

$\therefore$ Volume of the pen = $\pi r^2 h$

$$= \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{25}{\cancel{4}} \times \overset{5}{\cancel{\overset{\cancel{10}}{70}}}$$

$$= 1375 \text{ mm}^3$$

$$= 1.375 \text{ cm}^3$$

By data given

$1.375 \text{ cm}^3 \rightarrow 330$ words –

$$\frac{1}{5} \text{ of a litre} = \frac{1}{5}(1000 \text{ cm}^3)$$

$$\Rightarrow 200 \text{ cm}^3 \rightarrow x \text{ words}$$

$$\therefore x = \frac{200 \times 330}{1.375}$$

= 48000 words

2. A hemi-spherical tank of radius 1.75 m is full of water. It is connected with a pipe which empties the tank at the rate of 7 litre per second. How much time will it take to empty the tank completely?

Solution :

Radius of hemi-spherical tank = 1.75 m

$$r = \frac{7}{4} \text{ m}$$

$\therefore$ Volume of the tank

$$= \frac{2}{3}\pi r^3$$

$$= \frac{\cancel{2}}{3} \times \frac{\overset{11}{\cancel{22}}}{\cancel{7}} \times \frac{\cancel{7}}{\underset{2}{\cancel{4}}} \times \frac{7}{4} \times \frac{7}{4}$$

$$= \frac{539}{48}$$

$$= 11.229 \, m^3$$

$$= 11.229 \times 1000 \text{ litres}$$

$$= 11229 \text{ litres}$$

Water falls at the rate of 7 lrs/second

$\therefore$ Time taken by the pipe to empty the tank

$$= \frac{11229}{7} \text{ Sec}$$

= 1604 sec (approx)

$$= \frac{1604}{60} \text{ min}$$

= 27 min (approx)

3. Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r units.

Solution :

Given radius of solid hemisphere = r

Volume of a cone that can be carved

Out of hemisphere

$$= \frac{2}{3}\pi r^3 - \frac{1}{3}\pi r^2 h$$

But volume is maximum (given)

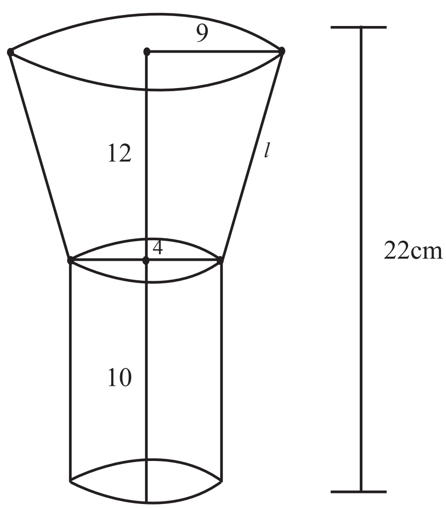
$\therefore$ h = r

$$\therefore \text{Required volume} = \frac{2}{3}\pi r^3 - \frac{1}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^3$$

4. An oil funnel of tin sheet consists of a cylindrical portion 10 cm long attached to a frustum of a cone. If the total height is 22 cm, the diameter of the cylindrical portion be 8cm and the diameter of the top of the funnel be 18 cm, then find the area of the tin sheet required to make the funnel.

Solution :



Area of tin sheet required to make the funnel

where R = 9 cm r = 4 cm, H = 10 cm

$$l = \sqrt{(R-r)^2 + h^2}$$

$$= \sqrt{25+144}$$

$$= \sqrt{169}$$

$$= 13$$

= CSA of Frustum + CSA of Cylinder

= π (R + r) l + 2πrH

= π [13 × 13 + 2 × 4 × 10]

$$= \frac{22}{7}[169+80]$$

$$= \frac{22}{7} \times 249$$

$$= \frac{5478}{7}$$

= 782.57 cm^3

5. Find the number of coins, 1.5 cm in diameter and 2 mm thick, to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Solution :

Given diameter of coin = 1.5 cm

(smaller cylinder)

$$\therefore \quad r = \frac{1.5}{2} = 0.75 \text{ cm}$$

h = 2 mm = 0.2 cm

Also, diameter of bigger cylinder = 4.5 cm

R = 2.25 cm

H = 10 cm

$\therefore$ Number of Coins =

$$\frac{\text{Volume of largest cylinder}}{\text{Volume of smallest cylinder}}$$

$$= \frac{\pi R^2 H}{\pi r^2 h}$$

$$= \frac{\frac{9}{4} \times \frac{9}{4} \times 10}{\frac{3}{4} \times \frac{3}{4} \times \frac{2}{10}}$$

$= 450$ coins

6. A hollow metallic cylinder whose external radius is 4.3 cm and internal radius is 1.1 cm and whole length is 4 cm is melted and recast into a solid cylinder of 12 cm long. Find the diameter of solid cylinder.

Solution :

Hollow Cylinder	**Solid Cylinder**
R = 4.3 cm	h = 12 cm
r = 1.1 cm	d = ?
H = 4 cm	

When hollow cylinder is melted & recast into a solid cylinder,

Volume of hollow cylinder = Volume of solid cylinder

$\Rightarrow \quad \not{\pi} H (R^2 - r^2) = \not{\pi} r^2 h$

$\Rightarrow \; 4[(4.3)^2 - (1.1)^2] = r^2 \times 12$

$\Rightarrow \quad r^2 = \frac{4(17.28)}{12}$

$r^2 = \frac{17.28}{3}$

$= 5.76$

$r = 2.4$

$\therefore$ Diameter of solid cylinder

= 2r

= 4.8 cm

7. The slant height of a frustum of a cone is 4 m and the perimeter of circular ends are 18 m and 16 m. Find the cost of painting its curved surface area at ₹100 per sq. m.

Solution :

In a frustum of a cone,

$l = 4$cm, $2\pi R = 18$, $2\pi r = 16$

$\Rightarrow \; R = \frac{9}{\pi} \qquad r = \frac{8}{\pi}$

$\therefore$ CSA of frustum of a cone

$= \pi l\,(R + r)$

$= \pi \times 4\left(\frac{9}{\pi} + \frac{8}{\pi}\right)$

$= 4 \times 17$

$= 68 \text{ m}^2$

$\therefore$ Cost of painting at ₹ 100/m^2

= 68 × 100 = ₹ 6800

8. A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is 14 cm. Find its thickness.

Solution :

In a hollow hemisphere,

Volume $= \frac{436\pi}{3}$ cm^3

D = 14 cm, R = 7 cm, t = ?

$\Rightarrow \; \frac{2}{3}\pi(R^3 - r^3) = \frac{436\pi}{3}$

$\Rightarrow \quad 7^3 - r^3 = 218$

$\Rightarrow \quad 343 - r^3 = 218$

$\therefore \quad r^3 = 125$

$\therefore \quad r = 5$ cm

$\therefore$ thickness, $t = R - r$

$= 7 - 5$

$= 2$ cm

9. The volume of a cone is $1005\frac{5}{7}$ cu. cm. The area of its base is $201\frac{1}{7}$ sq. cm. Find the slant height of the cone.

Solution :

Given volume of a cone = $1005\ {}^{5}\!/_{7}$ cm^3

& base area = $201\ {}^{1}\!/_{7}$ cm^2

$$\therefore \frac{1}{3}\pi r^2 h = \frac{7040}{7} \ \& \ \pi r^2 = \frac{1408}{7}$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{1408}{\not 7} \times h = \frac{7040}{\not 7}$$

$$\Rightarrow \quad h = \frac{7040}{1408} \times 3$$

$$\Rightarrow \quad h = 5 \times 3$$

$$\Rightarrow \quad h = 15$$

$$\text{Also}, \pi r^2 = \frac{1408}{7}$$

$$\Rightarrow \quad \frac{22}{7} \times r^2 = \frac{1408}{7}$$

$$\Rightarrow \quad r^2 = \frac{1408}{7} = 64$$

$$\therefore \quad r = 8$$

$$\therefore l = \sqrt{h^2 + r^2} = \sqrt{15^2 + 8^2} = \sqrt{225 + 64}$$

$$= \sqrt{289} = 17 \text{ cm}$$

$\therefore$ Slant height = 17cm

10. A metallic sheet in the form of a sector of a circle of radius 21 cm has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.

Solution :

Given radius of sector = 21 cm, $\theta = 216^0$

ie R = 21 = l (slant height of cone)

When the sector is made into a cone by bringing the radii together.

Length of arc of the sector = Perimeter of base of cone

$$\Rightarrow \quad \frac{\theta}{360} \times 2\pi R = 2\pi r$$

$$\Rightarrow \quad r = \frac{216}{360} \times 21$$

$$\Rightarrow \quad r = \frac{63}{5} = 12.6 \text{ cm}$$

$$\therefore \quad h = \sqrt{l^2 - r^2}$$

$$= \sqrt{21^2 - (12.6)^2}$$

$$= \sqrt{441 - 158.76}$$

$$= \sqrt{282.24}$$

$$= 16.8$$

$\therefore$ Volume of the cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{\not 3} \times \frac{22}{\not 7} \times \overset{4.2}{\not{12.6}} \times \overset{1.8}{\not{12.6}} \times 16.8$$

$$= 2794.176$$

$$\simeq 2794.18 \text{ cm}^3$$

PROBLEMS FOR PRACTICE

1. A girl empties a cylindrical bucket, full of sand of base radius 18 cm and height 32 cm on the floor, to form a conical heap of sand. If the height of this heap is 24 cm, find the slant height of cone. **(Ans : 43.27 cm)**

2. 12 cylindrical pillars of a building have to be cleaned. If the diameter of each pillar is 42 cm, height is 5m, What will be the cost of cleaning at the rate of Rs.5 per m^2.

   (Ans : Rs.396/-)

3. A conical tent of 56m base diameter requires 3080 m^2 of canvas for the cured surface. Find its height. **(Ans : 21m)**

4. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 19cm and the diameter of the cylinder is 7 cm. Find the surface area of the solid. **(Ans 418 m^2)**

5. A farmer connects a pipe of internal diameter 20 cm from a Canal into a cylindrical tank which is 10m in diameter and 2m deep. If the water flows through the pipe at the rate of 4 Km/hr, in how much time will the tank be completely filled ?

(Ans : 1 hr, 15min)

6. A toy is in the form of a cone of base radius 3.5 cm mounted on a hemisphere of base diameter 7 cm. If the total height of the toy is 15.5 cm, find the total surface area of the toy. **(Ans : 214.5cm^2)**

7. A cylindrical glass tube with radius 10 cm has water upto a height of 9 cm. A metal cube of 8 cm edge is immersed completely. By how much, the water level rise in the tube ? **(Ans : 1.63 cm)**

8. A vessel is in the form of a cone. Its height is 8 cm and radius of its top which is open is 5 cm. It is filled with water up to the rim. When lead shots, each of which is a sphere of diameter 1cm are droppd into vessel, one fourth of the water flows out. Find the number of lead shots dropped into the vessel.

(Ans : (100))

9. A hollow spherical shell has an inner radius of 8cm. If the volume of material is $\frac{1952\pi}{3}$ C.C, Find the thickness of the shell.

(Ans : 2cm)

10. Find the length of arc of the sector formed by opening out a cone of base radius 8cm. What is the central angle if the height of the cone is 6cm.

$$\left(Ans : 280^o,\ 50\frac{2}{7}\text{ cm} \right)$$

11. Find the capacity of a bucket having the radius of the top as 36cm and that of the bottom as 12cm, depth is 35cm.

(Ans 68640 cm^3)

12. Water flows through a cylindrical pipe of internal radius 3.5 cm at 5m per sec. Find the volume of water in litres discharged by the pipe in 1 min. **(Ans : 1155 litres)**

13. A rectangular sheet of metal foil with dimension 66 cm × 12 cm is rolled to form a cylinder of height 12cm. Find the volume of the cylinder. **(Ans : 4158 cm^3)**

14. Using clay, a student made a right circular cone of height 48cm and base radius 12cm. Another student reshapes it in the form of a sphere. Find the radius of sphere.

(Ans : 12 cm)

15. A solid sphere of diameter 28 cm is melted and recast into smaller solid cones each of diameter $4\frac{2}{3}$ cm and height 3cm. Find the number of cones so formed.

(Ans : 672)

OBJECTIVE TYPE QUESTIONS

1. The radii of 2 cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Then the ratio of their volumes is

a) 10 : 9 b) 20 : 27 c) 7 : 6

d) 5 : 2 **(Ans : (b))**

2. If the volume of sphere is $\frac{9}{16}$ cm^3, its radius is

$a)\ \frac{4}{3}$ cm $b)\ \frac{3}{4}$ cm

c) $\frac{3}{2}$ cm $d)\ \frac{2}{3}$ cm

(Ans : (b))

3. The height of a cone whose slant height 26cm and the diameter of the base is 10cm, is

a) 24.5 cm b) 26.5 cm c) 25.5 cm

d) 27.5 cm **(Ans : (c))**

4. The total surface area of a cylinder whose height is half the radius is

a) $6\pi r^2$ b) $8\pi r^2$ c) $2\pi r^2$

d) $3\pi r^2$ **(Ans : (d))**

5. The base area of a cone is 80cm^2. If its height is 9cm, then its volume is

a) 720cm^3 b) 720π cm^3

c) 240 cm^3 d) none **(Ans : (c))**

6. A well of diameter 2.1m is dug to a depth of 4m. The volume of the earth removed is

a) 4.4π m^3 b) 44.1 m^3

c) 0.441 m^3 d) 4.41 πm^3

(Ans : (d))

7. The volume of a hemisphere is 18π. Its radius is

a) 4cm b) 3cm c) 2cm

d) 6cm **(Ans : (b))**

8. If a rectangle ABCD is folded by bringing AB and CD together to form a cylinder, then the height of the cylinder is

a) BC b) AD c) AB

d) none **(Ans : (c))**

9. The CSA of a solid hemisphere if the TSA of the solid hemisphere is 12π cm^2, is

a) 8π b) 36π c) 6π d) 24π

(Ans : (a))

10. Ths CSA of a cone whose radius x cm, height y cm is

$a)\ \pi rl$ $b)\ \pi r\sqrt{x^2+y^2}$

c) $\pi y\sqrt{x^2+y^2}$ $d)\ \pi x\sqrt{x^2+y^2}$

(Ans : (d))