Chapter 8

Time and Distance

CHAPTER HIGHLIGHTS

- *General problems on Time, Speed, and Distance*
- *Speed*
- *Relative Speed*
- *Average Speed*
- *Boats and Streams*
- *Races and Circular Tracks*

Introduction

In this chapter, we will look at problems in the following different areas:

1. General problems on Time, Speed, and Distance
2. Relative Speed
3. Boats and Streams
4. Races and Circular Tracks

Before we look at problems in various areas, let us first look at some basic concepts pertaining to speed, time and distance.

Speed

Distance covered per unit time is called speed.

i.e. Speed = Distance/Time

The above relationship between the three variables distance, speed, and time can also be expressed as follows:

Distance = Speed × Time or Time = Distance/Speed

1. If two bodies travel with the same speed, distance covered ∝ time (Direct Variation).
2. If two bodies travel for the same period of time, distance covered ∝ speed (Direct Variation).
3. If two bodies travel the same distance,

$$\text{Time} \propto \frac{1}{\text{Speed}} \text{ (Inverse Variation).}$$

Distance is normally measured in kilometres, metres, or miles; time in hours or seconds and speed in km/hr (also denoted by kmph), miles/hr (also denoted by mph), or metres/second (denoted by m/s).

To convert speed in kmph to m/sec, multiply it with 5/18.

To convert speed in m/sec to kmph, multiply it with 18/5.

In the case of moving trains, three different situations need to be considered.

When a train passes a stationary point, the distance covered (in the passing) is the length of the train. If the train is crossing a platform (or a bridge), the distance covered by the train (in the crossing) is equal to the length of the train plus the length of the platform (or bridge). If two trains pass each other (travelling in the same direction or in opposite directions), the total distance covered (in the crossing or the overtaking, as the case may be) is equal to the sum of the lengths of the two trains.

Average Speed

Average speed of a body travelling at different speeds is defined as follows:

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Please note that the **Average speed** of a moving body is **Not equal** to the **Average of the speeds.**

If a body travels from point A to point B with a speed of p and back to point A (from point B) with a speed of q, then the average speed of the body can be calculated as $\mathbf{2pq/(p+q)}$. Please note that this does not depend on the distance between A and B.

If a body covers part of the journey at speed p and the remaining part of the journey at speed q and the distances of the two parts of the journey are in the ratio $m : n$, then the average speed for the entire journey is $\mathbf{(m+n)pq/(mq+np)}$.

Solved Examples

Example 1

Ashok covered a certain distance at a certain speed. If his speed was 20% more, he would take 10 minutes less to cover the same distance. Find the time he takes to cover the distance.

Solution

If his speed was 20% more, it would be 120%, i.e. $\frac{6}{5}$ times his actual speed.

$\therefore$ Time taken would be $\frac{5}{6}$ times his actual time.

Reduction in time $= \frac{1}{6}$ (actual time) = 10 minutes

$\therefore$ Actual time = 60 minutes.

Example 2

A car covered a certain distance at 90 kmph and returned back at 60 kmph. Find its average speed for the entire journey.

Solution

Let x km be the distance to be covered, each way.

Total time of travel (in hours)

$$= \frac{x}{90} + \frac{x}{60} = \frac{5x}{180} = \frac{x}{36}$$

Average speed (in km/hr)

$$= \frac{\text{Total disance travelled}}{\text{Total time taken}} = \frac{2x}{\frac{x}{36}} = 72.$$

Example 3

Find the time taken by a train, 100 m long, travelling at a speed of 63 kmph to cross a platform of length 250 m.

Solution

Speed of the train $= (63)\left(\frac{5}{18}\right) = \frac{35}{2}$ m/sec

Distance to be travelled by the train to cross the platform = length of the train + length of the platform.

Time taken to cross the platform

$$= \frac{100+250}{\frac{35}{2}} = 20 \text{ sec.}$$

Example 4

Ashok would reach his office 15 minutes early if he walked at 4 kmph from his house. He would reach it 45 minutes late if he walked at 3 kmph from his house. Find the distance between his house and office.

Solution

Let the distance be x km. Time taken by Ashok if he walked at 4 kmph $= \frac{x}{4}$ hours.

Time taken by Ashok if he walked at 3 kmph $= \frac{x}{3}$ hours.

In this case, he would take one hour more to reach his office compared to the time taken if he had walked at 4 kmph.

$$\therefore \quad \frac{x}{3} - \frac{x}{4} = 1$$

$$\Rightarrow \quad x = 12.$$

In general, if a person travelling between two points reaches p hours late travelling at a speed of u kmph and reaches q hours early travelling at v kmph, the distance between the two points is given by $\frac{vu}{v-u}(p+q)$.

Relative Speed

The speed of one (moving) body in relation to another moving body is called the relative speed of these two bodies, i.e. it is the speed of one moving body as observed, from the second moving body.

If two bodies are moving in the same direction, the relative speed is equal to the difference of the speeds of the two bodies.

If two bodies are moving in opposite directions, the relative speed is equal to the sum of the speeds of the two bodies.

Example 5

Find the time taken by a train 175 m long running at a speed of 54 kmph to overtake another train 75 m long running at a speed of 36 kmph.

Solution

Relative speed = 18 kmph = 5 m/sec

Time taken for the faster train to overtake the slower train

$$= \frac{(\text{Length of the faster train}) + (\text{Length of the slower train})}{\text{Their relative speed}}$$

$$= \frac{175+75}{5} = 50 \text{ sec}$$

Example 6

A train overtakes two persons, cyling at 9 kmph and 18 kmph in 40 seconds and 48 seconds, respectively. Find its length and speed.

Solution

Let the length and the speed of the train be ℓ *m* and *s* kmph, respectively.

$$\ell = 40(s-9)\frac{5}{18} = 48(s-18)\frac{5}{18}$$

$$\Rightarrow \quad \frac{s-9}{s-18} = \frac{48}{40}$$

$$\Rightarrow \quad s = 63$$

$$\therefore \quad \ell = 40\ (63-9)\times\frac{5}{18} = 600\ m.$$

Boats and Streams

Problems related to boats and streams are different in the computation of relative speed from those of trains/cars.

When a boat is moving in the same direction as the stream or water current, the boat is said to be moving **WITH THE STREAM OR CURRENT.**

When a boat is moving in a direction opposite to that of the stream or water current, it is said to be moving **AGAINST THE STREAM OR CURRENT.**

If the boat is moving with a certain speed in water that is not moving, the speed of the boat is then called the **SPEED OF THE BOAT IN STILL WATER.**

When the boat is moving upstream, the speed of the water opposes (and hence reduces) the speed of the boat.

When the boat is moving downstream, the speed of the water aids (and thus adds to) the speed of the boat. Thus, we have

Speed of the boat against stream = Speed of the boat in still water – Speed of the stream

Speed of the boat with the stream = Speed of the boat in still water Speed of the stream

These two speeds, the speed of the boat against the stream and the speed of the boat with the stream, are RELATIVE SPEEDS.

If u is the speed of the boat down the stream and v is the speed of the boat up the stream, then we have the following two relationships.

Speed of the boat in still water = $(u + v)/2$

Speed of the water current = $(u - v)/2$

In problems, instead of a boat, it may be a swimmer but the approach is exactly the same. Instead of boats/swimmers in water, it could also be a cyclist cycling against or along the wind. The approach to solving the problems still remains the same.

Example 7

A boat travels 30 km upstream in 5 hours and 100 km downstream in 10 hours. Find the speed of the boat in still water and the speed of the stream.

Solution

Upstream speed $= \frac{30}{5} = 6$ kmph

Downstream speed $= \frac{100}{10} = 10$ kmph

Speed in still water $= \frac{6+10}{2} = 8$ kmph

Speed of the stream $= \frac{10-6}{2} = 2$ kmph .

Example 8

Anand can row 20 km in 2 hours in still water. If the speed of the stream is 6 kmph, he would take 3.75 hours to cover a round trip journey. Find the distance that he would then cover each way.

Solution

Speed of the boat in still water $= \frac{20}{2} = 10$ kmph

Let the total distance covered be $2x$ km.

Given that, $$\frac{x}{10+6} + \frac{x}{10-6} = 3.75$$

$$x = 12$$

Races and Circular Tracks

When two persons P and Q are running a race, they can start the race at the same time or one of them may start a little later than the other. In the second case, suppose P starts the race and after 5 seconds, Q starts. Then we say P has a 'start' of 5 seconds. Alternatively, in a race between P and Q, P starts first and then when P has covered a distance of 10 metres, Q starts. Then we say that P has a 'start' of 10 metres.

In a race between P and Q where Q is the winner, by the time Q reaches the winning post, if P still has another 15 metres to reach the winning post, then we say that Q has won the race by 15 metres. Similarly, if P reaches the winning post 10 seconds after Q reaches it, then we say that Q has won the race by 10 seconds.

In problems on RACES, we normally consider a 100 m race or a 1 km race. The length of the track.

NEED NOT necessarily be one of the two figures mentioned above but can be as given in the problem.

When two or more persons running around a circular track (starting at the same point and at the same time), then we will be interested in two main issues:

1. When they will meet for the first time and
2. When they will meet for the first time at the starting point

To solve the problems on circular tracks, you should keep the following points in mind.

When two persons are running around a circular track in **OPPOSITE** directions

1. The relative speed is equal to the sum of the speeds of the two individuals and
2. From one meeting point to the next meeting point, the two of them TOGETHER cover a distance equal to the length of the track.

When two persons are running around a circular track in the **SAME** direction

1. The relative speed is equal to the difference of the speeds of the two individuals and
2. From one meeting point to the next meeting point, the faster person covers one COMPLETE ROUND more than the slower person.

We can now tabulate the time taken by the persons to meet for the first time ever or for the first time at the starting point in various cases.

When TWO people are running around a circular track

Let the two people A and B with respective speeds of a and $b(a > b)$ be running around a circular track (of length L) starting at the same point and at the same time. Then,

	When the two persons are running in the SAME direction	When the two persons are running in OPPOSITE directions
Time taken to meet for the FIRST TIME EVER	$\frac{L}{(a-b)}$	$\frac{L}{(a+b)}$
Time taken to meet for the first time at the STARTING POINT	LCM of $\left\{\frac{L}{a}, \frac{L}{b}\right\}$	LCM of $\left\{\frac{L}{a}, \frac{L}{b}\right\}$

Please note that when we have to find out the time taken by the two persons to meet for the first time at the starting point, what we have to do is to find out the time taken by each of them to complete one full round and then take the LCM of these two timings (L/a and L/b are the timings taken by the two of them respectively to complete on full round).

Example 9

In a 200 m race, A beats B by 10 m or 2 seconds. Find B's speed and A's speed.

Solution

A beat B by 10 m or 2 seconds.

⇒ When A reached the finishing line B was 10 m behind the finishing line and took 2 seconds to cover it.

$$\therefore \quad B\text{'s speed} = \frac{10}{2} = 5 \text{ m/sec}$$

Time taken by B to complete the race

$$= \frac{200 \text{ m}}{5 \text{ m/s}} = 40 \text{ seconds}$$

∴ Time taken by A to complete the race

$$= 38 \text{ seconds}$$

$$\text{A's speed} = \frac{200}{38} = \frac{100}{19} \text{ m/sec}$$

Example 10

Ramu is 50% faster than Somu. In a race, Ramu gave Somu a head start of 200 m. Both finished the race simultaneously. Find the length of the race.

Solution

Let the length of the race be x m.

$$\frac{x}{x-200} = \frac{150}{100}$$

$$\Rightarrow \quad x = 600 \text{ m.}$$

Example 11

In a 1200 m race, Ram beats Shyam by 300 m. In the same race, Shyam beats Tarun by 400 m. Find the distance by which Ram beats Tarun.

Solution

Let the speeds of Ram, Shyam, and Tarun be r m/sec, s m/sec, and t m/sec, respectively

$$\frac{r}{s} = \frac{1200}{1200-300} = \frac{4}{3}$$

$$\frac{s}{t} = \frac{1200}{1200-400} = \frac{3}{2}$$

$$\frac{r}{t} = \left(\frac{r}{s}\right)\left(\frac{s}{t}\right) = 2$$

⇒ By the time Ram covers 1200 $\overline{m}$, Tarun covers 600 $\overline{m}$.
∴ Ram beats Tarun by (1200 – 600), i.e. by 600 $\overline{m}$.

Exercises

Direction for questions 1 to 30: Select the correct alternative from the given choices.

1. Convert the following speeds into meters per second

(a) 36 km/hr
(A) 10 (B) 12 (C) 15 (D) 20

(b) 12.6 km/hr
(A) 3.5 (B) 4 (C) 0.35 (D) 6

(c) 252/35 km/hr
(A) 2.2 (B) 2.4 (C) 2 (D) 2.6

2. If a man runs at 6 metres per second, what distance (in km) will he cover in 3 hours and 45 minutes?
(A) 81 (B) 96 (C) 91 (D) 27

3. Travelling at $5/6^{th}$ of his usual speed a man is 10 minutes late. What is the usual time he takes to cover the same distance?
(A) 50 minutes (B) 70 minutes
(C) 1 hour (D) 75 minutes

4. X and Y are 270 km apart. At 9:00 a.m, buses A and B left X and Y for Y and X, respectively. If the speeds of A and B are 50 kmph and 40 kmph, respectively, find their meeting time.
(A) 11:00 a.m (B) 12:00 p.m
(C) 1: 00 p.m (D) 2:00 p.m

5. Car A left X for Y at 9:00 a.m. Car B left Y for X at 10:00 a.m. XY = 180 km. Speeds of A and B are 30 kmph and 20 kmph, respectively. Find their meeting time.
(A) 12:36 p.m. (B) 1:36 p.m.
(C) 1:00 p.m (D) 2:00 p.m

6. Ashok left X and reached Y in 4 hours. His average speed for the journey was 90 kmph. Find the distance between X and Y (in km).
(A) 180 (B) 360 (C) 720 (D) 900

7. Alok travelled from Hyderabad to Tirupati at 60 kmph and returned at 90 kmph. Find his average speed for the journey (in kmph).
(A) 72 (B) 75 (C) 66 (D) 78

8. What is the time taken by a train 650 $\overline{m}$ long travelling at 72 km/hr to cross a 750 $\overline{m}$ long platform?
(A) 60 sec (B) 65 sec
(C) 70 sec (D) 75 sec

9. What is the time taken by a 750 $\overline{m}$ long train travelling at 99 km/hr to cross a boy running at 9 km/hr towards the train?
(A) 30 sec (B) 33 sec
(C) 36 sec (D) 25 sec

10. In a 200 $\overline{m}$ race, Eswar gives Girish a start of 10 $\overline{m}$ and beats him by 10 $\overline{m}$. Find the ratio of their speeds.
(A) 1 : 1 (B) 9 : 10 (C) 10 : 9 (D) 19 : 20

11. In a 100 $\overline{m}$ race, Ganesh beats Harish by 10 $\overline{m}$ or 2 seconds. Find Harish's speed (in m/sec).
(A) 5 (B) $5\frac{5}{9}$
(C) $4\frac{1}{2}$ (D) 6

12. In a 100 $\overline{m}$ race, Akbar gives Birbal a start of 2 seconds. Birlbal covers 10 $\overline{m}$ by the time Akbar starts. If both of them finish together, find Akbar's speed. (in m/sec)
(A) 5 (B) $5\frac{5}{9}$
(C) $4\frac{1}{2}$ (D) 4

13. In a race, P beats Q by 20 seconds. Q beats R by 30 seconds. By how many seconds did P beat R?
(A) 44 (B) 25 (C) 50 (D) 36

14. In a 100 $\overline{m}$ race, A beats B by 10 $\overline{m}$ and B beats C by 20 m. Find the distance by which A beats C (in m).
(A) 30 (B) 28 (C) 32 (D) 36

15. Anand can row a boat in still water at a speed of 5 kmph. The speed of the stream is 3 kmph. Find the time taken by him to row 40 km downstream (in hours).
(A) 5 (B) 20 (C) 8 (D) 10

16. Ram, Shyam, and Tarun started cycling from a point on a circular track 600 $\overline{m}$ long with speeds of 10 m/sec, 15 m/sec, and 20 m/sec, respectively. Find the time taken by them to meet at the starting point for the first time (in seconds).
(A) 120 (B) 60 (C) 240 (D) 600

17. Ashwin and Bhaskar started running simultaneously from a point on a 300 $\overline{m}$ long circular track. They ran in opposite directions with speeds of 6 m/sec and 4 m/sec, respectively. After meeting for the first time, they exchange their speeds. Who will reach the starting point first?
(A) Ashwin
(B) Bhaskar
(C) Both reach simultaneously
(D) Cannot be determined

18. A man reaches his destination which is 16 km away, 9 min late, if he travels at 8 kmph. What should his speed be if he wishes to reach 15 minutes ahead of the right time?
(A) 10 kmph (B) 3 m/sec
(C) 20/9 m/sec (D) 12 kmph

19. The distance between two points P and Q is 84 km. Two persons start at the same time but one travelling from P towards Q and the other travelling from Q towards P. If their respective speeds are 36 kmph and 27 kmph, where do they meet each other?
(A) 48 km from Q (B) 24 km from P
(C) 36 km from P (D) 48 km from P

20. Towns P and Q are 80 km apart. Cars A and B are stationed at towns P and Q respectively. If they start simultaneously towards each other, they would meet in an hour. If both start simultaneously in the same direction, the faster car would overtake the slower car in 4 hours. Find the speed of the faster car (in kmph).
(A) 50 (B) 55 (C) 60 (D) 65

21. A cat on seeing a dog 100 m away turns around and starts running away at 24 kmph. The dog spots him one minute later and starts chasing the cat at a speed of 33 kmph. After how much time, from the start of the cat's run, will the chase end?
(A) 160 s (B) 220 s
(C) 260 s (D) 280 s

22. Train A starts at 6 a.m. from city P towards city Q at a speed of 54 kmph. Another train 'B' starts at 9 a.m. from P towards Q at 72 kmph. If the distance between P and Q is 1440 km, find at what distance from Q would the two trains meet each other?
(A) 648 km (B) 792 km
(C) 486 km (D) 954 km

23. Mahesh travelled from Hyderabad to Tirupati at a certain speed and returned at a certain speed. His average speed for the entire trip was the average of his onward and return speeds. He travelled a total distance of 1200 km in 12 hours. Find his onward speed (in kmph).
(A) 100 (B) 80 (C) 60 (D) 40

24. Two cars left simultaneously from two places P and Q, and headed for Q and P, respectively. They crossed each other after x hours. After that, one of the cars took y hours to reach its destination while the other took z hours to reach its destination. Which of the following always holds true?
(A) $x = \dfrac{y+z}{2}$ (B) $x = \dfrac{2yz}{y+z}$
(C) $x = \sqrt{yz}$ (D) $x = \dfrac{y^2+z^2}{y+z}$

25. A boat travels 30 km upstream in 5 hours and 24 km downstream in 3 hours. Find the speed of the boat in still water and the speed of the water current
(A) 7 kmph, 2 kmph (B) 14 kmph, 1 kmph
(C) 7 kmph, 1 kmph (D) 8 kmph, 2 kmph

26. Amar, Akbar, and Anthony start running in the same direction and from the same point, around a circular track with speeds 7 m/sec, 11 m/sec, and 22 m/sec, respectively. If Akbar can complete 5 revolutions around the track in 40 sec, when will they meet for the first time after they start?
(A) 56 s (B) 88 s (C) 118 s (D) 79 s

27. If Ashok travelled at $4/5^{th}$ of his usual speed, he would reach his destination 15 minutes late. By how many minutes would he be early if he travelled at $6/5^{th}$ of his usual speed?
(A) 12 (B) 10 (C) 15 (D) 20

28. In a 500 ft race, Habib beats Akram by 60 ft. If Habib takes 5 paces for every 4 paces taken by Akram, what is the ratio of the length of Habib's pace to that of Akram?
(A) 10 : 11 (B) 11 : 10
(C) 25 : 22 (D) 22 : 25

29. Girish takes 1 minute to complete a round around a circular track. Harish is twice as fast as Girish, Suresh is thrice as fast as Harish. All three start at the same point. Find the time taken by them to meet at the starting point for the first time (in minutes).
(A) 1 (B) 2
(C) 6 (D) 12

30. Two cars C and D start from a junction along two perpendicular roads at 8:00 a.m. and 9:00 a.m., respectively. If at 12 noon, the cars, which travel at the same speed, are 150 km apart, then, find the speed of each car.
(A) 15 kmph
(B) 45 kmph
(C) 60 kmph
(D) 30 kmph

Answer Keys

1. (a) A	**(b)** A	**(c)** C	**2.** A	**3.** A	**4.** B	**5.** C	**6.** B	**7.** A	**8.** C
9. D	**10.** C	**11.** A	**12.** B	**13.** C	**14.** B	**15.** A	**16.** A	**17.** C	**18.** A
19. D	**20.** A	**21.** C	**22.** B	**23.** A	**24.** C	**25.** C	**26.** B	**27.** B	**28.** A
29. A	**30.** D								