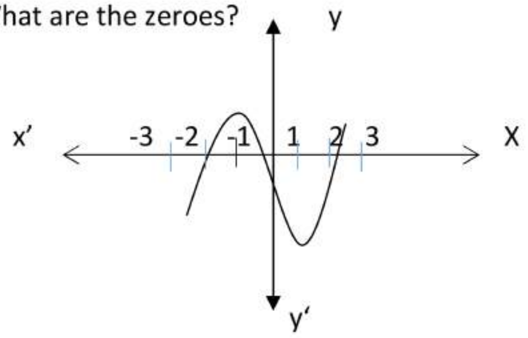
2. POLYNOMIALS

1. From the given graph $y = p(x)$. Find the number of zeroes of the polynomial $p(x)$. What are the zeroes?



2. What will be the nature of the graph of the following polynomials

(i) $ax^2 + bx + c$ when $a > 0$

(ii) $ax^2 + bx + c$ when $a < 0$

3. What is the relation between a and b, if sum of the zeroes of the quadratic polynomial $ax^2 + bx + c$ $(a \neq 0)$ is equal to the product of the zeroes.

4. What is the degree of the polynomial whose graph intersect the x – axis at four points.

5. If - 1 is one of the zeroes of the quadratic polynomial $ax^2 + bx + c$ $(a \neq 0)$, write at least one of its factor with justification.

6. If p and q are the zeroes of the quadratic polynomial $ax^2 + bx + c$ $(a \neq 0)$, find the value of $pq + (p+q)$

7. Find the zeroes of the polynomial $2x^2 - 3\sqrt{3}x + 3$

8. For what value of k, (-4) is a zero of the polynomial $x^2 - 2x - (3k + 3)$?

9. If α and β are the zeroes of the polynomial $x^2 - 4x - 12$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ without finding actual zeroes.

10. What should be subtracted from the polynomial of $p(x) = x^2 - 3ax + 3a - 7$ so that, $(x + 2)$ is a factor of the polynomial $p(x)$ and hence also find the value of a

11. If one of the zero of the polynomial $2x^2 - 4x - 2k$ is reciprocal of the other ,Find the value of k.

12. If α and β are the zeroes of the polynomial $2x^2 - 5x - 10$, then find the Value of $\alpha^{-2} + \beta^{-2}$ (by using algebraic identity)

13. Find the zeroes of the quadratic polynomial $2x^2 - 9 - 3x$ and verify the relationship between the zeroes and the coefficients

14. If two zeroes of the polynomial $x^3 - 4x^2 - 3x + 12$ are $\sqrt{3}$ and $-\sqrt{3}$, then find its third zero.

15. If α and β are the zeroes of the polynomial $f(x) = x^2 + px + q$ then form a polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

16. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, Find the value of p and q.

17. If α and β are the zeroes of the polynomial $f(x) = x^2 - 9x + a$, find the value of a if $5\alpha + 4\beta = 40$

18. If -2 and the 3 are the zeroes of the polynomial $ax^2 + bx - 6$, then find the value of a and b

19. If the polynomial $f(x) = x^3 + 2x^2 - 5x + 1$ is divided by another polynomial $x + 3$, then the remainder comes out to be $ax + b$. Find the values of a and b (without doing actual division)

20. If 2 and -3 are the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$,Then find the value of a and b.

ANSWER

1. 3 zeroes, Zeroes are -2, 0, 2
2. (i) a parabola opening upward (ii) a parabola opening downward
3. $\frac{-b}{a} = \frac{c}{a}$ i.e $b + c = 0$
4. Degree is 4(since it has 4 zeroes)
5. By Factor theorem $(x+1)$ will be one of its factors if $x + 1 = 0 \Rightarrow x = -1$
6. $pq + (p+q) = \frac{c}{a} + \frac{-b}{a} = \frac{c-b}{a}$
7. $2x^2-3\sqrt{3}x + 3 = (2x-\sqrt{3})(x-\sqrt{3})$
   Zeroes $x = \sqrt{3}/2$ and $x = \sqrt{3}$
8. $p(x) = x^2 - 2x - (3k+3)$
   $\therefore p(-4) = (-4)^2 - 2(-4) - (3k+3)$
   $\Rightarrow 0 = 21 - 3k$ So, $k = 7$
9. $p(x) = x^2 - 4x - 12$
   $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta = \frac{4}{-12} - 2(-12) = \frac{-4}{12} + 24 = \frac{-4+288}{12} = \frac{284}{12} = \frac{142}{6}$
10. If the polynomial $p(x) = x^2 - 3ax + 3a - 7$ is divided by $x + 2$, then by Remainder theorem, remainder is p(-2).
    $P(-2) = 9a - 3$
    $\therefore$ $9a - 3$ should be subtracted
    If $x + 2$ is a factor of $p(x)$, then by Factor theorem p(-2) = 0
    $\Rightarrow 9a - 3 = 0$
    So, $a = \frac{1}{3}$
11. Let α and $\frac{1}{\alpha}$ be the zeroes of the polynomial $p\,x) = 2x^2 - 4x - 2k$
    $\therefore \alpha \times \frac{1}{\alpha} = \frac{-2k}{2}$ so, $k = -1$
12. $p(x) = 2x^2 - 5x - 10$
    $\alpha^{-2} + \beta^{-2} = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\beta^2+\alpha^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{\left(\frac{5}{2}\right)^2 - 2\left(-\frac{10}{2}\right)}{\left(-\frac{10}{2}\right)^2}$
    $\frac{\frac{25}{4}+10}{\frac{100}{4}} = \frac{\frac{65}{4}}{\frac{100}{4}} = \frac{13}{20}$
13. $p(x) = 2x^2 - 3x - 9 = (2x+3)(x-3)$ [factorising by splitting of the middle term]
    Now, $p(x) = 0$ so, $x = \frac{-3}{2}$ and 3
    Sum of the zeroes $= \frac{-3}{2} + 3 = \frac{3}{2} = \frac{-\,coefficient\ of\ x}{coefficient\ of\ x^2}$
    Product if zeroes $= \frac{-3}{2} \times 3 = \frac{-9}{2} = \frac{constant\ tterm}{coefficient\ of\ x^2}$
14. As $\sqrt{3}$ and $-\sqrt{3}$ are the zeroes of the polynomial $x^3 - 4x - 3x + 12$,
    The quadratic polynomial forming by the given zeroes $= (x-\sqrt{3})((x+\sqrt{3})$
    $= x^2 - 3$
    Now, $x^3 - 4x - 3x + 12 = x^2(x-4) - 3(x-4) = (x^2-3)(x-4)$
    $= (x-\sqrt{3})((x+\sqrt{3})(x-4)$
    So the third zero of the given polynomial is 4
15. α and β are the zeroes of the polynomial $f(x) = x^2 + px + q$
    $\alpha + \beta = \frac{-p}{1} = -p$ and $\alpha \times \beta = \frac{q}{1} = q$
    so, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-p}{q}$ and $\frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{q}$

The required polynomial is $p(x) = k\,[x^2 - (\text{sum of zeroes})x + \text{product of zeroes}]$

$= k\,[x^2 + \frac{p}{q}x + \frac{1}{q}]$

Taking $k = q$ $\quad p(x) = q\,x^2 + p\,x + 1$

16. The zeroes of the polynomial $2x^2 - 5x - 3$ are given by

$2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$ so, $x = 3$ and $\frac{-1}{2}$

The zeroes of the polynomial $x^2 + px + q$ are 6 and – 1

Sum of the zeroes = 6+ (-1)

$-p = 5 \quad \therefore\ p = -5$

Product of the zeroes = 6× (-1)

$\therefore \quad q = -6$

17. α and β are the zeroes of the polynomial $f(x) = x^2 - 9x + a$

$\alpha + \beta = \frac{-(-9)}{1} = 9$

$5\alpha + 4\beta = 40 \quad \Rightarrow \alpha + 4\alpha + 4\beta = 40$

$\Rightarrow \alpha + 4(\alpha + \beta) = 40$

$\Rightarrow \alpha + 4\times 9 = 40 \quad \Rightarrow \alpha = 4$

Putting the value of α in $5\alpha + 4\beta = 40$ we get $\beta = 5$

So, product of the zeroes $\alpha \times \beta = \frac{a}{1} \quad \Rightarrow a = 4\times 5 = 20$

18. Let , $p(x) = ax^2 + bx - 6$

-2 and the 3 are the zeroes of the polynomial

Sum of the zeroes = - 2 + 3

$\Rightarrow \frac{-b}{a} = 1 \quad \Rightarrow a = -b$......................(i)

Product of the zeroes = -2×3

$\Rightarrow \frac{c}{a} = -6 \quad \Rightarrow \frac{-6}{a} = -6$ so , $a = 1$

From (i) we have $b = -1$

19. If $f(x) = x^3 + 2x^2 - 5x + 1$ is divided by another polynomial $x + 3$

So , by remainder theorem , remainder is $f(-3)$

Now, $\quad f(-3) = (-3)^3 + 2(-3)^2 - 5(-3) + 1$

$= -27 + 18 + 15 + 1$

$= -27 + 34$

Remainder = 7 .... (i)

But , remainder = $ax + b$ (given) .... (ii)

Comparing (i) and (ii) we have ,

$a\,x + b = 0.\,x + 7$ so, $a = 0$ and $b = 7$

20. Let , $p(x) = x^2 + (a+1)\,x + b$

2 and – 3 are the zeroes of $p(x)$

so , $p(2) = 0$

$\Rightarrow 2^2 + (a+1)\times 2 + b = 0$

$\Rightarrow 2a + b = -6$ .... (i)

and $p(-3) = 0$

$\Rightarrow (-3)^2 + (a+1)\times(-3) + b = 0$

$\Rightarrow -3a + b = -6$ .... (i)

Solving equation (i) and (ii) we have $a = 0$ and $b = -6$